Cooperative and Non-cooperative control for Slotted Aloha with random power level selections algorithms

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ABSTRACT

In this paper, we study the performance of Slotted Aloha under power differentiation schemes. We consider the uplink of a cellular system where m mobiles transmit over a common channel to a base station. In particular we analyze random sets possible transmission powers and further study the role of priorities given either to new arriving packet or to backlogged packets. We consider a general capture model where a mobile transmit successfully a packet if its instantaneous SINR is larger than the threshold. Under this capture model, we study both the cooperative team in which a common goal is jointly optimized as well as the noncooperative game problem in which mobiles try to optimize their own objectives. The performance metrics that we study are the throughput and the expected delay. Further we provide a stability analysis and show that schemes with power differentiation and power control can improve significantly the performance and could eliminate in some cases the bi-stable nature of Slotted Aloha.

1. INTRODUCTION

Aloha [2] and Slotted Aloha [14] have long been used as random distributed medium access protocols for radio channels. They are used in satellite networks and cellular telephone networks for the sporadic transfer of data packets. In these protocols, packets are transmitted sporadically by various users. If packets are sent simultaneously by more than one user then they collide. After a packet is transmitted, the transmitter receives the information on whether there has been a collision (and retransmission is needed) or whether it has been well received. All colliding packets are assumed to be corrupted which get backlogged and are retransmitted after some random time. We focus on the Slotted Aloha [7], in which time is divided into units. At each time unit a packet may be transmitted, and at the end of the time interval, the sources get the feedback on whether there was zero, one or more transmissions (collisions) during the time slot. A packet that arrives at a source is immediately transmitted.

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Packets that are involved in a collision are backlogged and are scheduled for retransmission after a random time. As in [5] we introduce new different schemes with multiple power levels. When several packets are sent simultaneously, one of them can often be successfully received due to the power capture effect. In this paper, we consider a general capture model where a mobile transmit successfully a packet if its instantaneous SINR is larger than the threshold. In the paper [5], the authors consider a particular capture model, in which if two or more packets are transmitted simultaneously with the same power, they assume that neither one can be captured. A similar capture model are already proposed in [9, 13, 15].

We study in this paper different schemes. In particular, we introduce the differentiation between new packets and backlogged packets allowing prioritization of one or the other in terms of transmitted power. We study and compare (1) a scheme with power diversity and without prioritization in transmission or retransmission; (2) a scheme in which a new packet is transmitted with the lowest power, and backlogged packets are transmitted at a random power selected among $N-1$ larger distinct levels; (3) a scheme in which a new packet is transmitted with the highest power, and backlogged packets are transmitted at a random power selected among $N-1$ lower distinct levels; (4) a scheme in which backlogged packets are retransmitted with the lowest power level and a new packet is transmitted at a random power selected among $N-1$ larger distinct levels; and (5) standard Slotted Aloha.

Interest has been growing in recent years in studying competition of networking in general, access to a common medium in particular, within the frame of non-cooperative game theory, see e.g. the survey paper [6]. Various game formulations of the standard Slotted Aloha (with a single power) have been derived and studied in [4, 3, 11, 12, 8] for the noncooperative choice of transmission probabilities. Several papers study Slotted Aloha with power diversities but without differentiating between transmitted and backlogged packets, and without the game formulation: In [13] it is shown that the system capacity could be increased from 0.37 to 0.53 if one class of terminals always used high power and the other always used low power level. In [9], power diversity is studied with the capture model that we use as well as with another capture model based on signal to noise ratio. [15] studies power diversity under three types of power distribution between the power levels and provides also stability analysis. [10] proposes a model and evaluates the throughput that can be achieved in a system of N nodes using generalized Aloha like protocols where nodes transmit data using a two-state decision system. For cooperative systems, it gives the throughput bounds and explores the trade-off between throughput and short-term fairness. In our proposal, we compute the effects of randomization in power levels in both cooperative and non-cooperative setup.

The capture model used in [4] is not always true. The authors in [4] assume that if only one user chooses the highest power, its transmission is succeeded independently of the other terminals and their choices. In fact, this assumption could not be always true. It's possible that the other mobiles jam the signal of the mobile whose power level is the highest, therefore no successful transmission exists. In this paper, a terminal succeeds its transmission if it chooses the most elevated power level comparing to the others mobiles and its *SINR* (signal to interferences and noise ratio) is greater than a given threshold ($SINR_{th}$).

Under more general capture model, we study the team problem in which we optimize transmission probabilities for the various schemes so as to achieve the maximum throughput or to minimize expected delay. We discover that in heavy load, the optimality is obtained at the expense of huge expected delay of backlogged packets (*EDBP*). We therefore consider the alternative objective of minimizing the *EDBP*. We study both the throughput as well as the delay performance of the global optimal policy. We also solve the multi-criteria problem in which the objective is a convex combination of the throughput and the *EDBP*. This allows in particular, to compute the transmission probabilities that maximize the throughput under a constraint on *EDBP*, which could be quite useful for delay-sensitive applications. We show that schemes with priority not only improve the average performances considerably but they are also able in some cases to eliminate the bi-stable nature of the Slotted Aloha.

We also studied the game problem in which each mobile chooses its transmission probability selfishly in order to optimize its own objective. This rise to a game theoretic model of which we study the equilibrium properties (Nash equilibrium). We show that the power diversity and the prioritization profit to mobiles also in this competitive scenario even if the advantage is less notorious than in the team's behavior.

The rest of this paper is organized as follows. In Section II, we describe the problem and model. In Section III, we discuss the team formulation of the problem. In Section IV, we formulate the game setting. And finally we discuss the performance of different schemes numerically in section V.

2. MODEL AND PROBLEM FORMULATION

We consider one central receiver and m mobiles without buffer. A mobile can transmit a packet using a power from N different levels. We consider a general capture model where a packet transmitted by a mobile is received successfully only if its $SINR$ ratio is larger than a given threshold $SINR_{th}$. Let P_i be the power level chosen by mobile i, σ^2 is the spectral density of the background noise. Let us denote the attenuation of the signal of mobile i by g_i. The expression of the *SINR* of mobile i is given by :

$$SINR_i = \frac{g_i \cdot P_i}{\sum_{j=1}^{k} g_j \cdot P_j + \sigma^2} \tag{1}$$

Hence, a packet of mobile i is transmitted successfully if its $SINR_i$ defined in (1) is greater than $SINR_{th}$.

We use a Markovian model such the one used in [4, 3, 5]. The arrival probability of the packets to the source i follows a Bernoulli process with parameter q_a (i.e. at each time slot, there is a probability q_a of a new arrival at a source, and all arrivals are independent). As long as there is a packet at a source (i.e. as long as it is not successfully transmitted) new packets to that source are blocked and lost (because we consider sources without buffer). The arrival processes to different sources are independent. A backlogged packet at source i is retransmitted with probability q_r^i and we should restrict in our control and game problems to simple policies in which q_r^i does not change in time. Since sources are symmetric, we should further restrict to find a symmetric optimal solution, that is, retransmission probabilities q_r^i that do not depend on i.

The state of the system is the number of backlogged packets in the beginning of a slot and we denote it by n. For any choice of values $q_r^i \in (0,1]$, the state process is a Markov chain that contains a single ergodic chain (and possibly transient states as well). Define q_r to be the vector of retransmission probabilities for all users (whose jth entry is q_r^j). Let $\pi(q_r)$ be the corresponding vector of steady state probabilities where its n^{th} entry, $\pi_n(q_r)$ denotes the probability of n backlogged nodes. When all entries of q_r are the same, say q, we shall write (with some abuse of notation) $\pi(q)$ instead of $\pi(q_r)$.

We introduce further notation. Assume that there are n backlogged packets, and all use the same value q_r as retransmission probability. Let $Q_r(i,n)$ be the probability that i out of the n backlogged packets retransmit at the slot. Then

$$Q_r(i,n) = \binom{n}{i}(1-q_r)^{n-i}(q_r)^i \tag{2}$$

Let $Q_a(i,n)$ be the probability that i unbacklogged nodes transmit packets in a given slot (i.e. that i arrivals occurred at nodes without backlogged packets). Then

$$Q_a(i,n) = \binom{m-n}{i}(1-q_a)^{m-n-i}(q_a)^i \tag{3}$$

Let $Q_r(i,0) = 0$ and $Q_a(i,m) = 0$.

3. THE TEAM PROBLEM

In this section we study Slotted Aloha as a team problem. All mobiles maximize the system throughput (or minimize delay). In the sequel, we assume that $g_i = g \; \forall i = 1, 2, .., m$. Therefore we analyze four different schemes.

3.1 Scheme 1 : Random power levels without priority scheme

In this approach, there is no preference between new packets or backlogged packets. A mobile can choose to transmit using a power level among N different levels in set $\mathcal{N} = \{1, 2, .., N\}$. In case all nodes use the same value q the transition probability of the system is given by $\bar{P}_{n,n+i} =$

$$\begin{cases} Q_a(m-n,n)\sum_{j=0}^{n} Q_r(j,n)(1-A_{j+m-n}), & i=m-n, i\geq 2 \\ Q_a(i,n)\sum_{j=0}^{n} Q_r(j,n)(1-A_{j+i}) & \\ \quad +Q_a(i+1,n)\sum_{j=0}^{n} Q_r(j,n)A_{j+i+1}, & 2\leq i< m-n \\ Q_a(1,n)\sum_{j=1}^{n} Q_r(j,n)(1-A_{j+1}) & \\ \quad +Q_a(2,n)\sum_{j=0}^{n} Q_r(j,n)A_{j+2}, & i=1 \\ Q_a(0,n)[Q_r(0,n)+\sum_{j=2}^{n} Q_r(j,n)(1-A_j)] & \\ \quad +Q_a(1,n)\sum_{j=0}^{n} Q_r(j,n)A_{j+1}, & i=0 \\ Q_a(0,n)\sum_{j=1}^{n} Q_r(j,n)A_j, & i=-1 \end{cases}$$

where the probability of a successful transmission among $k \geq 2$ is given by:

$$A_k = k\sum_{l=0}^{N-2}\sum_{k_1=0}^{k-1}\sum_{k_2=0}^{k-1}\cdots\sum_{k_{N-l-1}=0}^{k-1}[X_1^{k_1}\cdot X_2^{k_2} \cdots X_{N-l-1}^{k_{N-l-1}}\cdot X_{N-l}^{1}\cdot\delta(k-1-\sum_{s=1}^{N-l-1}k_s)\cdot u(\frac{P_{N-l}}{\sum_{s=1}^{N-l-1}P_s k_s+\sigma^2/g}-SINR_{th})] \quad (4)$$

with $A_0 = 0$ and $A_1 = 1$. In the equation (4), X_s is the probability that a packet (new arrival or backlogged) will choose power level P_s for transmission/retransmission. P_{N-l} is the power level chosen by the terminal whose transmission maybe potentially succeed (it's the highest power selected in this scenario). We denote by k_s the number of terminals that choose the power level P_s. $\delta(t)$ (Dirac distribution) and $u(t)$ (unite echelon) are as following :

$$\delta(t)=\begin{cases}1 & if \quad t=0\\ 0 & else\end{cases} \qquad u(t)=\begin{cases}1 & if \quad t\geq 0\\ 0 & else\end{cases} \quad (5)$$

3.2 Scheme 2 : Retransmission with more power

In this scheme, backlogged packets have more priority; a backlogged packet retransmits using a random power level among the N available (we use a uniform distribution when choosing a power level), while a new arrived use always the lowest level (P_1). Successful capture is occurred when one of the backlogged packets transmits with a power level which is larger than the one chosen by all others transmitters and its corresponding $SINR$ is greater than the $SINR_{th}$ when arriving at the AP or a single new arrival occurs and there is no retransmission attempt of any backlogged packet. The transition matrix is given by $\bar{P}_{n,n+i} =$

$$\begin{cases} Q_a(m-n,n)\sum_{j=1}^{n} Q_r(j,n)(1-B_{j,m-n}), & i=m-n, i\geq 2 \\ Q_a(i,n)\sum_{j=0}^{n} Q_r(j,n)(1-B_{j,i}) & \\ \quad +Q_a(i+1,n)\sum_{j=1}^{n} Q_r(j,n)B_{j,i+1}, & 2\leq i< m-n \\ Q_a(1,n)\sum_{j=1}^{n} Q_r(j,n)(1-B_{j,1}) & \\ \quad +Q_a(2,n)\sum_{j=1}^{n} Q_r(j,n)B_{j,2}, & i=1 \\ Q_a(0,n)[Q_r(0,n)+\sum_{j=2}^{n} Q_r(j,n)(1-B_{j,0})] & \\ \quad +Q_a(1,n)\sum_{j=0}^{n} Q_r(j,n)B_{j,1}, & i=0 \\ Q_a(0,n)\sum_{j=1}^{n} Q_r(j,n)B_{j,0}, & i=-1 \end{cases}$$

where the probability of a successful transmission among k retransmissions and k' new arrival packets when $k+k' \geq 2$ is given by:

$$B_{k,k'} = \sum_{l=0}^{N-2}\sum_{k_1=0}^{k-1}\cdots\sum_{k_{N-l-1}=0}^{k-1}[X_1^{k_1}\cdots X_{N-l-1}^{k_{N-l-1}}\cdot X_{N-l}^{1}]\cdot \delta(k-1-\sum_{s=1}^{N-l-1}k_s)\cdot u(\frac{P_{N-l}}{\sum_{s=1}^{N-l-1}P_s k_s+k'P_1+\sigma^2/g}-SINR_{th})$$

with $B_{0,0} = 0$,$B_{0,1} = 1$ and $B_{1,0} = 1$

3.3 Scheme 3 : Retransmission with less power

In this scheme, a new transmitted packet has the highest power. Backlogged packets attempt retransmissions with a random power choice among $N-1$ distinct lower power levels. The transition matrix is given by: $\bar{P}_{n,n+i} =$

$$\begin{cases} Q_a(i,n), & 2\leq i \\ Q_a(1,n)\sum_{j=1}^{n} Q_r(j,n)(1-C_{j,1}), & i=1 \\ Q_a(0,n)[Q_r(0,n)+\sum_{j=2}^{n} Q_r(j,n)(1-C_{j,0})] & \\ \quad +Q_a(1,n)\sum_{j=0}^{n} Q_r(j,n)C_{j,1}, & i=0 \\ Q_a(0,n)\sum_{j=1}^{n} Q_r(j,n)C_{j,0}, & i=-1 \end{cases}$$

where the probability of a successful transmission when $k \geq 2$ mobiles attempt retransmissions is given by:

$$C_{k,1} = k\sum_{k_1=0}^{k-1}\sum_{k_2=0}^{k-1}\cdots\sum_{k_{N-1}=0}^{k-1}[X_1^{k_1}\cdots X_{N-1}^{k_{N-1}}\cdot X_{N-l} \cdot\delta(k-\sum_{s=1}^{N-1}k_s)\cdot u(\frac{P_N}{\sum_{s=1}^{N-1}P_s k_s+\sigma^2/g}-SINR_{th})]$$

the probability of a successful retransmission among $k \geq 2$ is given by:

$$C_{k,0} = k\sum_{l=1}^{N-2}\sum_{k_1=0}^{k-1}\sum_{k_2=0}^{k-1}\cdots\sum_{k_{N-l-1}=0}^{k-1}[X_1^{k_1}\cdots X_{N-l-1}^{k_{N-l-1}}\cdot X_{N-l}^{1} \cdot\delta(k-1-\sum_{s=1}^{N-l-1}k_s)\cdot u(\frac{P_{N-l}}{\sum_{s=1}^{N-l-1}P_s k_s+\sigma^2/g}-SINR_{th})]$$

$$C_{k,k'} = 0 \quad if \quad K' \geq 2, \quad C_{0,1} = 1 \quad and \quad C_{1,0} = 1$$

3.4 Scheme 4 : Retransmission with the lowest power

In this proposal, a new transmitted packet uses a power among $N-1$ higher available power level. Backlogged packets retransmit with the lowest power level (P_1). The transition matrix of the Markov chain is given by $\bar{P}_{n,n+i} =$

$$\begin{cases} Q_a(m-n,n)\sum_{j=0}^{n} Q_r(j,n)(1-D_{j,m-n}), & i=m-n, i\geq 2 \\ Q_a(i,n)\sum_{j=0}^{n} Q_r(j,n)(1-D_{j,i}) & \\ \quad +Q_a(i+1,n)\sum_{j=0}^{n} Q_r(j,n)D_{j,i+1}, & 2\leq i< m-n \\ Q_a(1,n)\sum_{j=1}^{n} Q_r(j,n)(1-D_{j,1}) & \\ \quad +Q_a(2,n)\sum_{j=0}^{n} Q_r(j,n)D_{j,2}, & i=1 \\ Q_a(0,n)[Q_r(0,n)+\sum_{j=2}^{n} Q_r(j,n)(1-D_{j,0})] & \\ \quad +Q_a(1,n)\sum_{j=0}^{n} Q_r(j,n)D_{j,1}, & i=0 \\ Q_a(0,n)Q_r(1,n), & i=-1 \end{cases}$$

where $D_{k,k'}$ is the probability of a successful transmission among k backlogged packets and k' new packets such that $k'+k \geq 2$. The value $D_{k,k'}$is given by

$$k' \sum_{l=0}^{N-2} \sum_{k'_1=0}^{k-1} \cdots \sum_{k'_{N-l-1}=0}^{k'-1} [X_1^{k'_1} \cdot X_2^{k'_2} \cdots X_{N-l-1}^{k'_{N-l-1}} \cdot X_{N-l}^1]$$
$$\cdot\delta(k' - 1 - \sum_{pl=1}^{N-l-1} k'_{pl}) \cdot u(\frac{P_{N-l}}{\sum_{pl=1}^{N-l-1} P_l k'_{pl} + kP_1 + \sigma^2/g} - SINR_{th})]$$

where $D_{0,0} = 0$, $D_{0,1} = 1$ and $D_{1,0} = 1$

3.5 Performance metrics

To optimize in terms of either throughput or expected delay, we need to calculate the steady state of the system. Let's denote by $\pi_n(q_r)$ the equilibrium probability that the network is in state n (number of backlogged packets at the beginning of a slot). Hence the equilibrium state equations are:

$$\begin{cases} \pi(q) = \pi(q) \cdot P(q) \\ \sum_{n=0}^{m} \pi_n(q) = 1 \\ \pi_n(q) \geq 0, \qquad n = 0, 1, ..., m \end{cases} \tag{6}$$

When the steady state is achieved the average number of backlogged packets is given by equation :

$$S(q) = \sum_{n=0}^{m} \pi_n(q) \cdot n \tag{7}$$

The system throughput (defined as the sample average of the number of packets that are successfully transmitted) is given almost surely by the constant $thp(q) =$

$$\begin{cases} \sum_{n=0}^{m} \sum_{i=0}^{m-n} \sum_{j=0}^{n} \pi_n(q) Q_a(i,n) Q_r(j,n) A_{j+i} & \text{Scheme 1} \\ \sum_{n=0}^{m} \pi_n(q) [\sum_{i=0}^{m-n} \sum_{j=1}^{n} Q_a(i,n) Q_r(j,n) B_{j,i} + Q_a(1,n) Q_r(0,n)] & \text{Scheme 2} \\ \sum_{n=0}^{m} \pi_n(q) [Q_a(0,n) \sum_{j=1}^{n} Q_r(j,n) C_{j,0} + Q_a(1,n) \sum_{j=0}^{n} Q_r(j,n) C_{j,1}] & \text{Scheme 3} \\ \sum_{n=0}^{m} \pi_n(q) [\sum_{i=1}^{m-n} \sum_{j=0}^{n} Q_a(i,n) Q_r(j,n) D_{j,i} + Q_a(0,n) Q_r(1,n)] & \text{Scheme 4} \\ \sum_{n=0}^{m} \pi_n(q) [Q_a(0,n) Q_r(1,n) + Q_a(1,n) Q_r(0,n)] & \text{Same power} \end{cases} \tag{8}$$

The throughput satisfies (and thus can be computed more easily through)

$$thp(q) = q_a \sum_{n=0}^{m} \pi_n(q)(m-n) = q_a(m - S(q)) \tag{9}$$

Indeed, the throughput is the expected number of arrivals at a time slot (which actually enter the system), and this is expressed in the equation for $thp(q)$ by conditioning on n. The throughput should be equal to the expected number of departures (and thus the throughput) at stationary regime, which is expressed in (9). The expected delay of transmitted packets D, is defined as the average time, in slots, that a packet takes from its source to the receiver. Applying Little's result, this is given by:

$$D(q) = 1 + \frac{S(q)}{thp(q)} = 1 + \frac{S(q)}{q_a(m - S(q))} \tag{10}$$

Combining the equation (9) with 10) it follows that maximizing the global throughput is equivalent to minimizing the average delay of transmitted packets. We shall therefore restrict in our numerical investigation to maximization of the throughput. However, we shall consider the delay of backlogged packets as yet another objective to minimize.

Let Δ be the throughput of new arrivals, the throughput of the backlogged packets for each scheme is given by: $thp^c(q) = thp(q) - \Delta$ where Δ is calculated by:

$$\begin{cases} \sum_{n=0}^{m} \sum_{i=1}^{m-n} \sum_{j=0}^{n} \frac{i}{i+j} \pi_n(q) Q_a(i,n) Q_r(j,n) A_{i+j} & \text{Scheme 1} \\ \sum_{n=0}^{m} \pi_n(q) Q_a(1,n) Q_r(0,n) & \text{Scheme 2} \\ \sum_{n=0}^{m} \pi_n(q) Q_a(1,n) \sum_{j=0}^{n} Q_r(j,n) C_{j,1} & \text{Scheme 3} \\ \sum_{n=0}^{m} \sum_{i=1}^{m-n} \sum_{j=0}^{n} \pi_n(q) Q_a(i,n) Q_r(j,n) D_{j,i} & \text{Scheme 4} \\ \sum_{n=0}^{m} \pi_n(q) Q_a(1,n) Q_r(0,n) & \text{Same power} \end{cases} \tag{11}$$

The expected delay of backlogged packets D^c, which is defined as the average time, in slots, that a backlogged packet takes to go from the source to the receiver, can also be calculated by applying Little's result. Hence,

$$D^c(q) = 1 + \frac{S(q)}{thp^c(q)} \tag{12}$$

Team problem resolution. The solution of the team problem is therefore given as the solution of the following optimization problem:

$$\max_q \ objective(q) \ \ s.t. \begin{cases} \pi(q) = \pi(q) \cdot P(q) \\ \sum_{n=0}^{m} \pi_n(q) = 1 \\ \pi_n(q) \geq 0, \qquad n = 0, 1, ..., m \end{cases} \tag{13}$$

Stability. Another qualitative way to compare schemes is in the stability characteristics of the protocol. Slotted Aloha is known to have a bi-stable behavior, and we shall check whether this is also the case in our new schemes if answer is positive, under which conditions it happens?

Let us denote P_{succ} the expected number of successful transmissions in the slot, which is just the probability of a successful transmission and it is given by $P_{succ}(q) =$

$$\begin{cases} \sum_{i=0}^{m-n}\sum_{j=0}^{n} Q_a(i,n)Q_r(j,n)A_{j+i} & \text{Scheme 1} \\ \sum_{i=0}^{m-n}\sum_{j=1}^{n} Q_a(i,n)Q_r(j,n)B_{j,i} + Q_a(1,n)Q_r(0,n) & \text{Scheme 2} \\ Q_a(0,n)\sum_{j=1}^{n} Q_r(j,n)C_{j,0} + Q_a(1,n)\sum_{j=0}^{n} Q_r(j,n)C_{j,1} & \text{Scheme 3} \\ \sum_{i=1}^{m-n}\sum_{j=0}^{n} Q_a(i,n)Q_r(j,n)D_{j,i} + Q_a(0,n)Q_r(1,n) & \text{Scheme 4} \\ Q_a(0,n)Q_r(1,n) + Q_a(1,n)Q_r(0,n) & \text{Same power} \end{cases} \quad (14)$$

Define now the *drift* in state n, D_n, as the expected change in backlog from one slot to the next slot, which is the expected number of arrivals, $q_a(m-n)$ i.e. , less the expected number of successful departures P_{succ}, that is:

$$D_n = q_a(m-n) - p_{succ} \quad (15)$$

It has been observed for standard Slotted Aloha (see [5 sect.4]) that there are three equilibria. System equilibrium points occur where the curve, i.e.(P_{succ}) and the straight line, i.e.($q_a(m-n)$) intersect. When the drift, which is the difference between the straight line and the curve, is positive, the system state tends to increase, while it decreases when the drift is negative. This explains immediately why the middle equilibrium point is unstable and the two other are stable. A bi-stable situation as in the standard Aloha is hence undesirable since it means in practice that the system spends long time in each of the stable equilibria including in the one with large n corresponding to a congestion situation (low throughput and large delays). We shall study numerically the stability behavior of all schemes.

4. THE GAME PROBLEM

In fact Slotted Aloha system is usually a decentralized entity, therefore the team model does not hold anymore, so we shall formulate a game model. The decentralized model is more powerful and appropriate to Slotted Aloha. The equilibrium concept then replaces the optimality concept from the team problem. It possesses a robustness property: at equilibrium, no mobile has incentive to deviate.

Next, we formulate the game problem. For a given policy vector $\vec{q}_r$ of retransmission probabilities for all users (whose jth entry is q_r^j), define $([\vec{q}_r]^{-i}, \hat{q}_r^i)$ to be a retransmission policy where user j retransmits at a slot with probability q_r^j for all $j \neq i$ and where user i retransmits with probability $\hat{q}_r^i$. Each user i seeks to maximize his own throughput thp_i. The problem we are interested in is to find a symmetric equilibrium policy $\vec{q}_r^* = (q_r, q_r, .., q_r)$ such that for any user i and any retransmission probability q_r^i for that user,

$$thp_i(\vec{q}_r^*) \geq thp_i([\vec{q}_r^*]^{-i}, q_r^i). \quad (16)$$

Since we restrict to symmetric $\vec{q}_r^*$, we shall also identify it (with some abused of notation) with the actual transmission probability (which is the same for all users). Next we show how to obtain an equilibrium policy. We first note that due to symmetry, to see whether $\vec{q}_r^*$ is an equilibrium it suffices to check (16) for a single player. We shall thus assume that there are $m+1$ users all together, and that the first m users retransmit with a given probability $\vec{q}_r^{\,-(m+1)} = (q^o, .., q^o)$ and user $m+1$ retransmits with probability $q_r^{(m+1)}$. Define the set

$$\mathcal{Q}^{m+1}(\vec{q}_r^o) = argmax_{q_r^{(m+1)} \in [\epsilon,1]} \Big(thp_{m+1}([\vec{q}_r^o]^{-(m+1)}, q_r^{(m+1)}) \Big),$$

where $\vec{q}_r^o$ denotes (with some abuse of notation) the policy where all users retransmit with probability q_r^o, and where the maximization is taken with respect to $q_r^{(m+1)}$. Then q_r^* is a symmetric equilibrium if

$$q_r^* \in \mathcal{Q}_r^{m+1}(q_r^*).$$

To compute the performance measures of interest, we introduce again a Markov chain with a two dimensional state. The first state component corresponds to the number of backlogged packets among the users $1, ..., m$, and the second component is the number of backlogged packets (either 1 or 0) of user $m+1$. Due to lack of space, transition matrices of all schemes are given in Appendix of full paper [1].

Hence the average number of backlogged packets of source $(m+1)$ is given by:

$$S_{m+1}([\vec{q}_r^o]^{-(m+1)}, q_r^{m+1}) = \sum_{n=0}^{m} \pi_{n,1}([\vec{q}_r^o]^{-(m+1)}, q_r^{m+1}) \quad (17)$$

And the average throughput of user $(m+1)$ is given by:

$$thp_{m+1}([\vec{q}_r^o]^{-(m+1)}, q_r^{m+1}) = q_a \sum_{n=0}^{m} \pi_{n,0}([\vec{q}_r^o]^{-(m+1)}, q_r^{m+1}) \quad (18)$$

Hence the expected delay of transmitted packets of user $(m+1)$ for all schemes is given by:

$$D_{m+1}([\vec{q}_r^o]^{-(m+1)}, q_r^{m+1}) = 1 + \frac{S_{m+1}([\vec{q}_r^o]^{-(m+1)}, q_r^{m+1})}{thp_{m+1}([\vec{q}_r^o]^{-(m+1)}, q_r^{m+1})} \quad (19)$$

Let us denote the throughput of backlogged packets (i.e. of the packets that arrive and become backlogged) at source $(m+1)$ by:

$$thp_{m+1}^c(q_r^{m+1}) = \sum_{n=0}^{m}\sum_{n'=0}^{m} P_{(n,0),(n',1)}(q_r^{m+1})\pi_{n,0}(q_r^{m+1}) \quad (20)$$

Thus, the expected delay of backlogged packets at source $(m+1)$, is given by:

$$D_{m+1}(q_r^{m+1}) = 1 + \frac{S_{m+1}(q_r^{m+1})}{thp_{m+1}^c(q_r^{m+1})} \quad (21)$$

5. NUMERICAL INVESTIGATION

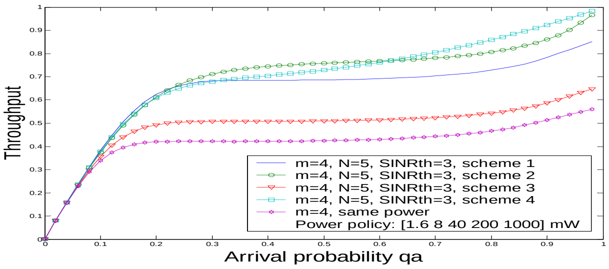
We describe next numerical investigation of the team and the game problems for the four schemes as well as standard slotted Aloha

5.1 Team problem: Maximizing the system throughput

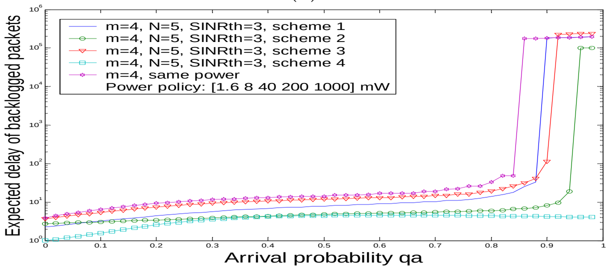
In this subsection we maximize the global throughput. In Fig 1(a) and Fig 1(b) we plot the throughput and expected delay of backlogged packets (EDBP) for all schemes for $m =$

4, $SINR_{th} = 3$, $N = 5$ and $P = [1.6\ 8\ 40\ 200\ 1000] mW$ as a power policy for all schemes. Slotted Aloha is then equivalent to scheme 1 with same power policy.

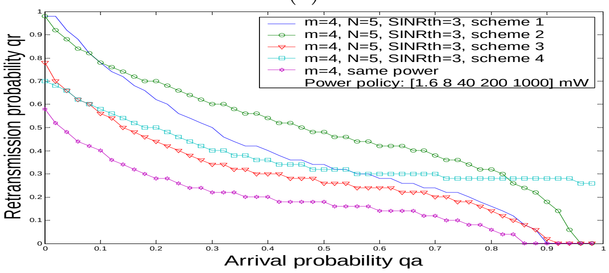
We observe that when load is very low $(0.1 < q_a)$ all schemes have nearly the same performance which is a linear function of q_a. In the average load, scheme 2 performs better than other schemes in terms of throughput. This is due to the fact that scheme 2 prioritizes the retransmission of backlogged packets operating the fact that there are few new comers. But at high load the throughput of scheme 4 is the highest because it prioritize the new arrivals. In fact new arrivals have an extended choice of power levels so its benefits from prioritization and power diversity. We remark that scheme 3 which is the same as the one used in [5] presents the lower performance comparing to other schemes with priority and power diversity, this is due to the negative effect of power randomization and power control given to backlogged packets, which directly influences the value of the instantaneous $SINR$. We note also that all schemes with random power selection outperform standard Aloha.



(a)



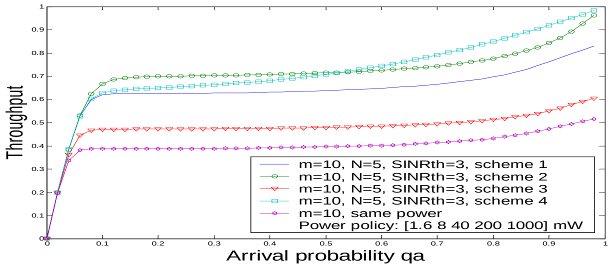
(b)



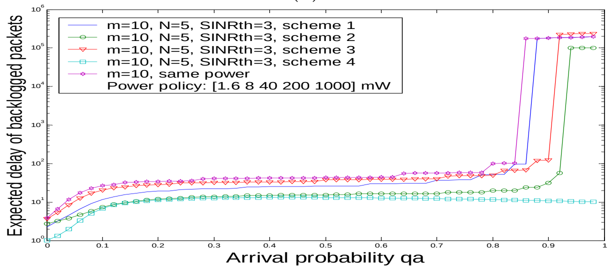
(c)

Figure 1: Performances for the team case vs. arrival probability q_a for all schemes when max. the system throughput for $m = 4$.

In term of expected delay of backlogged packets, we observe that maximizing throughput leads to the following results (see Fig 1(b)): at low load, scheme 4 performs better than other schemes. In the average and high load $(0.2 < q_a < 0.8)$ scheme 2 and 4 perform both well and are equivalent. But for $0.8 < q_a$, scheme 4 is the most interesting whereas scheme 1-3 and Slotted Aloha perform very bad.



(a)



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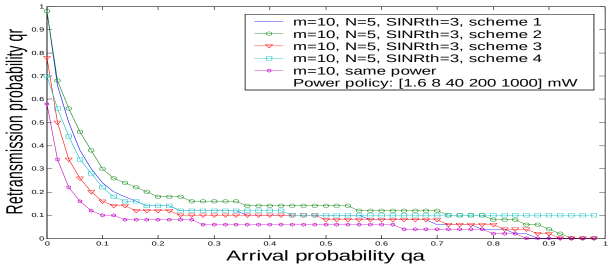


(c)

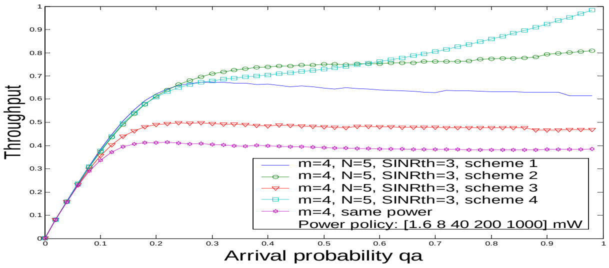
Figure 2: Performances for the team case vs. arrival probability q_a for all schemes when max. the system throughput for $m = 10$.

Next we plot the optimal retransmission probability versus arrival probability. We remark that for $m = 4$ (Fig 1(c)) all schemes optimal retransmission probability q_r decreases with q_a until to be semi-annulled ($q_r \simeq 10^{-4}$ because of δ-optimality) for schemes 1-3 and Slotted Aloha, when it keeps a constant value (about 0.3) for scheme 4 (for q_a over 0.5) because it prioritizes new packets and then it doesn't penalize too much from huge amount of backlogged packets and retransmission rate.

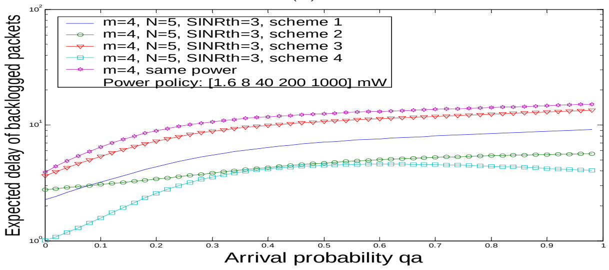
In figure 2, we consider 10 mobiles and 5 power levels. We observe similar trends in term of throughput and delay for all schemes. In fact even if the number of mobiles is wide, the performance is handled by decreasing retransmission probability so as to avoid extra collisions. We remark that at heavy load the system ask mobiles to decrease their retransmission probabilities to avoid collisions, therefore the system keeps a very good amount of successful departure, then an optimal value of throughput which is better compared to Slotted Aloha.

5.2 Team problem: Minimizing the delay

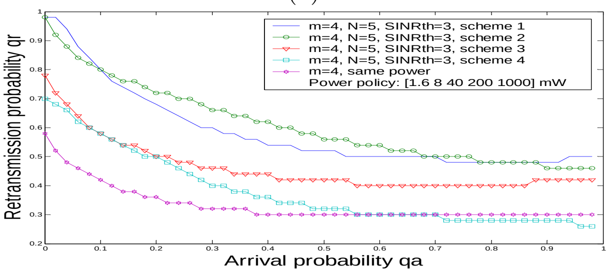
When maximizing the global throughput (figure 1 and 2) we observed a huge $EDBP$ under all schemes chiefly at heavy load except scheme 4 which handle a constant delay. This may be very harmful for many applications which are very sensitive to delay (real time applications). In figures Fig 3(a) and Fig 3(b), We shall investigate the problem of minimizing $EDBP$ and study the impact of this optimization on the throughput performance. We shall note in particular that throughput performance in the four schemes improves considerably with respect to Slotted Aloha. Scheme 1 is slightly better in terms of throughput only at light load, scheme 2 is almost better in medium load whereas scheme 4 outperforms remarkably the other schemes at high $(0.55 < q_a)$ and very high load. The case of 10 mobiles provides similar trends.



(a)



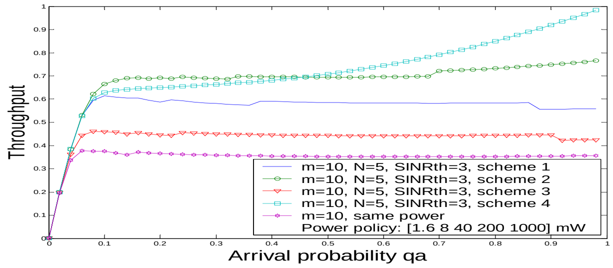
(b)



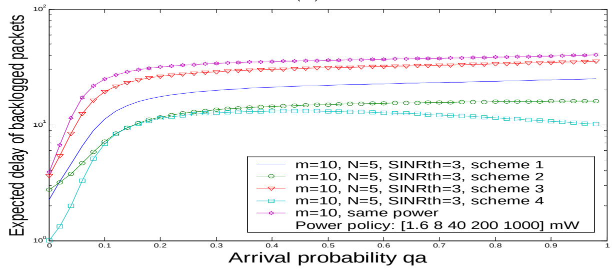
(c)

Figure 3: Optimal throughput, Expected delay of backlogged and retransmission probability for the team case as a function of arrival probability q_a for all schemes when minimizing the expected delay of backlogged packets for $m = 4$.

When $EDBP$ is minimized, for $m = 4$ and $N = 5$, retransmission probability decreases with q_a, so standard Aloha and scheme 4 have optimal retransmission probability of around 0.3 in heavy load whereas proposals 1-3 have much higher retransmission probabilities (Fig 3(c)). But when $m = 10$ and $N = 5$ (Fig 4(c)) we observe that optimal retransmission probability falls exponentially for all



(a)



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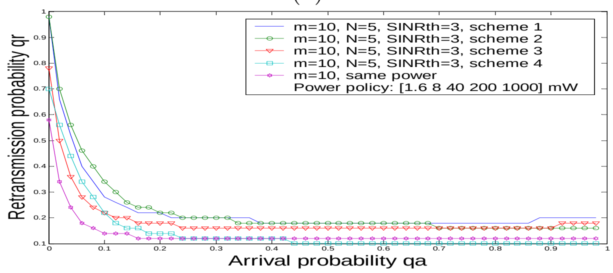


(c)

Figure 4: Optimal throughput, Expected delay of backlogged and retransmission probability for the team case as a function of arrival probability q_a for all schemes when minimizing the expected delay of backlogged packets for $m = 10$.

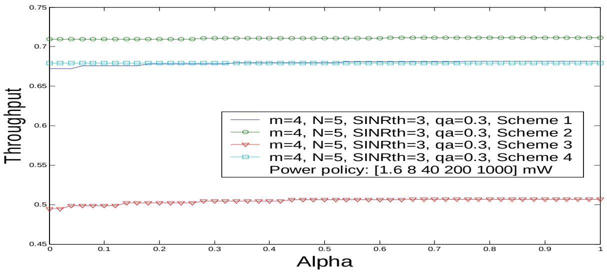
schemes. Therefore in high load, the retansmission probability for Slotted Aloha is around 0.13, for scheme 4 is around 0.1 and for scheme 1-3, is around 0.19.

Table 1 summarizes the performance of the team problem in terms of throughput and EDBP under throughput maximization. We can easily check the impact of m, q_a and N on system's performance as shown in previous subsection. This table is given in Appendix of full paper [1].

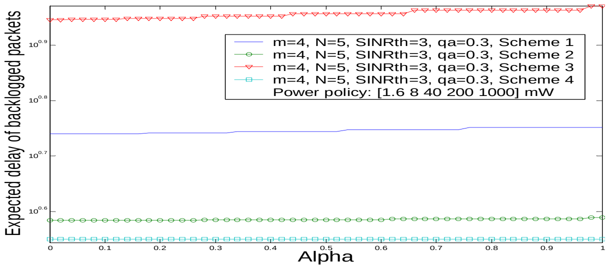
5.3 Team problem: Multi-criteria

In previous simulations we consider the extreme cases of maximizing independently the throughput or minimizing the $EDBP$. In practice it may be more interesting to have a multi-criteria optimization in which a convex combination of both the throughput and $EDBP$ are optimized. The objective is given by $\alpha thp(q) + (1-\alpha)/D^c(q), 0 \leq \alpha \leq 1$. This allows in particular handling QoS constraints: By varying α one can find appropriate trade-off between the throughput and delays.

At low load $(q_a = 0.3)$, for all schemes 1-4 (Fig 5(a and b)), the optimal throughput and $EDBP$ are slightly constants. In fact, its optimal retransmission probability under both objectives (maximizing throughput and minimizing delay) are so close. At high load (Fig 6(a and b)), we observe that the throughput (resp. $EDBP$) increases when α increases. Hence, there is a trade-off beteween throughput and $EDBP$ by changing α.

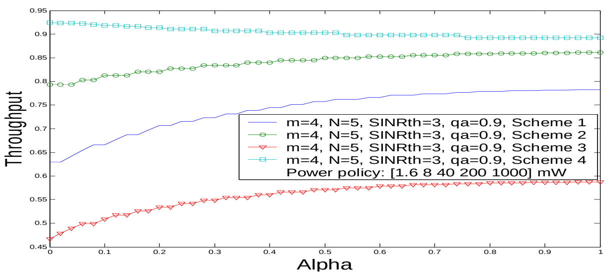


(a)

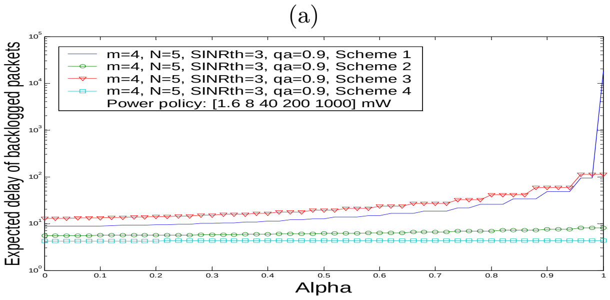


(b)

Figure 5: Throughput and delay as a function of α for all the schemes under light load ($q_a = 0.3$).



(a)



(b)

Figure 6: Throughput and delay as a function of α for all the schemes under high load ($q_a = 0.9$).

5.4 Team problem: Stability

In Fig 7, we illustrate the stability behavior for 40 mobiles ,$SINR_{th} = 3, q_a = 0.01$ and $q_r = 0.15$. The drift is the difference between the curves (representing the departure rate) and the straight line representing the arrival rate $q_a(m - n)$. We note that slotted Aloha is the only scheme that suffers from the bi-stability problem under $q_r = 0.15$. Except scheme 3 (more figures are provided in full paper [1]), all schemes suffer from the bi-stability problem at $q_r = 0.19$. Over this value of q_r all schemes suffer from bi-stability. We see that for standard slotted Aloha, the departure is at most $1/e \approx 0.37$ whereas for different power schemes it is interestingly higher.

The average number of backlogged packets (ABP) for different schemes which correspond to their equilibrium points are given in Table II (provided in full paper) with $m = 30$, $q_a = 0.01$ and N=5. This is compared to the expected number of backlogged packets. In the case of a single equilibrium and when $q_r < 0.5$, a good match is seen for schemes 1, 2 and 3, which means that the simple computation of the stable equilibrium can be used to approximate the expected number of backlogged packets. In standard Aloha we see that the congested stable equilibrium provides a very good approximation for the expected number of backlogged packets, which suggests that the system spends most of the time at that equilibrium. At high rate of retransmission there is the same behavior in all schemes when the retransmission probability increases (around 0.3). Then all schemes acquire a bi-stable behavior with $q_r = 0.3$, but contrary to standard aloha, we see from Table II that the expected number of backlogged packets for scheme 1, 2, 3 and 4 can be approximated by the desired stable equilibrium which is a very interesting feature. That means that in the bi-stability case for scheme 1, 2, 3 and 4, the system spends most of the time at the desired equilibrium. When mobiles tend to retransmit or become more aggressive (q_r around 0.5), we see that the congested (non desired) stable equilibrium provides a very good approximation for the expected number of backlogged packets in all schemes, which shows that the system spends most of the time at that equilibrium.

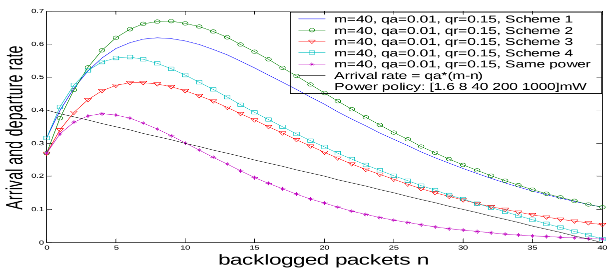


Figure 7: Probability of success transmission versus number of backlogged packets n for all the schemes

5.5 Game problem: Maximizing individual throughput

Next, we evaluate the performance in game problem. For m=3, i.e 4 mobiles altogether, analyzing Fig 8, we can already say that scheme 1-3 and slotted Aloha present nearly similar profile with some differences in terms of numerical values. In fact global equilibrium throughput is a concave function of arrival probability q_a, at low load it presents an increasing behavior until achieving a maximum throughput of $thp_{max} \simeq 0.34$ at $q_a \simeq 0.13$, $thp_{max} \simeq 0.56$ at $q_a \simeq 0.22$, $thp_{max} \simeq 0.68$ at $q_a \simeq 0.31$ and $thp_{max} \simeq 0.41$ at $q_a \simeq 0.16$

for respectively Slotted Aloha, scheme 1 (and 4), 2 and 3. This is due to the fact that mobiles are not very aggressive at low load; A possible explanation for this behavior is the following: If an individual tagged mobile was very aggressive (retransmission probabilities close to 1) in standard Aloha, algorithms 3 and 4 then eventually all other mobiles would become backlogged which could increase the collision rate and thus decrease the throughput of the tagged mobile. Hence for some values of arrival probabilities the equilibrium behavior of standard Aloha is not very aggressive. In contrast, schemes 1 and 2 suffer less from other mobiles becoming backlogged since they can reduce collisions due to the randomization and priorities. Hence increasing backlog of other mobiles does not penalize the tagged station anymore, so it has incentive to become more aggressive. The equilibrium transmission probabilities for schemes 1 and 2 are semi-constants as function of qa given by 0.997 (for 4 mobiles).

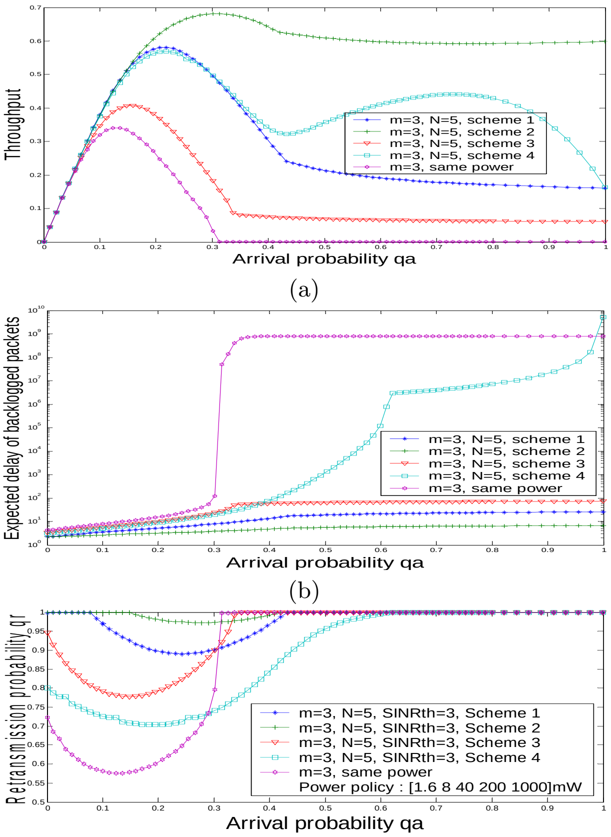


Figure 8: Aggregate throughput, EDBP and optimal retransmission probability versus arrival probability q_a for all schemes when maximizing the individual throughput and game setting with 4 mobiles.

A remarkable feature of the schemes 1-4 is that the equilibrium throughput is increasing in the arrival probabilities at low load, which is a similar behavior as the one we had in the team problem. In contrast, for high load the throughput decreases for Schemes 1-3 and it also contains a decreasing behavior in standard Aloha where it is going up for scheme 4. Thus the competition in the game formulation does not allow to benefit from increased input rates for standard Aloha and Scheme 1-3 (except for low values of q_a) whereas the new scheme 4 do benefit from that fact.

In term of EDBP, schemes 1-3 are insensitive to the value of q_a whereas scheme 4 and Slotted Aloha suffer from huge of EDBP because of high rate of collisions and retransmissions prioritization. For $m = 3$ scheme 2 provides the best performance whatever q_a.

For case with 10 mobiles Fig 9, we note that the equilibrium throughput vanishes for schemes 1-3 and Slotted Aloha at $q_a > 0.12$, whereas scheme 4 keeps an increasing behavior with q_a with a throughput collapse when q_a tends to 1.

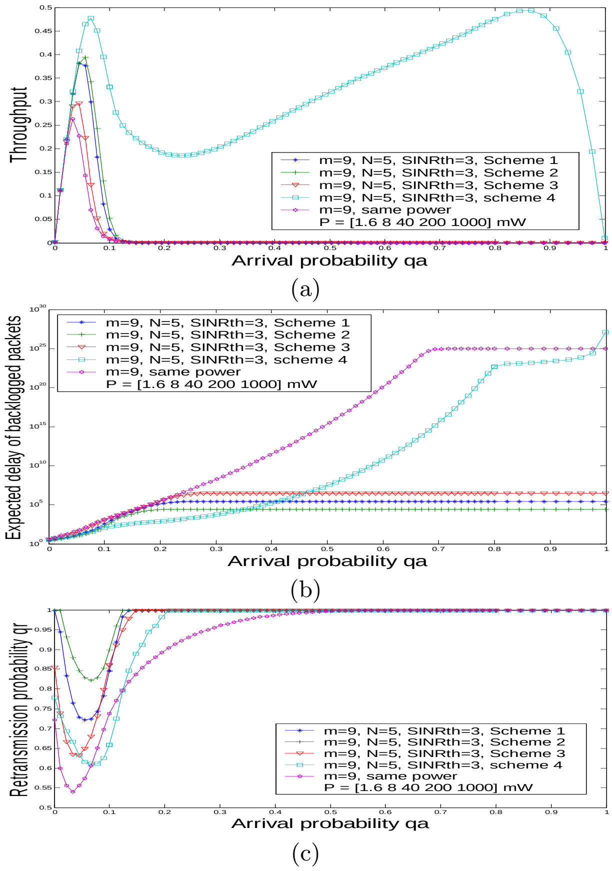


Figure 9: Aggregate throughput, EDBP and optimal retransmission probability versus arrival probability q_a, under individual throughput maximization for the game setting with 10 mobiles.

5.6 Game problem: Minimizing individual EDBP

Under delay minimization, we obtain nearly the same profile as the one obtained when maximizing individual throughput, which means that optimal retransmission probability that maximize the throughput is very close to the delay minimizer. A slight difference is seen in terms of retransmission probability at low load under schemes 2 and 4.

An interesting feature to note is that the throughput obtained when minimizing the EDBP is quite higher than the one obtained when maximizing the individual throughput; This is due to the fact that we are in a non-cooperative game setting, for which the equilibria are known not to be efficient (as is the case in the famous prisoners dilemma paradox).

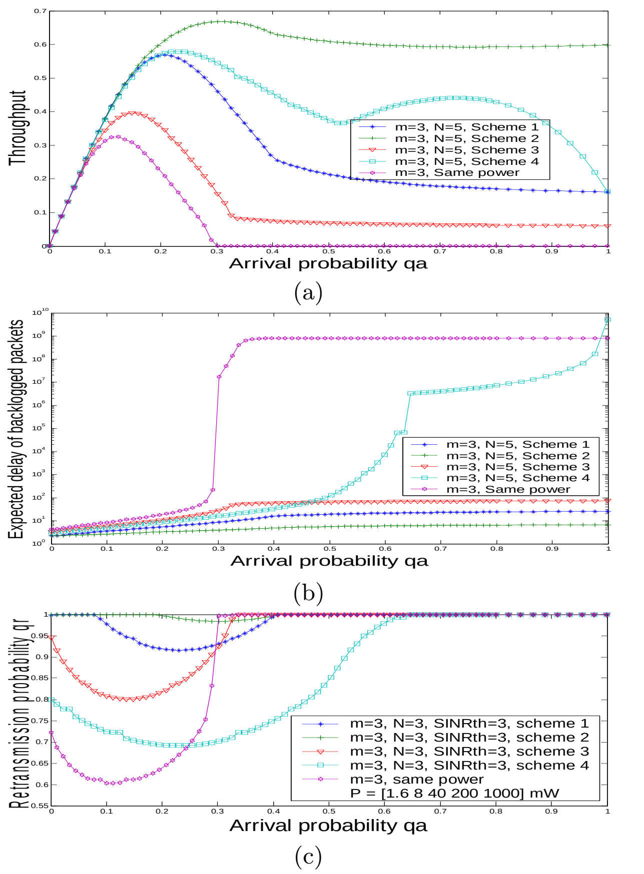


Figure 10: Aggregate throughput, EDBP and optimal retransmission probability versus arrival probability q_a for all schemes when minimizing the delay under game setting with 4 mobiles.

6. CONCLUSION

We have studied in this paper Slotted Aloha as a stochastic game with priority, power diversity and general power control. We also analyzed the system as a centralized system (team problem framework) and a decentralized form using a non-cooperative game formulation.

In the team case, we saw that scheme 2 (retransmission with more power) is the best in medium load and our new scheme 4 (retransmission with the least power) is the best in high and very high load, both when maximizing throughput or minimizing delay, whatever the number of mobiles. In contrast with the game formulation, scheme 2 offers the best performance either in term of throughput and delay on all loads when the number of mobiles is small, but with a greater number of users, either schemes 1, 2 and 3 suffer from the throughput collapse such Slotted Aloha, chiefly at heavy and very heavy load; whereas scheme 4 outperforms and tends to increase with arriving probability. At very high load (when q_a is close to 1) performances of scheme 4 decrease exponentially because mobiles become very aggressive, therefore more collisions occurs. Finally we confirm that algorithms presented in this paper provide better performance comparing to the implemented Slotted Aloha, and are more realistic comparing to previous works because they take into consideration the interferences problem and signal quality needed to decode correctly the captured signal.

7. REFERENCES

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