Utility Maximization for Resolving Throughput/Reliability Trade-offs in an Unreliable Network with Multipath Routing

Vladimir Marbukh
National Institute of Standards and Technology
100 Bureau Drive, Stop 8920, Gaithersburg,
MD 20899-8920, USA
Tel: 1 301 975 2235

E-mail: marbukh@nist.gov

ABSTRACT
This paper proposes a framework for balancing competing user (i.e., application) level requirements by resolving the corresponding trade-offs in a distributed system with limited resources. Assuming that each user's preferences are characterized by user-level utility function, the goal of balancing competing requirements for each user as well as across different users is to maximize the aggregate utility. The paper discusses this framework on an example of balancing user requirements for throughput and reliability in an unreliable network, where reliability is achieved through redundancy, e.g., using multipath routing.

Categories and Subject Descriptors
C.4 [**Performance of Systems**]: – *modeling techniques, performance attributes, reliability, availability, and serviceability.*

General Terms
Algorithms, Management, Performance, Design, Reliability, Theory.

Keywords
Distributed system, resource allocation, elastic user, multipath routing, throughput, reliability trade-offs, pricing, intelligent plane.

1. INTRODUCTION
Since network resources are shared by multiple users (i.e., applications) and performance of each user is typically characterized by multiple competing criteria, network management includes the following two major tasks: (a) making the best use of the allocated resources for each user by resolving the trade-offs among competing user criteria, and (b) sharing resources among different users. Framing the goal of network management as the aggregate utility maximization subject to the capacity constraints, where the aggregate utility is the sum of the individual user utilities, has been proposed in [1]. This framework is based on the concept of elastic users, capable of adjusting their behavior in response to congestion pricing signals. Papers [2]-[3] have developed a distributed scheme for aggregate utility maximization in a case when user utilities are expressed in terms of the link bandwidths. This scheme interprets Lagrange multipliers associated with capacity constraints as congestion costs of the corresponding resources. These costs are communicated to the elastic users, who adjust their resource requirements or willingness to pay for the resources by maximizing the individual net utilities. Figure 1 illustrates this scheme.

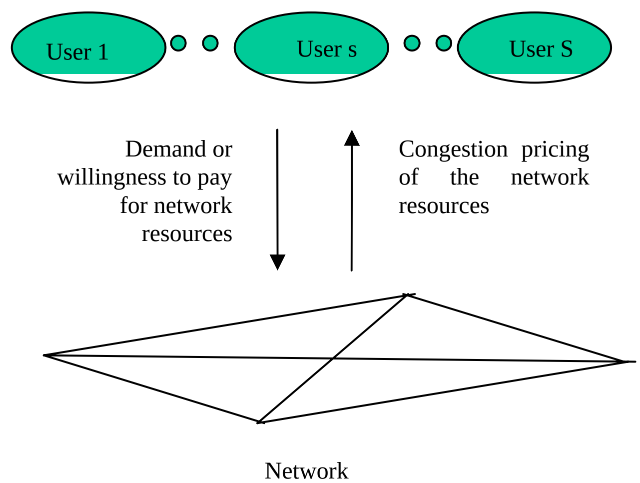

Fig. 1. Users directly responding to resource pricing

However, assumption [2]-[3] that user utilities are expressed in terms of the network resources may be too restrictive. Typically, users more naturally can express their preferences in terms of the user level requirements, such as rates and Quality of Service (QoS) parameters, rather than network level parameters, such as required bandwidth. Mapping user level requirements into network level resource requirements as well as mapping congestion resource pricing signals into pricing of the user level requirements depends on the specific network properties as well as specific implementation of the user level requirements. In the Internet with a dumb core and intelligent applications concentrated at the network edges this mapping can be performed by intelligent applications through probing.

Several recent proposals, starting with [4], argued in favor of relieving users from the burden of such probing by moving some intelligence to a separate "Intelligent Plane" (IntPlane). The IntPlane sits between the users and the network and hides the details of the network properties and user level requirements implementation mechanisms from the users. The advantages of such enhanced architecture include user convenience, possibility of optimization of the resource allocation and security considerations [4]. This paper proposes the functionality for the IntPlane as a mapping mechanism, which is shown on Figure 2.

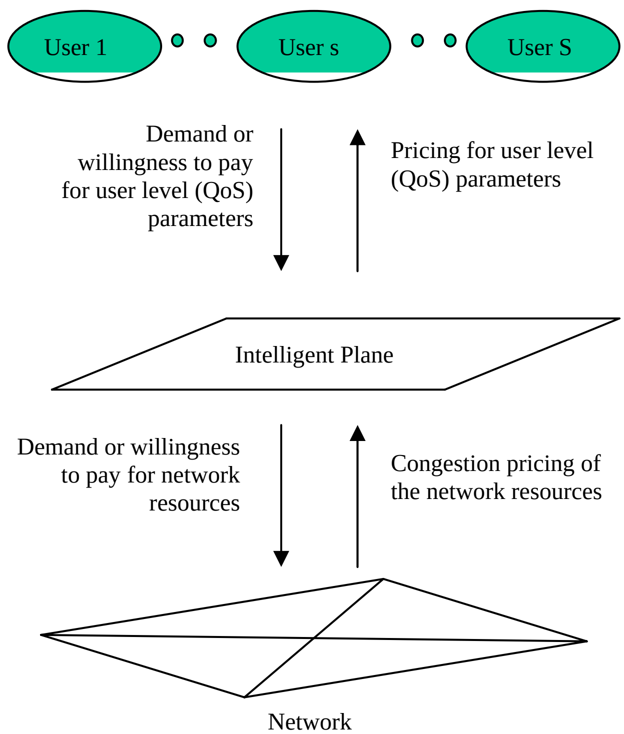

Fig. 2. Intelligent Plane as a mapping mechanism

Each elastic user attempting to maximize its individual net utility informs the IntPlane on its relative marginal utilities with respect to different user-level requirements and the “willingness to pay” for the network resource. The IntPlane performs the following tasks: (a) given the amount of the network resources allocated to each user, the IntPlane finds the optimal balance among competing user level requirements for each user, (b) maps user willingness to pay into payments for specific sets of the network resources, and (c) communicates to the user the aggregate congestion cost of the resources allocated to the user. Once the willingness to pay for the specific sets of resources is identified, the resources are allocated to users by a TCP-type algorithm. The “payments“ may either represent real funds, or be simply a parameter in the TCP-type protocol [5]. To ensure capability of this scheme to operate in a competitive (non-cooperative) environment, the resource allocation should be proportionally fair, meaning that resources are allocated to the users proportionally to the payments [2]-[3]. Proportional fairness ensures that both schemes, based on the direct user payments for the resources and user payments for the QoS, result in the same resource allocation and user payments [6].

This paper discusses possible implementation of the proposed enhanced architecture on an example of providing reliable services in an unreliable network. The reliability is achieved by reserving extra bandwidth on multiple paths. The packet level implementation of the redundancy scheme can be based on the route diversity coding [7]. Benefits of multipath routing for load balancing and protection against network element failures have been known for a long time [8]. However, research on load balancing, protection and restoration for wire-line and wireless networks has been mostly concentrated on evaluation of various performance and survivability metrics of certain multipath routing schemes [6]. While providing quantification of improving survivability with increase in redundancy through consuming more network resources, this research leaves aside the problem of balancing survivability and throughput for each user as well as across different users. Conventional practical solutions, which offer users a limited set of choices with respect to survivability, attempt to resolve these trade-offs within a centralized framework by assigning the corresponding service classes. A price based framework shifts choices regarding requested services, including survivability levels, to the users, assuming that users are aware of the available services and their prices [10]-[11].

The paper is organized as follows. Section 2 describes a model of the unreliable network and implementation of the reliable throughput. Section 3 introduces user utility of obtaining certain QoS, formulates the corresponding aggregate utility maximization framework, and discusses some possible approaches to bandwidth allocation intended to maximize the aggregate utility. Section 4 considers some examples and discusses the implication. Finally, conclusion briefly summarizes and proposes directions for future research.

2. MODEL

Subsection A defines two user $s \in S$ QoS parameters: the reliable throughput μ_s and the corresponding reliability exponent γ_s. Subsection B introduces a “fair” bandwidth sharing with controlled portions of link bandwidths allocated to different users. Subsection C describes an approximation for the reliability exponent used in the remainder of the paper.

2.1 User level parameters

Consider a network with link capacities c_l being subject to variability due to fading, mobility, node and link failures, etc. Each network user $s \in S$ is uniquely identified by its origin-destination and user level Quality of Service (QoS) requirements. Presence of several users with the same origin-destination models different types of applications with the same origin-destination, e.g., voice and video. We assume that link capacity fluctuations occur on such fast timescale that they cannot be completely absorbed by the network management actions. Due to these fluctuations, link capacities c_l are in effect random variables and thus it may be difficult or even impossible to guarantee a fixed bandwidth (throughput) to a user. Instead it may be more natural to view the instantaneous aggregate throughput x_s for a user $s \in S$ as a random variable. Due to possible large fluctuations in the instantaneous aggregate throughput x_s users may prefer to characterize their requirements in terms of the pair (μ_s, γ_s) of the "reliable" aggregate throughput μ_s and the corresponding reliability exponent γ_s quantifying the confidence level that the instantaneous throughput x_s does not deteriorate below μ_s, where

$$\gamma_s = -\log\left(\frac{P\{x_s \le \mu_s\}}{P\{x_s \le \tilde{x}_s\}}\right) \tag{1}$$

and the aggregate bandwidth reserved for user s is $\tilde{x}_s$. Figure 3 illustrates that "safety margin" $\Delta_s = \tilde{x}_s - \mu_s$ increases confidence that the instantaneous throughput x_s would not deteriorate below μ_s.

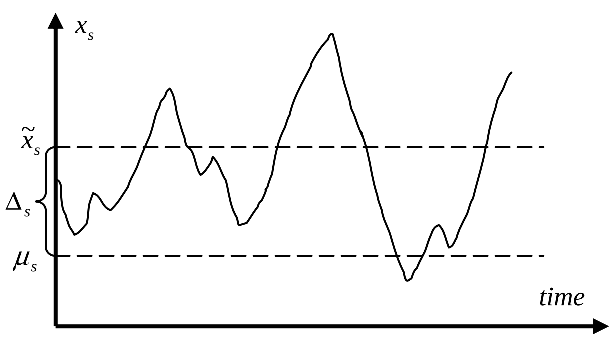

Fig. 3. Reliable aggregate throughput

This paper assumes that random link capacities c_l are jointly statistically independent for all links $l \in L$, each user s instantaneous aggregate throughput x_s can be approximated by a normally distributed random variable with average $\tilde{x}_s$ and standard deviation σ_s, and thus reliability exponent (1) is

$$\gamma_s = -\log\Phi\left(\frac{\tilde{x}_s - \mu_s}{\sigma_s}\right) \tag{2}$$

where

$$\Phi(\xi) = \sqrt{\frac{2}{\pi}}\int_{-\infty}^{\xi}\exp\left(-\eta^2/2\right)d\eta \tag{3}$$

Note that approximation (2)-(3) neglects small probability event that the bandwidth is negative. Further in the paper we use approximation

$$\gamma_s \sim \begin{cases} \frac{1}{2\sigma_s^2}(\tilde{x}_s - \mu_s)^2 & if \quad \tilde{x}_s > \mu_s \\ 0 & otherwise \end{cases} \tag{4}$$

which follows from (2) in a case of high reliability requirements: $\gamma_s \to \infty$.

2.2 Reliable Throughput

We assume that each user s is allocated a certain controlled portion ϕ_{ls} of the link l bandwidth c_l, or equivalently, the average bandwidth $\tilde{x}_{ls} = \tilde{c}_l\phi_{ls}$, where average capacity of a link l is $\tilde{c}_l = E[c_l]$, and $\phi_{l\Sigma} \overset{def}{=} \sum_{s\in S}\phi_{ls} \le 1$. The instantaneous bandwidth allocated to a user s on a link l is a random variable $x_{ls} = c_l\phi_{ls} = (c_l/\tilde{c}_l)\tilde{x}_{ls}$. In a case of small variability in the link capacities it is convenient to introduce "small" random variables $\xi_l = 1 - c_l/\tilde{c}_l$ with zero averages $E[\xi_l] = 0$, so that the instantaneous bandwidth allocated to a user s on a link l is

$$x_{ls} = (1-\xi_l)\tilde{x}_{ls} \tag{5}$$

In a particular case of a link failure model, when operational link l has capacity $c_l = \hat{c}_l$ and failed link has capacity $c_l = 0$ it is convenient to introduce binary random variables $\delta_l = 0$ if link l is operational and $\delta_l = 1$ otherwise, so that the instantaneous link l bandwidth is $c_l = (1-\delta_l)\hat{c}_l$, and $\xi_l = \delta_l - \bar{\delta}_l$, where $\bar{\delta}_l = E[\delta_l]$. In this particular case the instantaneous bandwidth (5) is $x_{ls} = (1-\delta_l)\hat{x}_{ls}$, where $\hat{x}_{ls} = \hat{c}_l\phi_{ls}$.

Given vector $X_s = (x_{sl}, l \in L)$, the maximum achievable user s instantaneous aggregate throughput is upper-limited by the capacity of the corresponding min-cut. This paper assumes a suboptimal implementation of the reliable throughput, based on the route diversity coding [4] and shown on Figure 4.

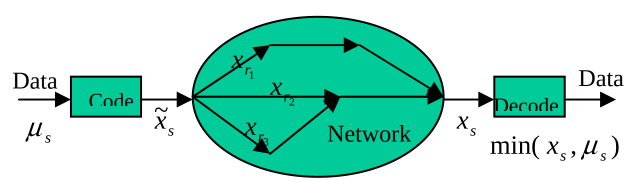

Fig. 4. Route diversity coding

In this implementation, after adding redundant bits and coding, user $s \in S$ data stream of rate μ_s is transformed into stream of higher rate $\tilde{x}_s \ge \mu_s$. This resulting stream is split into flows $\tilde{x}_{sr}$ over feasible routes $r \in R_s$ with the same origin-destination:

$$\tilde{x}_s = \sum_{r\in R_s}\tilde{x}_{sr}. \tag{6}$$

User s instantaneous throughput, i.e., rate of the user stream received at the destination, is

$$x_s = \sum_{r\in R_s} x_{sr} \,, \qquad (7)$$

where the instantaneous throughput over route $r \in R_s$ is

$$x_{sr} = (1-\xi_r)\tilde{x}_{sr} \qquad (8)$$

and the normalized variability of a route r capacity is characterized by random variable

$$\xi_r = 1 - \prod_{l\in r}(1-\xi_l)\,. \qquad (9)$$

The reliability exponent (4) quantifies the possibility of reconstructing user s data stream at the destination [4].

Calculation of the reliability exponent (4) is comparatively simple in a case when routes $r \in R_s$ do not have overlapping links. In this case the aggregate instantaneous throughput (7) is a sum of jointly statistically independent random variables since ξ_r are jointly statistically independent random variables for $r \in R_s$. When routes $r \in R_s$ do have overlapping links, calculation of the reliability exponent (4) is generally a difficult problem [9].

2.3 Approximation for Reliability Exponent

In (4) the user s average aggregate throughput is

$$\tilde{x}_s = \sum_{r\in R_s} \tilde{x}_{sr} \qquad (10)$$

and the variance of the aggregate throughput is

$$\sigma_s^2 = \sum_{r_1,r_2\in R_s} \theta_{r_1r_2}^2 \tilde{x}_{sr_1}\tilde{x}_{sr_2} \qquad (11)$$

where the "normalized correlation" between route $r_1, r_2 \in R_s$ capacities is characterized by

$$\theta_{r_1r_2}^2 = \sum_{l\in r_1\bigcap r_2} \theta_l^2 \qquad (12)$$

and the normalized variance of the link l capacity is $\theta_l^2 = E\left[\xi_l^2\right]$. Note that expression (4) can be obtained from a Gaussian link model in a large deviation regime of high reliability [12]. For a particular model of link failures the normalized variance of the link l capacity is $\theta_l^2 = \bar{\delta}_l\left(1-\bar{\delta}_l\right)$, where the probability of link l failure is $\bar{\delta}_l \in [0,1]$.

Reliability exponent (4) can be expressed as follows:

$$\gamma_s = \frac{1}{2}\left(1-\frac{1}{\omega_s}\right)^2 \left(\sum_{r_1,r_2\in R_s} \theta_{r_1r_2}^2 \alpha_{sr_1}\alpha_{sr_2}\right)^{-1} \qquad (13)$$

in terms of the user s redundancy factor, i.e., the number of bits transmitted per a bit of the "payload" [7],

$$\omega_s \overset{def}{=} \frac{1}{\mu_s}\sum_{r\in R_s} \tilde{x}_{sr} \qquad (14)$$

and portions of load routed on feasible paths $r \in R_s$ are

$$\alpha_{sr} \overset{def}{=} (\omega_s\mu_s)^{-1}\tilde{x}_{sr} \qquad (15)$$

where

$$\omega_s \ge 1 \qquad (16)$$

$$\sum_{r\in R_s} \alpha_{sr} = 1. \qquad (17)$$

Given load allocation vector $\alpha_s = (\alpha_{sr}, r \in R_s)$, the upper limit on reliability exponent (12), achieved as $\omega_s \to \infty$, is

$$\hat{\gamma}_s = \frac{1}{2}\left(\sum_{r_1,r_2\in R_s} \theta_{r_1r_2}^2 \alpha_{sr_1}\alpha_{sr_2}\right)^{-1} \qquad (18)$$

Given upper limit (18), the minimum redundancy (13) required to achieve reliability exponent γ for user s is

$$\omega = \left(1-\sqrt{\gamma/\hat{\gamma}_s}\right)^{-1} \qquad (19)$$

If routes $r \in R_s$ do not have overlapping links, formula (11) takes the following form

$$\sigma_s^2 = \sum_{r\in R_s} \left(\theta_r \tilde{x}_{sr}\right)^2 \qquad (20)$$

and thus, formula (13) simplifies as follows:

$$\gamma_s = \frac{1}{2}\left(1-\frac{1}{\omega_s}\right)^2 \left(\sum_{r\in R_s} \theta_r^2 \alpha_{sr}^2\right)^{-1} \qquad (21)$$

where $\theta_r^2 = \theta_{rr}^2$.

Given redundancy factor ω_s, one may attempt to maximize the reliability exponent (13):

$$\gamma_s^* = \max_{\alpha_{sr}\ge 0} \gamma_s \qquad (22)$$

subject to constraints (13)-(17).

Theorem 1. Given redundancy factor ω_s and network properties represented by matrix Θ_s, solution to optimization problem (22), (13)-(17) is

$$\gamma_s^* = \frac{1}{2}\left(1-\frac{1}{\omega_s}\right)^2 \left(\sum_{r_1,r_2\in R_s} t_{r_1r_2}^2\right)^{-1} \qquad (23)$$

and is achieved for load allocation

$$\alpha_{sr}^* = t_{r_1r_2}^2 \Big/ \sum_{r_1',r_2'\in R_s} t_{r_1'r_2'}^2 \qquad (24)$$

where symmetric and positive matrix $T_s = \left(t_{r_1r_2}^2\right)_{r_1,r_2\in R_s}$ is the inverse to Θ_s: $T_s = \Theta_s^{-1}$.

Proof is straightforward due to convexity. The following statements directly follow from Theorem 1.

Corollary 1. Given the network properties represented by matrix Θ_s, the upper limit on the reliability exponent (18), achieved as redundancy factor $\omega_s \to \infty$, is

$$\hat{\gamma}_s^* = \frac{1}{2}\left(\sum_{r_1,r_2\in R_s} t_{r_1 r_2}^2\right)^{-1} \qquad (25)$$

Corollary 2. If routes $r \in R_s$ do not have overlapping links, the maximal reliability exponents (23) is

$$\gamma_s^* = \frac{1}{2}\left(1-\frac{1}{\omega_s}\right)^2\left(\sum_{r\in R_s}\theta_r^{-2}\right)^{-1} \qquad (26)$$

the optimal load allocation (24) is

$$\alpha_{sr}^* = \theta_r^{-2} \Big/ \sum_{r'\in R_s}\theta_{r'}^{-2} \qquad (27)$$

and the upper limit (25) is

$$\hat{\gamma}_s^* = \frac{1}{2}\left(\sum_{r\in R_s}\theta_r^{-2}\right)^{-1} \qquad (28)$$

3. UTILITY MAXIMIZATION

Subsection A section introduces individual user utility of obtaining service parameters (μ_s, γ_s) and formulates the corresponding aggregate utility maximization framework. Subsection A discusses a situation of users adjusting their bandwidth requirements in response to bandwidth prices. Subsection B considers a situation of users adjusting their rate and QoS requirements in response to QoS pricing by IntPlane. Section C discusses a hybrid situation of users informing the IntPlane about their QoS requirements while adjusting their rates in response to pricing by IntPlane.

3.1 Utilities

Let $h_s(x,\mu)$ be a function, monotonously increasing in both arguments $0 \le \mu \le x < \infty$. Consider elastic user s whose satisfaction of obtaining service with parameters (μ,γ) is characterized by utility function

$$U_s(\mu,\gamma) = u_s(\mu)v_s(\gamma), \qquad (29)$$

where function $u_s(\mu)$ is a conditional average over the aggregate rate x_s:

$$u_s(\mu) = E_{x_s}\left[h(x_s,\mu)\middle| x_s > \mu\right], \qquad (30)$$

and function $v_s(\gamma)$ is monotonously increasing for $0 \le \gamma < \infty$.

Definition (29)-(30) is quite flexible, covering a wide range of possibilities. Consider some particular cases. User s having "hard" requirements on the reliability parameter $\gamma_s \ge \gamma_s^{\min}$ is characterized by utility function (32)-(33), where

$$v_s(\gamma) = \chi(\gamma - \gamma_s^{\min}), \qquad (31)$$

and step-wise function is $\chi(\gamma) = 1$ if $\gamma > 0$, and $\chi(\gamma) = 0$ if $\gamma \le 0$. A particular case of (29)-(31) with $\gamma_s^{\min} = 0$ and function $h_s(x,\mu) \equiv u_s(x)$ independent of the reliable throughput $\mu \in [0,\infty)$ describes an elastic user whose satisfaction is characterized by the average utility of the instantaneous aggregate throughput: $U_s = E[u_s(x_s)]$. A case when function $h_s(x,\mu) \equiv u_s(\mu)$ depends only on the reliable aggregate throughput $\mu \in [0,\infty)$ and $\gamma_s^{\min} = 0$ describes an elastic user concerned with the average throughput.

S. Shenker has proposed [1] aggregate utility maximization to be the objective of network management. In our particular case the aggregate utility maximization framework takes the following form:

$$\max \sum_s U_s(\mu_s,\gamma_s) \qquad (32)$$

with maximization over user level requirements $(\mu,\gamma) = (\mu_s,\gamma_s, s\in S)$ and vector $\tilde{X} = (\tilde{x}_{sr} : s\in S, r\in R_s)$ subject to constraints (4), link capacity constraints $\tilde{y}_l \le \tilde{c}_l$, flow non-negativity constraints: $\tilde{x}_{sr} \ge 0$ and constraints on the reliable throughput $0 \le \mu_s \le \tilde{x}_s$, $s \in S$, where the link l average load is

$$\tilde{y}_l = \sum_s \sum_{r:l\in r\subseteq R_s} \tilde{x}_{sr} \qquad (33)$$

Optimization problem (32) is equivalent to the following optimization problem

$$\max_{\mu,\gamma,\tilde{X}} W \qquad (34)$$

subject to the same constraints except the capacity constraints, where the "social welfare" is

$$W = \sum_s U_s(\mu_s,\gamma_s) - \sum_l f_l(\tilde{y}_l) \qquad (35)$$

and appropriately selected penalty functions $f_l(y)$ may quantify the congestion penalty in terms of delays or packet loss as link utilization approaches link capacities [3]. For packet networks it is often assumed [14] that

$$f_l(y) = y/(\tilde{c}_l - y). \qquad (36)$$

3.2 Bandwidth Pricing

This subsection assumes that each $s \in S$ (a) is aware of the network properties quantified by matrix Θ_s, and (b) capable of finding the optimal balance (μ_s^*, γ_s^*) between competing requirements for the reliable throughput μ_s and the corresponding reliability exponent γ_s by maximizing the individual utility, given allocated bandwidths $\tilde{X}_s = (\tilde{x}_{sr}, r\in R_s)$:

$$\tilde{U}_s(\tilde{X}_s) = \max_{\gamma_s} u_s(\tilde{x}_s - \sigma_s\sqrt{2\gamma_s})v_s(\gamma_s) \quad (37)$$

Once individual utilities (37) with respect to the network resources $\tilde{X}_s = (\tilde{x}_{sr}, r \in R_s)$ are identified, the aggregate utility maximization problem (34)-(35) becomes

$$\max_{\tilde{X}} \left\{ \sum_s \tilde{U}_s\left(\tilde{X}_s\right) - \sum_l f_l\left(\sum_s \sum_{r:l\in r\subset R_s} \tilde{x}_{sr} \right) \right\} \quad (38)$$

Consider the following individual optimization problem for a user s attempting to maximize its individual net utility:

$$\max_{\tilde{x}_{sr}\geq 0} \max_{\gamma\geq 0} \left\{ u_s(\tilde{x}_s - \sigma_s\sqrt{2\gamma})v_s(\gamma) - \sum_{r\in R_s} d_r\tilde{x}_{sr} \right\} \quad (39)$$

where the route r price is:

$$d_r = \sum_{l\in r} f_l'(\tilde{y}_l), \quad (40)$$

the link l price $f_l'(\tilde{y}_l)$ is a derivative of the congestion penalty function for this link $f_l(\tilde{y}_l)$, and the link load $\tilde{y}_l$ is given by (33). Solving individual optimization problem (49)-(50) by each user $s \in S$ also maximizes the aggregate utility (46) if the link prices are "right", meaning that derivatives $f_l'(\tilde{y}_l)$ are calculated at the optimal link l load $\tilde{y}_l = \tilde{y}_l^{opt}, \ \forall l$.

Kuhn-Tucker necessary conditions for a vector $\tilde{X}_s = (\tilde{x}_{sr}, r \in R_s)$ to solve (38) are as follows [13]:

$$\sqrt{2\gamma}\sigma_s^{-1} \sum_{r'\in R_s} \theta_{rr'}^2 \tilde{x}_{sr'} = 1 - \frac{d_r}{u_s'} \ \ if \ d_r \leq u_s' \quad (41)$$

$$\tilde{x}_{sr} = 0 \ \ if \ d_r > u' \quad (42)$$

where $u_s'(\mu) = du_s(\mu)/d\mu$ is the derivative of the user s utility at the point of this user reliable throughput $\mu = \mu_s$ and σ_s is given by (11). If user utilities $\tilde{U}_s(\tilde{X}_s)$ are concave, (41)-(42) are also the corresponding sufficient conditions [13]. In this case, user s optimal response to the pricing signals d_r is requesting bandwidth vector $\tilde{X}_s = (\tilde{x}_{sr}, r \in R_s)$, which solves system (41)-(42) and thus maximizes its individual net utility (39)-(40).

Generally, optima in (38) and (39) are achieved when some flows are zero: $\tilde{x}_{sr} = 0$ for some $r \in R_s$, $s \in S$. In fact, this situation is typical in presence of "high cost", e.g., highly congested or very "long" routes, when optimal solution is not to use these "expensive" routes. For example, conventional shortest path routing uses only one, "optimal" route. Given $\mu \geq 0$, define a subset of feasible routes participating in user $s \in S$ transmission:

$$R_s(\mu) = \{r : u_s'(\mu) > d_r, r \in R_s\} \quad (43)$$

Consider two routes $r_1, r_2 \in R_s(\mu)$, which do not have overlapping link with each other or with any other route $\forall r \in R_s(\mu)$: $r_i \bigcap r = \varnothing$, $i = 1,2$. In this case we have from (41):

$$\frac{\tilde{x}_{sr_1}}{\tilde{x}_{sr_2}} = \left(\frac{\theta_{r_2}}{\theta_{r_1}}\right)^2 \frac{1 - d_{sr_1}/u_s'}{1 - d_{sr_2}/u_s'} \quad (44)$$

It follows from (44) that if two routes $r_1, r_2 \in R_s(\mu)$ have the same cost: $d_{r_1} = d_{r_2}$, then the user transmission rate on these routes should be inversely proportional to the variances of the fluctuating bandwidths of the corresponding routes:

$$\tilde{x}_{sr_1}/\tilde{x}_{sr_2} = \left(\theta_{r_2}/\theta_{r_1}\right)^2 \quad (45)$$

This conclusion that load allocation among several routes of the same cost should send more traffic on the better quality routes while preserving routing diversity is intuitively plausible.

In a case of hard reliability constraints (31) when feasible routes $r \in R_s$ do not have overlapping links, the optimal flow vector $\tilde{X}_s = (\tilde{x}_{sr}, r \in R_s)$ can be identified explicitly. Indeed, in this case we obtain the following expression for the flows $\tilde{x}_{sr}, r \in R_s(\mu)$:

$$\tilde{x}_{sr} = \frac{\mu}{\sum_{r'\in R_s(\mu)} \frac{1}{\theta_{r'}^2}\left(1 - \frac{d_{r'}}{u_s'}\right) - 2\gamma} \left(1 - \frac{d_r}{u_s'}\right)\frac{1}{\theta_r^2} \quad (46)$$

Substituting (46) into right-hand side of the following necessary condition for optimality

$$u_s' = \frac{1}{\mu}\sum_{r\in R_s} \tilde{x}_{sr} \quad (47)$$

we obtain a quadratic algebraic equation for the derivative u_s', yielding the reliable throughput $\mu = \mu_s$. Then, flows are determined by (46).

3.3 QoS Pricing

Consider user $s \in S$ individual optimization problem

$$\max_{\mu,\gamma\geq 0} \{U_s(\mu,\gamma) - \mu D_s(\gamma)\} \quad (48)$$

where the marginal price of the reliable throughput is

$$D_s = \frac{\tilde{d}_s}{1 - \sqrt{\gamma/\hat{\gamma}_s}}, \quad (49)$$

the price of a unit of the average throughput is

$$\tilde{d}_s = \sum_{r\in R_s} d_r\alpha_{sr}, \quad (50)$$

the upper limit on the reliability exponent $\hat{\gamma}_s$ is given by (18), the cost of a route r is d_r and vector $\alpha_s = (\alpha_{sr}, r \in R_s)$ characterizes user s traffic split among feasible routes.

Given split $\alpha = (\alpha_s, s \in S)$, maximization of the individual net utility (48) by each user $s \in S$ also maximizes the aggregate utility:

$$\sum_s U_s(\mu_s, \gamma_s) - \sum_l f_l\left(\sum_s \frac{\mu_s}{1-\sqrt{\gamma_s/\hat{\gamma}_s}} \sum_{r:l\in r\subset R_s} \alpha_{sr}\right) \quad (51)$$

over user level requirements $(\mu_s, \gamma_s : s \in S)$ if the route costs are

$$d_r = \sum_{l\in r} f_l'\left(\sum_s \frac{\mu_s}{1-\sqrt{\gamma_s/\hat{\gamma}_s}} \sum_{r:l\in r\subset R_s} \alpha_{sr}\right) \quad (52)$$

The problem of joint maximization of the aggregate utility (51) over user level parameters $(\mu_s, \gamma_s : s \in S)$ and split $(\alpha_{sr}, r \in R_s, s \in S)$, which characterizes implementation of user-level (QoS) parameters, can be decomposed into (a) maximization of individual net utility (48) by each user $s \in S$, and (b) minimization of the cost of implementation of user $s \in S$ requirements by the IntPlane:

$$\tilde{D}_s^* = \min_{\alpha_r \ge 0} \frac{1}{1-\sqrt{2\gamma \sum_{r_1,r_2\in R_s} \theta_{r_1r_2}^2 \alpha_{sr_1}\alpha_{sr_2}}} \sum_{r\in R_s} d_r \alpha_{sr} \quad (53)$$

subject to constraints (17).

Cost minimization (53) subject to constraint (17) can be carried out as follows. Consider optimization problem:

$$\tilde{\theta} = \min_{\alpha_{sr}\ge 0} \sum_{r_1,r_2\in R_s} \theta_{r_1r_2}^2 \alpha_{sr_1}\alpha_{sr_2} \quad (54)$$

subject to constraints

$$\sum_{r\in R_s} d_r \alpha_{sr} \le \tilde{d} \quad (55)$$

and constraints (17). Note that this optimization problem intends to maximize the bound on the reliability exponent (18) subject to upper constraint on the average route cost, or, equivalently, to minimize the average route cost subject to lower bound on the reliability exponent (18).

Optimization problem (54)-(55), (17) is convex and it can be shown that solution to this problem reduces to solving system of two algebraic equations for the corresponding Lagrange multipliers. Due to limited space we only consider two particular cases. In a case of a user s concerned only with the average throughput: $\gamma_s \to 0$, solution to (54)-(55), (17) sends entire traffic on minimum cost routes. If there are several minimum cost routes, a situation of minimum equal cost multipath arises. The optimal load split among minimum cost routes is

$$\alpha_k = \left(\sum_{i,j=1}^{K_1} t_{1ij}^2\right)^{-1} \sum_{j=1}^{K_1} t_{1kj}^2 \quad if \quad k = 1,..,K_1$$
$$\alpha_k = 0 \quad otherwise \quad (56)$$

where matrix $T_m = \left(t_{mij}^2\right)_{i,j=1}^{K_m}$ is inverse to the matrix $\Theta_m = \left(\theta_{ij}^2\right)_{i,j=1}^{K_m}$, and the redundancy factor is $\omega = 1$. In another extreme case of very reliability sensitive user s: $\gamma \to \hat{\gamma}_s^* - 0$, the optimal load split among feasible routes $r \in R_s$ is given by (24), and redundancy factor is given by (19).

3.4 Pricing Reliable Bandwidth

Consider the following scheme. Each user $s \in S$ informs the IntPlane on the part of the utility function $v_s(\gamma)$ expressing user preferences with respect to the reliability exponent. Given the reliability exponent γ_s and amount λ_s each user $s \in S$ is charged for a unit of reliable throughput μ_s, user s determines the total amount it is willing to pay w_s by solving its individual optimization problem:

$$w_s = \arg\max_{w\ge 0}\{u_s(w/\lambda_s)v_s(\gamma_s) - w\} \quad (57)$$

Given $v_s(\gamma)$ and w_s for all users $s \in S$, IntPlane allocates user-level (QoS) parameters (μ_s, γ_s) and selects the implementation, i.e., the bandwidth allocation, $\tilde{X} = (\tilde{x}_{sr}, r \in R_s, s \in S)$ by solving the following optimization problem:

$$\max_{(\mu,\gamma)} \max_{\tilde{X}} \sum_s w_s \ln \mu_s - \sum_l f_l\left(\sum_s \sum_{r:l\in r} \tilde{x}_{sr}\right) \quad (58)$$

subject to constraints (4).

If charges λ_s are "right", solution to optimization problems (58) also maximizes the aggregate utility (34)-(35). Indeed, formal differentiation in ( ) with respect to w yields:

$$\lambda_s = u_s'(\mu_s)v_s(\gamma_s) \quad (59)$$

Charge per unit of reliable throughput (59) maximizes the aggregate utility (35) if

$$\lambda_s = D_s \quad (60)$$

where D_s is given by (59). It is directly follows from ( ) that this pricing scheme is proportionally fair.

Assuming that given reliable throughputs (μ_s) and user willingness to pay (w_s), the IntPlane allocates the reliability exponents (γ_s) and bandwidths $\tilde{X} = (\tilde{x}_{sr}, r \in R_s, s \in S)$ by solving optimization problem

$$\max_{\gamma}\max_{\tilde{X}}\sum_s w_s \ln \mu_s - \sum_l f_l\left(\sum_s \sum_{r:l\in r} \tilde{x}_{sr}\right), \qquad (61)$$

the optimal allocation of the reliable throughputs can be achieved as follows. Assuming that each user monitors its reliable rate μ_s and smoothly adjusts its willingness to pay to maximize the individual net utility ( ), we have:

$$w_s = \mu_s u_s'(\mu_s) v_s(\gamma_s) \qquad (62)$$

Thus adjusting the reliable throughputs as follows

$$\dot{\mu}_s = k(w_s - \mu_s D_s) \qquad (63)$$

maximizes the aggregate utility.

Note that this pricing scheme assumes that users truthfully reveal their preferences with respect to the reliability. It can be shown that the truthfulness is the optimal strategy for the users.

4. EXAMPLES

This Section discusses benefits of multi-path routing. While Subsection A considers a simple case of three feasible routes with one overlapping link, Subsection B considers a general case of feasible routes without overlapping links.

4.1 Benefits of Multi-path Routing

In a case of a single-path routing, when user traffic must be routed on a single path, the optimal route and the corresponding price of a unit of the reliable throughput are determined by solution to the following optimization problem

$$D_{r_s^*(\gamma)} = \min_{r\in R_s} D_r(\gamma) \qquad (64)$$

where the price of a unit of the reliable route r throughput is

$$D_r(\gamma) = d_r \Big/ (1-\theta_r\sqrt{2\gamma}) \qquad (65)$$

Figure 7 sketches the price of a unit of the reliable throughput on a fixed route (65), the price of optimal single-route implementation (64) (fat curve), and the price of optimal implementation using multipath routing (49) as functions of the reliability parameter γ.

Figure 7 assumes a typical situation, when higher quality routes are more congested due to higher demand: $d_{r_1} < d_{r_2} < d_{r_3}$, while $\theta_{r_1} > \theta_{r_2} > \theta_{r_3}$.

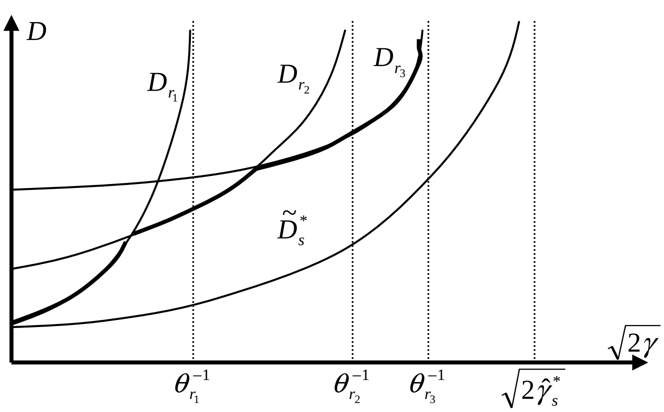

Fig. 7. Price of a unit of the reliable throughput

In a case of a single-path routing, when user reliability requirements for γ are low, the least congested, low quality route r_1 should be used. As user reliability requirements increase, the user traffic should be carried on more congested, higher quality route r_2. As user reliability requirements keep increasing, the user traffic should be shifted to the most congested route r_3 having the highest quality. Sufficiently high user reliability requirements cannot be met with a single-path routing.

Since, according to (64)-(65), maximal reliability exponent user s can achieve with a single path routing is

$$\breve{\gamma}_s^* = (1/2)\max_{r\in R_s} \theta_r^{-2}, \qquad (66)$$

it follows from (28) that this user can increase its reliability exponent with multi-path routing without overlapping links up to

$$\Gamma_s = \left(\sum_{r\in R_s} \theta_r^{-2}\right) \min_{r\in R_s} \theta_r^2 > 1 \qquad (67)$$

times. Gain (67) increases with increase in the routing diversity. Beneficial effect of multi-path routing on load balancing manifests itself in reduction of the average price of the unit of reliable throughput. Generally, this beneficial effect increases with increase in the user reliability requirements. Note that multi-path routing does not have beneficial effect for a user not concerned with reliability $(\gamma = 0)$, since in this case optimal implementation is based on the minimum congestion cost routing.

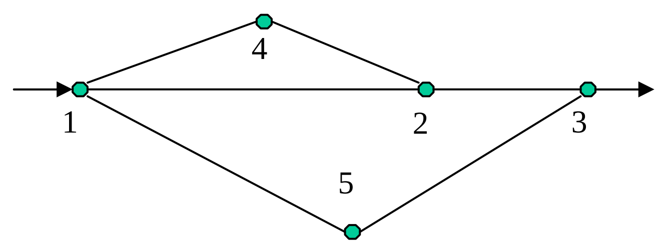

Fig. 8. Network topology

To get feeling of equal cost multi-path routing consider a network shown on Figure 8. The network has three feasible routes $r_1 = (1,2,3)$, $r_2 = (1,4,2,3)$, and $r_3 = (1,5,3)$ with the same congestion costs: $d_1 = d_2 = d_3 = d$, and matrix

$$\Theta = \begin{pmatrix} \theta^2 & \chi\theta^2 & 0 \\ \chi\theta^2 & \theta^2 & 0 \\ 0 & 0 & \theta^2 \end{pmatrix} \qquad (68)$$

where parameter $\chi \in [0,1]$ characterizes overlapping between routes r_1 and r_2. In this case the optimal load split is as follows:

$$\alpha_1 = \alpha_2 = \frac{1}{3+\chi}, \quad \alpha_3 = \frac{1+\chi}{3+\chi}.$$

If $\chi = 0$, i.e., equal cost routes r_1, r_2 and r_3 do not overlap, the optimal allocation splits load equally among these three routes: $\alpha_1 = \alpha_2 = \alpha_3 = 1/3$. If $\chi = 1$, i.e., matrix (68) describes a network with just two equal cost routes $r \equiv r_1 \equiv r_2$ and r_3, the optimal loads allocation splits load equally among these two routes: $\alpha_r = \alpha_3 = 1/2$.

4.2 Routes without Overlapping Links

To illustrate our results, consider a case of K feasible routes without overlapping links: $\Theta = diag\left(\theta_1^2, \theta_2^2, .., \theta_K^2\right)$, where without loss of generality we assume that $\theta_1 \geq \theta_2 \geq \theta_K$, i.e., route r_1 has lower quality than route r_j if $1 \leq i < j \leq K$.

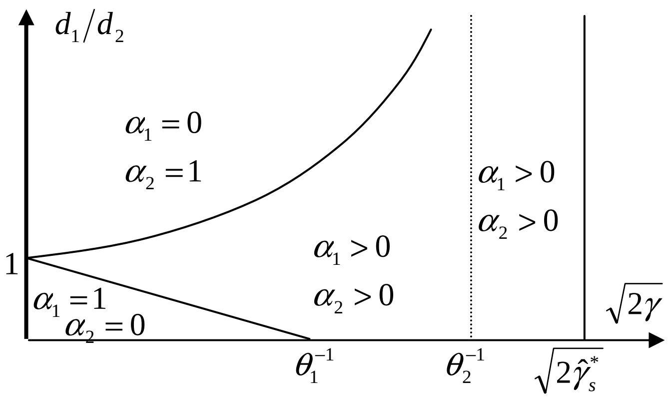

Fig. 9. Optimal route mixture, given route costs

Figure 9 sketches the phase diagram, given the route costs d_k, $k = 1,2$ and reliability exponent γ in a case of $K = 2$ feasible routes. This diagram shows three qualitatively different region with respect to the optimal route mixture (α_1, α_2), where α_k is the portion of the user traffic to be routed on path r_k, given route relative congestion costs d_1/d_2 and user reliability requirements γ. In the region $\alpha_1 = 1,\ \alpha_2 = 0$ entire user traffic should be sent over route r_1. In the region $\alpha_1 = 0,\ \alpha_2 = 1$ entire user traffic should be sent over route r_2. In the region $0 < \alpha_1, \alpha_2 < 1$ user traffic should be split between routes r_1 and r_2. Also note that the part of Figure 9, where $d_1/d_2 \leq 1$ represents a typical situation when lower quality route is less congested.

It is instructive to analyze the optimal route mixture as user reliability requirements γ or relative route congestion cost d_1/d_2 changes. Not reliability conscious user should use the minimum cost route. As user reliability requirements γ increase, multi-path routing becomes preferable. Consider change in optimal connectivity as low quality route r_1 becomes more congested, i.e., as d_1/d_2 increases from zero to infinity. In this case optimal connectivity for not reliability sensitive user should change from single route r_1 to multi-path routing $r_1 \bigcup r_2$, and eventually to single high quality, less congested route r_2. Connectivity for moderately reliability sensitive user should change from multi-path routing $r_1 \bigcup r_2$ to single route r_2 since low quality route r_1 alone cannot provide required transmission reliability. Highly reliability sensitive user should be always connected over both routes: r_1 and r_2, since neither route alone can guarantee required transmission reliability. Generalization to case of an arbitrary number of feasible routes without overlapping links is straightforward.

5. CONCLUSION

This paper has discussed possible approaches to balancing completing user-level (QoS) requirements for each user as well as across different users by maximizing the aggregate utility. The ultimate goal is a distributed optimization which isolates users from the network layer. This isolation can be naturally achieved by assuming that elastic users communicate their QoS requirements to the Intelligent Plane, which implements these requirements by allocating the network resources and informs the users on the congestion cost of this implementation. Developing distributed algorithms capable of maximizing the aggregate, user-level utility in a realistic environment is a difficult and to a large degree unexplored problem.

Our conjecture is that the pricing scheme proposed in section 4.3 may be applicable in a general situation of maximizing aggregate, user-level utility in a distributed environment, with utilities $U_s = u_s(x_s)v(q_s)$, where user s rate is x_s and vector q_s characterizes the user-level (QoS) requirements. In this general situation each user s determines the total amount it is willing to pay w_s by solving its individual optimization problem:

$$w_s = \arg\max_{w \geq 0} \{u_s(w/x_s)v_s(q_s) - w\} \tag{69}$$

Given $v_s(q)$ and w_s for all users $s \in S$, IntPlane allocates user rates and QoS parameters (x_s, q_s) and allocates resources by solving the following optimization problem:

$$\max_{(x,q)} \sum_s w_s \ln x_s \tag{70}$$

subject to the capacity constraints.

In the conclusion note that to be stable in a non-cooperative, e.g., commercial, environment the resource allocation algorithm should be proportionally fair, i.e., user payments should reflect the resource usage. It is known [3] that in a case of users directly requesting network resources the aggregate utility can be maximized with proportionally fair pricing. In a case of users paying for the QoS requirements, proportional fairness should be considered as an additional constraint on the pricing scheme to ensure that users have no incentive to deviate from the pricing scheme or mislead the IntPlane about their QoS prefences.

6. ACKNOWLEDGMENTS

Author appreciates illuminating comments by A. Nakassis.

7. REFERENCES

[1] S. Shenker, "Fundamental design issues for the future Internet," *IEEE JSAC*, 13 (1995) 1176-1188.

[2] F.P. Kelly, "Charging and rate control for elastic traffic," European Transactions on Telecommunications, Vol. 8, 1997, pp. 33-37.

[3] F.P. Kelly, A.K. Maulloo, and D.H.K. Tan, "The rate control for communication networks: shadow prices, proportional fairness and stability," *Journal of the Operational Research Society*, pp. 237-252, vol. 409, 1998.

[4] D. Clark, C. Partridge, J.C. Ramming, and J. Wroslawski, "A knowledge plane for the Internet," *ACM SIGCOMM*, 2003.

[5] S. H. Low, F. Paganini and J. C. Doyle, "Internet congestion control," *IEEE Control Systems Magazine*, 22(1), Feb. 2002, 28-43.

[6] V. Marbukh and R.E. Van Dyck, "On aggregate utility maximization by greedy ASs competing to provide Internet services," *Forty-Second Annual Allerton Conference on Communication, Control, and Computing*, Illinois, 2004.

[7] A. Tsirigos and Z.J. Haas, "Analysis of multipath routing – part I: the effect on the packet delivery ratio," IEEE Trans. *On Wireless Communications*, Vol. 3, No. 1, January 2004, pp. 138-146.

[8] N.F. Maxemchuk, "Dispersity routing, Proc. IEEE ICC, San Francisco, CA, 1975.

[9] A. Tsirigos and Z.J. Haas, "Analysis of multipath routing – part II: mitigation of the effects of frequently changing network topologies," *IEEE Trans. On Wireless Commun*, Vol. 3, No. 2, 2004, 500-511.

[10] J. MacKie-Mason, "What does economics have to do with survivability?" InfoSurv, June 1997. available at http://www-personal.umich.edu/~jmm/presentations/darpa-econ-tutorial.pdf.

[11] V. Marbukh, "Towards market approach to providing survivable services," *Conf. on Inf. Sciences and Syst.* Princeton Univ., 2004.

[12] F.P. Kelly, "Notes on effective bandwidth," In "Stochastic Networks: Theory and Applications" (*Editors F.P. Kelly, S. Zachary and I.B. Ziedins*) Royal Statistical Society Lecture Notes Series, 4. Oxford University Press, 1996. 141-168.

[13] S. Boyd and L. Vandenberge, Convex Optimization, Cambridge University Press, 2004.

[14] D. Bertsekas and R. Gallger, Data Networks, Prentice-Hall, 1992.

[15] C. Courcoubetis and R. Weber, Pricing Communication Networks, Wiley, 2003.

[16] V. Marbukh, "A knowledge plane as a pricing mechanism for aggregate, user-centric utility maximization," *ACM Performance Evaluation Review*, Volume 32, Issue 2, pp. 22-24, 2004

[17] J. Crowcroft, R. Gibbens, F. Kelly and S. Ostring, "Modeling incentives for collaboration in mobile ad hoc networks," *Performance Evaluation*, 57, pp. 427-439, 2004.

[18] P. Marbach and Ying Qiu, " Cooperation in Wireless Ad Hoc Networks: A Market-Based Approach," available at http://www.cs.toronto.edu/~marbach/publ.html.

[19] V. Marbukh, "On aggregate utility maximization based network management: challenges and possible approaches," *IEEE ICC 2004*