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Non Local Effects in the Sznajd Model: Stochastic resonance aspects

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Abstract

The Sznajd model is basically an Ising spin model, widely used in sociophysics studies as a simple mechanism for predicting decision-making in a closed community through interactions among the nearest neighbors. In the present work we aim to deepen our understanding of this model by analyzing not only local or first neighbor interactions but also long range ones. Besides, we consider the system as being subjected to two signals, a stochastically social internal one, mimicking "social temperature", and an external periodic signal playing the role of the effects of fashion or propaganda. Under these conditions, we show the occurrence of a double stochastic resonance phenomenon when depicting signal-to-noise ratio as a function of both, the social temperature and the non local interaction parameters.

Keywords: Sznajd model, stochastic resonance.

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1. Introduction

The study of complex systems has recently attracted the attention of theoretical physicists. Statistical physics has in particular been extended beyond its usual limits, to tackle sociological problems, leading to the so called *sociophysics* [1–5]. In such a context, the study of the evolution of public opinion through simple mathematical models is a favorite topic. Some of these models involve a population, where each member of the group can adopt an "opinion" value chosen between two possible alternatives (say +1 or −1). These values may represent the position for or against a particular topic or the preference for one or other candidate or political party. That individual agent's opinion can evolve according to some simple rules. A widely studied case is the so called *Sznajd model* [6]. In the present work we analyze a modified version of this model that was initially proposed in [7]. The model was modified so as to include *contrarians* [8], namely people who are non-conformist opposition. This was achieved by introducing a certain probability, depending on a *social temperature*, a parameter that corresponds to social turmoil, and which adds a fluctuating component to the system [9, 10].

Although several two-dimensional lattice models and even small world networks were studied [12–14], we have considered the agents as being arranged in a simple one-dimensional array of L components. The present model also involves the presence of an external field, representing the fashion or propaganda, which may induces alternate changes in the agent's opinion. We have studied the phenomenon of *stochastic resonance* (SR) [11], which consists in an enhancement of the system's response when subjected to the action of a periodical external field (the fashion or propaganda) when the internal noise (in this case, the social temperature) reaches an optimal value. Within this social context, SR was previously studied by several authors [13, 15–18]. As a complement to a previous work [20], we also analyze the effect of introducing some degree of non locality in the selection of those agents whose opinion is modified at each time step. To the best of our knowledge, it is the first time that non local interactions are considered in the Sznajd model, so the systems's response to this new factor remains unknown.

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The paper is organized as follows: in Section 2 the model and simulation method are introduced, Section 3 presents the main results and discussions, and finally some conclusions are put forward in Section 4.

2. Model and simulation method

2.1. The model

The model consists of a one dimensional array of L components, each one representing an agent, which can take two possible opinion values ($s_i = \pm 1$). Periodic boundary conditions are considered. The Sznajd model with *social temperature* was considered in previous works [9, 20]. Here we consider four possibilities when choosing the interacting agents.

2.2. Interaction agent rules

Each trial consists in choosing one agent at random, e. g. agent i, and then fixing another agent $i+1$. Let call the value of that agent's opinion s_i ($s_i = \pm 1$), and i and $i+1$ *discussion agents*, i.e., the ones who first meet each other and exchange their opinions; agents i and $i+1$ interact with agents j and k. These agents are called *modification agents*, i.e., those who react after the discussion agents decision has been made.
Interaction A: we take $j = i-1$ and $k = i+2$ (this model is also studied in a related work [20]).
Interaction B: we take $j = i-n$ and $k = i+1+n$, where $n-1$ is the number of neighbors to be skipped. This model attempt to take into account long-range interactions.
Interaction C: it is a variant of model B. Here we randomly choose agents j and k: the former, between $i-1$ and $i-n$ and agent the latter, between $i+2$ and $i+1+n$, where n is the number of neighbors among whom the choice is made. With the present model we attempt to study the effect of accidental encounters on the system's response.
Interaction D: in this variant, we take the four agents at random (i, l-not necessarily $i+1$, j and k).

Once we have defined the interactions, the Sznajd rules are set in the following way:

- If $s_i \times s_{i+1} = 1$, then s_j y s_k take the same values of s_i and s_{i+1}.
- If $s_i \times s_{i+1} = -1$, then s_i takes the value of s_j and s_{i+1}, takes the value of s_k.

(in case D, consider s_l instead of s_{i+1}).

Let call R_1 the rule as indicated before. This rule, which is a variant of the Sznajd model, has already been used in [7]. In previous versions of the Sznajd rule, when $s_i \times s_{i+1} = -1$, the modification agents either adopts antiferromagnetic values, or nothing happens. In the present version, each discussion agent aligns with its corresponding modification agent.

2.3. Fashion and social temperature

The effect of fashion is modeled by taking into account the signal of a periodic external field [20], and it is introduced as follows. An agent i is chosen and R_1 is applied to the four agents (i, $i+1$, j and k). For each agent modified according to rule R_1, we calculate the following probability:

$$p = \Lambda \exp(\frac{\alpha + q \times H}{T}), \tag{1}$$

where q denotes the new value of the opinion of the considered agent recently modified according to rule R_1 (every agent's opinion is considered separately), α is a fixed parameter related to the strength of nearest-neighbor interactions, which just defines the units for temperature is measurement (we use natural units in what follows $\alpha = 1$), T is the *social temperature* and H is a periodic function of the form:

$$H = H_0 \times \sin(\omega t), \tag{2}$$

where H_0 is the field amplitude (fashion) (we take $0 \leq H_0 < \alpha$) and $\omega = 2\pi/P$ (P = period). The normalization constant Λ takes the form:

$$\Lambda^{-1} = \exp(\frac{\alpha + q \times H}{T}) + \exp(-\frac{(\alpha + q \times H)}{T}). \tag{3}$$

We then choose a random number u ($0 \leq u \leq 1$).

(i) If $u < p$, the agent retains the position resulting from the application of R_1.

(ii) If $u > p$, the agent considered adopts the opposite opinion to that established by R_1 (the sign is inverted).

Note that, when q and H have the same direction ($q \times H > 0$), the probability p of retaining the new value (obtained according to R_1) is increased. On the other hand, when $q \times H < 0$, the probability p decreases and the selected agents's opinion is more likely to shift to the opposite sense. In both cases, then, the factor $q \times H$ contributes to aligning the agents in the direction of field H.

Note that $p \to 1$ when $T \to 0$ and $p \to 0,5$ when $T \to \infty$. If temperature tends to 0, the effect of fashion disappears.

3. Results and discussion

A time step consists of L trials, where L is the size of the system. At each simulation time t, we define the mean opinion m as:

$$m = \frac{\sum_i s_i}{L}. \tag{4}$$

Hence, we have that $-1 \leq m \leq 1$.

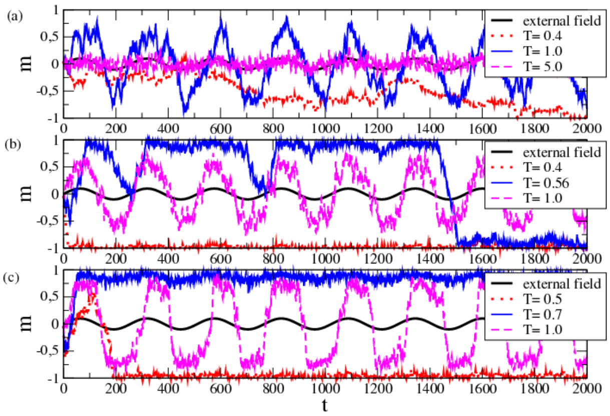

Figure 1. External field (black line) and mean opinion m, as a function of time, t, for $L = 256$, $H_0 = 0.1$, period $P = 256$ and three different social temperatures for interactions A (a), B with $n = 14$ (b) and C with $n = 14$ (c)

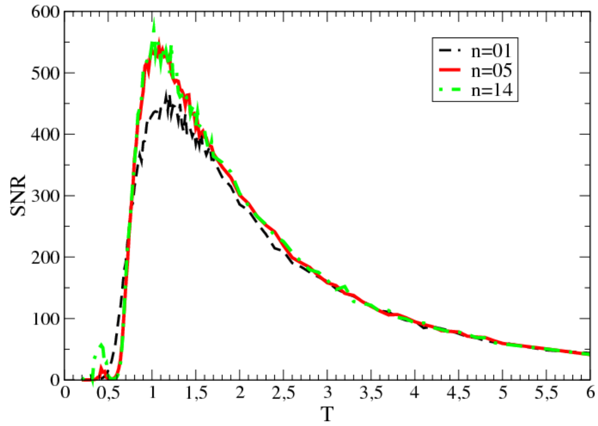

Figure 2. SNR as a function of T for interaction B and different values of n, for the case of $L = 256$, $H_0 = 0.1$ and period $P = 256$.

Figure 1 shows the external field (black line) and the mean opinion as a function of time, for different social temperatures T and for three of the interactions considered. In all these cases, the system size was $L = 256$, the field amplitude was set as $H_0 = 0.1$, and the field period was $P = 256$. The effect of different system sizes, L, and different field amplitudes, H_0, is analyzed in other work [20].

For interaction A (Fig. 1(a)) we can notice that at a low temperature the system's response (mean opinion) does not follow the external field. For intermediate temperatures ($T = 1$) it follows the field with a short delay, an aspect that has already been analyzed elsewhere ([20]). For high temperatures, m follows the field too, but oscillating close to zero.

Something peculiar occurs for interactions B and C. At relatively low temperatures ($T = 0.4$ for interaction B and $T = 0.5$ for model C) the value of m converges to a full polarization (-1 in these cases), that is, people's opinion becomes almost uniform (i.e. a kind of consensus is reached), but a small oscillation is observed around this value, driven by the external field. At higher temperatures ($T = 0.56$ for interaction B and $T = 0.7$ for interaction C), the value of m also goes close to an extreme value, oscillating around it, but from time to time it changes the sign in an aperiodic form, almost randomly. At higher temperatures ($T = 1$) it follows the external field.

We have also calculated the Fourier transform for m as a function of time in the presence of the external field, and then, the signal-to-noise ratio (SNR) for the frequency corresponding to the driving field, defined as

$$SNR = \frac{\int_{w_0-\delta}^{w_0+\delta} S(w)dw}{\int_{w_0-\delta}^{w_0+\delta} S_{back}(w)dw} \quad (5)$$

where

$$S(w) = lim_{\tau\to\infty} \int_{-\infty}^{\infty} < m(t)m(t+\tau) > exp(-iw\tau)d\tau \quad (6)$$

where S_{back} means the value of S in the background of that region. We have calculated the value of SNR for each temperature, averaging over 1000 realizations.

Figures 2 and 3 show SNR vs. T for interactions B and C for different values of n, for the case of $L = 256$, $H_0 = 0.1$ and $P = 256$. Interaction A has already been studied for different system's sizes, L, frequencies and field amplitudes, H_0 in another paper ([20]). For interaction A (or interaction B or C with $n = 1$), only one peak is observed, whose maximum is the fingerprint of the stochastic resonance phenomenon. That means there is a temperature value at which the system's response is optimal, that is, the value of m nicely follows the fashion (or the periodical external field). For interactions B and C, as the value of n grows, we have found that a second resonant peak occurs, at a temperature lower than that corresponding to the main peak. As n grows, the secondary peak remains relatively small in the case of B, while in the case of C, it grows for higher values of n (see figure 3). This means that the introduction of some randomness in the choice of the interacting neighbors results in the formation of an additional resonant peak at a lower temperature and a of valley between the two peaks. However the meaning of this new peak seems to

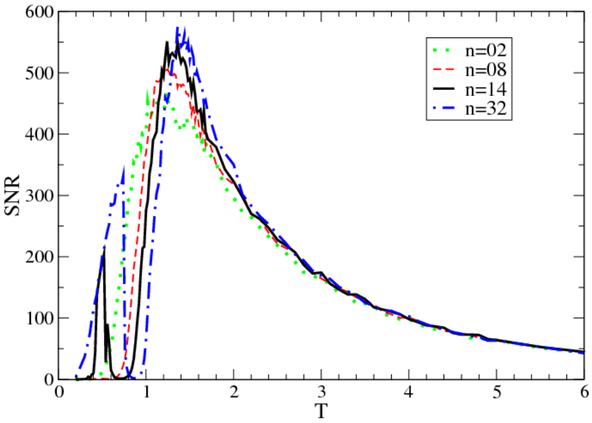

Figure 3. SNR as a function of T for interaction C and different values of n, for the case of $L = 256$, $H_0 = 0.1$ and period $P = 256$.

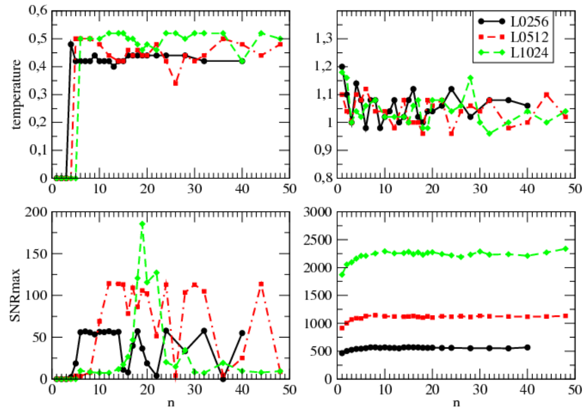

Figure 4. Values of T_{max} (up) and SNR_{max} (down) for the maximum in the graph of SNR vs T (see figure 2) for the first (left) and the second peak (right) for interaction B.

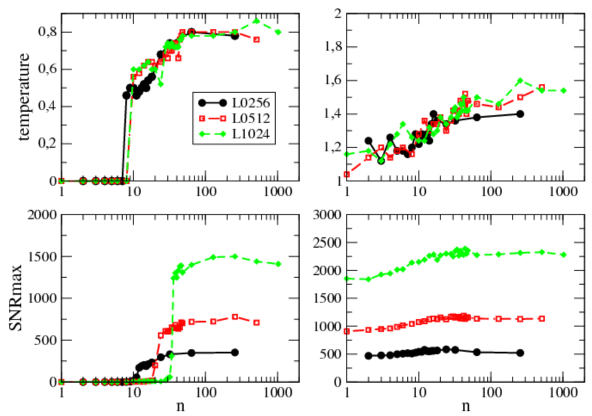

Figure 5. Values of T_{max} (up) and SNR_{max} (down) for the maximum in the graph of SNR vs T (see figure 3) for the first (left) and the second peak (right) for interaction C.

be similar to the analogous one indicated in [19]. It is evident that the modified Sznajd model exhibits such a phenomenon.

Such a secondary peak can be explained by observing Fig. 1. The case of $n = 14$ was analyzed. The temperature corresponding to the maximum of the small peak is approximately $T = 0.4$ for B and $T = 0.5$ for model C. The population's opinion converges to an almost uniform value ($m \approx -1$ in these cases) and presents small oscillations around this value. That results in a significant signal in the value of the SNR measured at the frequency of the fashion, but there is no effective change in the sign of m following the field. The valley in SNR vs. T, corresponds approximately to $T = 0.56$ for B and 0.7 for C. In these cases the periodicity is broken because of the changes of sign of m at relatively random moments. Finally, the mean peak, for $T \approx 1.0$ corresponds to the change of sign of m following the fashion with the same periodicity.

We have also studied the values of SNR and T corresponding to the maximum of the first and the second peak (main peak) as a function of n for B and C, and for three different systems' sizes ($L = 256, 512, 1024$). The results are shown in Figures 4 (interaction B) and 5 (interaction C).

In the case of B (Fig. 4), for small values of n there is no secondary peak (peak 1) and from a certain value of n ($4-6$, depending on L) the peak appears with a value of T approximately between 0.4 and 0.5. The height of the peak (SNR_{max}) is variable for $L = 256$ and $L = 512$ but it presents a maximum value for $n = 19$ in the case of $L = 1024$. Another resonant-like behavior seems to be present in this case.

With respect to the main peak (peak 2), the value of T is approximately constant between $T = 1.0$ and $T = 1.2$. For low values of n, the value of SNR_{max} increases up to a saturation value which is proportional to L.

In the case of C (Fig. 5), there is no peak 1 for low values of n. The small peak appears around $n = 8$. The temperature corresponding to the maximum of this peak, increases up to a saturation value of approximately $T = 0.8$. The height of that peak, SNR_{max} also increases up to a saturation value that is proportional to L. For the second peak (the main one), the value of T increases almost linearly approximately

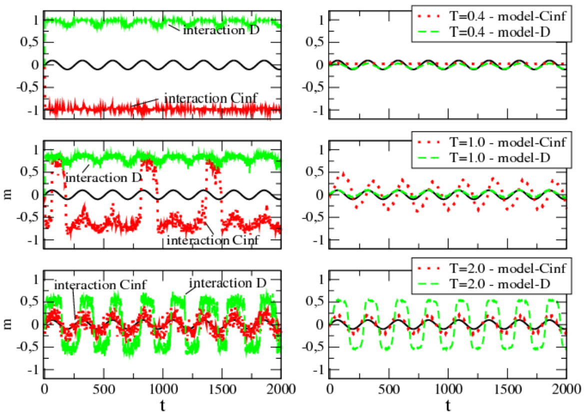

Figure 6. External field (black line) and mean opinion m as a function of time t for $L = 256$, $H_0 = 0.1$, period $P = 256$ and three different social temperatures (up: $T = 0.4$, middle: $T = 1.0$, down: $T = 2.0$) for interactions C_{inf} (dotted) and D (dashed). The continuous line represent the external signal. Left side: one single simulation. Right side: average over 500 realizations.

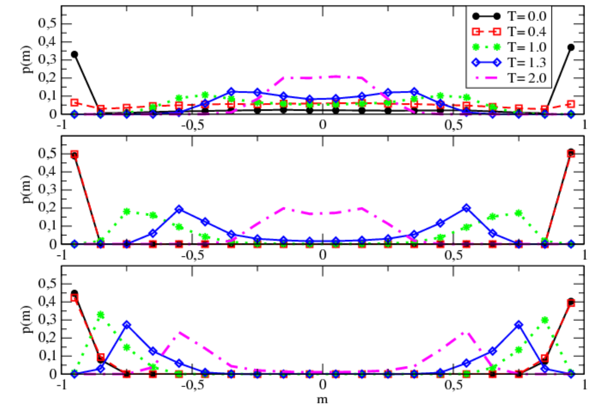

Figure 7. Histograms of distribution probabilities of m for $H_0 = 0.1$, $P = 256$, $L = 512$ and five different temperatures. Average over 1024 simulations. Up: interaction A. Middle: interaction C_{inf}. Down: interaction D.

from $T = 1.0$ to $T = 1.6$. The value of SNR_{max} also increases initially linearly up to a saturation value proportional to L.

When C is considered with range n equivalent to the size of the system this means that we randomly select three agents: i, then we take agent $i + 1$, j (excluding i and $i + 1$) and k (excluding agents i, $i + 1$ and j). This particular case of model C will be called model C_{inf}. We will now compare models A, C_{inf} and D (with the four agents selected at random).

Figure 6 depicts the evolution of the mean opinion m with time for interactions C_{inf} and D, at three different temperatures. On the left side one single simulation and on the right side the average over 500 realizations can be seen. At low temperatures ($T = 0.4$ in this case), the mean opinion takes the value 1 or -1; for D it presents an oscillation following the external signal, while for model C_{inf} the oscillations near -1 (in this case) seem to be very noisy. At intermediate temperatures ($T = 1.0$ in this case) the C_{inf} model shows on one side oscillations following the signal which, from time to time, it jumps to the other side (with respect to 0), in a seemingly random way. For model D, m oscillates following the signal but at one side with respect to 0 (near 1 in this case). At relatively high temperatures ($T = 2.0$ in this case), m oscillates around 0 following the external signal, for both models.

Figure 7 depicts the histograms for the values of m for interactions A (up), C_{inf} (middle) and D (down) at five different temperatures. At $T = 0$, m takes the value 1 or -1 in the three cases, but for D we can see that there is a probability of finding m near ± 0.9 due to the periodic oscillations of the mean opinion. As temperature increases, the distribution of m tends to concentrate around 0, specially for A. Meanwhile, for interaction C_{inf} the tendency is the same but $|m|$ is higher than for A at each temperature. For model D, $|m|$ is higher than for C_{inf} at each temperature and, not even for high temperatures, does it concentrate close to 0. The histograms are symmetrical around 0 in all cases.

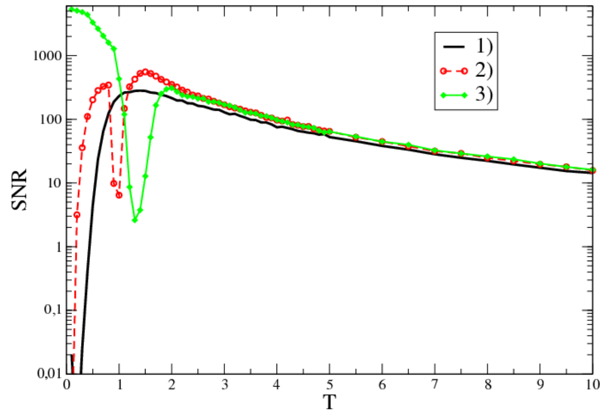

Figure 8. SNR as a function of T for interactions A (1), C_{inf} (2) and D (3), for the case of $L = 256$, $H_0 = 0.1$ and period $P = 256$.

Finally, figure 8 illustrates SNR as a function of T for interactions A, C_{inf} and D. Note the logarithmic scale in the vertical axes. For A, at low temperatures the SNR is very low and it increases up to a maximum value for $T = 1.4$ approximately. After that, it starts to decrease monotonically but with a slope smaller than the one it previously increases with. For C_{inf} the tendency is similar, but two maximums with a valley between them can be observed, with a minimum at $T = 1.0$ approximately and the two peaks at $T = 0.8$ and $T = 1.5$. For interaction D, the SNR is very high at low temperatures and then it starts to decrease until a minimum value for $T = 1.3$. Then it increases until a maximum for $T = 2.0$ and then it starts to decrease again, following for high values of T, the same tendency than interactions A and C_{inf}.

4. Conclusions

To summarize, we have studied a modified Sznajd model, where the presence of contrarians, is introduced in a probabilistic way by means of a *social temperature*. The model also includes the effect of fashion or propaganda, introduced as an external periodic field, which influence the agents'opinion in one or other direction.

Each selected agent has two possibilities, i.e. to adopt some particular opinion according to rule R_1, or to do exactly the opposite (that is, to be a "contrarian"). In each case, the probability of being a contrarian depends on the social temperature and the external field and gives to the model some random behavior that allows the possibility for the stochastic resonance phenomenon to exist.

When studying the SNR of the Fourier transform for the system's response as a function of temperature, we found a stochastic resonance phenomenon. This finding means that the signal-to-noise curve presents a peak, at a given social temperature for which the SNR is maximum.

A modification to the Sznajd rule was introduced in the sense that the selected agents were choosen not only in a consecutive way, but also within some range in the lineal array. In other words, we have studied what happens to the Sznajd model when non local interactions between agents take place. This modification induces a new resonance behavior, namely the occurrence a second peak in the curves of SNR vs. T, indicating a kind of double stochastic resonance phenomenon. To the best of our knowledge it is the first time thet such a phenomenon is reported within an opinion formation framework.

The relationship between the peaks and the distance between the neighbors was also studied, finding that the main resonant social temperature (peak 2) is independent of it, while, for the case of the second resonant peak (peak 1), such a dependence is only weak. Our results seem to indicate that, even in the case of a society which is not highly polarized, with little social turmoil, an adequate level of propaganda could result in an even stronger support to the majority's opinion.

Other aspects of non local effects on the modified Sznajd model behavior will be the subject of further work.

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