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Effective user selection algorithm for quantized precoding in massive MIMO

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Abstract

The downlink of a multi-user massive MIMO wireless system is considered, where the base station equipped with a large number of antennas simultaneously serves multiple users. In this paper, an effective user selection algorithm is proposed for quantized precoding in massive MIMO systems. The algorithm aims at minimizing the correlation of precoders among users by relaxing the optimal problem to be convex and solving it using the Primal Newton's Barrier Method. The complexity of the proposed algorithm is relatively low and the performance shown by the numerical results is close to the exhaustive search method. The advantage of the proposed algorithm increasingly shows up as the transmit antennas increase significantly.

Keywords: massive MIMO, user selection, quantized precoding

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1. Introduction

Very-large multiple-input multiple-output (MIMO), also called massive MIMO, is a new technique that can potentially offer large network capacities in multi-user scenarios, where the base stations are equipped with a large number of antennas simultaneously serving multiple users. As is known to us, MIMO channels, created by deploying antenna arrays at the transmitter and receiver, offers the opportunity to upscale the spectral and energy efficiencies by order of magnitude, realizing the vision of high-performance green mobile radio. Thus, massive MIMO has become an promising technology to overcome the explosive rate data demand and has attracted much attention [1]. While the advantages of massive MIMO are conceptually straightforward, realizing massive MIMO in the network level is far from simple and several fundamental deadlocks remains.

When the transmitter in base station (BS) equipped with multiple antennas sends different and independent messages or data symbols to each user simultaneously, it promises high capacity and high-quality wireless communication links by spatial multiplexing and diversity. Under the condition that transmitter and receiver have perfect knowledge of the channel state information (CSI),the theoretical sum-capacity of K users grows linearly with the minimum number of transmit antennas (N_t) and users which is impractical. The inter-user interference should be dealt to achieve the maximize sum-capacity in the system. Here, the CSI at transmitter (CSIT) as an essential component for precoding plays a key role in mitigating the interference. The transmitter need to utilize the CSIT to preprocess or precode the messages before transmission to reduce the inter-user interference and provide multiplexing gain.

CSIT can be acquired by estimating uplink signals in time division duplex (TDD) systems and receiving feedback signals from users in frequency division duplex (FDD) systems. In TDD systems, channel reciprocity can be utilized for pilot training in the uplink to acquire the complete CSIT, but the performance of the system is constrained by the length of pilot and pilot contamination. In FDD systems, the feedback signals are generated based on the fact that each user estimates the CSI via demodulating pilot signals. According to the obtained CSI, the receiver selects the optimal precoder by transversing the whole codebook which is shared by the receiver and the transmitter, then reports the Precoding Matrix Indicator (PMI) to the transmitter via the limited feedback channel. Afer that the transmitter selects a precoder from the codebook as a function of PMI to accomplish the precoding process for the active users.

The performance of a multi-user MIMO(MU-MIMO) system depends on user selection and power allocation schemes under a given quantized codebook, since the

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inter-user interferences restrict the sum rate of the system. The users subset with minimal spatial channel correlation among each other will be scheduled to mitigate the interference. That means the maximum correlation among selected users need to be minimized in all possible user combinations. Several user selection algorithms have been proposed for downlink MU-MIMO system and the basic principle is to maximize the total throughput of the paired users. The exhaustive enumeration with capacity criticism is optimal, but requires infeasible complexity. As a result, several suboptimal user selection algorithms are studied in [3–7]. These suboptimal user selection algorithms can be briefly categorized as capacity-based user selection scheme and Frobenius norm-based user selction scheme. The principle of capacity-based user selection scheme is to select the first user with highest throughput and greedily select the candidate user which provides the highest total throughput together with those selected users which requires lots of singular value decompositions(SVDs)[3, 4]. Meanwhile, the Frobenius norm-based selection algorithms have been proposed which either considers the norm of the channel gain vector[5] or the orthogonality between the selected channel vectors[6] and even jointly considered[7]. But all these mentioned suboptimal selection methods are based on the whole channel matrix which is unreachable at the transmitter in a limited feedback precoding system. Although the Frobenius norm-based user selection can be extended to the limited feedback system, such as the use on IEEE 802.16m and LTE systems, it requires frequent Gram-Schmidt Orthogonalization procdures(GSOs) to calculate null space and aggregate correlation at each iteration. Thus, the complexity of these algorithms are rather high for massive MIMO with hundreds of transmit antennas(proportional to the cube of the number of transmit antennas).

In this paper, we propose an effective user selection algorithm for the quantizied precoding system with the convex optimization theory. In brief, in order to get the approximate orthogonal user subset, we minimize the correlation of precoders obtained by the feedback PMIs. The complexity of our proposed user selection algorithm increases only linearly with the growth of the transmit antennas, and with the extensive attention of massive MIMO technology, this user selection algorithm will have potential application. Simulation results show that the performance of our proposed convex optimization user selection algorithm outperforms the greedy algorithm, and approaches to the performance of the exhaustive search method.

The rest of the paper is organized as follows. In Section II, we briefly describe the MU-MIMO system and summarize the feedback procedures for the user scheduling and precoding in the cellular systems. We in Section III present the convex optimization user selection algorithm in detail. Section IV shows the simulation results in comparison with the performance of others. And finally, Section V concludes the paper.

2. System Model

2.1. Massive MIMO System Model

In this paper, we focus on the downlink transmission of a massive MIMO system, i.e., the base station (BS) with massive antennas as the transmitter and several user equipment (UE) as the receiver, and we adopt the multi-user MIMO transmission architecture with codebook based precoding mechanism, as shown in Fig1. The BS is equipped with N_t transmit antennas, and N_u UEs with N_r receive antennas each is served by the BS in the cell; and the maximum number of spatial multiplexing users is K which are chosen from N_u distributed users by an user scheduling algorithm. In this paper, we assume $N_r = 1$, $N_t >> K$, and $K \geq 1$. The overall channel is given by $\mathbf{H} = [\mathbf{H}_1^T, \cdots, \mathbf{H}_K^T]^T$, where $\mathbf{H}_k \in \mathbb{C}^{1\times N_t}$ is the channel matrix between the base station and the kth user. Let $\mathbf{s} = [s_1, s_2, ..., s_K] \in \mathbb{C}^{K\times 1}$, $\mathbf{P} = [\mathbf{W}_{s(1)}, \mathbf{W}_{s(2)}, ..., \mathbf{W}_{s(K)}] \in \mathbb{C}^{N_t\times K}$, and $\mathbf{x} = \mathbf{P}s \in \mathbb{C}^{N_t\times 1}$ denote the modulation symbol vector,the precoding matrix and the transmit symbol vector, respectively, where s_k and $\mathbf{W}_{s(k)}$ are for the kth user. After $\mathbf{x}$ passing the channel and being added the noise $\mathbf{z}$ where $\mathbf{z}$ is the additive white complex Gaussian noise(AWGN)vector with covariance matrix $\sigma^2\mathbf{I}$, we will get the received signal $\mathbf{y}$:

$$\mathbf{y} = \mathbf{HPs} + \mathbf{z} \tag{1}$$

For UE k, the received signal y_k is

$$\begin{aligned} y_k &= \mathbf{H}_k\mathbf{x}_k + \sum_{i\neq k}\mathbf{H}_k\mathbf{x}_i + z_k \\ &= \mathbf{H}_k W_k s_k + \sum_{i\neq k}\mathbf{H}_k W_i s_i + z_k \end{aligned} \tag{2}$$

For the equation (2), the first item is a desired signal for the kth user and the second term is inter-user interference which caused the significant degradation of the performance. After the channel matrix $\hat{\mathbf{H}}_k$ is obtained through the channel estimation, the received signal y_k is demodulated as:

$$\hat{s}_k = d_k y_k \tag{3}$$

If the interference is unaware to the receiver, which means the interference is treated as part of the noise, the matched filter(MF) is usually adopted:

$$d_k = (\hat{\mathbf{H}}_k\mathbf{W}_k)^* \tag{4}$$

Each user chooses one of codewords from a pre-design codebook and feedbacks the CSI from the

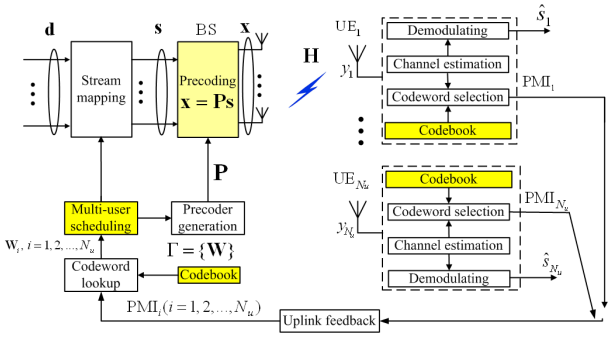

Figure 1. The feedback precoding model of the multi-user MIMO system.

receiver to the transmitter via a limited feedback channel. A codebook containing a list of codewords $\mathbf{W_i}$ which is shared by the the transmitter side and the receiver side is given, with each of codewords reflecting one state of the channel at a certain time, the receiver searches and reports an precoding matrix index within the codebook, called PMI, to the transmitter according to certain rules; If N_u users are waiting to be scheduled at the transmitter, the scheduler will determine $K(< N_u)$ users according to a scheduling algorithm based on the feedback from the users. At the same time, the scheduler will select a precoding matrix $\mathbf{P}$ from the codebook and multiply the signals to accomplish the precoding process for the scheduled K users before transmission.

2.2. Codebook Construction

In Frequency Division Duplexing (FDD) systems, where the fading channel reciprocity cannot be exploited, the performance is achieved by using instantaneous CSI feedback from the users to the BS, between the downlink training phase and the data transmission phase. Hence, the codebook should be pre-designed. Many codebooks have been proposed, such as Kerdock codebook [8], codebooks based on vector quantization [9], Grassmannian packing [10], discrete Fourier transform (DFT)[11], and quadrature amplitude modulation [12]. Codebooks based on vector quantization have taken the channel distribution into account, but have to be re-designed as the change of channel distribution. For uncorrelated channels, the Grassmannian is nearly the optimal codebook, but it has no systematic construction, requiring numerical iterations and high storage. Kerdock codebook, it can provide good system performance with reduced storage requirements and selection computational requirements due to the characteristics of quaternary alphabet and systematic construction. However, in massive MIMO systems, closely-spaced ($\lambda/2$) antenna arrays are favorable to be adopted, and the channels are usually correlated, hence, the Grassmannian codebook and the Kerdock codebook are not optimized for such correlated channels. Thus, the DFT codebook that is able to trace the substantial correlation is expected to be adopted for massive MIMO system equipped with the closely-spaced antenna array.

2.3. Codeword Search

We can choose the optimal codeword from codebook $\Gamma = \{\mathbf{W}_1, \mathbf{W}_2, ..., \mathbf{W}_N\}$ based on the estimated channel. The codebook is shared by the transmitter and receiver. Codeword selection criteria: the capacity selection criterion is adopted [13], which is to maximize the channel gain as a function of the channel matrix $\hat{\mathbf{H}}$ by traversing the whole codebook Γ:

$$\mathbf{W}_k{}^{\text{opt}} = \mathbf{W}^{\Gamma}(\hat{\mathbf{H}}_k) = \max_{\mathbf{W}\in\Gamma} \left\|\mathbf{W}\hat{\mathbf{H}}_k\right\|_2^2 \tag{5}$$

where $\mathbf{W}_k{}^{\text{opt}}$ denotes a $N_t \times 1$ matrix of the optimal precoding matrix for the kth user. All users shall report their own requested codewords $\mathbf{W}_i$ to BS.

3. User Selection Algorithm

When multiple users are using the same resources at the same time, there would be severe interference between their signals. Each user should be capable of decoding his respective stream by reducing the interference due to other stream. This can be achieved by pairing users whose procedures are orthogonal to each other in a dataregion and precoding them appropriately at transmitter and post-processing at the receiver so that each user sees only its own information. For the limited feedback systems, the user selection algorithm neither capacity-based user selection algorithm nor Frobenius norm-based user selection algorithm can be employed for massive MIMO, so we propose an effective the user selection method according to the orthogonalization property of precoding matrix. In this section, we formulate the problem of user selection as a constrained convex optimization problem[15] that can be solved efficiently using numerical methods such as interior-point algorithms.

The received signal for user k can be expressed as (2).With formula (4) the demodulated signal can be written as

$$\begin{aligned}\hat{s}_k &= (\mathbf{H}_k\mathbf{W}_k)^* y_k \\ &= |\mathbf{H}_k\mathbf{W}_k|^2 s_k + (\mathbf{H}_k\mathbf{W}_k)^* \sum_{i\neq k} \mathbf{H}_k\mathbf{W}_i s_i + (\mathbf{H}_k\mathbf{W}_k)^* z_k\end{aligned} \tag{6}$$

The second term of the equation (6) is the inter-user interference. And the interference power of the

demodulated signal to the user k is

$$P_{\text{int}}^{k} = \sum_{i \neq k} \left| (\mathbf{H}_k \mathbf{W}_k)^* \mathbf{H}_k \mathbf{W}_i \right|^2 \mathrm{E}\left[s_k^* s_k \right] = \sum_{i \neq k} \left| (\mathbf{H}_k \mathbf{W}_k)^* \mathbf{H}_k \mathbf{W}_i \right|^2 \mathrm{E}\left[s_k^* s_k \right] \tag{7}$$

Thus, to minimize P_{int}^{k} at each step, a candidate user which is orthogonal to previously selected users to be chosen. However, such a user maybe do not exist in practical systems. So under the given precoding technique, we instead select a user with the precoder orthogonal, by greatest extent, to the matrix formed by the precoders of previously selected users. We release our goal to minimize the preceding vector correlation to get the desired user subset S. i.e.

$$\min \left\| \mathbf{P}_{(S)}^{H} \mathbf{P}_{(S)} \right\|_1 \tag{8}$$

where S denotes the desired user subset and $\|\cdot\|_1$ is a matrix norm which returns the value of maximum column sum.

In order to select a suboptimal user subset, we define $\Delta_i (i = 1, \cdots, Nu)$ as the user selection variable for each user as the selection index such that,

$$\Delta_i = \begin{cases} 1 & i^{th}\ UEselected \\ 0 & otherwise \end{cases} \tag{9}$$

Now consider a $Nu \times Nu$ diagonal matrix $\mathbf{\Delta}$ used for user selection at the BS, which has Δ_i as its diagonal entries. The diagonal matrix $\mathbf{\Delta}$ is represented as

$$\mathbf{\Delta} = \begin{pmatrix} \Delta_1 & & 0 \\ & \ddots & \\ 0 & & \Delta_{Nu} \end{pmatrix}_{Nu \times Nu} \tag{10}$$

where Δ_i is defined as in (9).Then , the selected users' precoding matrix is $\mathbf{P\Delta}$. Using (8), the modified correlation equation for the massive MIMO can be rewritten as

$$\begin{aligned} &\min_{\mathbf{\Delta}} \left\| (\mathbf{P\Delta})^H \mathbf{P\Delta} \right\|_1 \\ &subjectto: \\ &\Delta_i \in \{0,1\} \rightarrow condition1 \\ &trace(\mathbf{\Delta}) = \sum_{i=1}^{N_u} \Delta_i = K \rightarrow condition2 \end{aligned} \tag{11}$$

The equation in (8) when the substitute $\mathbf{P}_{(S)} = \mathbf{P\Delta}$ is concave in Δ_i. Since the variables Δ_i are constrained to be binary integer, this renders the users selection problem in massive MIMO NP-hard. In order to solve this problem, the concept of linear programming relaxation is used[16]. Linear programming relaxation of the 0-1 integer program is adopting a weaker constraint such that each variable is a real number within the interval [0,1] rather than the constraint that each variable must be 0 or 1 , i.e., for each constraint of the form, one uses a pair of linear constraints $0 \leq \Delta_i \leq 1$ instead of $\Delta_i = \{0, 1\}$ of original integer program. With the linear programming method, we transforms the NP-hard optimization problem of integer programming into a linear program that can be solved in polynomial time. We can prove that the problem with relaxation is an convex optimization problem that the objective function is concave and the constrains is linear inequalities. The solution to the relaxed linear problem can be used to gain information about the solution of original integer problem. Applying this technique, the user selection problem in the massive MIMO can be expressed as

$$\begin{aligned} &\min_{\mathbf{\Delta}} \left\| (\mathbf{P\Delta})^H \mathbf{P\Delta} \right\|_1 \\ &subjectto: \\ &0 \leq \Delta_i \leq 1 \rightarrow condition1 \\ &trace(\mathbf{\Delta}) = \sum_{i=1}^{N_u} \Delta_i = K \rightarrow condition2 \end{aligned} \tag{12}$$

Thus we relaxing a combinatorial optimization problem to a solvable problem with a convex objective function. This optimization problem yields a fractional solution, from which the K largest Δ_i are selected and their indices represent the optimal users.

4. Numerical Results

Based on the selection schemes, we present some simulation results in this section. The simulation procedure follows the system model in Fig 1. Without loss of generality, we take the case of 64 transmit antennas as an example for the performance evaluation of the user selection scheme based on DFT codebook.

For comparison, we show the performances of the massive MIMO system with various user selection schemes: the optimal selection (exhaustive search), the proposed selection scheme, the Frobenius norm-based selection scheme, the greedy selection scheme and the random selection. The greedy selection method is that we iteratively selected the candidate user whose precoding vector is of minimizing correlation together with those selected users. We also shows the theoretical curve with perfect CSIT as the upper bound. Table 1 gives the parameter configuration for the following simulations.

We evaluate the capacity as obtained with our selection algorithm, as well as that obtained by exhaustive search, for practical system parameters. Fig 2 depicts the achievable sum rate of the exhaustive search, the proposed selection scheme,the Frobenius

Table 1. Parameters Configuration

Parameter	Value
Channel model	Winner II
BS antenna setup	64 co-polarized antennas,$\lambda/2$ spacing
UE antenna setup	Single antenna
System frequency (GHz)	2.1
Number of channel realizations	1000
Number of UEs	10, 20
Codebook size (bit)	10

Table 2. The complexity order of different user selection scheme

Schemes	Proposed	Frobenius norm-based	Greedy
Complexity	$o(N_u^{2.5}N_t)$	$o(N_u N_t^3)$	$o(N_u^2 N_t)$

norm-based selection scheme, the greedy and the random selection for six scheduled UEs ($K = 6$) as a function of SNR under 10 UEs. The results indicate that the scheme applied to the DFT-URA deployment has remarkable performance gain compared to the random selection and also outperforms the Frobenius norm-based and greedy scheme. It is because the proposed scheme brings in a better orthogonality of selected UEs than the random selection, suitable for multi-user MIMO in the case where UEs are separated spatially in angular domain. Besides, we show the performance of eight scheduled UEs under 30 UEs in Fig 3 and the performace of greedy scheme is even inferior for the reason that the larger the number of the total user, the more likely it would fall into local optimum.

Optimal selection involves an exhaustive search over all possible $\binom{N_u}{K}$ subsets of UEs requiring $\binom{N_u}{K}K$ additions/multiplications, which grows exponentially with Nu for $K \approx N_u/2$.This can be seen using Stirling's approximation for the factorial. At the same time,the greedy scheme with the complexity of $o(N_u^2 N_t)$. when we use the Primal Newton's Barrier Method(PNB) to solve convex relaxation(12), For the barrier method , the number of Newton steps is upper bounded by $\sqrt{Nu}$ [16]. Each Newton step has a complexity $o(N_u^2 N_t)$. Thus, the total complexity is $o(N_u^{2.5} N_t)$ which is a significant improvement to exhaustive search. We summarize the complexity order of the scheme in table 2. We remark that the complexity of the proposed algorithm is linearly of the number of transmit antennas, which has potential advantages in massive MIMO.

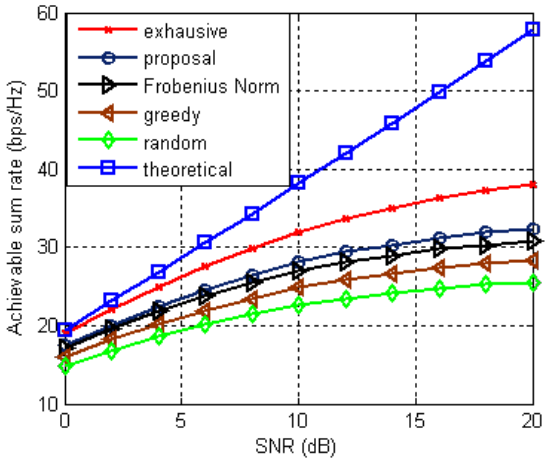

Figure 2. Achievable sum rate with K = 6,Nu=10

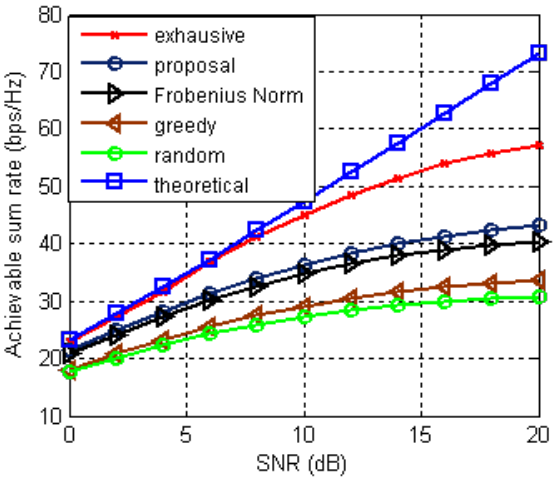

Figure 3. Achievable sum rate with K = 8,Nu=30

5. Conclusion

In this paper, the UE selection scheme for the massive MIMO systems with limited feedback precoding techniques is discussed. Under the codebook design for the URA deployment of the numerous closely-spaced antennas, which would be probably adopted by Massive MIMO, an effective UE selection scheme is proposed. The simulation results show that the proposed scheme outperforms the random selection in terms of the achievable sum rate under the scenario via simulations. The proposed UE selection scheme can contribute to multi-user scheduling for massive MIMO, which would be probably applied in future Beyond 4G systems. Considering the update of 3D channel model, advanced jointing scheduling and antenna selection algorithms should be further studied.

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