Recursive Estimation and Identification of Wireless Ad Hoc Channels from Measurements

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ABSTRACT

This paper is concerned with a time varying wireless ad hoc channel modeling, its parameter estimation and system identification from received signal measurement data. The channel model is represented in state space form, while the expectation maximization algorithm and Kalman filtering are used in the channel parameter and state estimation, respectively. The proposed algorithm is recursive, and therefore the inphase and quadrature components of the ad hoc channel and its parameters are estimated online from received signal measurements. The proposed algorithm is tested using measurement data collected from moving wireless sensor nodes, and the results are presented.

Keywords

Ad hoc channel, expectation maximization algorithm, Kalman filter, stochastic state space representation.

1. INTRODUCTION

Time varying (TV) wireless ad hoc channel models capture both the space and time variations of wireless systems, such as wireless sensor networks. These variations are due to the relative mobility of the receiver, transmitter and/or scatterers [1], [2]. This paper is concerned with the development of TV wireless channel models based on system identification algorithms to extract various ad hoc channel parameters using received signal measurement data.

The majority of research papers in this field such as in [3], [4] use time-invariant (static) models for wireless channels. In time-invariant models, channel parameters are random but do not depend on time, and remain constant throughout the observation and estimation phase. This contrasts with TV models, where the channel dynamics become TV stochastic processes [1], [2].

In [1] and [2], the TV channel parameters are estimated from approximating the Doppler power spectral density (DPSD) of the wireless ad hoc channel. However, in reality one can not have

access to the TV DPSD at all times during the estimation process. We propose to estimate channel parameters as well as the inphase and quadrature components directly from the received signal measurements, which are usually available or easy to obtain in any wireless ad hoc or sensor network. The expectation maximization (EM) algorithm [6] and Kalman filtering [7] are employed in the estimation process. The proposed algorithm is recursive, and therefore can be implemented in real time. The proposed models and estimation algorithm are tested using measurement data collected from moving Crossbow's TelosB wireless sensor nodes. The experimental setup and results are presented.

These models are important in the development of a practical channel simulator that replicates wireless channel characteristics, and produces outputs that vary in a similar manner to the variations encountered in a real-world channel environment.

The paper is organized as follows. In Section 2, the TV wireless ad hoc channel mathematical model is introduced. In Section 3, the EM algorithm together with the Kalman filter, to estimate the channel parameters as well as the inphase and quadrature components from received signal measurements, is developed. In Section 4, experimental setup and numerical results are presented. Finally, Section 5 provides the conclusion.

2. MATHEMATICAL MODELS FOR TV WIRELESS AD HOC CHANNELS

The general TV model of a wireless channel is typically represented by the band-pass impulse response [5]:

$$H(t;\tau) = \sum_{j=1}^{J(t)} \left(I_j(t,\tau)\cos(\omega_c t) - Q_j(t,\tau)\sin(\omega_c t) \right) \delta\left(\tau - \tau_j(t)\right) \quad (1)$$

where $H(t;\tau)$ is the band-pass response of the channel at time t, due to an impulse applied at time $t-\tau$, $J(t)$ is the random number of multipath components, ω_c is the carrier frequency, and the set $\left\{ I_j(t,\tau), Q_j(t,\tau), \tau_j(t) \right\}_{j=1}^{J(t)}$ describes the random TV inphase component, quadrature component, and arrival time of the different paths, respectively. Let $s_l(t)$ be the low pass equivalent representation of the transmitted signal, then the band pass representation of the received signal is given by:

$$y(t) = \sum_{j=1}^{J(t)} \left(I_j(t,\tau)\cos(\omega_c t) - Q_j(t,\tau)\sin(\omega_c t) \right) s_l\left(t - \tau_j(t)\right) \quad (2)$$
$$+ v_I(t)\cos(\omega_c t) - v_Q(t)\sin(\omega_c t)$$

where $\{v_I(t)\}_{t\geq 0}$ and $\{v_Q(t)\}_{t\geq 0}$ are two independent and identically distributed (iid) white Gaussian noise processes, with density $\mathcal{N}(0;\sigma_v^2)$.

It is shown in [1] that the DPSD of a wireless ad hoc channel, denoted by $S(s)$, can be approximated by an even, stable, rational, and factorizable transfer function, $\tilde{S}(s) = H(s)H(-s)$, where $H(s)$ is:

$$H(s) = \frac{b'_{n-1}s^{n-1}+...+b'_1 s+b'_0}{s^n + a'_{n-1}s^{n-1}+...+a'_1 s+a'_0} \quad (3)$$

Consequently, the inphase and quadrature components can be realized using the following stochastic observable canonical form (OCC) state space representation:

$$dX_{I,j}(t) = A_I X_{I,j}(t)dt + B_I dW_j^I(t)$$
$$I_j(t) = C_I X_{I,j}(t) + f_j^I(t)$$
$$dX_{Q,j}(t) = A_Q X_{Q,j}(t)dt + B_Q dW_j^Q(t) \quad (4)$$
$$Q_j(t) = C_Q X_{Q,j}(t) + f_j^Q(t)$$

where

$$X_{I,j}(t) = \left[X_{I,j}^1(t), X_{I,j}^2(t),...,X_{I,j}^n(t) \right]^T,$$
$$X_{Q,j}(t) = \left[X_{Q,j}^1(t), X_{Q,j}^2(t),...,X_{Q,j}^n(t) \right]^T,$$
$$A_I = A_Q = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a'_0 & -a'_1 & -a'_2 & \cdots & -a'_{n-1} \end{bmatrix},$$
$$\quad (5)$$
$$B_I = B_Q = \begin{bmatrix} b'_{n-1} \\ \vdots \\ \vdots \\ b'_1 \\ b'_0 \end{bmatrix}, \quad C_I = C_Q = \begin{bmatrix} 1 & 0 \cdots 0 \end{bmatrix}$$

$\{W_j^I(t)\}_{t\geq 0}$ and $\{W_j^Q(t)\}_{t\geq 0}$ are two independent standard Brownian motions, which correspond to the inphase and quadrature components of the jth path respectively, $f_j^I(t)$ and $f_j^Q(t)$ are arbitrary functions representing the line-of-sight (LOS) of the inphase and quadrature components for the jth path respectively, and T is vector or matrix transpose. Without loss of

generality, we consider the case of flat fading, in which the fading channel has purely a multiplicative effect on the signal and the multipath components are not resolvable. Thus, it can be considered as a single path [5]. We also consider the non-line-of-sight (NLOS) case, i.e., $f_j^I(t) = f_j^Q(t) = 0$, which represents an environment with large obstructions.

The channel model in (4) assumes that mobile nodes move at constant velocities. But in reality, the propagation environment varies continuously due to movement of the nodes at variable speeds and directions, the angle of arrival of the wave upon the receiver can vary continuously, and objects or scatters move in between the transmitter and the receiver resulting in appearance or disappearance of existing paths from one instant to the next. As a consequence of these variations, the statistical characteristics of the received signal vary continuously, giving rise to a DPSD which varies from one observation instant to the next. Therefore, the channel parameters A_I, A_Q, B_I, and B_Q in (4) are time varying and represented as $A_I(t)$, $A_Q(t)$, $B_I(t)$, and $B_Q(t)$. Solution of the time varying version of the state space model in (4) is given by:

$$X_L(t) = \Phi_L(t,t_0)X_L(t_0) + \int_{t_0}^t \Phi_L(t,u)B_L(u)dW_L(u) \quad (6)$$

where $L = I$ or Q, $\Phi_L(t,t_0)$ is the fundamental matrix, and $\dot{\Phi}_L(t,t_0) = A_L(t)\Phi_L(t,t_0)$. Therefore, the mean of $X_L(t)$ is:

$$E\left[X_L(t) \right] = \Phi_L(t,t_0)E\left[X_L(t_0) \right] \quad (7)$$

and the covariance matrix of $X_L(t)$ is:

$$\Sigma_L(t) = \Phi_L(t,t_0)Var\left[X_L(t_0) \right]\Phi_L^T(t,t_0)$$
$$+ \int_{t_0}^t \Phi_L(t,u)B_L(u)B_L^T(u)\Phi_L^T(t,t_0)du \quad (8)$$

For the time invariant case, $A_L(t) = A_L$ and $B_L(t) = B_L$, then (6), (7), and (8) simplify to:

$$X_L(t) = e^{A_L(t-t_0)}X_L(t_0) + \int_{t_0}^t e^{A_L(t-u)}B_L dW_L(u)$$
$$E\left[X_L(t) \right] = e^{A_L(t-t_0)}E\left[X_L(t_0) \right]$$
$$\Sigma_L(t) = e^{A_L(t-t_0)}Var\left[X_L(t_0) \right]e^{A_L^T(t-t_0)} \quad (9)$$
$$+ \int_{t_0}^t e^{A_L(t-u)}B_L B_L^T e^{A_L^T(t-u)}du$$

It can be seen in (7) and (8) that the mean and variance of the inphase and quadrature components are functions of time. Thus the statistics of the inphase and quadrature components, and therefore the statistics of the ad hoc channel, are times varying. Indeed, these stochastic state space models reflect the TV characteristics of the ad hoc channel.

Similarly, following the state space representation in (4), the ad hoc channel can be represented using general stochastic state space representation of the form:

$$dX(t) = A(t)X(t)dt + B(t)dW(t)$$
$$y(t) = C(t)X(t) + D(t)v(t) \qquad (10)$$

where

$$X(t) = \begin{bmatrix} X_I(t) & X_Q(t) \end{bmatrix}^T,$$
$$A(t) = \begin{bmatrix} A_I(t) & 0 \\ 0 & A_Q(t) \end{bmatrix}, B(t) = \begin{bmatrix} B_I(t) & 0 \\ 0 & B_Q(t) \end{bmatrix},$$
$$C(t) = \begin{bmatrix} \cos(\omega_c t)C_I & -\sin(\omega_c t)C_Q \end{bmatrix}, \qquad (11)$$
$$D(t) = \begin{bmatrix} \cos(\omega_c t) & -\sin(\omega_c t) \end{bmatrix},$$
$$dW(t) = \begin{bmatrix} dW^I(t) & dW^Q(t) \end{bmatrix}^T$$
$$v(t) = \begin{bmatrix} v_I(t) & v_Q(t) \end{bmatrix}^T$$

In this case, $y(t)$ represents the received signal measurement, $X(t)$ is the state variable of the inphase and quadrature components, and $v(t)$ is the measurement noise, which is assumed to be Gaussian.

In [1] and [2], the channel parameters $\{a'_{n-1},...,a'_0, b'_{n-1},...,b'_0\}$ are obtained from approximating the DPSD. However, in reality one can not have access to the DPSD at all times during the estimation process. In this paper, the channel parameters as well as the inphase and quadrature components are estimated directly from received signal measurements, which are usually available or easy to obtain in any wireless network. The EM algorithm and Kalman filtering are employed in the channel parameter and state estimation, respectively. These algorithms are introduced in the next section.

3. WIRELESS AD HOC CHANNEL ESTIMATION VIA THE EM ALGORITHM AND KALMAN FILTERING

This section describes the procedure employed to estimate the channel model parameters and states associated with the state space model in (10), using the EM algorithm [6] together with Kalman filtering [7]. However, for simplicity we consider the discrete-time version of (10) given by:

$$x_{t+1} = A_t x_t + B_t w_t$$
$$y_t = C_t x_t + D_t v_t \qquad (12)$$

where $x_t \in \mathfrak{R}^n$ is a state vector, $y_t \in \mathfrak{R}^d$ is a measurement vector, $w_t \in \mathfrak{R}^m$ is a state noise, and $v_t \in \mathfrak{R}^d$ is a measurement noise. The noise processes w_t and v_t are assumed to be independent zero mean and unit variance Gaussian processes.

The unknown system parameters $\theta_t = \{A_t, B_t, C_t, D_t\}$ as well as the system states x_t are estimated through the received signal measurement data, $Y_N = \{y_1, y_2,...,y_N\}$. The methodology employed is recursive and based on the EM algorithm together with the Kalman filter. The Kalman filter is introduced next.

3.1 Channel State Estimation: The Kalman Filter

The Kalman filter estimates the channel states x_t for given system parameter θ_t and measurements Y_t. It is described by the following equations [7]:

$$\hat{x}_{t|t} = A_t \hat{x}_{t-1|t-1} + P_{t|t} C_t^T D_t^{-2} \left(y_t - C_t A_t \hat{x}_{t-1|t-1} \right)$$
$$\hat{x}_{t|t-1} = A_t \hat{x}_{t-1|t-1}, \ \hat{x}_{0|0} = m_0 \qquad (13)$$

where $t = 0,1,2,...,N$, and $P_{t|t}$ is given by:

$$\bar{P}_{t|t}^{-1} = P_{t-1|t-1}^{-1} + A_t^T B_t^{-2} A_t$$
$$P_{t|t}^{-1} = C_t^T D_t^{-2} C_t + B_t^{-2} - B_t^{-2} \bar{P}_{t|t} A_t^T B_t^{-2}$$
$$P_{t|t-1} = A_t P_{t-1|t-1} A_t^T + B_t^2 \qquad (14)$$

The channel parameters $\theta_t = \{A_t, B_t, C_t, D_t\}$ are estimated using the EM algorithm which is introduced next.

3.2 Channel Parameter Estimation: The EM Algorithm

The EM algorithm uses a bank of Kalman filters to yield a maximum likelihood (ML) parameter estimate of the Gaussian state space model. Let $\theta_t = \{A_t, B_t, C_t, D_t\}$ denotes the system parameters in (12), P_0 denotes a fixed probability measure; and $\{P_{\theta_t}; \theta_t \in \Theta\}$ denotes a family of probability measures induced by the system parameters θ_t. If the original model is a white noise sequence, then $\{P_{\theta_t}; \theta_t \in \Theta\}$ is absolutely continuous with respect to P_0 [6]. Moreover, it can be shown that under P_0 we have:

$$P_0: \quad \begin{cases} x_{t+1} = w_t \\ y_t = v_t \end{cases} \qquad (15)$$

The EM algorithm is an iterative scheme for computing the ML estimate of the system parameters θ_t, given the data Y_t. Specifically, each iteration of the EM algorithm consists of two steps: The expectation step and the maximization step.

The expectation step evaluates the conditional expectation of the log-likelihood function given the complete data, which is described by:

$$\Lambda(\theta_t, \hat{\theta}_t) = E_{\theta_t}\left\{\log\frac{dP_{\theta_t}}{dP_{\hat{\theta}t}} \mid Y_t\right\} \tag{16}$$

where $\hat{\theta}_t$ denotes the estimated system parameters at time step t. The maximization step finds:

$$\hat{\theta}_{t+1} \in \arg\max_{\theta_t \in \Theta} \Lambda(\theta_t, \hat{\theta}_t) \tag{17}$$

The expectation and maximization steps are repeated until the sequence of model parameters converge to the real parameters.

For the system model in (12), the EM algorithm is described by the following equations [6]:

$$\hat{A}_t = E\left(\sum_{k=1}^{t} x_k x_{k-1}^T \mid Y_t\right) \times \left[E\left(\sum_{k=1}^{t} x_k x_k^T \mid Y_t\right)\right]^{-1}$$

$$\hat{B}_t^2 = \frac{1}{t}E\left(\sum_{k=1}^{t}\left((x_k - A_k x_{k-1})(x_k - A_k x_{k-1})^T\right) \mid Y_t\right)$$

$$= \frac{1}{t}E\left(\sum_{k=1}^{t}\left((x_k x_k^T) - A_k(x_k x_{k-1}^T)^T - (x_k x_{k-1}^T)A_k^T + A_k(x_{k-1}x_{k-1}^T)A_k^T\right) \mid Y_t\right) \tag{18}$$

$$\hat{C} = E\left(\sum_{k=1}^{t} y_k x_k^T \mid Y_t\right) \times \left[E\left(\sum_{k=1}^{t} x_k x_k^T \mid Y_t\right)\right]^{-1}$$

$$\hat{D}_t^2 = \frac{1}{t}E\left(\sum_{k=1}^{t}\left((y_k - C_k x_k)(y_k - C_k x_k)^T\right) \mid Y_t\right)$$

$$= \frac{1}{t}E\left(\sum_{k=1}^{t}\left((y_k y_k^T) - (y_k x_k^T)C_k^T - C_k(y_k x_k^T)^T + C_k(x_k x_k^T)C_k^T\right) \mid Y_t\right)$$

where $B_t^2 = B_t B_t^T$, $D_t^2 = D_t D_t^T$, and $E(\cdot)$ denotes the expectation operator. The system parameters $\{\hat{A}_t, \hat{B}_t^2, \hat{C}_t, \hat{D}_t^2\}$ can be computed from the following conditional expectations [6]:

$$L_t^{(1)} = E\left\{\sum_{k=1}^{t} x_k^T Q x_k \mid Y_t\right\},$$

$$L_t^{(2)} = E\left\{\sum_{k=1}^{t} x_{k-1}^T Q x_{k-1} \mid Y_t\right\},$$

$$L_t^{(3)} = E\left\{\sum_{k=1}^{t}\left[x_k^T R x_{k-1} + x_{k-1}^T R^T x_k\right] \mid Y_t\right\}, \tag{19}$$

$$L_t^{(4)} = E\left\{\sum_{k=1}^{t}\left[x_k^T S y_k + y_k^T S^T x_k\right] \mid Y_t\right\}$$

where Q, R and S are given by:

$$Q = \left\{\frac{e_i e_j^T + e_j e_i^T}{2}\right\}, R = \left\{\frac{e_i e_j^T}{2}\right\}, i, j = 1, 2, \dots n$$

$$S = \left\{\frac{e_i e_l^T}{2}\right\}; i = 1, 2, \dots n; l = 1, 2, \dots d \tag{20}$$

in which e_i is the unit vector in the Euclidean space; that is $e_i = 1$ in the ith position, and 0 elsewhere. For instance, consider the case $n = d = 2$, then $E\left(\sum_{k=1}^{t} x_k x_{k-1}^T \mid Y_t\right)$ is described by:

$$E\left(\sum_{k=1}^{t} x_k x_{k-1}^T \mid Y_t\right) = \begin{bmatrix} L_t^{(3)}(R_{11}) & L_t^{(3)}(R_{21}) \\ L_t^{(3)}(R_{12}) & L_t^{(3)}(R_{22}) \end{bmatrix} \tag{21}$$

where $R_{ij} = \{e_i e_j^T / 2; i, j = 1, 2\}$. The other terms in (18) can be computed similarly.

The conditional expectations $\{L_t^{(1)}, L_t^{(2)}, L_t^{(3)}, L_t^{(4)}\}$ can be estimated from measurements Y_t as follows:

1) Filter estimate of $L_t^{(1)}$:

$$L_t^{(1)} = E\left\{\sum_{k=1}^{t} x_k^T Q x_k \mid Y_t\right\} =$$

$$-\frac{1}{2}Tr\left(N_t^{(1)} P_{t|t}\right) - \frac{1}{2}\sum_{k=1}^{t} Tr\left(N_{k-1}^{(1)} \bar{P}_{k|k}\right) \tag{22}$$

$$-\frac{1}{2}\sum_{k=1}^{t}\begin{pmatrix} -2x_{k|k}^T P_{k|k}^{-1} r_k^{(1)} + 2x_{k|k-1}^T P_{k|k-1}^{-1} r_{k-1}^{(1)} - x_{k|k}^T N_k^{(1)} x_{k|k} \\ +x_{k|k-1}^T B_k^{-2} A_k \bar{P}_{k|k} N_{k-1}^{(1)} \bar{P}_{k|k} A_k^T B_k^{-2} x_{k|k-1} \end{pmatrix}$$

where $Tr(\cdot)$ denotes the matrix trace. In (22), $r_k^{(1)}$ and $N_k^{(1)}$ satisfy the following recursions:

$$\begin{cases} r_k^{(1)} = \left(A_k - P_{k|k} C_k^T D_k^{-2} C_k A_k\right) r_{k-1}^{(1)} + 2P_{k|k} Q x_{k|k-1} \\ \quad - P_{k|k} N_k^{(1)} P_{k|k} C_k^T D_k^{-2}\left(y_k - C_k x_{k|k-1}\right) \\ r_{k|k-1}^{(1)} = A_k r_k^{(1)} \\ r_0^{(1)} = 0_{m\times 1} \end{cases} \tag{23}$$

$$\begin{cases} N_k^{(1)} = B_k^{-2} A_k \bar{P}_{k|k} N_{k-1}^{(1)} \bar{P}_{k|k} A_k^T B_k^{-2} - 2Q \\ \quad N_0^{(1)} = 0_{m\times m} \end{cases}$$

2) Filter estimate of $L_t^{(2)}$:

$$L_t^{(2)} = E\left\{\sum_{k=1}^{t} x_{k-1}^T Q x_{k-1} \mid Y_t\right\} =$$

$$E_\theta\left\{x_0^T Q x_0 \mid Y_t\right\} + E_\theta\left\{\sum_{k=1}^{t} x_k^T Q x_k \mid Y_t\right\} - E_\theta\left\{x_t^T Q x_t \mid Y_t\right\} \tag{24}$$

Therefore, $L_t^{(2)}$ can be obtained from $L_t^{(1)}$.

3) Filter estimate of $L_t^{(3)}$:

$$
\begin{aligned}
L_t^{(3)} &= E\left\{ \sum_{k=1}^{t} \left(x_k^T R x_{k-1} + x_{k-1}^T R^T x_k \right) \mid Y_t \right\} = \\
&\quad -\frac{1}{2} Tr\left(N_t^{(3)} P_{t|t} \right) - \frac{1}{2} \sum_{k=1}^{t} Tr\left(N_{k-1}^{(3)} \bar{P}_{k|k} \right) \\
&\quad -\frac{1}{2} \sum_{k=1}^{t} \left(\begin{array}{l} -2 x_{k|k}^T P_{k|k}^{-1} r_k^{(3)} + 2 x_{k|k-1}^T P_{k|k-1}^{-1} r_{k|k-1}^{(3)} - x_{k|k}^T N_k^{(3)} x_{k|k} \\ + x_{k|k-1}^T B_k^{-2} A_k \bar{P}_{k|k} N_{k-1}^{(3)} \bar{P}_{k|k} A_k^T B_k^{-2} x_{k|k-1} \end{array} \right)
\end{aligned}
\tag{25}
$$

In this case, $r_k^{(3)}$ and $N_k^{(3)}$ satisfy the following recursions:

$$
\begin{cases}
r_k^{(3)} = \left(A_k - P_{k|k} C_k^T D_k^{-2} C_k A_k \right) r_{k-1}^{(3)} - P_{k|k} N_k^{(3)} P_{k|k} C_k^T D_k^{-2} \\
\qquad \left(y_k - C_k x_{k|k-1} \right) + \left(2 P_{k|k} R + 2 P_{k|k} B_k^{-2} A_k \bar{P}_{k|k} R^T A_k \right) x_{k-1|k-1} \\
r_{k|k-1}^{(3)} = A_k r_k^{(3)} \\
r_0^{(3)} = 0_{m \times 1}
\end{cases}
\tag{26}
$$

$$
\begin{cases}
N_k^{(3)} = B_k^{-2} A_k \bar{P}_{k|k} N_{k-1}^{(3)} \bar{P}_{k|k} A_k^T B_k^{-2} - 2 R \bar{P}_{k|k} A_k^T B_k^{-2} \\
\qquad - 2 B_k^{-2} A_k \bar{P}_{k|k} R^T \\
N_0^{(3)} = 0_{m \times m}
\end{cases}
$$

4) Filter estimate of $L_t^{(4)}$:

$$
\begin{aligned}
L_t^{(4)} &= E\left\{ \sum_{k=1}^{t} \left(x_k^T S y_k + y_k^T S^T x_k \right) \mid Y_t \right\} \\
&= \sum_{k=1}^{t} \left(x_{k|k}^T P_{k|k}^{-1} r_k^{(4)} - x_{k|k-1}^T P_{k|k-1}^{-1} r_{k|k-1}^{(4)} \right)
\end{aligned}
\tag{27}
$$

where $r_k^{(4)}$ satisfy the following recursions:

$$
\begin{cases}
r_k^{(4)} = (A_k - P_{k|k} C_k^T D_k^{-2} C_k A_k) r_{k-1}^{(4)} + 2 P_{k|k} S y_k \\
r_{k|k-1}^{(4)} = A_k r_k^{(4)} \\
r_0^{(4)} = 0_{m \times 1}
\end{cases}
\tag{28}
$$

Using the filters for $L_t^{(i)}$ ($i = 1,2,3,4$) and the Kalman filter described earlier, the system parameters $\theta_t = \{A_t, B_t, C_t, D_t\}$ can be estimated through the EM algorithm described in (18). Experimental results that show the applicability of the above algorithm in estimating the channel parameters as well as the inphase and quadrature components from measurements are discussed next.

4. EXPERIMENTAL RESULTS

In this section, we perform experiments to measure the received signal power of moving sensors in wireless sensor network platform, and then the developed EM algorithm together with Kalman filtering is used to estimate the ad hoc channel parameters as well as the inphase and quadrature components from received signal measurements. The wireless sensors used in our experiment are Crossbow's TelosB sensor nodes. A single

TelosB sensor node is shown in Figure 1. It has the following specifications: IEEE 802.15.4 Compliant, data rate is 250 kbps, carrier frequency is 2.4 GHz, and has USB interface. This sensor is implemented with a Chipcon CC2420 RF transceiver chip which provides a built-in received signal strength indicator (RSSI). This indicator is averaged over 8 symbol periods (128 micro second). The RSSI has a dynamic range of 100 dB and is accurate to +/- 6 dB [8].

Experimental setup consists of two moving transceivers (sensors 1 and 2) and one passive receiver (sensor 3) connected to a workstation as shown in Figure 2. At each time step, sensors 1 and 2 each broadcast a packet containing a source address and the
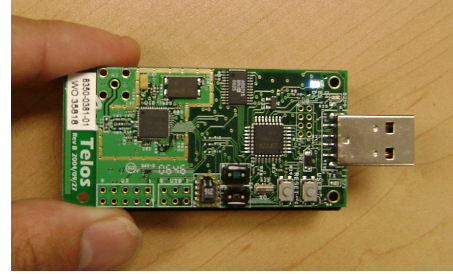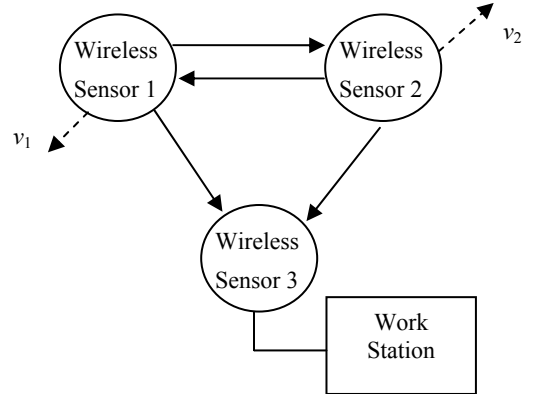

Figure 1. Crossbow TelosB sensor node.

Figure 2. Experimental setup: Two moving transceivers (sensors 1 and 2) and one fixed receiver (sensor 3).

Figure 3. Indoor environment considered in our experiment.

Figure 4. Outdoor environment considered in our experiment.

RSSI of the most recently received packet from the other sensor. Sensor 3 never transmits; rather, it forwards packets from sensors 1 and 2 to a workstation for analysis. The ad hoc channel between sensor 1 and 2 is time varying since both sensors move in different (variable) velocities and directions. Indoor and outdoor environments are considered. In our experiment, the indoor environment is the laboratory room shown in Figure 3, while the outdoor environment is shown in Figure 4.

In the estimation and identification process, a 4th order ad hoc channel model as described in (10) and (11) is considered. Thus, the system parameters $\theta_t = \{A_t, B_t, C_t, D_t\}$ can be represented as:

$$A_t = \begin{bmatrix} 0 & 1 & 0 & 0 \\ a_1 & a_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & a_3 & a_4 \end{bmatrix}, B_t = \begin{bmatrix} b_1 & \delta_{12} & \delta_{13} & \delta_{14} \\ b_2 & \delta_{22} & \delta_{23} & \delta_{24} \\ \delta_{31} & \delta_{32} & b_3 & \delta_{34} \\ \delta_{41} & \delta_{42} & b_{43} & \delta_{44} \end{bmatrix},$$
$$C_t = \begin{bmatrix} \cos(\omega_c t) & 0 & -\sin(\omega_c t) & 0 \end{bmatrix}, D_t = \begin{bmatrix} d_1 & d_2 \end{bmatrix} \quad (29)$$

The estimation includes the channel parameters, inphase and quadrature components, and the received signal, which are then compared to the ones obtained from measurement data. It is assumed that the received signal measurement data are corrupted by white noise sequences. Figures 5 and 6 show respectively the indoor and outdoor measured and estimated received signal using the EM algorithm together with Kalman filter for 500 sampled data taken from measurements between sensor 1 and 2. The indoor system parameters are estimated as:

$$\hat{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -0.3066 & 0.0016 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -0.5940 & 0.0059 \end{bmatrix},$$
$$\hat{B}^2 = \begin{bmatrix} 0.8531 & -0.0369 & -0.0284 & 2.8531*10^{-4} \\ -0.3962 & 0.0649 & 0.0032 & -0.0193 \\ -0.0742 & 0.0074 & 0.0753 & 0.0021 \\ 3.1804*10^{-4} & -0.0532 & 0.0064 & 0.0853 \end{bmatrix}, \quad (30)$$
$$\hat{C} = \begin{bmatrix} \cos(\omega_c t) & 0 & -\sin(\omega_c t) & 0 \end{bmatrix}, \hat{D}^2 = \begin{bmatrix} 2.0262 \end{bmatrix}.$$

while the outdoor system parameters are estimated as:

$$\hat{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -0.7151 & 0.0037 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -0.1500 & 0.0515 \end{bmatrix},$$
$$\hat{B}^2 = \begin{bmatrix} 0.5824 & -0.0735 & -0.0735 & 1.5395*10^{-4} \\ -0.7452 & 0.0846 & 0.0083 & -0.0375 \\ -0.0864 & 0.0365 & 0.0454 & 0.0264 \\ 1.8643*10^{-4} & -0.0643 & 0.0820 & 0.0753 \end{bmatrix}, \quad (31)$$
$$\hat{C} = \begin{bmatrix} \cos(\omega_c t) & 0 & -\sin(\omega_c t) & 0 \end{bmatrix}, \hat{D}^2 = \begin{bmatrix} 1.1725 \end{bmatrix}.$$
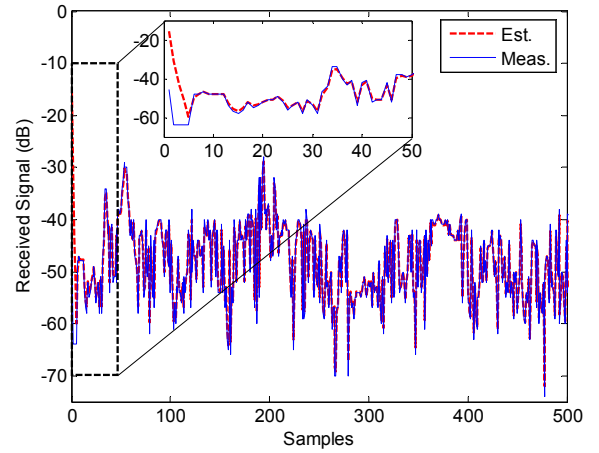

Figure 5. Indoor measured and estimated received signal from sensor 2 by using a 4th order ad hoc channel model.
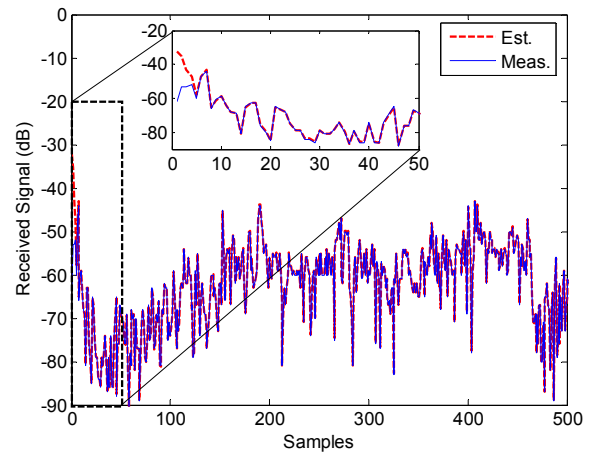

Figure 6. Outdoor measured and estimated received signal from sensor 2 by using a 4th order ad hoc channel model.
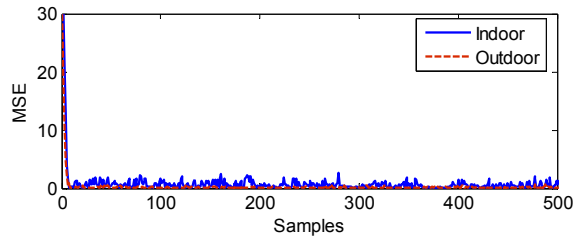
Figure 7. Received signal estimates RMSE for indoor and outdoor environments using the EM algorithm together with the Kalman filter.

From Figures 5 and 6, it can be noticed that the received signals from indoor and outdoor wireless ad hoc channels have been estimated with very high accuracy. It takes few iterations (about 5 iterations) for the estimation algorithm to converge. The root mean square errors (RMSE) for indoor and outdoor environments are shown in Figure 7. It can be seen that indoor RMSE is higher than the one for outdoor because of reflections from walls and objects in indoor environment.

5. CONCLUSION

This paper describes a general scheme for extracting time varying mathematical ad hoc channel models from noisy received signal measurements. The channel models are represented in stochastic state space form. The proposed algorithm consists of filtering based on the Kalman filter to remove noise from data, and identification based on the expectation maximization (EM) algorithm to determine the parameters of the model which best describe the measurements. Indoor and outdoor experimental setups are considered. Experimental results indicate that the measured data can be generated through a simple 4th order discrete-time stochastic differential equation.

6. ACKNOWLEDGMENTS

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