1

Estimation of the Ricean *K* Factor in the High Speed Railway Scenarios

Linyi Gao, Zhangdui Zhong, Bo Ai, Lei Xiong State Key Laboratory of Rail Traffic Control and Safety Beijing Jiaotong University, 100044, Beijing, China E-mail: GaoLinyipaper@gmaill.com

Abstract—As a very important parameter in link budget and channel modeling, the Ricean K factor in the viaduct and cutting scenarios along the high speed railway is estimated by using a moment-based estimator. The practical measurement is taken in the train at a speed of more than 250 km/h. The measured distributions are compared with the Ricean distributions and it's seen that the estimation of K is accurate. Channel conditions of the two special scenarios are analyzed based on the measurement and estimation results.

Index Terms—Ricean factor estimation, viaduct, moment-based estimator, cutting.

I. INTRODUCTION

 $R_{\rm line}$ along straight railway embankments, viaducts, cuttings and tunnels. The base stations are 10 to 30 meters away from the track and most of the antenna heights are more than 30 meters, so there is always a line of sight (LoS) between the transmitter and the receiver. The received signal consists of a direct LoS path and several diffuse paths, whose envelope is known for having the Ricean distribution. The ratio of the power of the LoS component to the diffuse components is the Ricean factor, always represented by K. From [1] we know that the GSM high velocity problem can be solved either through increasing the SNR by 2dB or through increasing the Rice parameter by about 6dB. In the situation of high speed, the rise of BER can be compensated through increasing the Ricean factor, which is also a significant parameter in link budget and channel modeling. Therefore the estimation of the Ricean factor is very important.

The estimation of the Ricean factor has been studied widely. In [2], a general class of moment-based estimators are proposed. The authors also introduce a novel estimator that relies on the I/Q components of the received signal. In [3]-[5], the performance of moment-based estimators are analyzed, and a conclusion is drawn that the estimator based on the first and the second moments is more efficient but complex while the second and fourth order moment-based estimator has a simple closed-form expression. The maximum likelihood estimator (MLE) is derived in [6], which is shown to have requirement of a cumbersome inversion of a nonlinear function of K. In [7], an alternative Bayesian approach is proposed to tackle the twoparameter estimation problem of the Ricean distribution and the performance of the proposed Bayesian estimator is demonstrated through simulations and analysis.

In this paper, two moment-based estimators are presented and the Ricean K factor is estimated in the viaduct and cutting scenarios along the Zhengzhou-Xi'an high speed railway. To evaluate the performance of the estimation, the measured distributions are compared with the Ricean distributions. Channel conditions of the two special scenarios are analyzed based on the estimation results.

II. TWO MOMENT-BASED ESTIMATORS

In the railway wireless communications, the received signal envelop R(t) has a Ricean distribution with corresponding probability density function given by [2]

$$p_R(\mathbf{r}) = \frac{\mathbf{r}}{\sigma^2} \exp(-\frac{\mathbf{r}^2 + s^2}{2\sigma^2}) I_0(\frac{\mathbf{r} \cdot s}{\sigma^2})$$
(1)

where $I_n(\cdot)$ is the *n*th-order modified Bessel function of the first kind, *r* is the envelope of the received signal, σ^2 is the variance of the diffuse components, and s^2 is the power of the LoS path. The *K* factor is defined as

$$K = \frac{s^2}{2\sigma^2} \tag{2}$$

The moment of the Ricean distribution can be expressed as $\mu_n = E \Big[R^n(t) \Big] = (2\sigma^2)^{n/2} \exp(-K) \Gamma \Big(n/2 + 1 \Big)_1 F_1(n/2 + 1; 1; K) \Big]$

where $\Gamma(\cdot)$ is the gamma function and ${}_{1}F_{1}(\cdot;\cdot;\cdot)$ is the confluent hypergeometric function. From (3) we know that the moments depend only on two unknown parameters K and σ . Thus to estimate K we should know at least two different moments of R(t). For $n \neq m$ we define the following functions of K [2]:

$$f_{n,m}(K) = \frac{\mu_n^m}{\mu_m^n} \tag{4}$$

Since that $f_{n,m}(K)$ depends only on K, we can invert the

This work is supported by the Joint Funds of State Key Program of NSFC (Grant No. 60830001), the program for Changjiang Scholars and Innovative Research Team in University (Grant No. IRT0949) and the State Key Laboratory of Rail Traffic Control and Safety (Contract No. RCS2008ZZ006), Beijing Jiaotong University.

corresponding $f_{n,m}(K)$ to solve K. Hence, an estimation of K can be expressed as

$$\hat{K}_{n,m} = f_{n,m}^{-1}(\frac{\mu_n^m}{\mu_n^m}), \hat{\mu}_k = \frac{1}{N} \sum_{l=0}^{N-1} R^k(lT_s)$$
(5)

where N is the number of available samples and T_s is the sampling period.

From (3) we can get that [7]:

$$\mu_{1} = \frac{\sqrt{\pi}}{2} (2\sigma^{2})^{\frac{1}{2}} \exp(-K)_{1} F_{1}(\frac{3}{2};1;K)$$
(6)

$$\mu_2 = s^2 + 2\sigma^2 = 2\sigma^2(K+1) \tag{7}$$

$$\mu_4 = 8\sigma^4 + 8\sigma^2 s^2 + s^4 \tag{8}$$

Substituting (6, 7, 8) to (4), we obtain:

$$f_{1,2}(K) = \frac{\pi e^{-K}}{4(K+1)} \left[(K+1)I_0(\frac{K}{2}) + KI_1(\frac{K}{2}) \right]^2$$
(9)

$$f_{2,4}(K) = \left[\frac{(K+1)^2}{K^2 + 4K + 2}\right]^2$$
(10)

The corresponding estimator $\hat{K}_{1,2}$ involves the complex numerical procedure of calculating the inverse function of (9), which is always difficult to get the result. However, this problem can be solved with a lookup table. The left side of (9), $f_{1,2}(K)$, can be estimated according to (4), and the right side can be calculated by increasing K from 0 to a suitable value which can make both sides of (9) close to each other. In contrast with $\hat{K}_{1,2}$, the estimation of $\hat{K}_{2,4}$ has a closed-form expression:

$$\hat{K}_{2,4} = \frac{-2\hat{\mu}_2^2 + \hat{\mu}_4 - \hat{\mu}_2\sqrt{2\hat{\mu}_2^2 - \hat{\mu}_4}}{\hat{\mu}_2^2 - \hat{\mu}_4}$$
(11)

From (11) we know that $\hat{K}_{2,4}$ can be estimated directly by the moments of the received signal envelope, so the estimation can be implemented in real time using a sliding window approach. The performance of the two estimators has been compared in [2-5], though the asymptotic variance of the estimator $\hat{K}_{2,4}$ is a little larger than that of $\hat{K}_{1,2}$, we choose the estimator $\hat{K}_{2,4}$ to estimate the *K* factor because it's easier to implement in practice.

III. MEASUREMENT SYSTEM AND SCENARIOS

To estimate the K factor in the high speed railway scenarios, we take wireless field strength tests on the Zhengzhou-Xi'an Passenger Dedicated Line where a part of the whole, i.e., Zhengzhou to Luoyang is selected.

The test system consists of the GSM-R base stations and the vehicle test system. The GSM-R base stations are always located less than 30m away from the track, and the transmitter antenna is directional. The interval of every two base stations is about 3 km. However, the frequencies of the adjacent base stations are different, and they are re-used every seven stations according to the theory of cell reuse. In our tests, we just use the

stations whose frequency is 930.2 MHz, shutting down other stations.

The vehicle test system contains a special test antenna for GSM-R, a Willteck 8300 Griffin fast measurement receiver, a distance sensor, a field strength test software and a computer to record the test data, as Fig. 1 shows.



Fig. 1. The vehicle test system

To reduce the sample interval, the Willteck 8300 surveys just one channel each time. The distance sensor sends pulse to the pulse adapter, and the Griffin samples the signal received by the special GSM-R test antenna once every 10 cm according to the output of the pulse adapter, then the computer records the sampling data from the Griffin and the geographic information from the GPS instrument, finally we can use the field strength test software to analyze the data.

The parameters of the test system are shown in TABLE I.

TABLE I Measurement Parameters

Train speed	270—300 km/h		
Testing frequency	930.2 MHz		
Sampling interval	10 cm		
Configuration of antennas	Transmitting antenna	Receiving antenna	
Location	10-30m to the track	on the train roof	
Gain	17 dBi	-	
Polarization mode	Cross Polarization (+/- 45)	vertical	
Output power	40 dBm	-	

The typical scenarios of a high speed railway contain viaducts, cuttings and tunnels. In this paper, we select viaducts and cuttings to estimate the K factors.

Viaduct is one of the special scenarios for high speed trains. To make the train run smoothly at a high speed up to 350 km/h, we should try to reduce the slope and curvature of the railway line. One of the solutions is viaduct. There are lots of viaducts on the Zhengzhou-Xi'an high speed railway line as Fig. 2 shows.

Another typical scenario which should be paid more attention to is the cutting, as shown in Fig. 3. In this scenario the diffuse components of the received signal increase, and the radio propagation environment is more complex.





Fig. 2. The viaduct scenario

Fig. 3. The cutting scenario

IV. MEASUREMENT RESULTS

In the paper, we choose four viaducts of the same type and four similar cuttings on the Zhengzhou-Xi'an high speed railway to estimate the K factors. The locations of these scenarios are shown in TABLE II.

TABLE II LOCATIONS OF THE SCENARIOS

Scenario	Locations(m)	Scenario	Locations(m)
Viaduct 1	K662.625-K663.625	Cutting 1	K600.08-K601.08
Viaduct 2	K674.523-K676.523	Cutting 2	K601.2-K602.2
Viaduct 3	K681-K682.24	Cutting 3	K647.45-K648.477
Viaduct 4	K683.1-K684.822	Cutting 4	K648.54-K649.4

In TABLE II, K stands for kilometer and the numbers after K mean the distance from the viaducts or cuttings to the reference point defined in the engineering.

To determine the K factor of the received signal, the effects of the path loss and shadowing (known as large scale fading) have to be removed first. In order to extract the fading envelope, the received signal is normalized to its local average power. For the received sample $R(x_i)$, the local average power is given by:

$$R(x) = \sum_{i=W/2}^{i+W/2} \frac{R(x_i)}{W}$$
(12)

where W is the number of samples. This local average power estimation is computed for each individual sample in a sliding window. It has been suggested by [8] that a window length of 40 wavelengths is used for macrocells. For the sampling rate used in our measurements, this distance translates to approximately 130 samples per window.

The steps of our estimation are as follows:

1) Collecting the measurement data of the power of the received signal with 10 cm sampling interval.

2) Removing local average power from the measurement data.

3) Estimating $\hat{\mu}_2$ and $\hat{\mu}_4$ according to (5) with a sliding window, and the number of samples in the window is 100.

4) Calculating Ricean parameters $\hat{K}_{2,4}$, σ^2 and s^2 by using (11), (7) and (2).

In order to evaluate the performance of our estimation, we will compare the Ricean CDF (Cumulative distribution function) drawn from estimated parameters σ^2 and s^2 with CDF of the measurement data in the following subsections.

A. The K Factor Estimation in Viaduct Scenario

To get the K factor in viaduct scenario, we make tests on the four different viaducts listed in TABLE II and analyze the measurement data. K is estimated every 10 meters along each viaduct, and the statistical properties, i.e., mean and standard deviation (Std) are shown in TABLE III.

TABLE IIISTATISTICS OF THE K FACTOR IN VIADUCT SCENARIO

Scenario	Max (dB)	Min (dB)	Mean(dB)	Std (dB)	
Viaduct 1	11.6721	3.6656	7.9351	1.8921	
Viaduct 2	10.6982	0.1772	6.6178	1.6255	
Viaduct 3	11.9127	0.9911	6.1567	2.0736	
Viaduct 4	12.6781	0.5390	6.5936	2.0476	

TABLE III presents that in viaduct scenario, mean of the K factor is more than 6 dB and the Std is about 2 dB. So in viaduct scenario, the channel experiences a slight fading and the LOS path is strong.

The measured distributions are compared with the Ricean distributions in Fig. 4-7. In these figures, the blue lines stand for the measured distributions. They are the CDFs of the measurement data. The red lines are the Ricean CDFs with K determined by the moment-based estimator.



Fig. 4. CDF comparison for viaduct 1



Fig. 5. CDF comparison for viaduct 2



Fig. 6. CDF comparison for viaduct 3



Fig. 7 CDF comparison for viaduct 4

These figures show that the Ricean distributions are very close to the measured distributions, which means that the estimation of the K factor is accurate.

B. The K Factor Estimation in Cutting Scenario

Tests are carried out in the four cuttings listed in TABLE II. The statistical properties of the K factors are calculated and shown in TABLE IV.

TABLE IV Statistics of The K Factor in Cutting Scenario

Scenario	Max (dB)	Min (dB)	Mean(dB)	Std (dB)
Cutting 1	9.2385	-8.8938	2.1788	4.2031
Cutting 2	10.1947	-6.0082	2.4274	4.0767
Cutting 3	7.1405	-13.8883	-1.1371	4.2253
Cutting 4	9.3814	-14.5148	0.3075	4.1976

TABLE IV presents that in cutting scenario, mean of the K factor is below 2.5 dB and the Std is more than 4 dB, which suggests that the channel changes fast and the diffuse components increase significantly.

The comparisons of distributions are shown in the following figures. The definition of lines is the same as that in Fig. 4.



Fig. 8. CDF comparison for cutting 1



Fig. 9. CDF comparison for cutting 2



Fig. 10. CDF comparison for cutting 3



Fig. 11. CDF comparison for cutting 4

Fig.8 to Fig.11 display that the moment-based estimator performs well and the estimation of the K factor is close to the actual value.

V. CONCLUSION

In this paper, two traditional moment-based estimators are introduced. The power of the received radio signal is measured when the train operates at a speed of more than 250 km/h along the Zhengzhou-Xi'an high speed railway. The K factors of viaduct and cutting scenarios are estimated by using the moment-based estimator, which employs the second and the fourth moments of the signal envelope. The comparison between the measured distributions and the Ricean distributions indicates that the estimation of K is accurate. Based on the measurement and analysis, it is revealed that in viaduct scenario, the value of K varies within a very small range with the mean over 6 dB, which means that there always exists a strong LoS path and the channel fluctuates slowly. While in cutting scenario, K varies within a relatively large range with the Std more than 4 dB, which implies that the channel varies rapidly.

ACKNOWLEDGMENT

We would like to thank to Dr. Xu Rongtao and the test team for their help and support to complete this paper.

REFERENCES

- Manfred Goller, "Application of GSM in high speed trains: Measurement and simulations," IEE Colloquium on Radio communications in Trans., 1995.
- [2] Cihan Tepedelenlioglu, Georgios B.Giannakis, "The Ricean K factor: Estimation and performance analysis," IEEE Trans. on Wirless Commun., vol. 2, pp.799-810, July 2003.
- [3] Ali Abdi, Cihan Tepedelenlioglu, Mostafa Kaveh, and Georgios Giannakis, "On the estimation of the K parameter for the Rice fading distribution," IEEE Commun. Lett., vol. 5, pp.92-94, March 2001.
- [4] Cihan Tepedelenlioglu, Ali Abdi, Georgios B.Giannakis and Mostafa Kaveh, "Performance analysis of moment-based estimation for the K parameters of the Rice fading distribution," in Proc. IEEE Int. Conf. Acoust., Speech, Signal Processing, vol. 4, pp.2521-2524, 2001.
- [5] Wang Zhen, Wang Ping and Xiong Haitao, "Estimation and measurement of K parameter for Rice fading channel," J. of Data Acquisition& Processing, vol. 24, pp. 109-113, Jan.2009.
- [6] Jan Sijbers, Arnold J. den Dekker, Paul Scheundersand Dirk Van Dyck, "Maximum-Likelihood estimation of Rician distribution parameters," IEEE Trans. on Medical Imaging, vol.17, pp.357-361, 1998.
- [7] Lieve Lauwers, Kurt Barbé, Wendy Van Moer and Rik Pintelon, "Estimating the parameters of a Rice distribution: a Bayesian approach," IEEE Instrumentation and Measurement Technol. Conf., pp.114-117, 2009.
- [8] W. C. Y. Lee, "Estimate of local average power of a mobile radio signal," *IEEE Trans. Veh. Technol.*, vol. VT-34, no. 1, Feb. 1985.