A Highly Scalable Perfect Hashing Algorithm

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Where is Belo Horizonte?

Pampulha's Church
Oscar Niemeyer

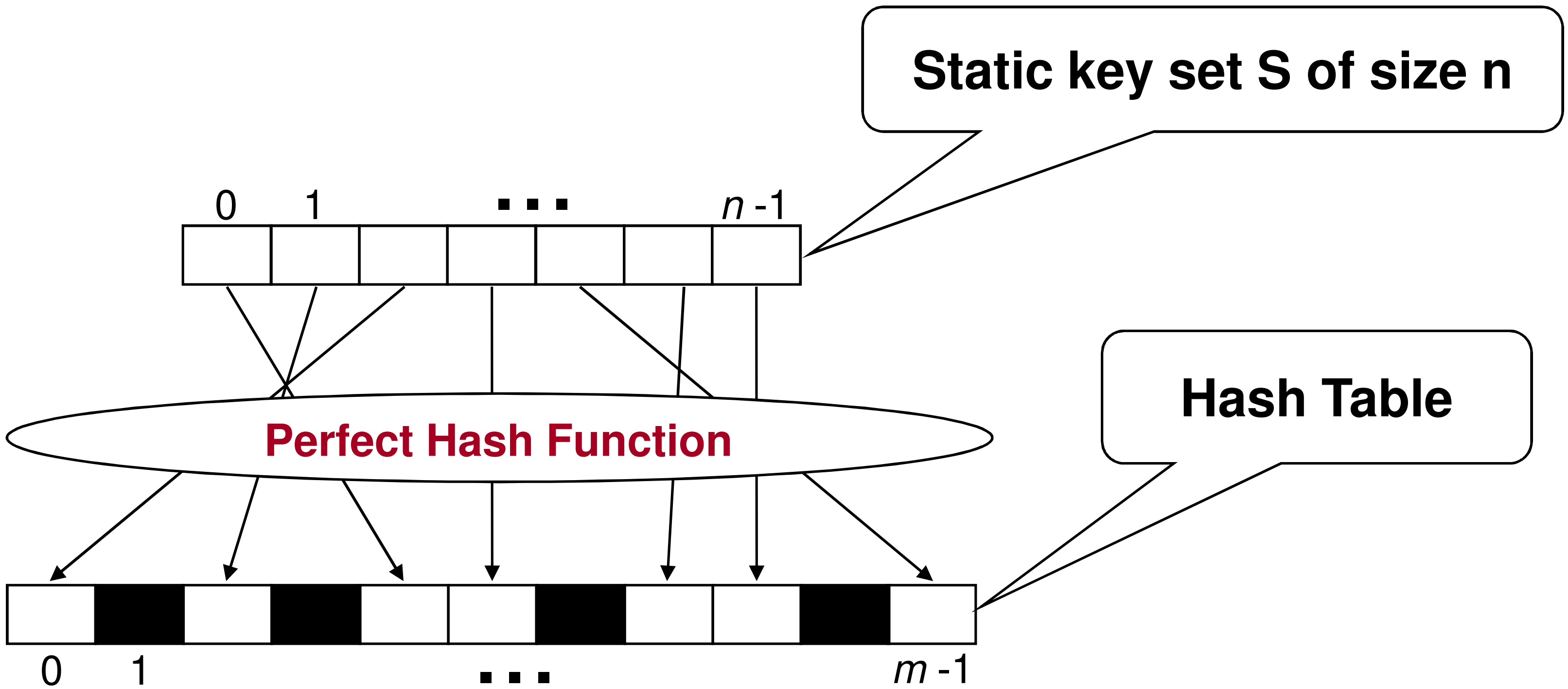
Objective of the Presentation

Present a perfect hashing algorithm:

- Sequential construction of the function
- Distributed construction of the function
- Description and evaluation of the function:
  - *Centralized* in one machine
  - *Distributed* among the participating machines

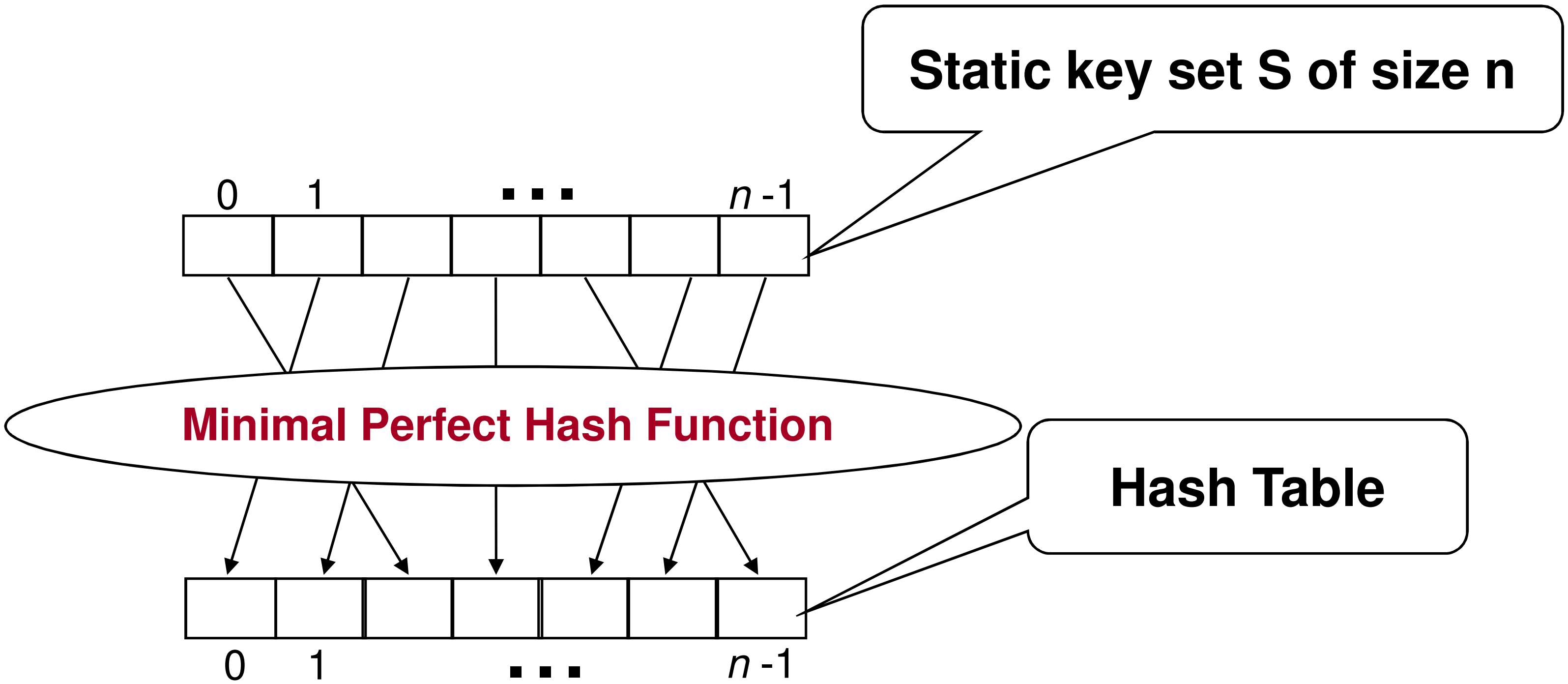
Algorithm is highly scalable, time efficient and near space-optimal

Perfect Hash Function



$$S \subseteq U, \text{ where } |U| = u$$

Minimal Perfect Hash Function



$$S \subseteq U, \text{ where } |U| = u$$

The Algorithm

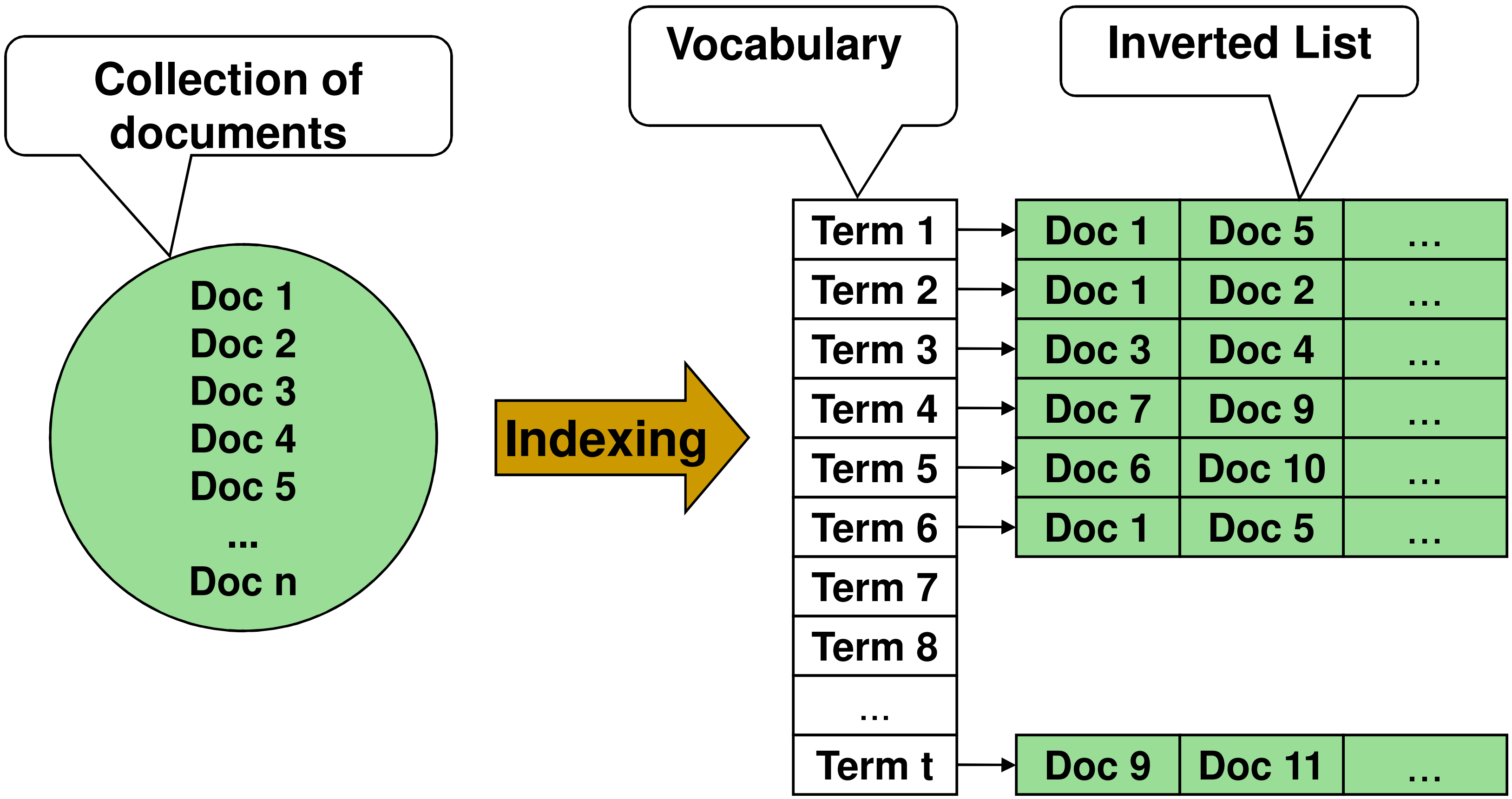
A perfect hashing algorithm that uses the idea of partitioning the input key set into small buckets:

- Key set fits in the internal memory
  - **I**nternal **R**andom **A**ccess memory algorithm
- Key set larger than the internal memory
  - **E**xternal **C**ache-**A**ware memory algorithm

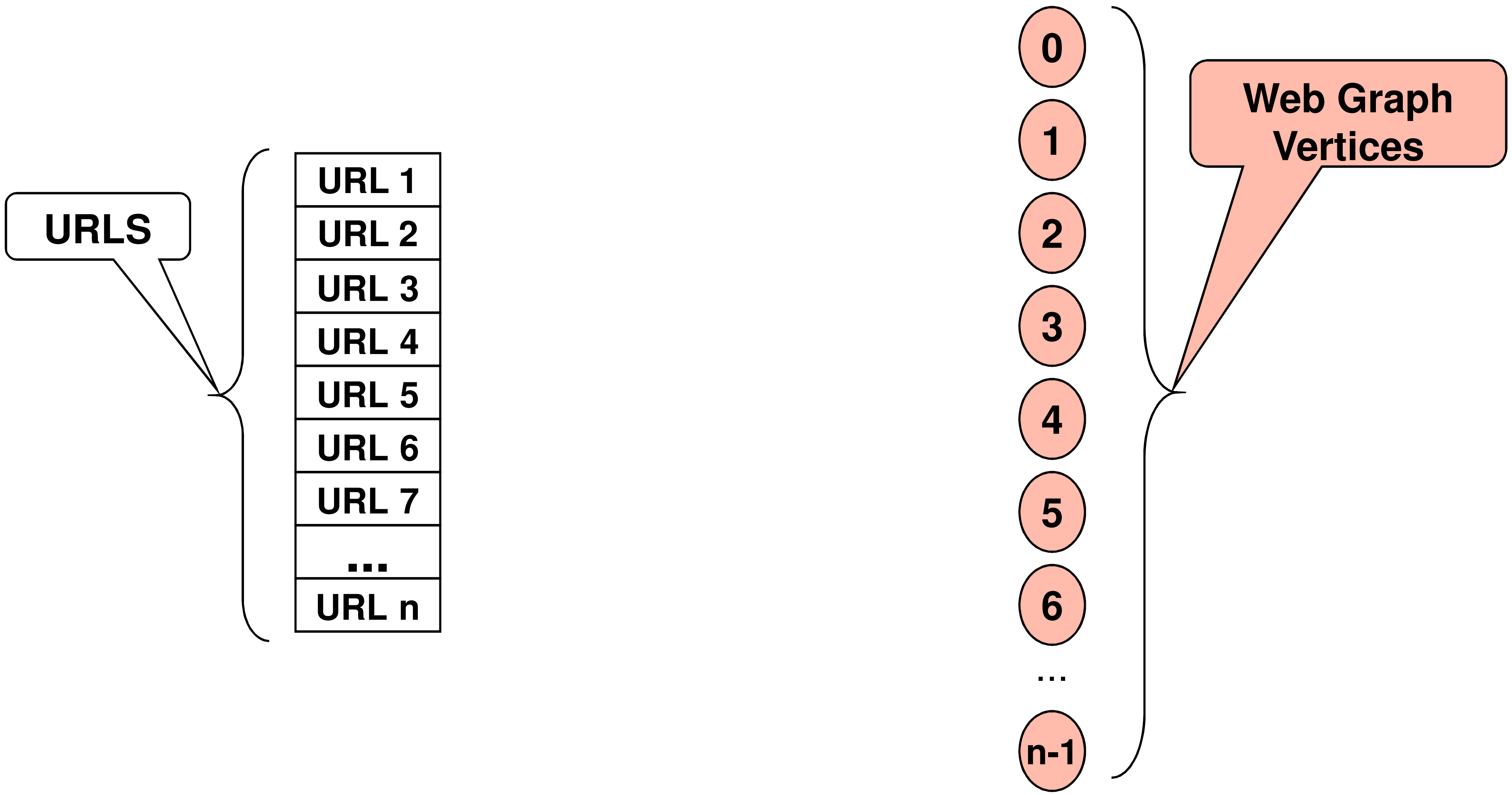
Where to use a PHF or a MPHF?

- Access items based on the value of a key is ubiquitous in Computer Science
- Work with huge static item sets:
  - In data warehousing applications:
    - On-Line Analytical Processing (OLAP) applications
  - In Web search engines:
    - Large vocabularies
    - Map long URLs in smaller integer numbers that are used as IDs

Indexing: Representing the Vocabulary



Mapping URLs to Web Graph Vertices



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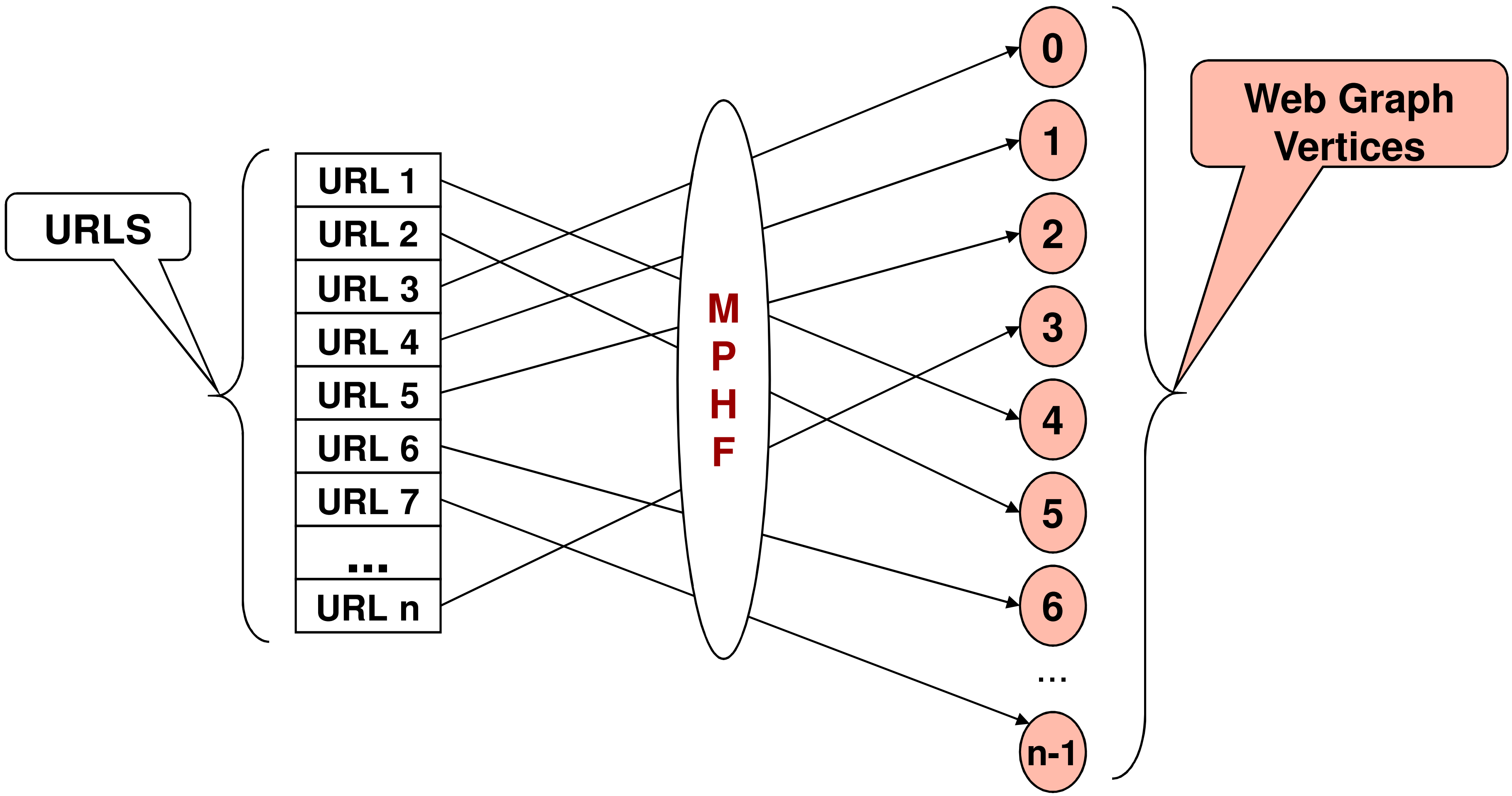


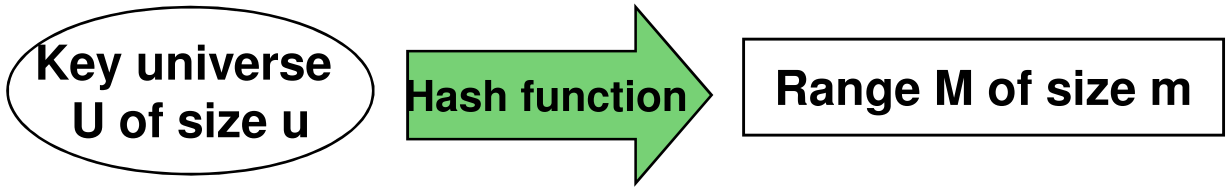
Information Theoretical Lower Bounds for Storage Space

- PHFs (m ≈ n): $\text{Storage Space} \geq \frac{n^2}{m} \log e$
- MPHFs (m = n): $\text{Storage Space} \geq n \log e$

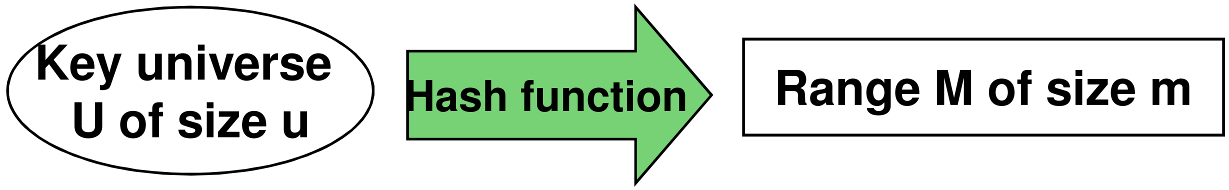
m < 3n

$\log e \approx 1.4427$

Uniform Hashing Versus Universal Hashing



Uniform Hashing Versus Universal Hashing



Uniform hashing

- # of functions from U to M?

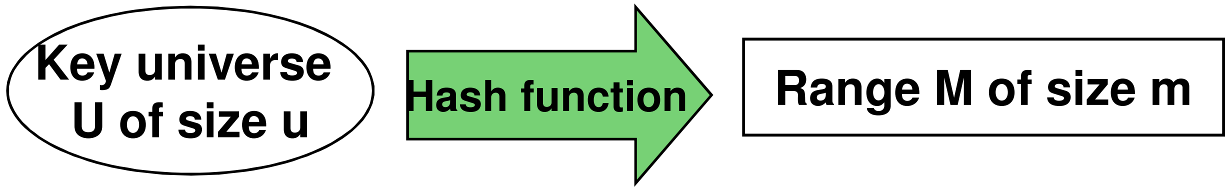
$$m^u$$

- # of bits to encode each fucntion

$$u \log m$$

- Independent functions with values uniformly distributed

Uniform Hashing Versus Universal Hashing



Uniform hashing

- # of functions from U to M?

$$m^u$$

- # of bits to encode each fucntion

$$u \log m$$

- Independent functions with values uniformly distributed

Universal hashing

- A family of hash functions $\mathcal{H}$ is universal if:
  - for any pair of distinct keys (x_1, x_2) from U and
  - a hash function **h** chosen uniformly from $\mathcal{H}$ then:

$$\Pr(h(x_1) = h(x_2)) \leq \frac{1}{m}$$

Intuition Behind Universal Hashing

- We often lose relatively little compared to using a completely random map (uniform hashing)
- If S of size n is hashed to n^2 buckets, with probability more than ½, no collisions occur
  - Even with complete randomness, we do not expect little $o(n^2)$ buckets to suffice (the birthday paradox)
  - So nothing is lost by using a universal family instead!

Related Work

- Theoretical Results

  (use uniform hashing)

- Practical Results

  (use universal hashing - assume uniform hashing for free)

- Heuristics

Theoretical Results

Use Complete Randomness (Uniform Hash Functions)

Work	Gen. Time	Eval. Time	Size (bits)
Mehlhorn (1984)	Expon.	Expon.	O(n)
Hagerup and Tholey (2001)	O(n+log log u)	O(1)	O(n)

Theoretical Results

Use Complete Randomness (Uniform Hash Functions)

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Botelho & Ziviani (CIKM 2007)	**O(n)**	**O(1)**	**O(n)**

Practical Results

Assume Uniform Hashing for Free (Use Universal Hashing)

Work	Gen. Time	Eval. Time	Size (bits)
Czech, Havas & Majewski (1992)	**O(n)**	**O(1)**	**O(n log n)**
Majewski, Wormald, Havas & Czech (1996)	**O(n)**	**O(1)**	**O(n log n)**

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Empirical Results

Work	Application	Gen. Time	Eval. Time	Size (bits)
Fox, Chen & Heath (1992)	**Index data in CD-ROM**	**Exp.**	**O(1)**	**O(n)**
Lefebvre & Hoppe (2006)	**Sparse spatial data**	**O(n)**	**O(1)**	**O(n)**

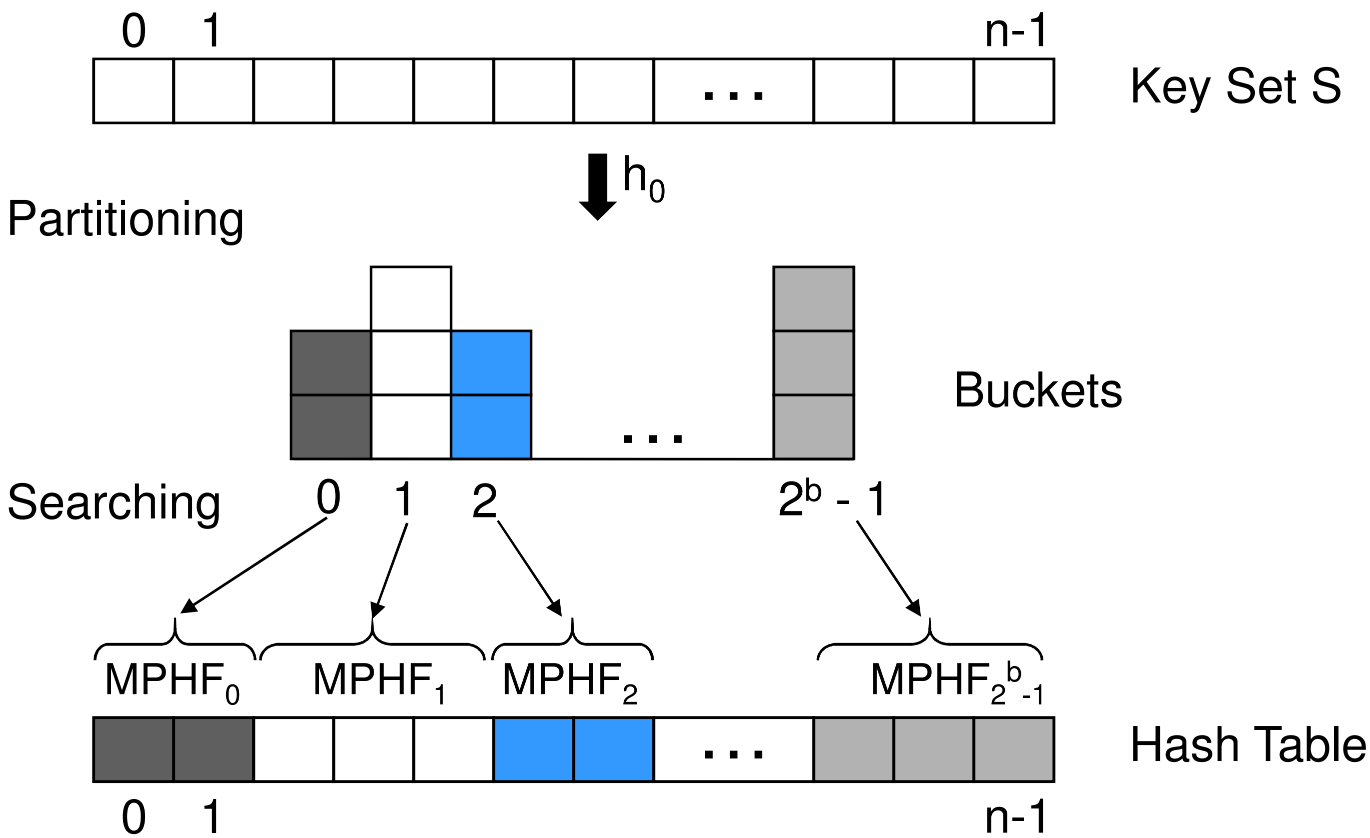
The Sequential

External Cache-Aware Algorithm...

External Cache-Aware Memory Algorithm

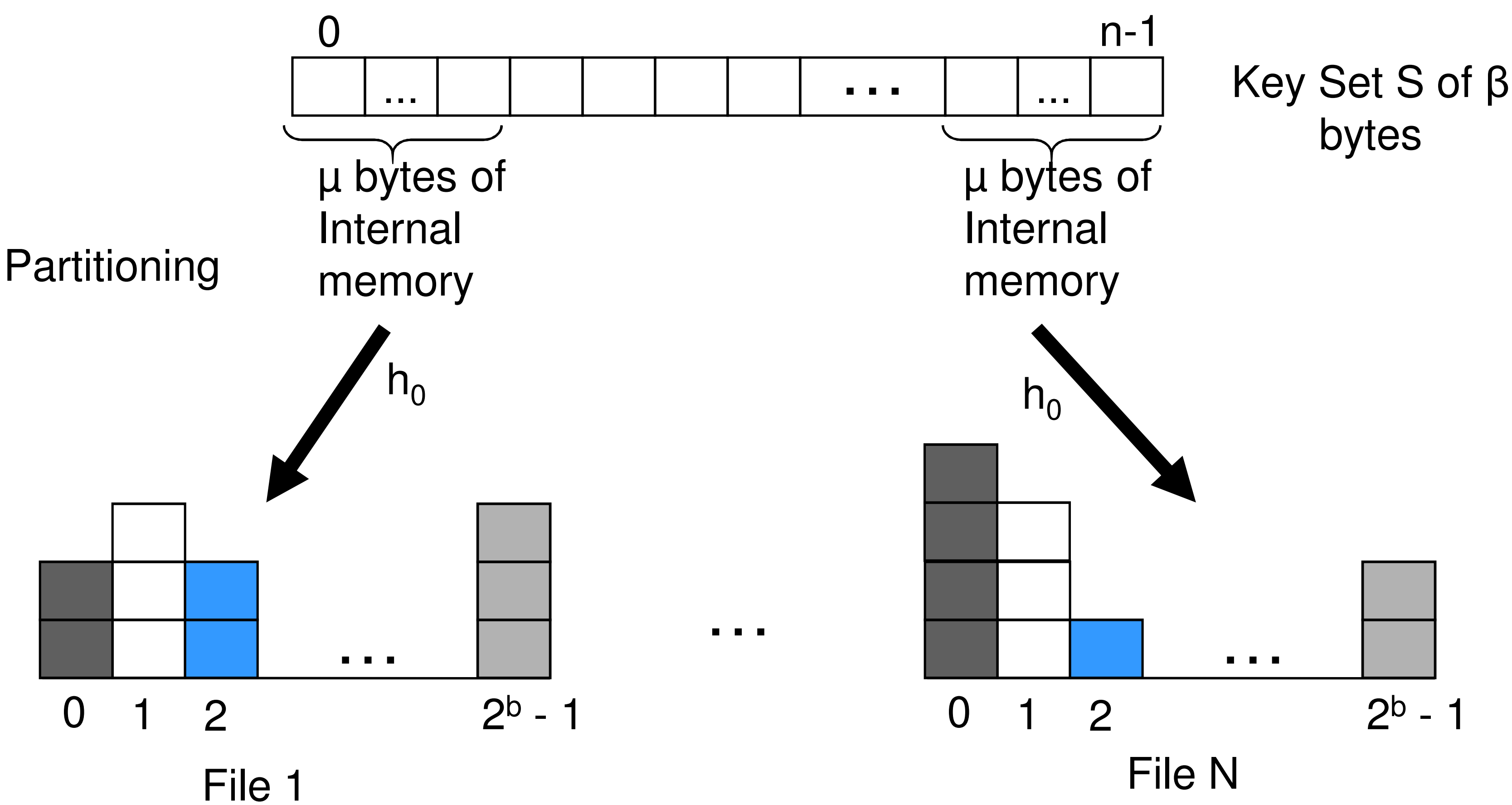
- First MPHF algorithm for very large key sets (in the order of billions of keys)
- This is possible because
  - Deals with external memory efficiently
  - Works in linear time
  - Generates compact functions (near space-optimal)
    - MPHF ($m = n$): $3.3n$ bits
    - PHF ($m = 1.23n$): $2.7n$ bits
    - Theoretical lower bound:
      - MPHF: $1.44n$ bits
      - PHF: $0.89n$ bits

Sequential External Perfect Hashing Algorithm



$$MPHF(x) = MPHF_i(x) + offset[i];$$

Key Set Does Not Fit In Internal Memory



$N = \beta/\mu$ b = Number of bits of each bucket address Each bucket ≤ 256

Important Design Decisions

- We map long URLs to a fingerprint of fixed size using a hash function
- Use our linear time and near space-optimal algorithm to generate the MPHF of each bucket
- How do we obtain a linear time complexity?
  - Using internal radix sorting to form the buckets
  - Using a heap of N entries to drive a N-way merge that reads the buckets from disk in one pass

Algorithm Used for the Buckets:

Internal Random Access Memory Algorithm...

Internal Random Access Memory Algorithm

- Near space optimal
- Evaluation in constant time
- Function generation in linear time
- Simple to describe and implement
- Known algorithms with near-optimal space either:
  - Require exponential time for construction and evaluation, or
  - Use near-optimal space only asymptotically, for large *n*
- Acyclic random hypergraphs
  - Used before by Majewski et all (1996): O(n log n) bits
- We proceed differently: O(n) bits

  (we changed space complexity, close to theoretical lower bound)

Random Hypergraphs (r-graphs)

- 3-graph:

0 1

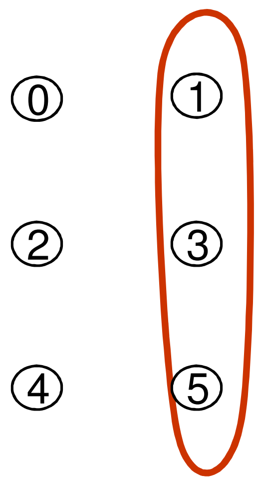
2 3

4 5

- 3-graph is induced by three uniform hash functions

Random Hypergraphs (r-graphs)

- 3-graph:

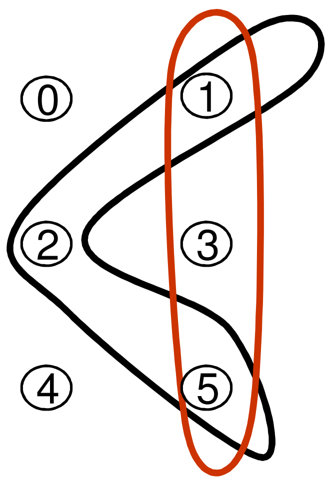


$h_0(jan) = 1 \quad h_1(jan) = 3 \quad h_2(jan) = 5$

- 3-graph is induced by three uniform hash functions

Random Hypergraphs (r-graphs)

- 3-graph:



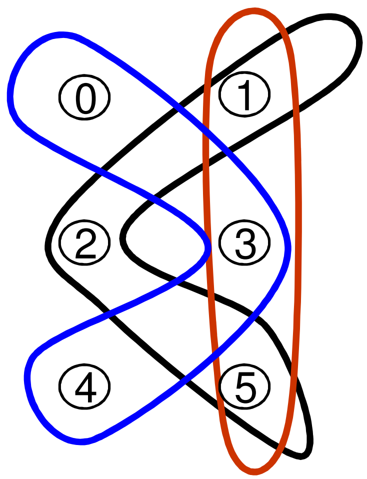
h_0(jan) = 1 h_1(jan) = 3 h_2(jan) = 5

h_0(feb) = 1 h_1(feb) = 2 h_2(feb) = 5

- 3-graph is induced by three uniform hash functions

Random Hypergraphs (r-graphs)

- 3-graph:



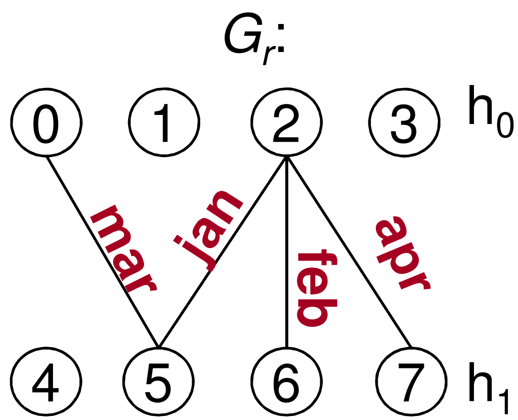
h_0(jan) = 1 h_1(jan) = 3 h_2(jan) = 5

h_0(feb) = 1 h_1(feb) = 2 h_2(feb) = 5

h_0(mar) = 0 h_1(mar) = 3 h_2(mar) = 4

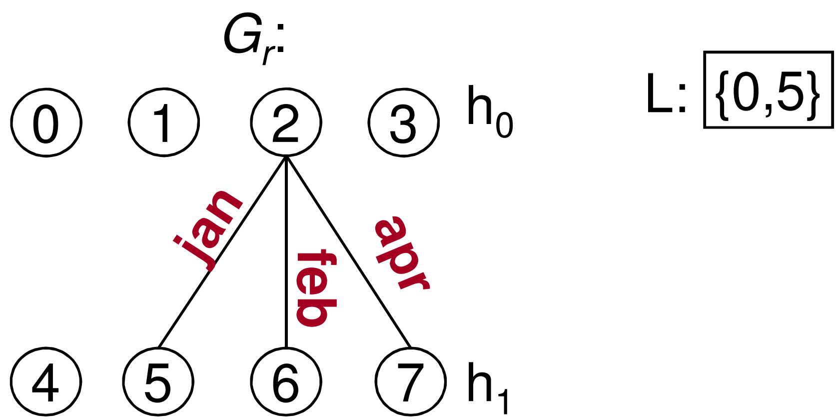
- 3-graph is induced by three uniform hash functions
- Our best result uses 3-graphs

Acyclic 2-graph

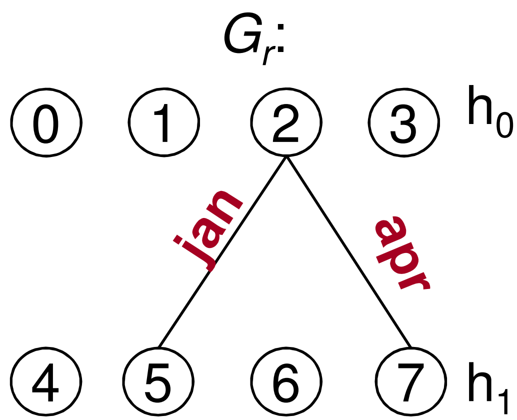


L:Ø

Acyclic 2-graph

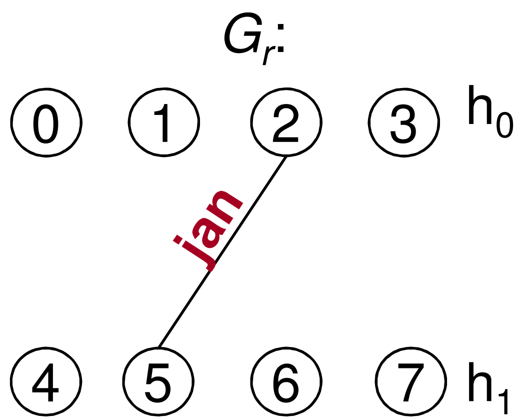


Acyclic 2-graph



	0	1
L:	{0,5}	{2,6}

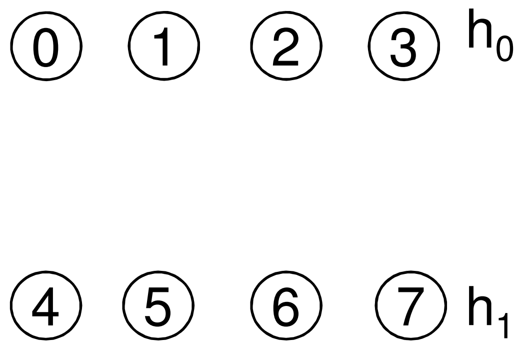
Acyclic 2-graph



	0	1	2
L:	{0,5}	{2,6}	{2,7}

Acyclic 2-graph

G_r:



G_r is acyclic

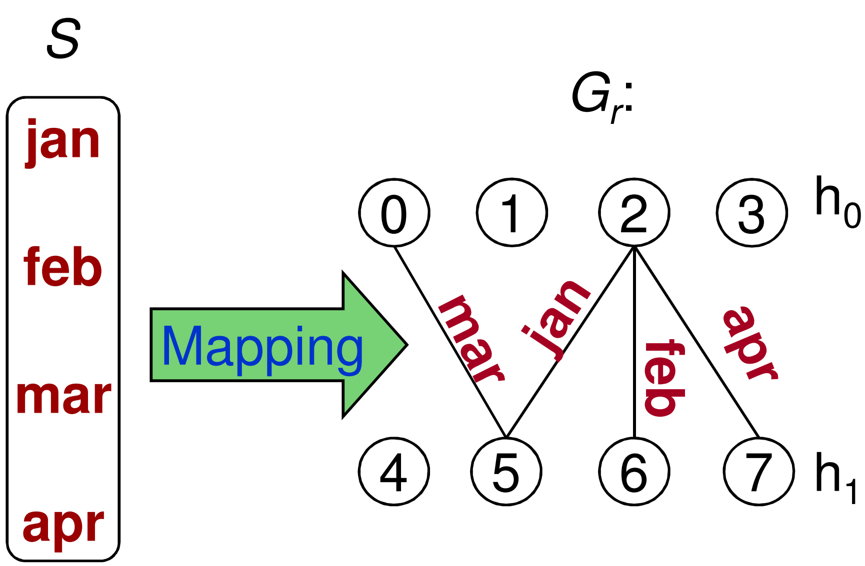
	0	1	2	3
L:	{0,5}	{2,6}	{2,7}	{2,5}

Internal Random Access Memory Algorithm (r=2)

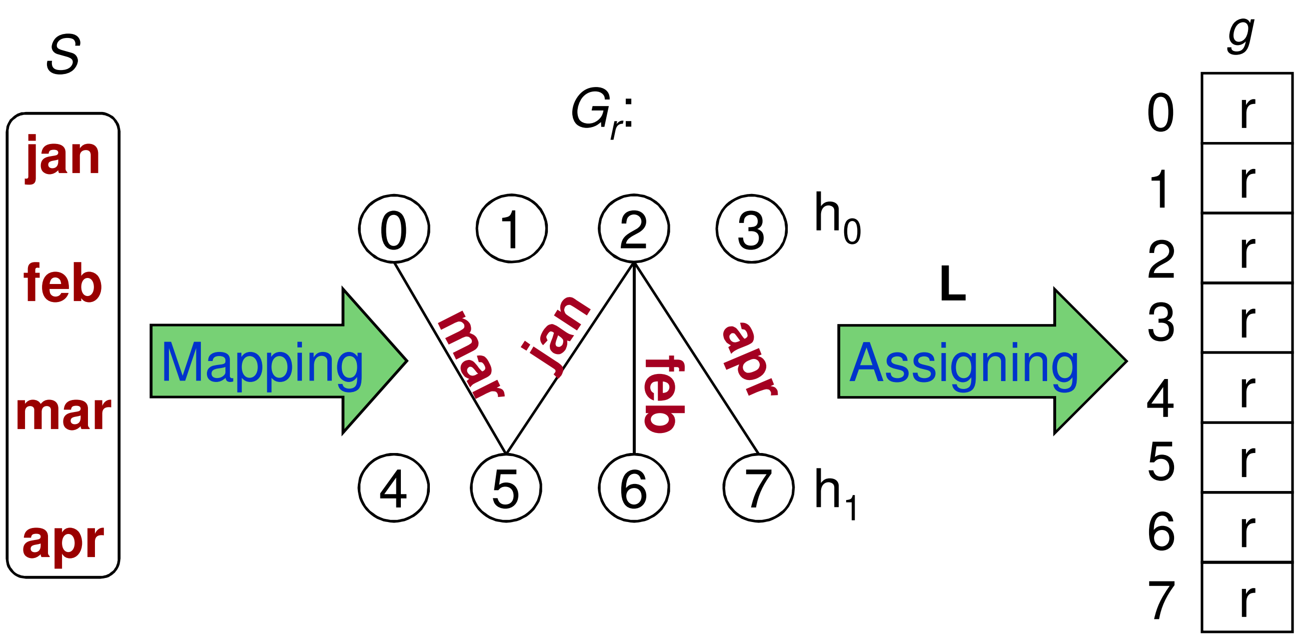
S

jan
feb
mar
apr

Internal Random Access Memory Algorithm (r=2)

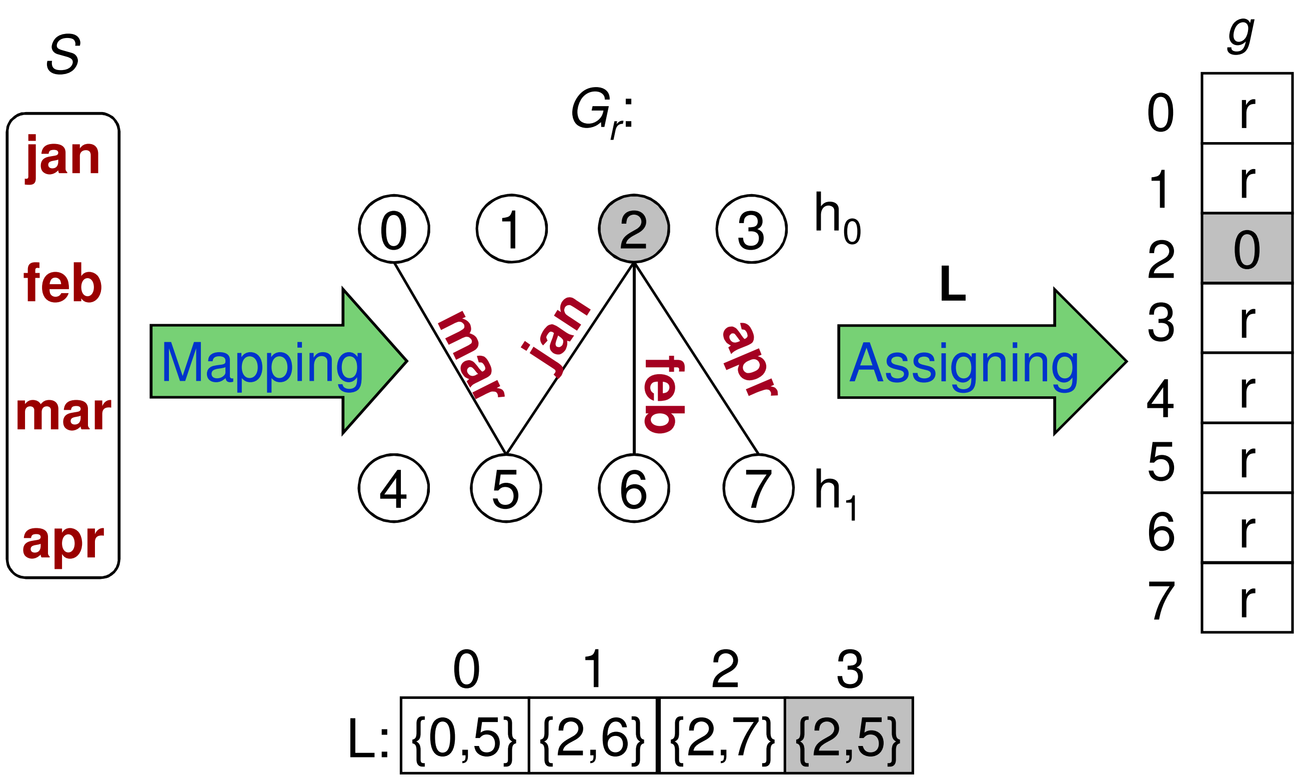


Internal Random Access Memory Algorithm (r=2)



	0	1	2	3
L:	{0,5}	{2,6}	{2,7}	{2,5}

Internal Random Access Memory Algorithm (r=2)



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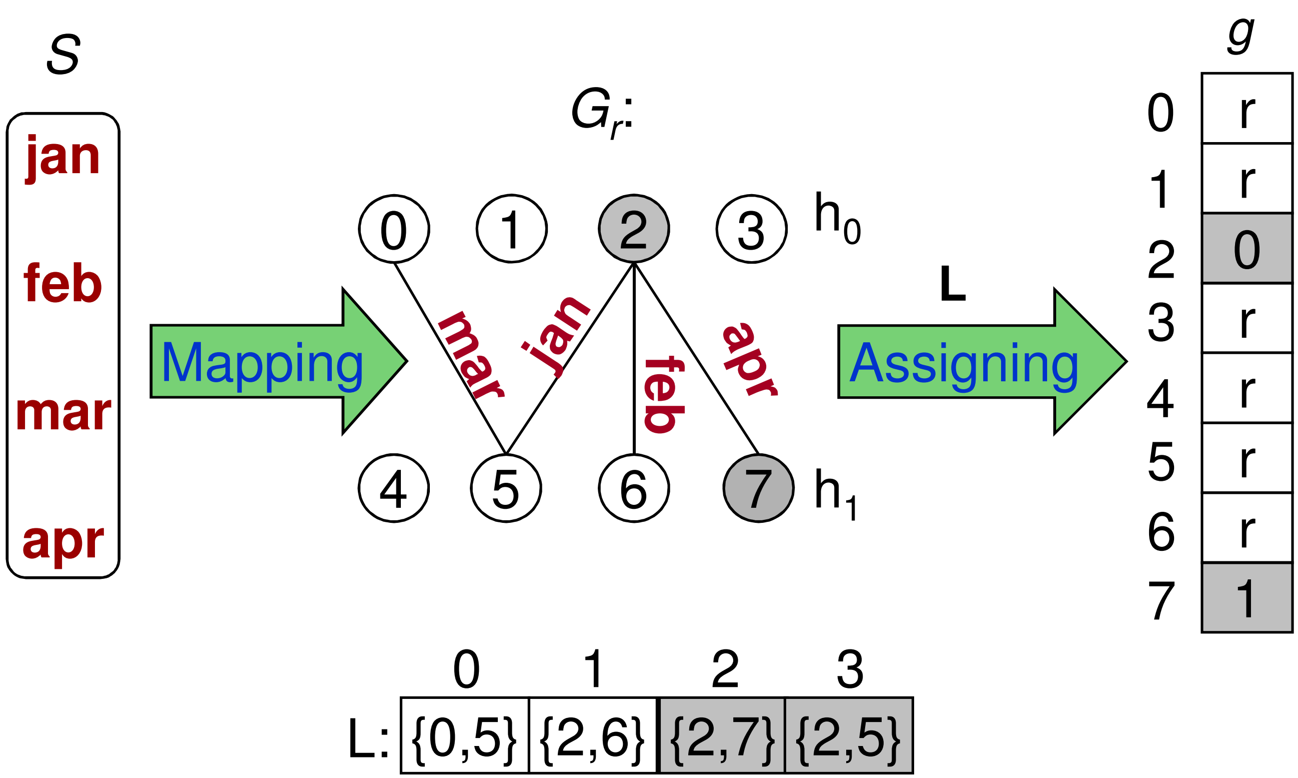

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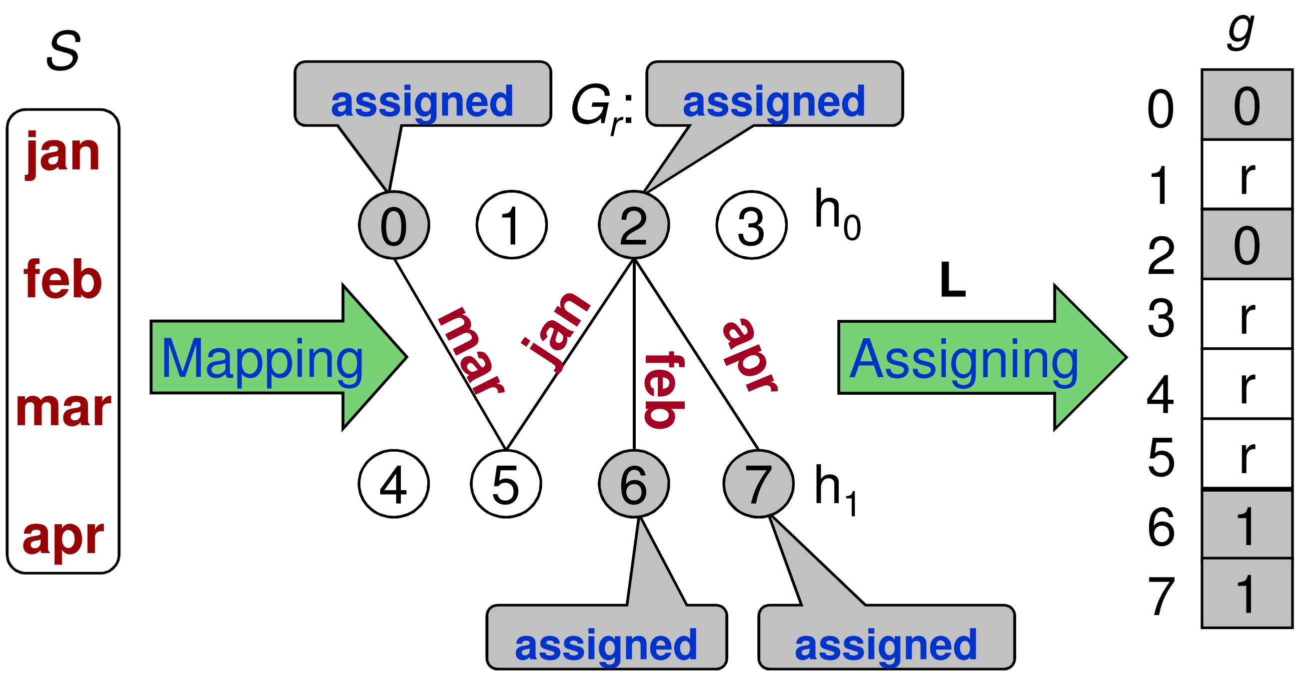
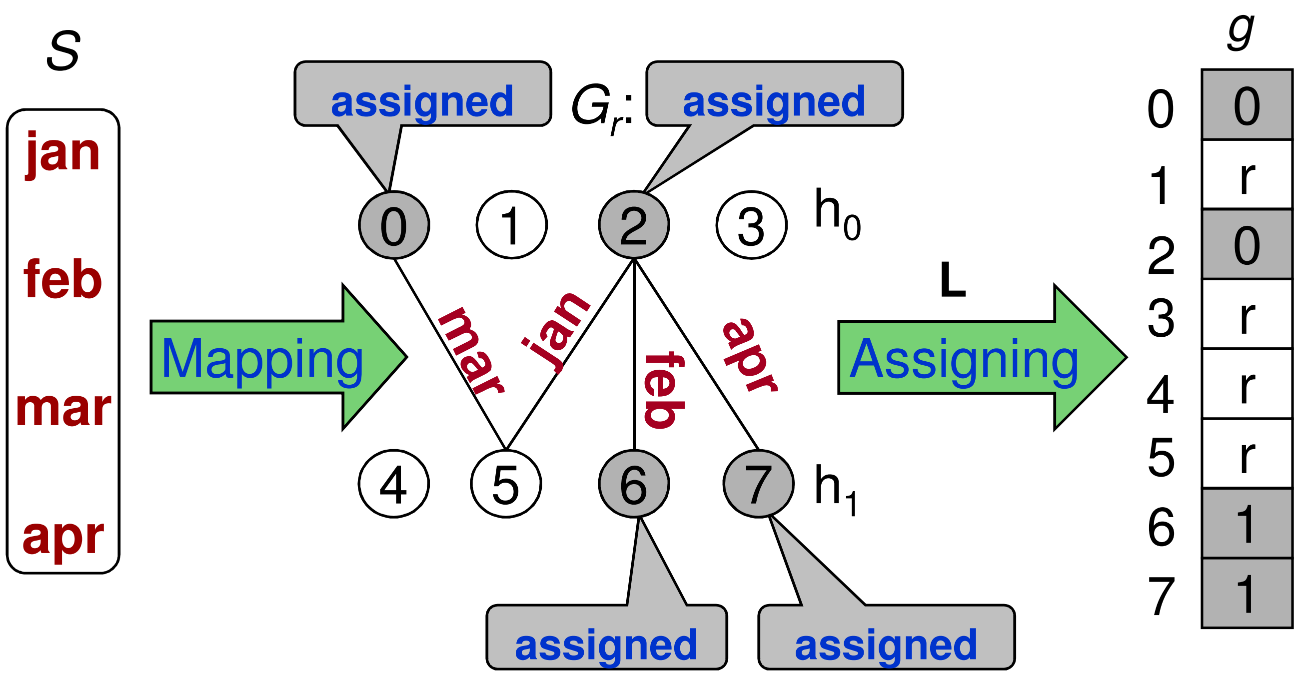
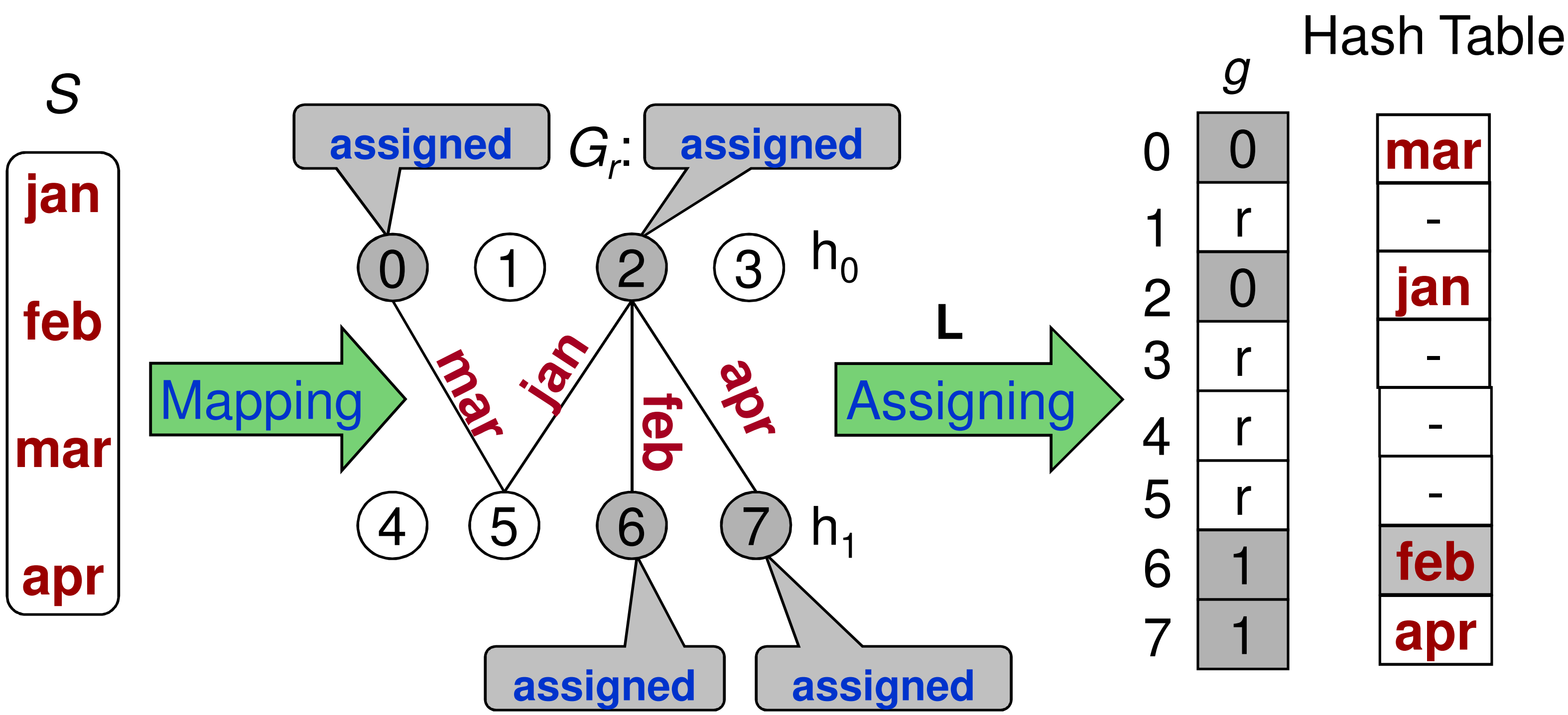


Internal Random Access Memory Algorithm (r=2)



$i = (g[h_0(\text{feb})] + g[h_1(\text{feb})]) \bmod \mathbf{r} = (g[2] + g[6]) \bmod \mathbf{2} = 1$

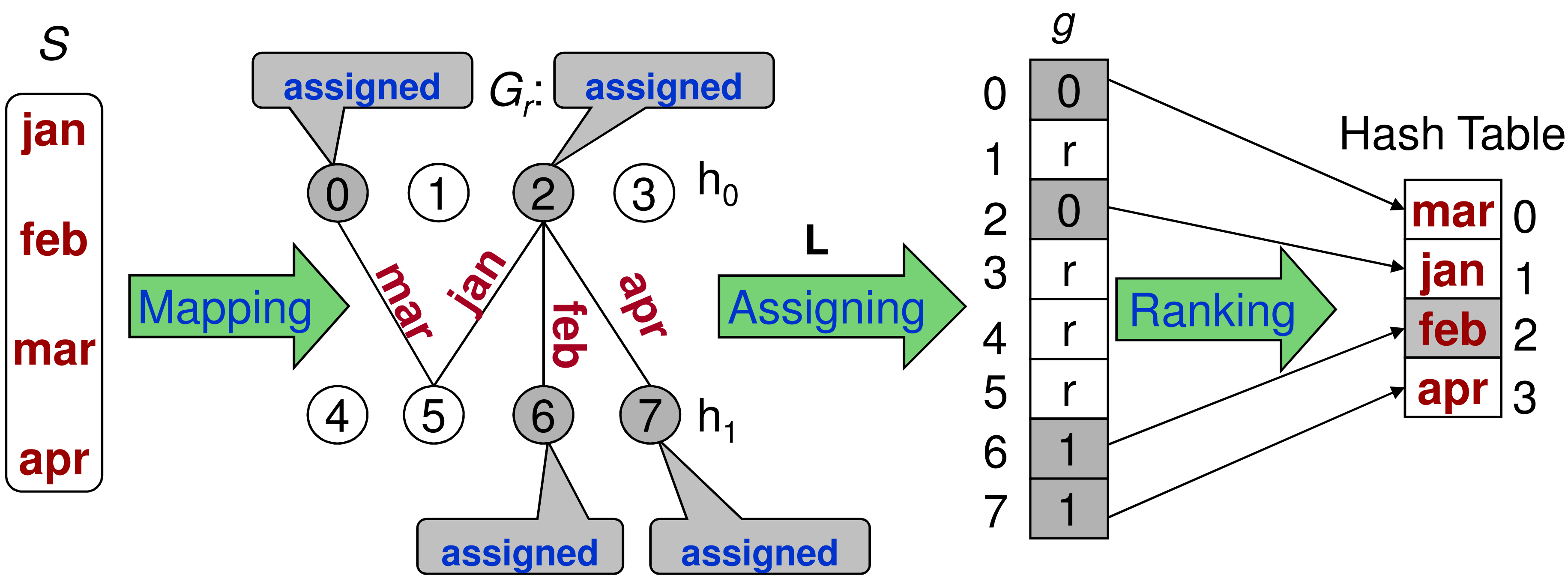
Internal Random Access Memory Algorithm: PHF



$i = (g[h_0(feb)] + g[h_1(feb)]) \bmod \mathbf{r} = (g[2] + g[6]) \bmod \mathbf{2} = 1$

$phf(feb) = h_{i=1}\,(feb) = 6$

Internal Random Access Memory Algorithm: MPHF

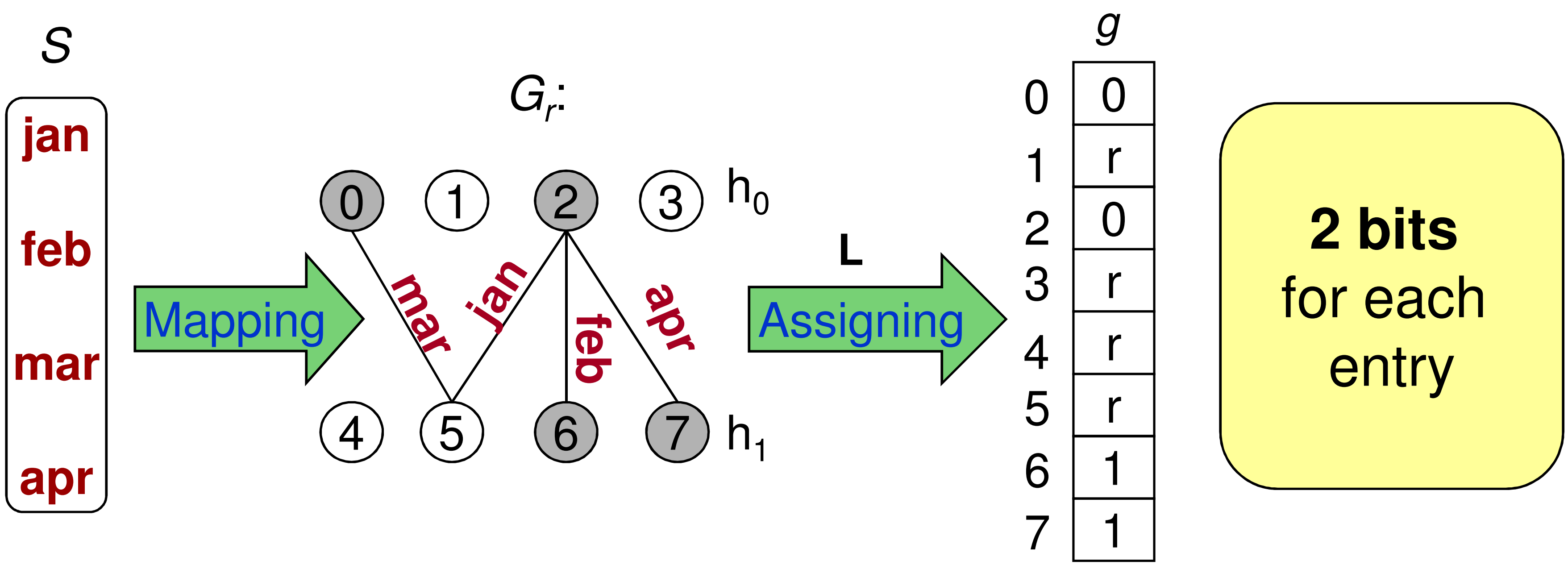


$i = (g[h_0(\text{feb})] + g[h_1(\text{feb})]) \bmod \mathbf{r} = (g[2] + g[6]) \bmod \mathbf{2} = 1$

$\text{phf}(\text{feb}) = h_{i=1}(\text{feb}) = 6$

$\text{mphf}(\text{feb}) = \text{rank}(\text{phf}(\text{feb})) = \text{rank}(6) = 2$

Space to Represent the Function



Space to Represent the Functions (r = 3)

- PHF g: [0,m-1] $\rightarrow$ {0,1,2}
  - m = cn bits, c = 1.23 $\rightarrow$ **2.46** n bits
  - (log 3) cn bits, c = 1.23 $\rightarrow$ **1.95 n** bits (arith. coding)
  - Optimal: **0.89n** bits
- MPHF g: [0,m-1] $\rightarrow$ {0,1,2,3} (ranking info required)
  - 2m + εm = (2+ ε)cn bits
  - For c = 1.23 and ε = 0.125 $\rightarrow$ **2.62 n** bits
  - Optimal: **1.44n** bits.

Use of Acyclic Random Hypergraphs

- Sufficient condition to work
- Repeatedly selects h_0, h_1..., h_{r-1}
- For $r = 3$, $m = 1.23n$: Pr_a tends to 1
- Number of iterations is $1/Pr_a = 1$

Experimental Results

- Metrics:
  - Generation time
  - Storage space for the description
  - Evaluation time
- Collection:
  - 1.024 billions of URLs collected from the web
  - 64 bytes long on average
- Experiments
  - Commodity PC with a cache of 4 Mbytes
  - 1.86 GHz, 1 GB, Linux, 64 bits architecture

Generation Time of MPHFs (in Minutes)

n (millions)	32	128	512	1024
Sequential ECA	0.95 ± 0.02	5.1 ± 0.01	22.0 ± 0.13	46.2 ± 0.06

Related Algorithms

- Fox, Chen and Heath (1992) – FCH
- Majewski, Wormald, Havas and Czech (1996) – MWHC

All algorithms coded in the same framework

Generation Time

Algorithms	Generation Time (sec)
Internal (r = 3)	6.7 ± 0.01
External	6.3 ± 0.11
MWHC	7.18 ± 0.01
FCH	2,400.1 ± 711.6

3,541,615 URLs

Generation Time and Storage Space

Algorithms	Generation Time (sec)	Space (bits/key)
Internal (r = 3)	6.7 ± 0.01	2.6
External	6.3 ± 0.11	3.1
MWHC	7.18 ± 0.01	26.76
FCH	2,400.1 ± 711.6	4.2

3,541,615 URLs

Generation Time, Storage Space and Evaluation Time

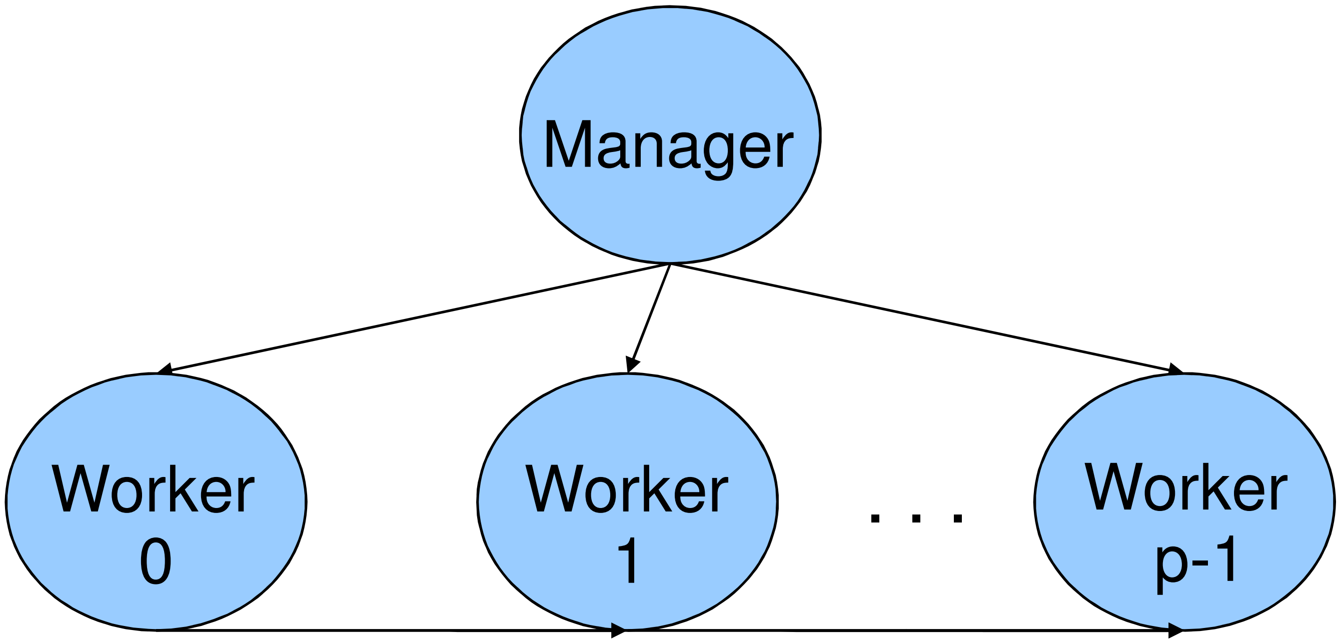
Algorithms	Generation Time (sec)	Space (bits/key)	Evaluation time (sec)
Internal (r = 3)	6.7 ± 0.01	2.6	2.1
External	6.3 ± 0.11	3.1	2.7
MWHC	7.18 ± 0.01	26.76	2.46
FCH	2,400.1 ± 711.6	4.2	1.6

3,541,615 URLs

Key length = 64 bytes

The Distributed External Memory Based Algorithm ...

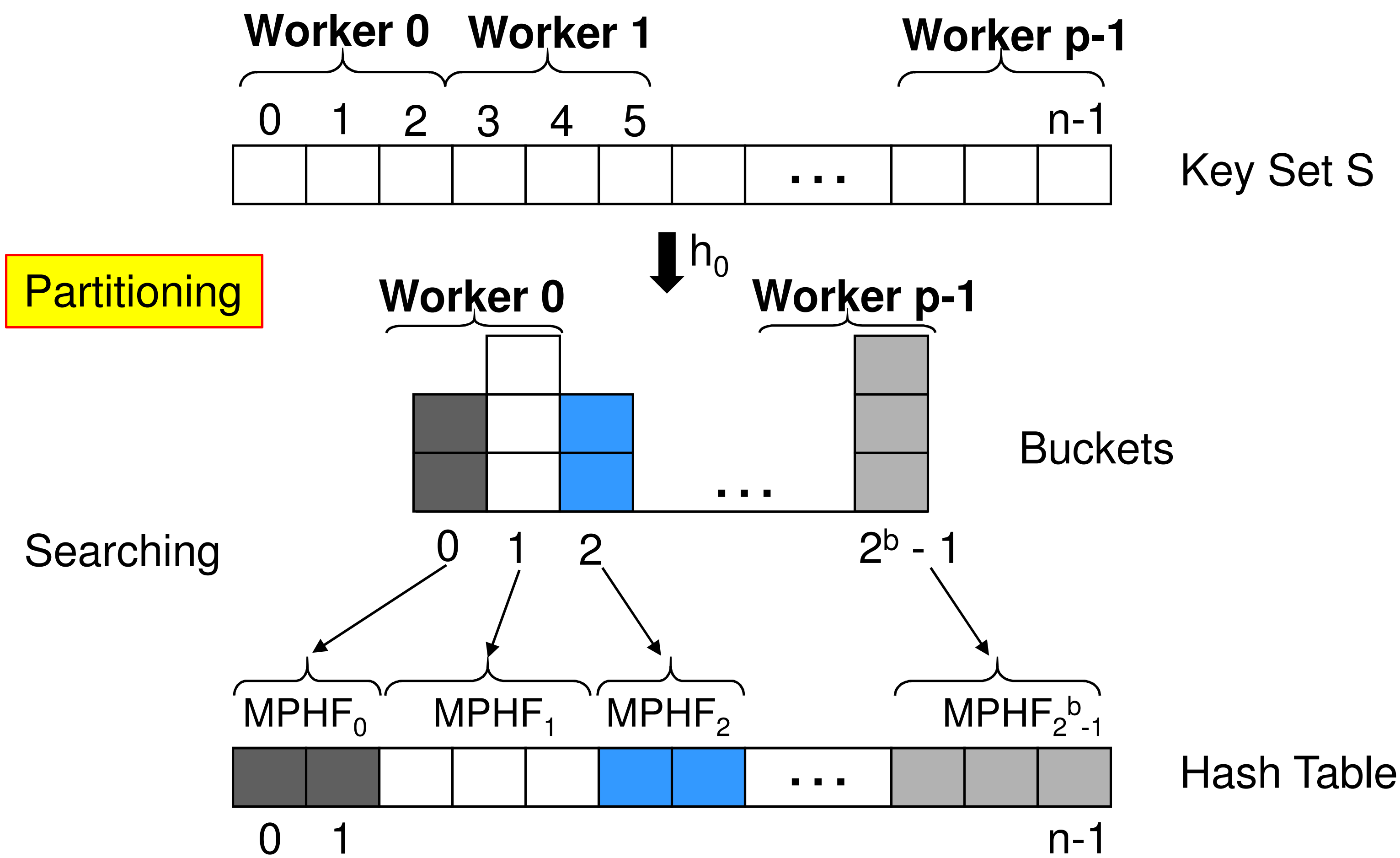
Distributed Construction of the MPHFs



Distributed Construction of MPHFs

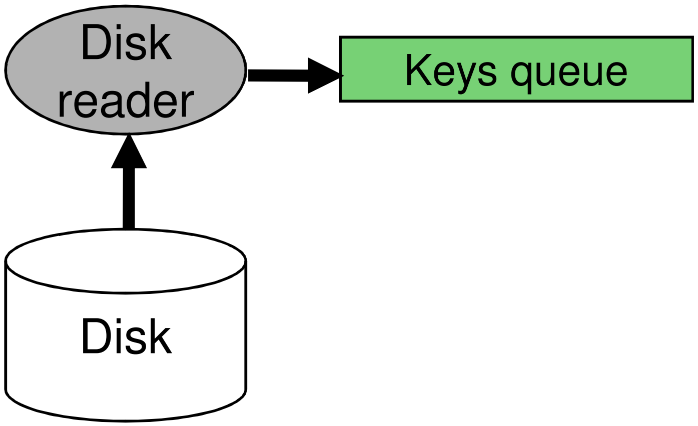
- Manager :
  - Assign tasks to workers
  - Determine global values during execution
  - Dump resulting MPHFs to disk
- Worker :
  - Has a partition of the keys on disk
  - Creates its buckets from the keys ($B_{pw} = N_b/p$)
    - Migrate data whenever necessary
  - Constructs a MPHF for each bucket

Sequential External Perfect Hashing Algorithm



$$MPHF(x) = MPHF_i(x) + offset[i];$$

Partitioning Step in Each Worker



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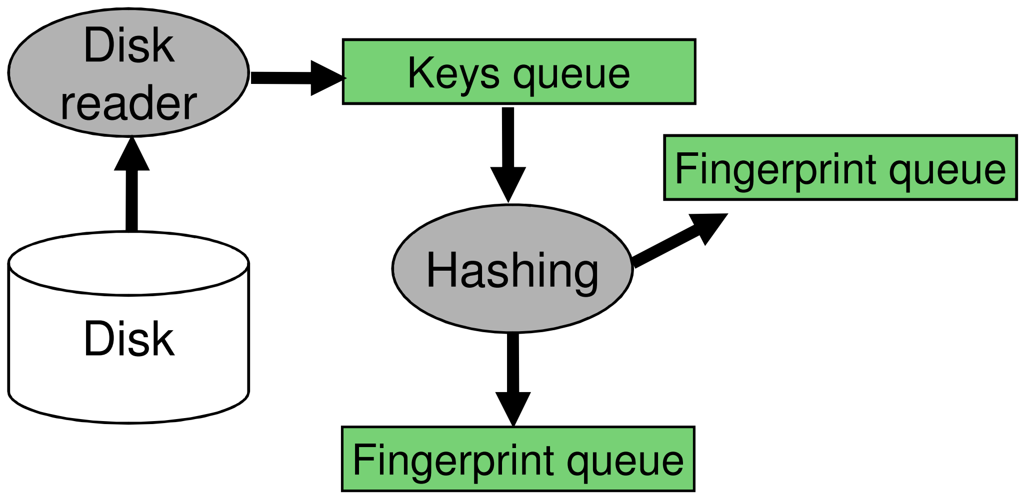

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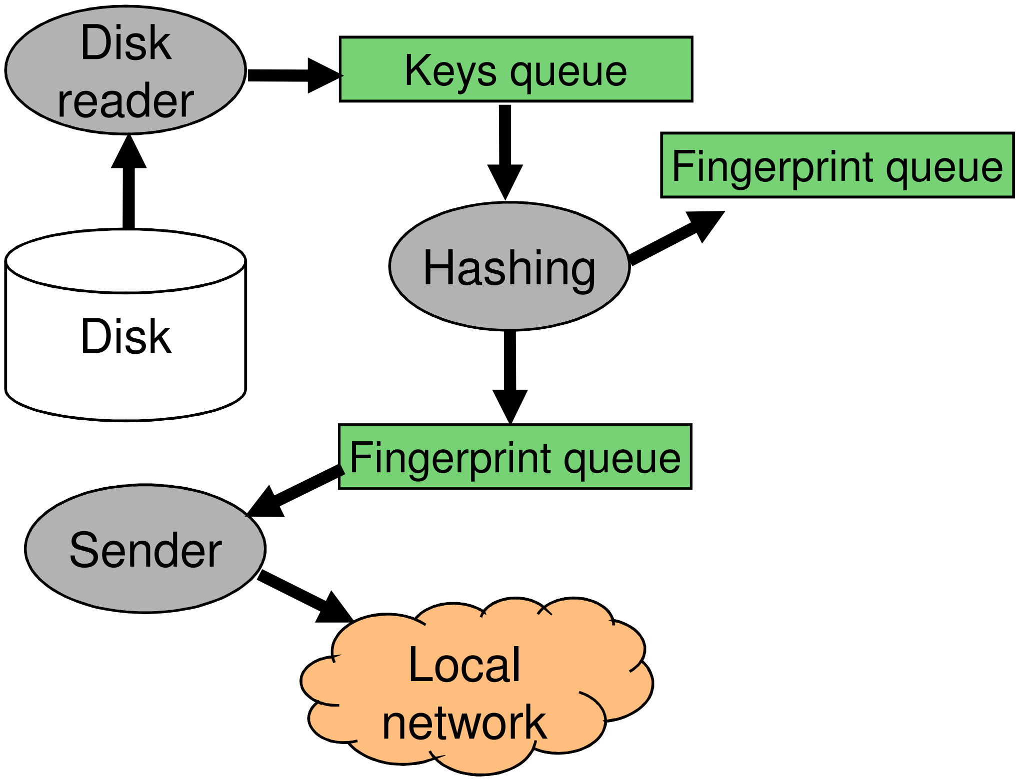

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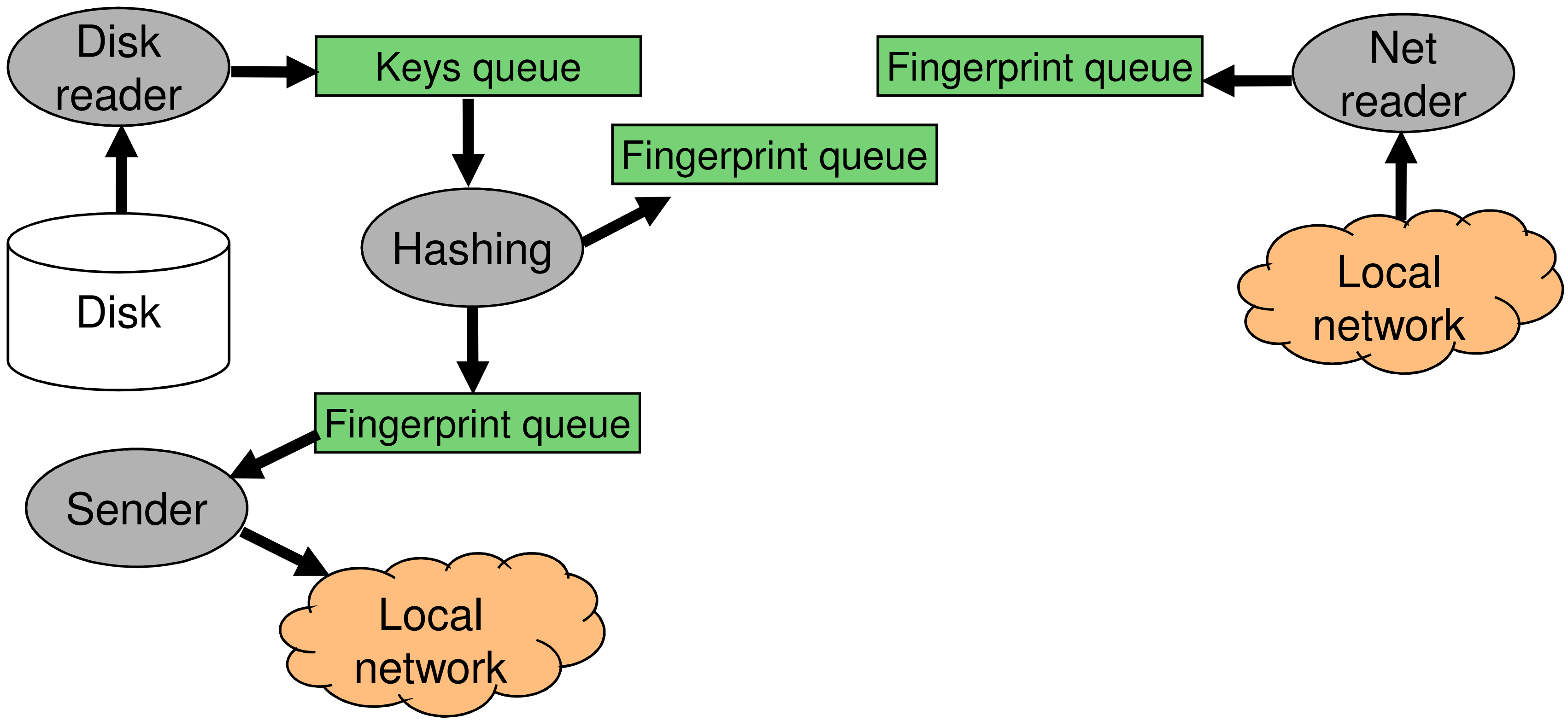

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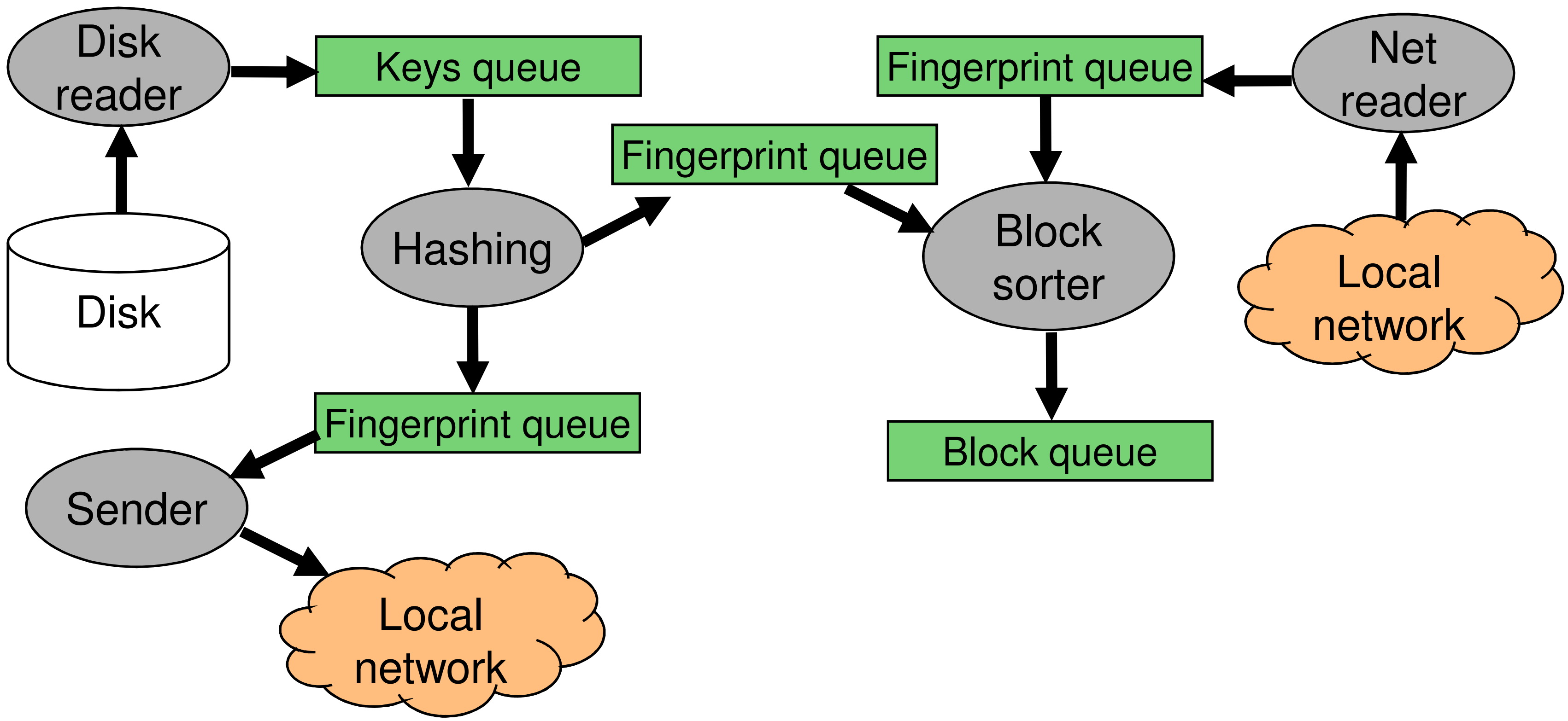
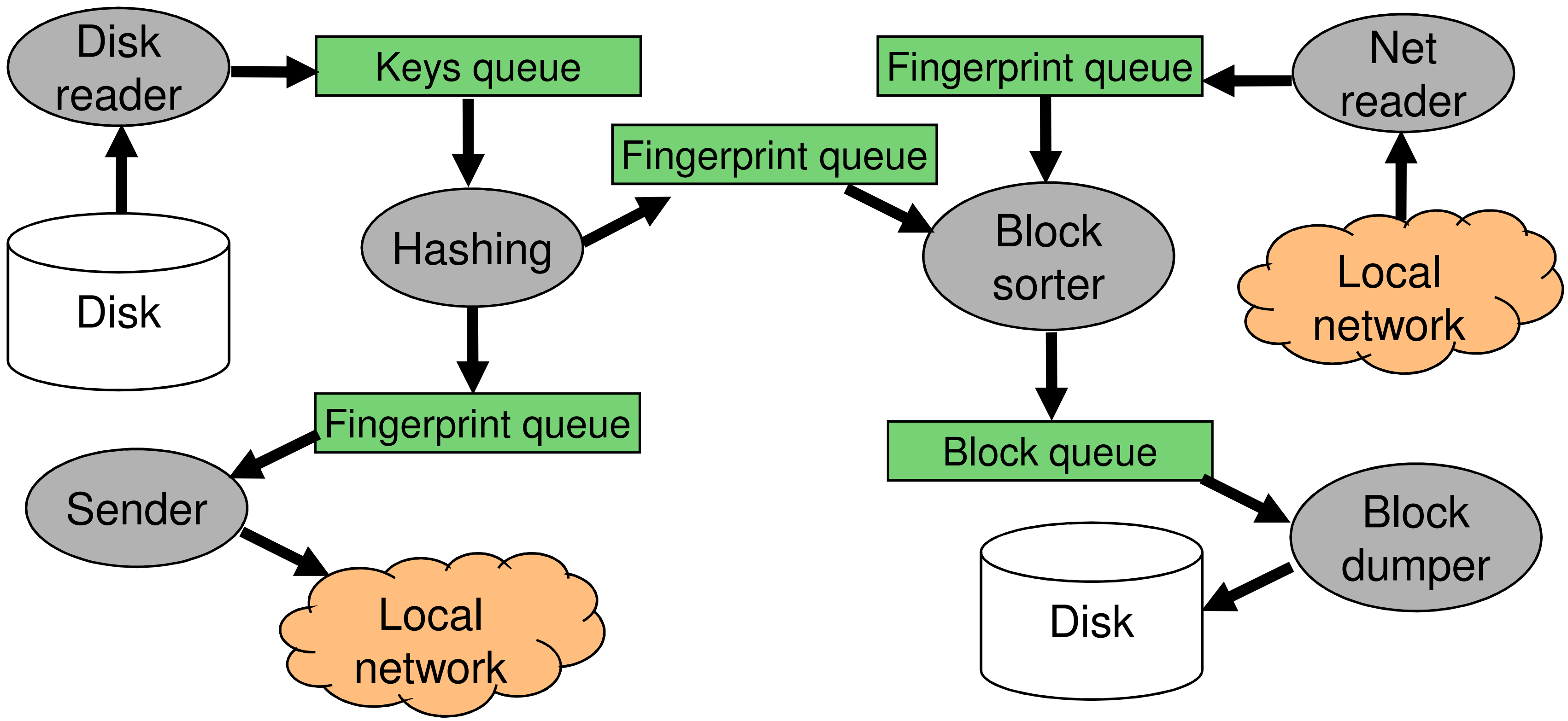
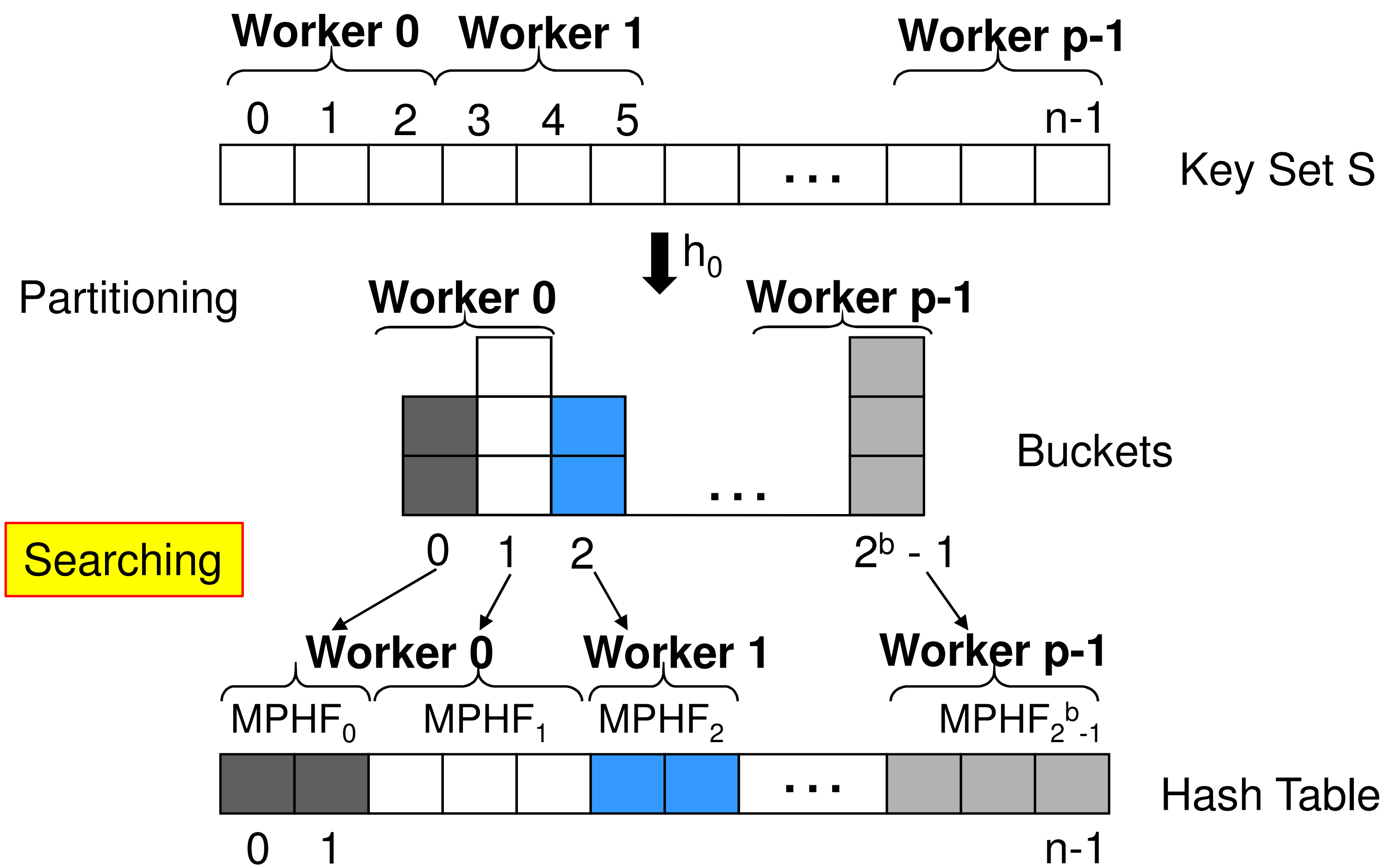


Partitioning Step in Each Worker

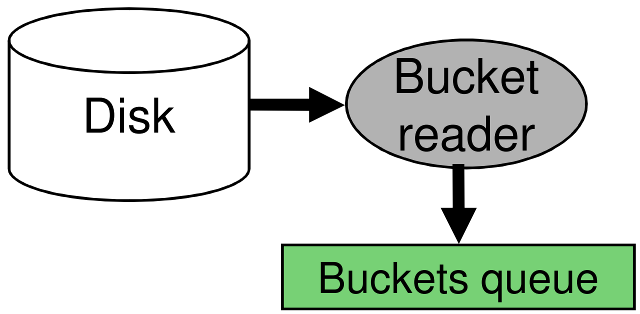


Sequential External Perfect Hashing Algorithm

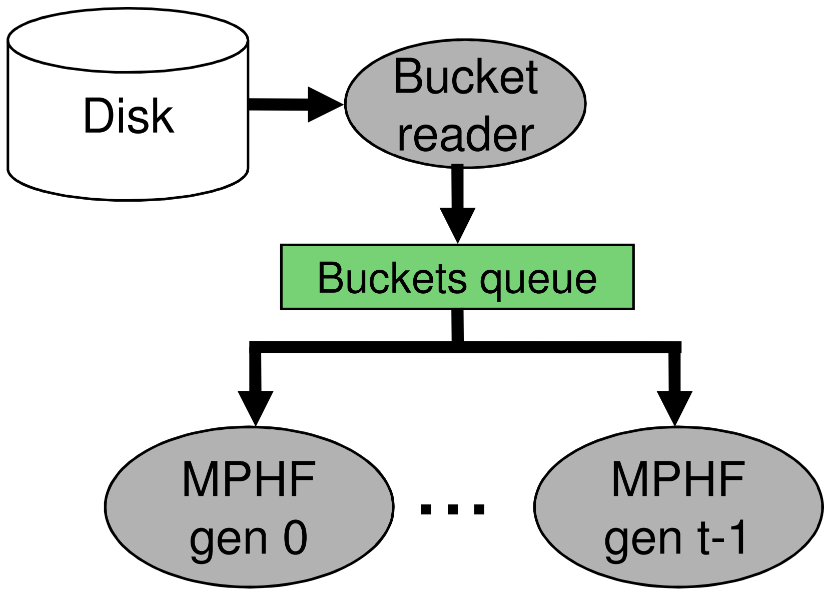


$MPHF(x) = MPHF_i(x) + offset[i];$

Searching Step in Each Worker



Searching Step in Each Worker



Description and Evaluation of a MPHF

- *Centralized* in one machine
- *Distributed* among the participating machines

Centralized Description and Evaluation of MPHFs

- End of the partitioning step:
  - Worker sends size of each bucket to manager
  - Manager calculates the *offset* array
- End of searching step (construction of MPHFs):
  - Worker sends *MPHFs* of its buckets to manager
  - Manager writes sequentially final *MPHF* to disk
- $MPHF(x) = MPHF_i(x) + offset[i]$

Distributed Description and Evaluation of MPHFs

- Description of MPHFs of a bucket
  - Stays in the bucket
- Evaluation of a MPHF
  - Locate the key inside the bucket:
    $MPHF_{partial}(k) = MPHF_i(k) + localoffset[i]$
  - Add this to the number of keys before worker *w:*
    $MPHF(k) = MPHF_{partial}(k) + globaloffset[w]$
  - A key stream is evaluated in parallel

Advantages of Distributed Evaluation of MPHFs

- No need to send the MPHFs of a bucket to manager
- They are written to disk in parallel by the workers
- Final function is stored in a distributed way
  - Size of the description of the MPHF grows linearly with the size of the input key

Communication Overhead

- On average, the number of keys τ sent through the net during the execution is:

$$\tau = \frac{n(p-1)}{p}$$

p	Keys sent by a worker to the net		
	Max (%)	Min (%)	τ (%)
4	75.008	74.994	75.000
10	90.009	89.991	90.000
14	92.864	92.849	92.857

Experimental Setup

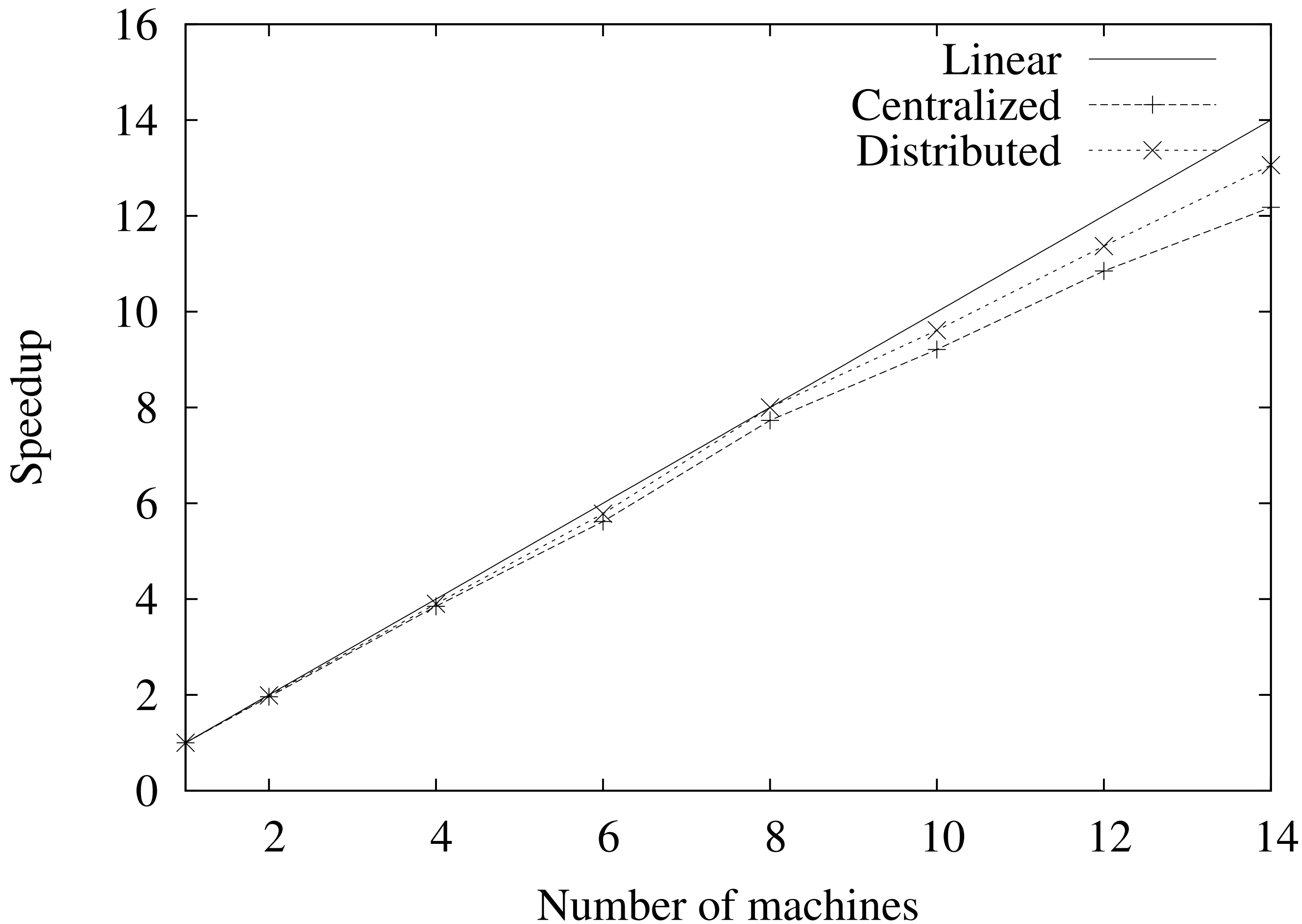
- Three collections

Collection	Avg. Key Size	*n* (billions)
URLs	64	1.024
Random Integers	16	1.024
	8	1.024

- Cluster with 14 equal 64 bits single core machines
  - 2 gigabytes of main memory
  - 2.13 gigahertz
  - Linux operating system version 2.6

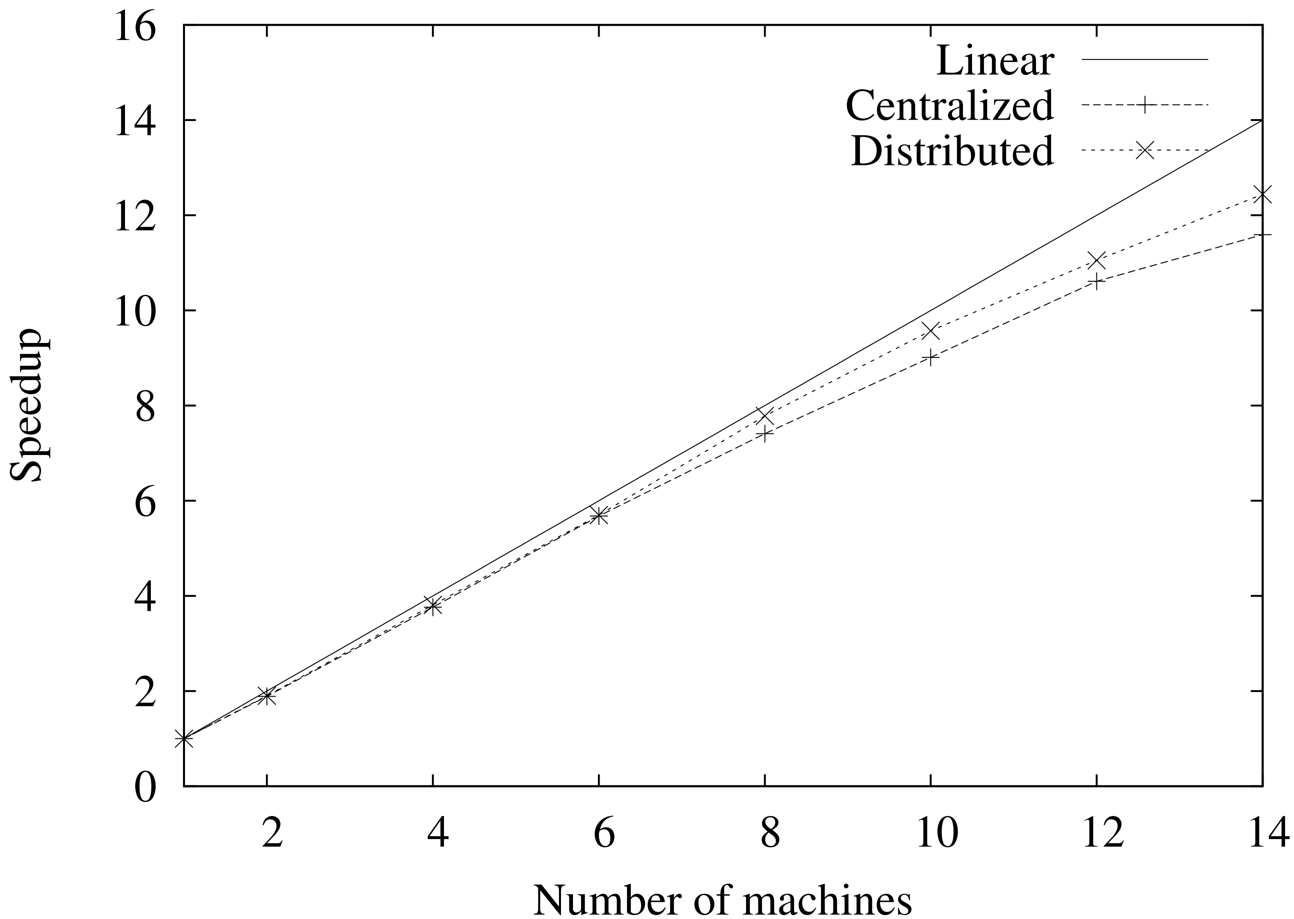
Speedup

- 64 bytes URLs



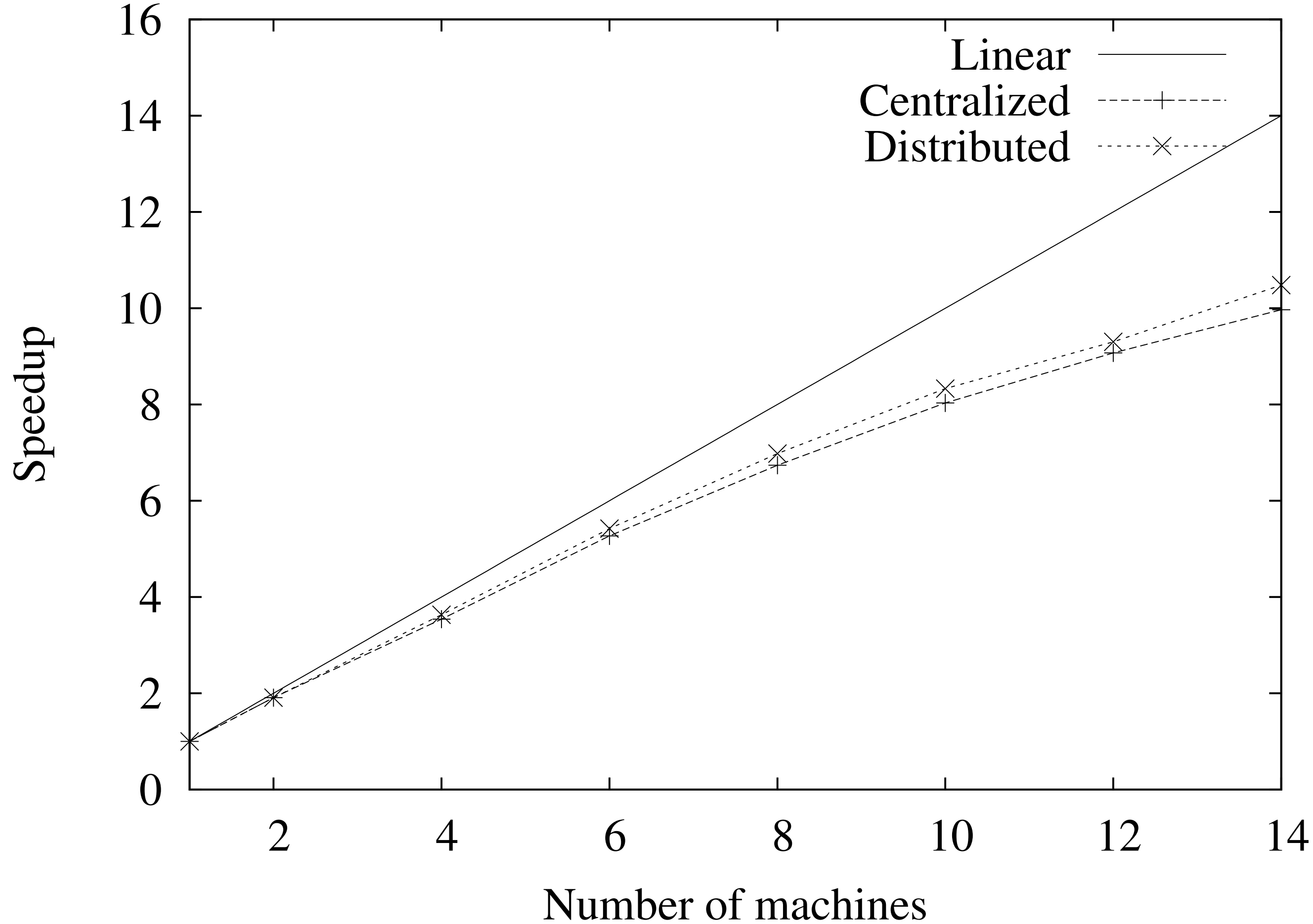
Speedup

- 16 bytes random integers



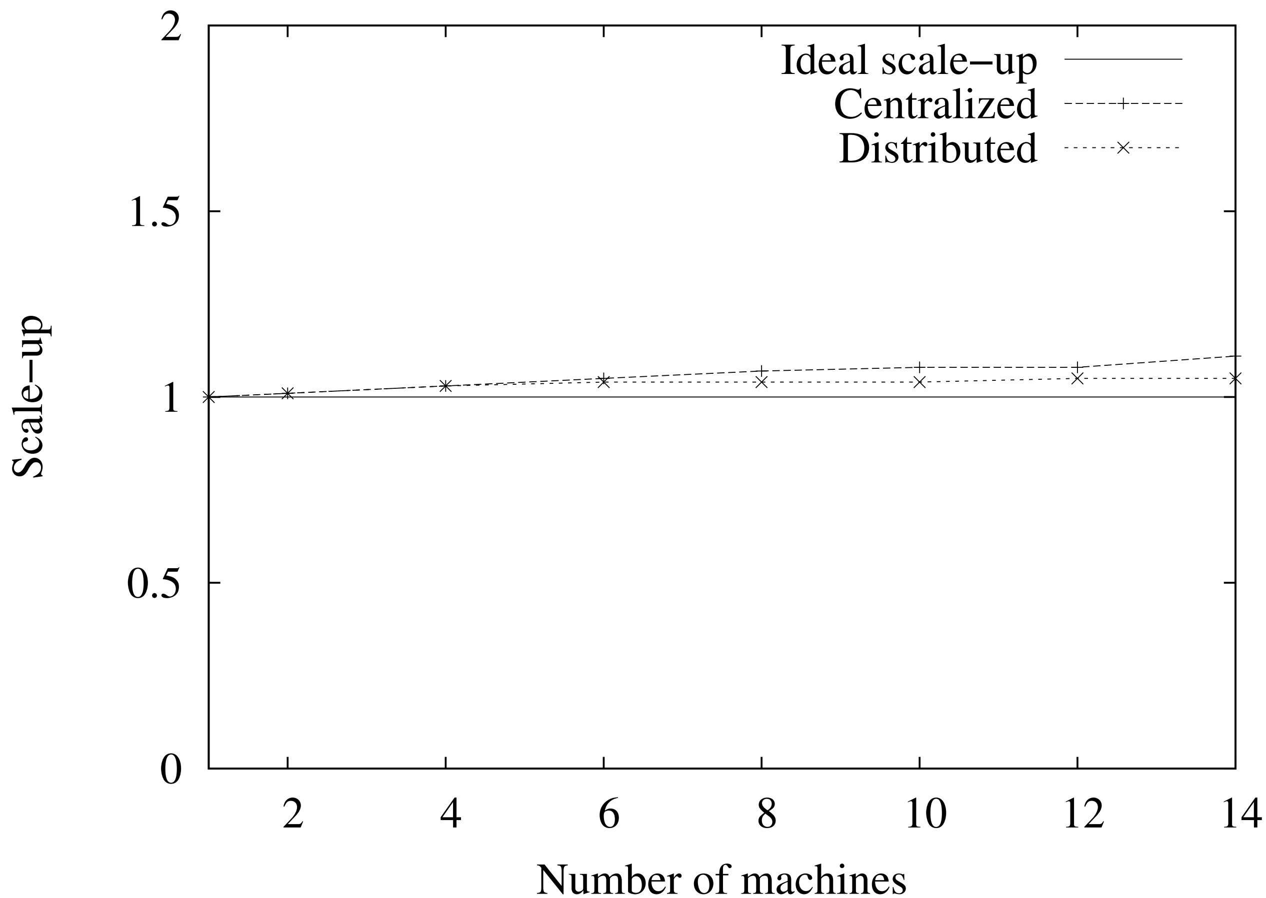
Speedup

- 8 bytes random integers



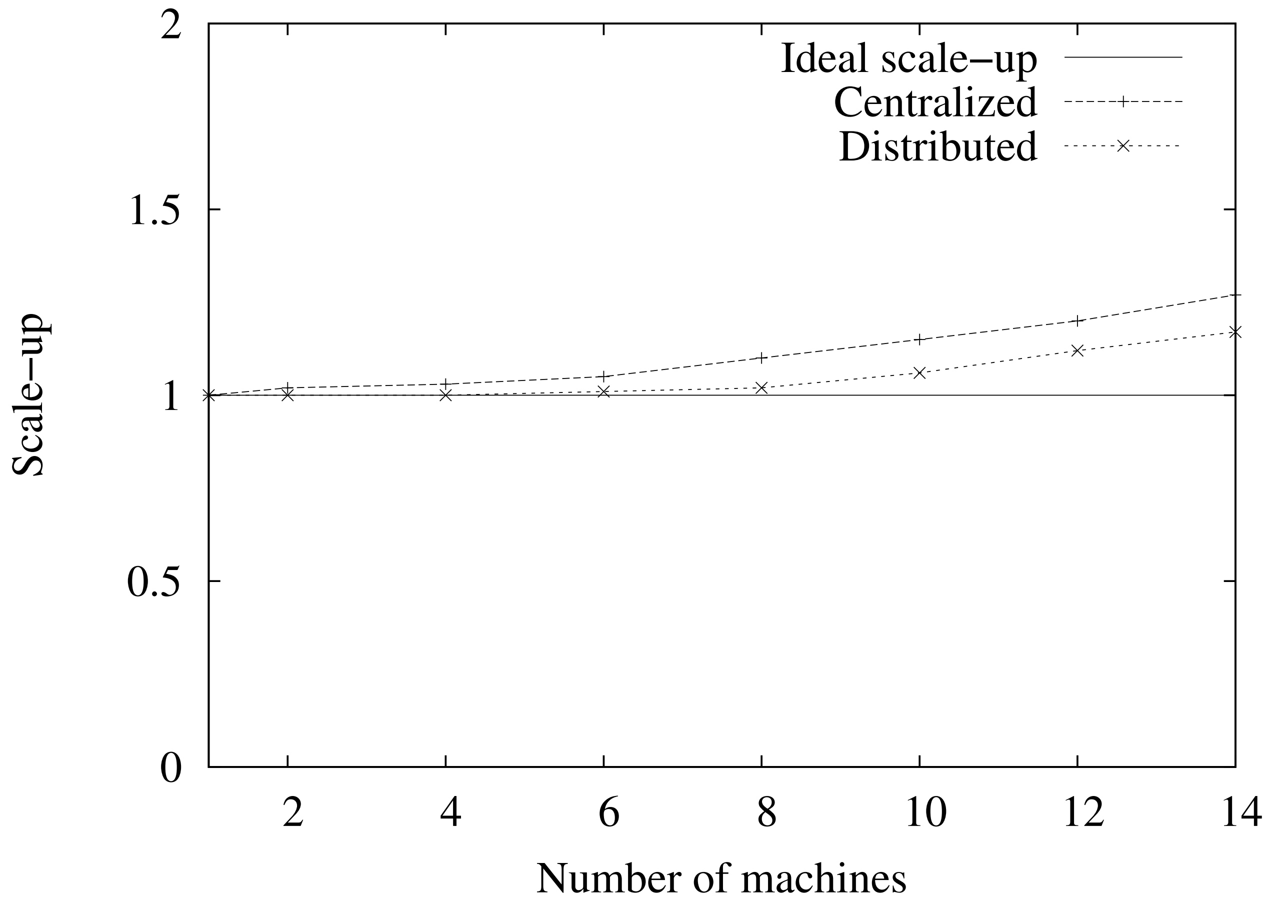
Scale-up

- 64 bytes URLs



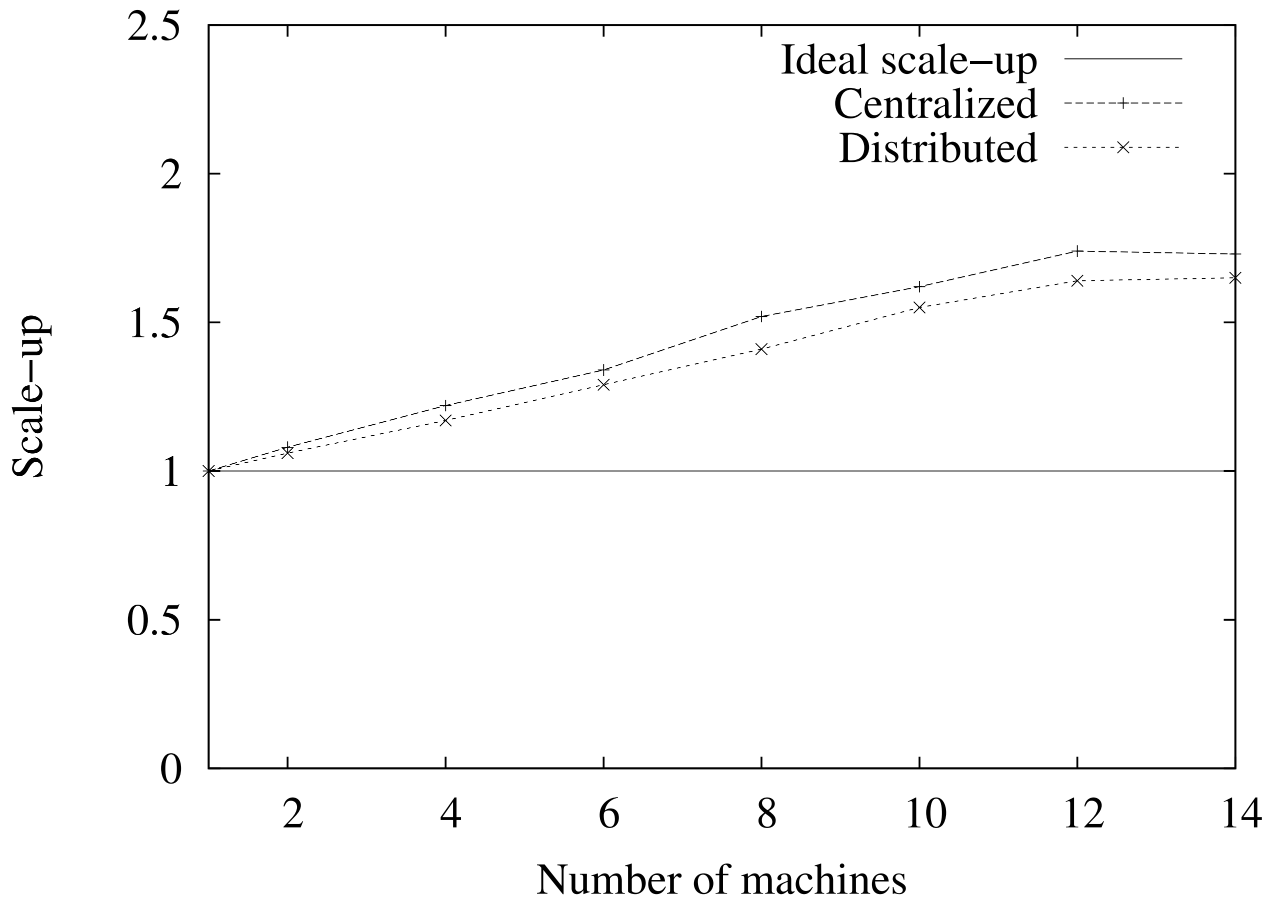
Scale-up

- 16 bytes random integers



Scale-up

- 8 bytes random integers



Scale-up

- 14.336 billion random integers
- 14 machines (each with 1.024 billion keys)

n (billions)	Random Integer Collections	Construction time		
		Seq.	Distrib.	U_p
14.336	16-byte	41.17	49.5	1.20
	8-byte	34.58	58.00	1.68

Load Balancing

- Execution time: fastest minus slowest (in minutes)

p	t_{fw}	t_{sw}	$t_{sw} - t_{fw}$
4	12.78	12.86	0.09
10	4.32	4.40	0.07
14	3.76	3.84	0.08

Distributed Evaluation

- 1.024 billion key stream taken at random (minutes)

Collection	Evaluation time (min)	
	Centr. Eval.	Distrib. Eval.
64-byte URLs	33.1	21.7
16-byte Integers	24.5	11.5
8-byte Integers	18.2	10.1

C Minimal Perfect Hashing Library

- Why to build a library?
  - Lack of similar libraries in the free software community
  - Test the applicability of our algorithm out there
- Feedbacks:
  - 2,243 downloads (until May 27th, 2008)
  - Incorporated by Debian
- Library address: http: //cmph.sourceforge.net

Conclusions

- Sequential and parallel perfect hashing algorithm
- Near space-optimal functions in linear time
- Function evaluation in time O(1)
- The algorithms are simpler and has much lower constant factors than existing theoretical results
- Outperforms the main practical general purpose algorithms found in the literature

Conclusions

- Construction time: 14 machines, 1 billion URLs
  - Sequential algorithm: 50 minutes
  - Parallel algorithm: 4 minutes
- Speedup > 90% for keys with more than 16 bytes
- Description and evaluation of MPHF:
  - Centralized
  - Distributed: fast evaluation for key streams