Model For Sharing Femto Access

Mariem Krichen
PRiSM, University of Versailles
45 avenue des Etats-Unis
78035 Versailles, France
mariem.krichen@prism.uvsq.fr

Johanne Cohen
PRiSM, University of Versailles
45 avenue des Etats-Unis
78035 Versailles, France
johanne.cohen@prism.uvsq.fr

Dominique Barth
PRiSM, University of Versailles
45 avenue des Etats-Unis
78035 Versailles, France
dominique.barth@prism.uvsq.fr

ABSTRACT

In wireless access network optimization, today's main challenges reside in traffic offload and in the improvement of both capacity and coverage networks. The mobile operators are interested in solving their localized coverage and capacity problems in areas where the macro network signal is not able to serve the demand for mobile data. Thus, the major issue for mobile operators is to find the best solution at reasonable expanses. The femto cell seems to be the answer to this problematic. In this work[1], we focus on the problem of sharing femto access between a same mobile operator's customers. A paradigm for bandwidth sharing management added to a TBAS model for exchanging connectivity is proposed for a fair sharing connectivity system ensuring QoS. This paper focuses on an economic model based on FON model and considers the sharing femto access problem as a problem divided into to 2 levels: a game restricted to service requesters customers (SRCs) and a second game restricted to service providers customers (SPCs). We consider that SRCs are static and have some similar and regular connection behavior. We also note that each SPC and each SRC have a software embedded respectively on its femto access, user equipment (UE) on which A_{Dist} algorithm is running to learn the best strategy increasing its gain using only local information.

We will try to answer the following questions for a game with N SRCs and P SPCs: how many connections are necessary for each SRC/SPC in order to learn the strategy maximizing its gain? Does exist an algorithm converging to a stable state? If yes, does this state a Nash Equilibrium?

Keywords-component: game theory, sharing femto access, TBAS, Nash Equilibrium, distributed learning algorithm, stable state.

1. INTRODUCTION

Today, one of the biggest issues for Mobile Operators is to provide acceptable indoor coverage for wireless networks. Among the several in-building solutions, the femto cell is the one which is gaining significant interest. A *femto cell* is a small cellular base station characterized by low transmission power, limited access to a closed user group designed for residential or small business use. But its expansive buying cost is not motivating to purchase it. This solution could help operators solve localized coverage problems and extend their network. Indeed, in some areas where the macro network signal is weak, a network of open femto cells access would significantly improve the voice quality and data connectivity. This would be feasible if access points owners accept to be part of a Club where each member is willing to open up its access point to other members. This idea of sharing part of its bandwidth started with FON[2], a club where members share their WiFi connection and inspired us to propose a Club where members share their femto accesses with bandwidth guarantees. A femto club member could share its 3G/LTE signal securely with other club members. Incentives for an owner of an access point to be member of such a club can be not only to share a part of the cost of the access point but also to make advertisement or to share some information through a specific social network associated to the club. These incentives would logically lead the club to manage by itself only such bandwidth exchange, but since this technology uses licensed spectrum, the only model that could for the moment be adopted for sharing femto access is the one where the mobile operator is also participating. This work presents the sharing femto access model. The specifications for designing this model include an Eco system sharing, a fair sharing and QoS guarantees. This work takes into account these specifications and presents a paradigm for bandwidth sharing management and a Token Based Accounting System for exchanging connectivity. Sharing femto access is a service proposed by the Mobile Operator to its clients. These customers are divided into *service providers customers (SPCs)* and *service requesters customers (SRCs)* : *SPCs* are the owners of femto cells accesses for which they have contracted with a *Mobile operator* denoted by MO. *SRCs* are customers using a mobile terminal in an area covered by some SPCs access points and requesting to use these access points. Note that a user can be both a SPC and

[1]This work is supported by the COMET project AWARE. http://www.ftw.at/news/project-start-for-aware-ftw

[2]FON is a for-profit company incorporated and registered in the U.K. FON was created in Madrid, Spain, by Martin Varsavsky, an Argentine/Spanish entrepreneur and founder of many companies in the last 20 years.

VALUETOOLS 2011, May 16-20, Paris, France

DOI 10.4108/icst.valuetools.2011.246071

a SRC. Dynamic femto spectrum sharing is a challenging problem for all the actors. Indeed the amount of requested bandwidth by SRCs as well as the amount of bandwidth shared by SPCs and pricing should be determined such that the utility of all the agents is maximized. Since the interests of all the actors could be antagonist, especially between many SRCs requesting a same SPC, we model our system as a game to determine equilibria of such situations.

Related works.

The Token-based Accounting for P2P-Systems is presented in [10]. The scheme uses tokens as proof of resource or service usage and tokens are issued using a decentralized mechanism. The problem of sharing bandwidth and pricing has been already addressed by Dusit Niyato et al [9], then modeled as a game. The challenging problem in this context is that bandwidth sharing requires a âĂIJpeacefulâĂİ co-existence of both primary and secondary users.

The potential games introduced by Rosenthal [15] are classical games having at least one pure Nash equilibrium. These games have a potential function such that each of its local optimums corresponds to a pure Nash equilibrium. This property has been used for congestion game in general (see [13] for a survey), with Resource Reuse in a wireless context (see [7]) and for a real-time spectrum sharing problem with QoS provisioning [17].

A decentralized learning algorithm of Nash equilibria in multi-person stochastic games with incomplete information has been presented by M.A.L. Thathachar et all. In the considered game, the distribution of the random payoff is unknown to the players and further none of the players know the strategies or the actual moves of other players. It is proved that all stable stationary points of the algorithm are Nash equilibrium for the game [14]. The study presented in this article will use this algorithm in the game restricted to SRCs where each SRC will learn the strategy maximizing its gain using only local information. We will check whether if the stationary point the algorithm converges to is a pure Nash equilibrium.

Our contribution..

Section 2 defines the specifications to be taken into account in the model of sharing femto access. The sharing femto access is defined in Section 3 as a problem divided into 2 levels: a Game restricted to SRCs and a Game restricted to SPCs. Section 4 studies equilibrium existence in the Game restricted to SRCs. Section 5 give details about the algorithm of sharing femto access with its 2 levels and gives some simulation results. Finally, Section 6 draws a general conclusion and gives some perspectives.

2. GENERAL MODEL FOR SHARING FEMTO ACCESS

In this section, we will describe the actors involved in the model presented in its second part.

2.1 Actors Description

The different actors interacting in this model use the same mobile resources and are of two types. First the owners of femtocells accesses for which they have contracted with a Mobile Operator (we talk about S*ervice Provider Customers* or SPC). Secondly customer using a mobile terminal in an area covered by some SPCs (we talk about *Service Requested Customers* or SRC). Note that a user can be both a SPC and a SRC. We consider here that SRC and SPC lead to a social network accepting to share/use femto access and should define their profiles. We now give some details on SPC and SRC. We note that the names for categories Club members were inspired from FON Model for sharing WiFi connection.

2.1.1 SPC Actor

A SPC is a customer of the MO possessing a femto access with indoor and possible outdoor coverage. The SPC proposes to share an amount of its bandwidth with SRCs for a price per bandwidth unit which depends on the type of connection (described later). Note that some SPCs share their access for economic reasons while some others share access for further eventual needs.

- A SPC belonging to the first category is a SPC which access point is well situated (near a restaurant, station or a bar) and its signal has an outdoor well accessibility. Sharing access will allow him to generate some gain. That would be a good motivation for SPCs to pay for femtocell due to its expansive cost. We denote by *Bill* the SPC belonging to this category. Usually, this category of SPCs share a big part of bandwidth to get more revenues. These revenues are shared with the Mobile Operator.

- A SPC belonging to the second category represents SPCs possessing femtocells for their own needs: this could be because the 3G indoor signal is very bad or all the householders have contract with the same mobile operator and want to have some discount from their mobile communications initiated at home. The SPCs belonging to this category are generally mobile SPCs having big QoS needs either at home or outdoor: they want to share part of their bandwidth at home because they would require some outdoor. We denote by *Linus* the SPC belonging to this category. Usually, this category of SPCs does not share a big part of bandwidth because they may need it for their proper applications. Regardless of their profiles, SPCs get free roaming at other femto cells access points.

The SPC is characterized by its *sensitivity to Gain* denoted by $\mu \in [0, 1]$ and its *sensitivity to its own connection QoS* denoted by $\Gamma \in [0, 1]$. These two parameters are dual: $\mu + \Gamma = 1$.

The *Gain sensitivity* parameter indicates its sensitivity degree to the price of the connection shared while the *QoS sensitivity* parameter indicates the SPC's tolerance degree towards preemption risk (defined later). When $\mu > 1/2$, we say that the SPC is sensitive to gain. Otherwise, the SPC is considered as sensitive to its access QoS.

2.1.2 SRC Actor

A SRC is a customer of the MO in need of good QoS at reasonable price while moving. In the following, we will define two categories of SRCs:

- The first category represents SRCs getting free roaming. To be part of this category, a SRC should be registered as a Bill or a Linus. In other words, this SRC should also be a SPC: a customer accepting to share part of its bandwidth (for free or against money) with the club members.
- The second category of SRCs represents the MO's customers not having femto access at home. This could be due to its expansive cost or because these SRCs have a good 3G indoor signal. However, they need QoS outdoor at reasonable price. We will denote by Alien a SRC belonging to this category of SRCs. In this case, Aliens pay to get roaming; the Mo will receive all the gain if the SPC is a Linus and will share the revenues if the SPC is a Bill.

A SRC is characterized by a *QoS sensitivity* parameter α and a *price sensitivity* parameter β. The SRC wants to use a connection that would be provided by one SPC. The *QoS sensitivity* parameter indicates the SRC's tolerance degree towards the QoS degradation of the connection while the *price sensitivity* parameter indicates the SRC's tolerance degree towards the cost of the connection. These two parameters are dual: $\alpha + \beta = 1$.

The more one SRC is sensitive to QoS, the less its tolerance degree towards the QoS degradation will be. The more one SRC is sensitive to price, the less he is able to pay for the connection. When $\alpha > 1/2$, the SRC is considered as sensitive to QoS. Otherwise, the SRC is sensitive to price.

2.2 Specifications

A model with an Eco-System sharing.

The proposed model should ensure that no profit cases between Club members would happen. Two factors could lead to a such kind of situations: femto access point outdoor accessibility and the amount of bandwidth shared by the SPCs.

An example of a profit case is when a SPC uses while roaming amounts of shared bandwidth much lesser than what he shares with SRCs. Such a case could happen when a SPC has a femto access point with well outdoor accessibility (near a restaurant ,a station or a bar). But, each time this SPC needs to get roaming, he only finds femto accesses with very small parts of shared bandwidth. The proposed model should be designed in a way to incent SPCs to let their femto cell easily accessible from the street: SPCs should be aware that they will have the right to consume other SPCs bandwidth as much as they consume its bandwidth. So, the more a SPC shares bandwidth, the more he would be able to receive further services in its future moving. Thus, bandwidth shared should be enough to cover its further needs in term of QoS.

A fair model for sharing bandwidth.

The proposed model should ensure some fairness between SRCs.

- The first form of fairness is Billing fairness. SRCs should pay just for the amount of bandwidth used.
- The second form of fairness is access fairness. Indeed, all the SRCs should have initially the same right of allocated bandwidth for their moves. Then, depending on the amount of bandwidth shared and their accessibility, they will have more or less access rights than other SRCs.

Type of Tokens	Sub-type of Tokens	Abbrv	Token Provider	Token Receiver
Own Tokens	Free	FOT	MO	Bill/Linus
	Paying	POT	MO	Alien
Foreign Tokens	Free	FFT	Bill/Linus	Bill/Linus
	Paying	PFT	Alien	Bill/Linus

Table 1: Different types of tokens

Since both of the two categories of SPCs (Bills and Aliens) share access, so both will have free roaming. The only difference is that the first category(Bills) gets some revenues for that while the second category (Aliens) do not. A solution should be proposed to ensure some kind of fairness between the two categories of SPCs.

A paradigm for bandwidth sharing management with QoS guarantees..

On one hand, the paradigm for bandwidth sharing should guarantee a certain access speed and a data transmission speed. On another hand, some connections (that would be more expansive than others) should be guaranteed to SRCs: this means that these connections will never be canceled by their owners.

2.3 Proposed Model

The purpose here is to propose a model of sharing access resources between SPCs and SRCs giving QoS guarantees and economic added value. This model is a distributed mechanism mainly managed by the members of the club (SRCs and SPCs) and not by the mobile operator. The operator only provides the network services supporting the mechanism. As it is studied in some social networks, security aspects are mainly insured by the identification of all the members of the club.

TBAS model for exchanging connectivity.

Token Based Accounting System (TBAS) [10] for a peer to peer network is a model of exchanging services between peers that avoids profit cases. This model involves two actors: a service requestor and a service provider.

Each service requestor (or SRC) has its own tokens provided by a central entity (called the banker) and that he may spend for services provided. In the case of sharing femto access, the banker could be the MO. Whenever a service requestor (SRC) needs a connection, that could be provided by a service provider (SPC), he sends some of its own tokens to the Service provider. The service provider will receive these tokens as foreign ones that, for cryptographic reasons, could not be used for its own needs (as they were its own tokens). These foreign tokens will be sent to the banker to exchange them against new own ones or against money.

Figure 1 highlights the TBAS process in Peer to Peer (P2P) network and its application to sharing femto access model. 4 types of tokens are considered and described in Table 1.

With this model, we notice that:

- Aliens pay exactly for what they consume in terms of

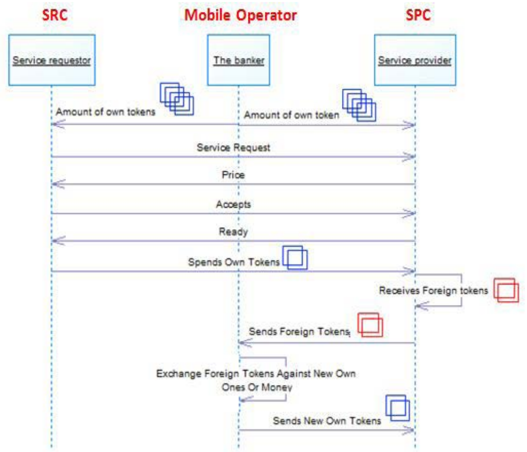

Figure 1: Application of TBAS in P2P network to sharing femto access model

bandwidth.

- Linuses exchange all the foreign tokens against new own ones. So the more they give connection to other members, the more they are allowed to use other members connections.
- Bills exchange only the free foreign tokens (received by by other Bills/Linuses) against new own ones. The paying foreign tokens (received by Aliens) are exchanged against money. So, if a Bill is visited only by Aliens, he will not be allowed to use more than the amount of bandwidth corresponding to the own tokens provided by the banker initially.
- Linuses who share connection for free to support community spirit will have more free roaming than Bills who receive some gain instead of that. Some fairness could thus be achieved between Linuses and Aliens.
- No profit cases could be possible since the bandwidth shared is controlled thanks to tokens.

To summarize, TBAS applied to the problem of sharing femto access is a model representing many advantages among which profit case avoidance, unfair situations avoidance, better network coverage and capacity for the MO and thus more generated revenues.

A paradigm for bandwidth sharing management with QoS guarantees.

the following paradigm proposes a solution to QoS issues. Our work considers several SPCs and several SRCs. We will denote by X a given SPC. We assume that the SPC's resource reserved for sharing is an amount of bandwidth denoted by $B_S(X)$. Then, we will consider that each SRC Y requests connection from X. Actually, the bandwidth $B_S(X)$ of SPC X is divided into two parts:

$$B_S(X) = B_{S_G}(X) + B_{S_Y}(X)$$

- $B_{S_G}(X)$ is the part of bandwidth in which SRCs communications can never be preempted. It is kind of a guaranteed QoS allocated to SRCs.

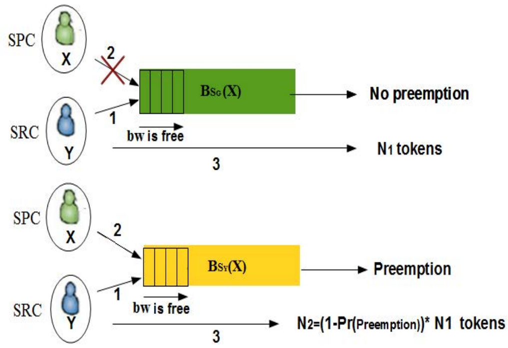

Figure 2: Bandwidth sharing process and billing process

- $B_{S_Y}(X)$ is the part of bandwidth in which SRCs communicatiosn can be preempted. This preemption is due to the fact that the SPC has priority on this part of bandwidth. Thus, a communication allocated in $B_{S_Y}(X)$ is characterized by a risk of preemption.

Figure 2 introduces the sharing bandwidth process and the billing process in both cases of a *Green* and a *Yellow* connection allocation: let's consider a SRC Y who needs an amount of bandwidth equal to bw. bw will be allocated to Y only if it is free. If X will need bw, he will not be able to use it if the connection he allocated to Y is green. However, he will be able to preempt the connection allocated to Y if it is a yellow one.

Token Based Accounting Protocol is used to exchange services between SRCs and SPCs against tokens (representing money). The billing process depends on whether the SRC has used a green connection or a yellow one.

In the case of a Green connection, the SRC will spend N_1 tokens corresponding to the used bandwidth bw. In the case of a Yellow connection, the SRC will spend N_2 ($N_2 < N_1$) tokens if the connection succeed. Indeed, the yellow connection is cheaper than the green one due to the risk of preemption. If the yellow connection given to the SRC has failed, this connection will be free.

The paradigm for bandwidth sharing management added to the TBAS model for exchanging connectivity constitutes a general model for a fair sharing connectivity ensuring QoS.

Interaction between SPCs actors and SRCs actors.

In general context, SRCs will request for some amount of bandwidth from SPCs depending of their profiles. Each SPC will treat the SRCs requests for a fixed bandwidth split.

1. For SRCs, the utility depends on their requests, the other SRCs requests as well as the SPC's decision (bandwidth allocated and type of connection) for a fixed SPC's bandwidth split. So, the SRC's utility depends mainly on its competition with other SRCs to receive some bandwidth from the same SPC.

2. For SPCs, the utility depends on their bandwidth split for fixed SRCs requests.

3. GAME PRESENTATION

Sharing femto access could be seen as a problem divided into two levels: a first level modeled as a game restricted to SRCs and a second level modeled as a game restricted to SPCs. The first level considers fixed SPCs strategies while the second one considers fixed SRCs strategies. This could be motivated by the fact that learning methods are reliable only when the surrounding environment is invariant.

Game restricted to SRCs.

The game restricted to SRCs where N SRCs are the players is the first level of sharing femto access problem. It considers the competition between SRCs and assumes that all SPCs bandwidth splits are fixed. We do not consider the mobility of SRCs. However, we assume that SRCs have some regular and similar connection behavior: each SRC requests some SPCs connections in nearly same time slots with almost invariant needs in terms of QoS. This means that requesting the SPCs femto access becomes almost routine for SRCs. This could be seen as repeated games.

Along requested connections, each SRC will learn, thanks to an algorithm running in a software embedded in its user equipment, the best strategy to be played to maximize its gain.

The game restricted to SRCs is defined as follows: given fixed SPCs bandwidth splits (into green part and yellow part), what would be the best strategy to be played by SRCs in order to have a stable situation where the strategy of each SRC player is optimal for him considering the other SRCs strategies. This situation corresponds to a pure Nash equilibrium in game theory. Recall that a *pure Nash equilibrium* of a game is a situation where, for each player, there is no unilateral strategy deviation that increases its utility [12].

Game restricted to SPCs.

The game restricted to SPCs where P SPCs are the players is the second level of sharing femto access problem. It considers the competition between SPCs and assumes that all SRCs bandwidth requests are fixed.

Along allocated connections, each SPC will learn, thanks to an algorithm in its femto access, the best strategy to be played to maximize its gain.

The game restricted to SPCs is defined as follows: given fixed SRCs bandwidth requests, what would be the best strategy to be played by SRCs in order to have a stable situation where the strategy of each SPC player is optimal for him considering the other SPCs strategies.

3.1 Game restricted to SRCs

As mentioned in the previous section, the game restricted to SRCs assumes that SPCs bandwidth splits are fixed. Let's consider SPC_j a fixed SPC. Let B_S^j be the total bandwidth SPC_j is agree to share and let Ψ_S^j be the proportion of $B_{S_G}^j$ regarding B_S^j.

3.1.1 SRCs QoS needs

Each SRC's QoS needs depend on the type of application he requests. Requesting for femto access is equivalent to request an amount of bandwidth. Fixing this amount of bandwidth depends on the following parameters: the type of application (real time, elastic), the QoS parameter that the applications requires (delay, time transfer file, ...), the type of connection (Green or Yellow) and the SRC's profile (QoS sensitive SRC or price sensitive SRC).

Our work takes into account the File Transfer Application. QoS is defined as the time transfer file that we will denote by t. The SRC's QoS satisfaction is related to t. For each SRC, the time transfer file should be between T_1 and T_2 and is defined as follows:

- Case $t = T_1$: BW_{Max} corresponds to the required bandwidth to download a file in $t = T_1$. If a SPC provides an amount of bandwidth equal to BW_{Max}, then the SRC's QoS satisfaction is at the top.
- Case $t = T_2$: BW_{Min} corresponds to the required bandwidth to download a file in $t = T_2$. If a SPC provides an amount of bandwidth equal to BW_{Min}, then the SRC's QoS satisfaction is minimal.

Each SRC will request for a minimum amount of bandwidth and a maximum amount of bandwidth in Green and Yellow depending on its profile and on its user equipment Signal-Strength towards the SPC's femto cell.

All the SRCs requests can not be accepted. In fact, since SPCs bandwidths are limited and since several SRCs could request for the same SPC's connection at the same time, one possible response that a SRC could receive is a deny one. Requesting for a Minimum and a Maximum amount of bandwidth will decrease the chances to receive such a response. Besides, requesting a bandwidth interval generalizes the fact of requesting a fixed amount of bandwidth. In this way, a SRC could receive an amount of bandwidth which may be different from its optimal request but would avoid him to have no payoff.

The minimum and the maximum amount of bandwidth are fixed depending on the SRC's profile. Note that the parameters caracterizing a SRC's profile are real in $[0, 1]$. We aim at translating these parameters into intervals of bandwidth requests which are actually integers. So, we introduce a parameter ε representing the discretization of the bandwidth resquested. Let S_{SRC_i} be the set of possible strategies of SRC_i. In the following, we focus on the request of a SRC denoted by SRC_i according to its profile.

1. We consider the case where SRC_i has its QoS sensitivity parameter α_i greater than 1/2. SRC_i fixes a revenue threshold under which he denies any proposed connection (high QoS degradation). This threshold denoted by Rev_Th_i corresponds to a minimum amount of bandwidth to be requested. This parameter depends on the QoS sensitivity α_i of the SRC. $Rev\_Th_i = \alpha_i - \kappa$ where $\kappa \in [0, 1]$ is the allowed variation from the QoS degradation tolerance fixed according to the SRC's profile (more specifically α_i).

   $$S_{SRC_i} = \{Rev\_Th_i, Rev\_Th_i + \varepsilon, Rev\_Th_i + 2\varepsilon, \dots, 1\}.$$

2. We consider the case where SRC_i has its QoS sensitivity parameter α_i less than 1/2. This implies that its price sensitivity parameter β_i is greater than

1/2. SRC_i fixes a cost threshold denoted by $Cost\_Th_i$ above which he denies any proposed connection (the cost is beyond what he is able to pay). This threshold corresponds to a maximum amount of bandwidth to be requested. This parameter depends on the QoS sensitivity of the SRC and is defined as follows: $Cost\_Th_i = \alpha_i + \kappa$.

$$S_{SRC_i} = \{Cost\_Th_i, Cost\_Th_i - \varepsilon, Cost\_Th_i - 2\varepsilon, \ldots, 0\}.$$

The definition of S_{SRC_i} presented above takes only into account intrinsic information. In fact, the strategies are fixed following the parameters characterizing the SRC's profile (α, β). Now, to formalize a request that could be more easily interpreted by the SPC, for each element s_i of S_{SRC_i}, we define an interval of bandwidth to be requested from the SPCs. Since two types of connection are proposed by SPCs, we will propose the two following solutions for bandwidth request:

1. Each SRC sends to all the SPCs the same interval of bandwidth to be requested corresponding only to Green connection request. Indeed, since no preemption exists for this type of connection, the interval of Green bandwidth requested does not depend on SPCs. Once a SPC receives the interval of green bandwidth requested by a given SRC, he will translate this interval of bandwith into an interval of bandwidth that the SRC needs in Yellow, using the probability of preemption of its yellow connection. On one hand, the advantage of this solution is that the bandwidth request is formulated once and sent to all the SPCs. On another hand, the drawback of this solution is that the SPC could cheat while computing the interval of the amount of bandwidth needed by the SRC in Yellow and thus propose an amount of bandwidth which in reality does not correspond with SRC's needs. In this solution, for each element s_i in S_{SRC_i}, we define a single interval of bandwidth to be requested to all SPCs. This represents a couple of integers :$(g_i = [m_i^G, M_i^G])$.

2. Each SRC requests from each SPC to send him the information of the risk of preemption taken in the case of yellow allocated connection. In this solution, the SRC computes by himself the interval of bandwidth to be requested in Yellow which differs from one SPC to another following the preemption risk characterizing its yellow connection. We denote by SPC_j a given SPC. Let δ^j be the probability of preemption of its yellow connection. On one hand, this solution is more reliable than the previous one since SRCs are sure not to receive an amount of bandwidth which is not in the interval of bandwidth needed. The drawback of this solution is that it makes the bandwidth request process more complicated. In fact, this solution supposes that SRCs have the knowledge of some extrinsic information. Besides, the SRC's bandwidth request is specific to each SPC regarding its yellow connection probability of preemption. Another drawback of this solution is that SPCs could lie about the information sent concerning the probability of preemption to incent SRCs to request more bandwidth in yellow. In this solution, for each element s_i in S_{SRC_i}, we define two intervals of bandwidth to be requested for each SPC. This represents a set of couple of couple integers:

$$\{(g_i^1 = [m_i^{G_1}, M_i^{G_1}], y_i^1 = [m_i^{Y_1}, M_i^{Y_1}]);$$
$$(g_i^2 = [m_i^{G_2}, M_i^{G_2}], y_i^2 = [m_i^{Y_2}, M_i^{Y_2}]); \ldots;$$
$$(g_i^P = [m_i^{G_P}, M_i^{G_P}], y_i^P = [m_i^{Y_P}, M_i^{Y_P}])\}$$

The parameters $m_i^{X_j}, M_i^{X_j}$ represent respectively the minimum and the maximum amount of bandwidth to be requested from the SPC_j in X connection where $X \in \{G, Y\}$. They are defined as follows :

1. Case where SRC_i has its QoS sensitivity parameter α_i greater than 1/2:
   (a) $m_i^{G_j} = \frac{BW_{max}}{s_i}$ and $m_i^{Y_j} = \frac{BW_{max}}{s_i} \times (1 - \delta^j)$
   (b) $M_i^{G_j} = BW_{max}$ and $M_i^{Y_j} = BW_{max}$

2. Case where SRC_i has its QoS sensitivity parameter α_i less than 1/2.
   (a) $M_i^{G_j} = \frac{BW_{max}}{s_i}$ and $M_i^{Y_j} = \frac{BW_{max}}{s_i} \times (1 - \delta^j)$
   (b) $m_i^{G_j} = BW_{min}$ and $m_i^{Y_j} = BW_{min}$

3.1.2 SPCs bandwidth allocation

Each SRC sends a bandwidth request to SPCs using either the first solution or the second solution presented in the previous subsection. Once all the SRCs requests received, each SPC decides the way its bandwidth is allocated to SRCs. The request of each SRC_i is represented by one element in S_{SRC_i}. According to a set Π of SRCs requets $\Pi =< s_1, s_2, \ldots, s_N >$ where s_i corresponds to the request of SRC_i, for any i, $1 \leq i \leq N$, each SPC_j gives an answer to each SRC_i represented by a triple (G_i^j, Y_i^j, bw_i^j) defined as follows:

- bw_i^j represents the amount of bandwidth proposed by SPC_j to SRC_i.
- $G_i^j = 1$ (resp. $Y_i^j = 1$) means that SRC_i has received a Green (resp. Yellow) connection proposition from SPC_j. Note that the case where $G_i^j = 1$ and $Y_i^j = 1$ is not possible.
- If $G_i^j = 0$ and if $Y_i^j = 0$, then SRC_i has received no connection proposition from SPC_j.

Let $config^j(\Pi)$ be the set of all answers (one answer per SRC) of SPC_j to Π. In other words,

$$config^j(\Pi) =< (G_1^j, Y_1^j, bw_1^j), (G_2^j, Y_2^j, bw_2^j), \ldots, (G_N^j, Y_N^j, bw_N^j) >$$

The answers respect the two following properties.

1. SPC_j gives SRC_i an amount of bandwidth equal to bw_i^j where bw_i^j is in the interval requested. More formally, if $(G_i^j = 1)$ then $bw_i^j \in g_i^j$ or if $(Y_i^j = 1)$ then $bw_i^j \in y_i^j$.

2. SPC_j provides bandwidth to SRCs in the limits of its bandwidth availability in Green and Yellow. Thus:

$$\sum_{i=0}^{N} G_i^j \times bw_i^j < \Psi_S^j B_S^j \text{ and}$$
$$\sum_{i=0}^{N} Y_i^j \times bw_i^j < (1-\Psi_S^j) B_S^j.$$

Moreover, SPC_j allocates its bandwidth in a way maximizing this outcome function:

$$U_{SPC}(config^j(\Pi)) = \sum_i Prop(bw_i^j)(\mu^j - \Gamma^j)\left(G_i^j + Y_i^j(1-\delta^j)\right) \quad (1)$$

Note that SPC_j's outcome function considers only its own profile described in Section 2.1.1. Given a SRC strategy set, each SPC proposes to allocate to each SRC a connection in Green and Yellow such as its outcome function is maximized.

3.1.3 *Formalization of Game restricted to SRCs.*

Game theory models the interactions between players competing for a common resource. In our system, the formulation of this noncooperative game $G = \langle \mathcal{N}, S, U \rangle$ can be described as follows:

- The set of players is $\mathcal{N}$. Each player is a SRC. There are N SRCs.

  $$\mathcal{N} = \{SRC_1, SRC_i, .., SRC_N\}$$

- The space of pure strategies S formed by the Cartesian product of each set of pure strategies

  $$S = S_{SRC_1} \times S_{SRC_i} \times ... \times S_{SRC_N}$$

  Note that for each SRC_i, the set S_{SRC_i} is defined in Section 3.1.1. A pure strategy s_i for which corresponds the SRC's request.

- A set of utility functions U that quantifies the SRCs outcomes from each SPC.

  $$U = \{(U_1^1, U_1^2, ..., U_1^P), (U_i^1, U_i^2, ..., U_i^P), ..., (U_N^1, U_N^2, ..., U_N^P)\}$$

  According to a set Π of SRCs requets $\Pi =< s_1, s_2, \ldots, s_N >$ where s_i corresponds to the strategy of SRC_i, for any i, $1 \le i \le N$, each SRC's utility regarding each SPC is determined through the SPC's allocation proposition. Since several allocation decisions could maximize a SPC's outcome function given by Equation (1), the SRC's utility regarding a SPC corresponds to the mean of all these allocation decisions. Let $sol^j(\Pi)$ be the set of $config^j(\Pi)$ that maximizes SPC_j's outcome function with $M = |sol^j(\Pi)|$.

  The utility $U_i^j(\Pi)$ of SRC_i (normalized $\in [0,1]$) from $sol^j(\Pi)$ is expressed as follows:

  $$U_i^j(\Pi) = \sum_{c \in sol^j(\Pi)} \frac{gain_i^j(c)}{M}. \quad (2)$$

  The gain of SRC_i from SPC_j's allocation decision c in $sol(\Pi)^j$ is expressed as follows:

  $$gain_i^j(c) = Rev_i^j(c) - Cost_i^j(c)$$

  Where c represents a triple (G_i^j, Y_i^j, bw_i^j) to SRC_i.

  - $Rev_i^j(c) \in [0,1]$ represents the SRC's QoS satisfaction from connection c provided by SPC_j and is expressed as follows:

    $$Rev_i^j(c) = Rev(bw_i^j) G_i^j \alpha_i + Rev(bw_i^j) Y_i^j (1-\delta^j)\alpha_i$$

    where $Rev(bw_i^j) \in [0,1]$

  - $Cost_i^j(c) \in [0,1]$ represents the cost of connection c provided by SPC_j and is expressed as follows:

    $$Cost_i^j(c) = Cost(bw_i^j) G_i^j \beta_i + Cost(bw_i^j) Y_i^j (1-\delta^j)\beta_i$$

    where $Cost(bw_i^j) \in [0,1]$

SRC_i selects the SPC following one of these two proposed solutions:

1. In the first solution, SRC_i selects the SPC_j where

   $$j = Arg_j\{Max(U_i^j(\Pi))\}; j = \{1, ..., P\}$$

   The drawback of this solution is that the connection proposition(s) $(sol^j(\Pi))$ the SPC_j sends to each SRC_i are computed considering that all the SRCs requests and thus could discourage some SRCs to take SPC_j's connection: due to high competition, some SRCs will not choose SPC_j's connection. Since several SPCs are existing, SRCs have thus several possibilities for connection acceptance. So, SPC_j's proposition could be much more interesting to each SRC_i if some other SRCs do not to accept SPC_j's connection.

2. The second solution takes into account the point presented in the first solution. Instead of sending only one per SRC taking into consideration all SRCs requests, SPC_j sends P connection propositions. Each connection proposition considers a number l of competing SRCs with $1 \le l \le P$. In this solution, SRC_i will choose the SPC_j for which its mean utility over all the connection propositions is the highest.

3.2 Game restricted to SPCs

The Game restricted to SPCs is triggered when the game restricted to SRCs stops. It assumes that SRCs requests are fixed as well as their SPCs attachement decisions.

The game restricted to SPCs is described as follows: for a fixed SRCs request denoted by Π^* as well as their SPCs attachement decisions, the aim of each SPC_j is to determine the optimal Ψ_S^j for which its utility from $sol^j(\Pi^*)$ is maximum, taking into consideration only the SRCs who have accepted its connection proposition. So, Ψ_{S_j} represents the strategy of each SPC_j.

SPC's strategy is denoted by $s_j \in [0,1]$. Let's denote by S_{SPC_j} the set of possible strategies of the SPC_j.

$$S_{SPC_j} =< 0, ..., \Psi_S^j - \varepsilon, \Psi_S^j, \Psi_S^j + \varepsilon, ..., 1 > .$$

4. EQUILIBRIUM IN GAME RESTRICTED TO SRCS.

Since each SRC_i has a finite set of strategies, this game has a mixed Nash equilibrium [12]. In the following, we study the existence of a pure Nash equilibrium in the Game restricted to SRCs using the properties of potential games. The definition of potential game could be found in [15].

THEOREM 1. *Each instance of the game restricted to SRCs with only type of connections requests, admits at least one pure Nash equilibrium.*

The proof of this theorem is detailed in [11].

Sketch of proof.
For the case where all SPCs propose one type of connection, in [11], an algorithm is given to compute a pure Nash equilibrium. The algorithm is based on building some alliances so that, the allocations of SPCs bandwidth is optimal. Let C be an arbitrary pure profile. We denote by $F(C)$ the total free bandwidth among all SPCs. We will show that the Best Response Dynamic in this game converges to a pure Nash equilibrium. The Best Response dynamic algorithm corresponds to compute a sequence of profiles. Let C be an arbitrary profile. If no player has incentive to change its strategy in C, then C is a pure Nash equilibrium and this algorithm stops. If at least one player has incentive to change its strategy in C, then this player changes its strategy by choosing its best response and there is a new profile C'. Now, the algorithm applies the same process for C' and so on. If some players have incentive to change their strategy, then $F(C) > F(C')$. Thus the Best Response Dynamic algorithm will end up since at each step the free bandwidth is reduced. Note that function F correspond to a potential function and that this game is a potential game.

4.1 Distributed Algorithm

Considered Algorithm 1.

1. At every time step, each player (SRC) chooses an action according to his current Action Probability Vector (APV). Thus, the i^{th} player selects strategy $s = a_i(k)$ at instant k with probability $q_{i,s}(k)$.

2. Each player obtains a payoff based on the set of all actions. We note the reward to player i at time k: $gain_i(k)$.

3. Each player updates his APV according to the rule:

$$q_i(k+1) = q_i(k) + b \times gain_i(k) \times (e_{a_i(k)} - q_i(k)), i = 1, ..., n, \tag{3}$$

where $0 < b < 1$ is a parameter and $e_{a_i(k)}$ is a unit vector of dimension m with $a_i(k)^{th}$ component unity.

It is easy to see that decisions made by players are completely decentralized, at each time step, player i only needs $gain_i$ and q_i, respectively his payoff and strategy, to update his APV. Notice, that componentwise, Equation (3) can be rewritten:

$$q_{i,s}(k+1) = \begin{cases} q_{i,s}(k) & -b(gain_i(k))q_{i,s}(k) & \text{if} \quad a_i \neq s \\ q_{i,s}(k) & +b(gain_i(k))(1 - q_{i,s}(k)) & \text{if} \quad a_i = s \end{cases} \tag{4}$$

Let K be the space of mixed profiles. Let $Q[k] = (q_1(k), \ldots, q_N(k)) \in K$ denote the state of the player team at instant k. Our interest is in the asymptotic behavior of $Q[k]$ and its convergence to a Nash Equilibrium. Clearly, under the learning algorithm specified by Equation (3), $\{Q[k], k \geq 0\}$ is a Markov process. Observe that this dynamic can also be put in the form

$$Q[k+1] = Q[k] + b \cdot G(Q[k], a[k], gain[k]), \tag{5}$$

where $a[k] = (a_1(k), ..., a_N(k))$ denotes the actions selected by the player team at k and $gain[k] = (gain_1(k), ..., gain_N(k))$ their resulting payoffs, for some function $G(.,.,.)$ representing the updating specified by equation (3), that does not depend on b. Consider the piecewise-constant interpolation of $Q[k], Q^b(.)$, defined by

$$Q^b(t) = Q[k], t \in [kb, (k+1)b], \tag{6}$$

where b is the parameter used in (3). $Q^b(.)$ belongs to the space of all functions from R into K. These functions are right continuous and have left hand limits. Now consider the sequence $\{Q^b(.) : b > 0\}$. We are interested in the limit $Q(.)$ of this sequence as $b \to 0$. The following is proved in [3]:

PROPOSITION 1 ([3]). *The sequence of interpolated processes $\{Q^b(.)\}$ converges weakly, as $b \to 0$, to $Q(.)$, which is the (unique) solution of Cauchy problem*

$$\frac{dQ}{dt} = \phi(Q), Q(0) = Q_0 \tag{7}$$

where $Q_0 = Q^b(0) = Q[0]$, and $\phi : K \to K$ is given by

$$\phi(Q) = E[G(Q[k], a[k], gain[k])|Q[k] = Q],$$

where G is the function in Equation (5).

Recall that a family of random variable $(Y_t)_{t \in R}$ weakly converges to a random variable Y, if $E[h(X_t)]$ converges to $E[h(Y)]$ for each bounded and continuous function h. This is equivalent to convergence in distributions. The proof of Proposition 1 in [3], that works for general (even with stochastic payoffs) games, is based on constructions from [8], in turn based on [1], i.e. on weak-convergence methods, nonconstructive in several aspects, and does not provide error bounds. Using (4), we can rewrite $E[G(Q[k], a[k], gain[k])]$ in the general case as follows $E[G(Q[k], a[k], gain[k])]_{i,s}$

$$\begin{aligned} &= q_{i,s}(1 - q_{i,s})E[gain_i|Q(k), a_i = s]) \\ &\quad - \textstyle\sum_{s' \neq s} q_{i,s'} q_{i,s} E[gain_i|Q(k), a_i = s'] \\ &= q_{i,s}[\textstyle\sum_{s' \neq s} q_{i,s'} E[gain_i|Q(k), a_i = s] \\ &\quad - \textstyle\sum_{s' \neq s} q_{i,s'} E[gain_i|Q(k), a_i = s'] \\ &= q_{i,s} \textstyle\sum_{s'} (E[gain_i|Q(k), a_i = s] - q_{i,s'} E[r_i|Q(k), a_i = s']), \end{aligned} \tag{8}$$

using the fact that $1 - q_{i,s} = \sum_{s' \neq s} q_{i,s'}$. Let $h_{i,s}$ be the expectation of the payoff for i if player i plays pure strategy s, and players $j \neq i$ play (mixed) strategy q_j. Formally,

$$h_{i,s}(q_1, \cdots, q_{i-1}, q_{i+1}, \cdots, q_n) = E[gain\ for\ i\ |Q(k), a_i = s].$$

Let $\overline{h_i}(Q)$ the mean value of $h_{i,s}$, in the sense that

$$\overline{h_i}(Q) = \sum_{s'} q_{i,s'} h_{i,s'}(Q).$$

We obtain from (8),

$$E[G(Q[k], a[k], c[k])]_{i,s} = q_{i,s}(h_{i,s} - \overline{h_i}(Q)). \tag{9}$$

$$\frac{dq_{i,s}}{dt} = q_{i,s}(h_{i,s} - \overline{h_i}(Q)). \quad (10)$$

This is a replicator equation, that is to say a well-known and studied dynamics in evolutionary game theory [18, 6]. In this context, $h_{i,s}$ is interpreted as player i's fitness for a given game, and $\overline{h_i}(Q)$ is the mean value of the expected fitness in the above sense. In particular, solutions are known to satisfy the following theorem (sometimes called Evolutionary Game Theory Folk Theorem) [18, 3].

THEOREM 2 (SEE E.G. [18, 3]). *The following are true for the solutions of the replicator equation* (10)*:*

- *All corners of space K are stationary points.*
- *All Nash equilibria are stationary points.*
- *All strict Nash equilibria are asymptotically stable.*
- *All stable stationary points are Nash equilibria.*

From this theorem, we can conclude that the dynamics (10), and hence the learning algorithm when b goes to 0, will never converge to a point in K which is not a Nash equilibrium. However, for general games, there is no convergence in the general case [3]. We will now show that for ours games, there is always convergence. It will then follow that the learning algorithm we are considering here converges towards Nash equilibria, i.e. solves the learning problem for our games .

THEOREM 3. *In the game where all SPCs propose green connections, the learning algorithm, for any initial condition in K (except borders), always converges to a Nash Equilibrium.*

PROOF. Let Q be an arbitrary mixed profile. We denode $F(Q)$ be the average total free bandwidth among all SPCs if SRCs play according to Q. We claim that $F(.)$ is a Lyapunov function of the dynamics, i.e. that F is monotone along trajectories.

Indeed,

$$\begin{aligned}
\frac{dF(Q(t))}{dt} &= \sum_{i,s} \frac{\partial F}{\partial q_{i,s}} \frac{dq_{i,s}}{dt} \\
&= \sum_{i,s} \frac{\partial F}{\partial q_{i,s}}(Q) q_{i,s} \sum_{s'} q_{i,s'}[h_{i,s}(Q) - h_{i,s'}(Q)] \\
&= -\sum_{i,s} h_{i,s}(Q) q_{i,s} \sum_{s'} q_{i,s'}[h_{i,s}(Q) - h_{i,s'}(Q)] \\
&= -\sum_i \sum_s \sum_{s'} q_{i,s} q_{i,s'}[h_{i,s}(Q)^2 - h_{i,s}(Q) h_{i,s'}(Q)] \\
&= -\sum_i \sum_s \sum_{s'>s} q_{i,s} q_{i,s'}[h_{i,s}(Q) - h_{i,s'}(Q)]^2 \\
&\leq 0
\end{aligned} \quad (11)$$

Thus F is decreasing along the trajectories of the ODE and, due to the nature of the ODE (10), for initial conditions in K will be confined to K.

Hence from the Lyapunov Stability theorem (see e.g. [5] page 194), asymptotically all trajectories will be in the set $K' = \{Q^* \in K : \frac{dF(Q^*)}{dt} = 0\}$.

Now, from (11), we know that $\frac{dF(Q^*)}{dt} = 0$ implies $q_{i,s} q_{i,s'}[h_{i,s}(Q) - h_{i,s'}(Q)] = 0$ for all i, s, s', hence that Q^* is a stationary point of the dynamics.

Since from Theorem 2, all stationary points that are not Nash equilibria are unstable, the theorem follows. □

5. SHARING FEMTO ACCESS PROBLEM: ALGORITHM AND SIMULATIONS

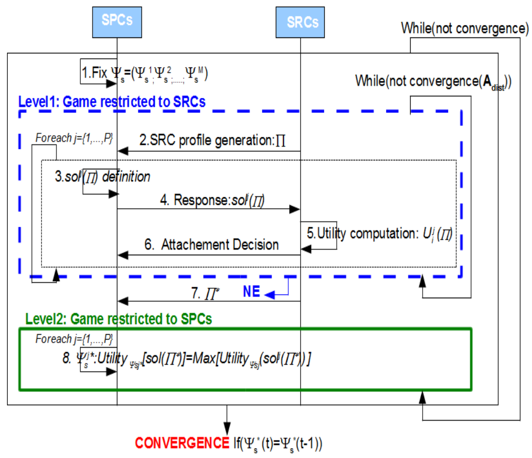

Figure 3: Algorithm principle of the sharing femto access problem

In Section 3, we have seen that the sharing femto access problem is divided into two levels: Level 1 representing the game restricted to SRCs and level 2 representing the game restricted to SPCs. The algorithm principle of sharing femto access problem with its two levels is detailed in Figure 3. Then we will present some simulations results.

5.1 Algorithm principle

1. Each SPC fixes its bandwidth split.
   $\Psi_S = \{\Psi_S^1, \Psi_S^2, \ldots, \Psi_S^P\}$
2. Each SRC sends to all SPCs a request using a specific distributed algorithm denoted by A_{dist}.
3. Each SPC defines its connection allocation $sol(\Pi)$.
4. Each SPC sends its connection allocation proposition to all SRCs.
5. Each SRC computes its utility following $sol(\Pi)$.
6. Each SRC sends to each SPC a positive or a negative response to its proposition.
7. When the Algorithm A_{Dist} converges, a signal is sent to SPCs to trigger the game restricted to SPCs.
8. Each SPC computes its optimal bandwidth split Ψ_S^{j*} considering the fixed Π^* and the SRCs who accepted its connection proposition.

The whole algorithm stops when Ψ_S^* is the same in two consecutive iterations of the whole algorithm.

5.2 Simulations

In this section, we will present one simulation with N SRCs and 1 SPC: each SRC_i is characterized by α_i (the QoS sensitivity parameter of SRC_i) and is requesting for the SPC's connection characterized by μ to download a file.

We will only focus on the cases of a same category SRCs and more specifically QoS sensitive SRCs and a gain sensitive SPC. The N SRCs want to download a file of $1Mbyte$.

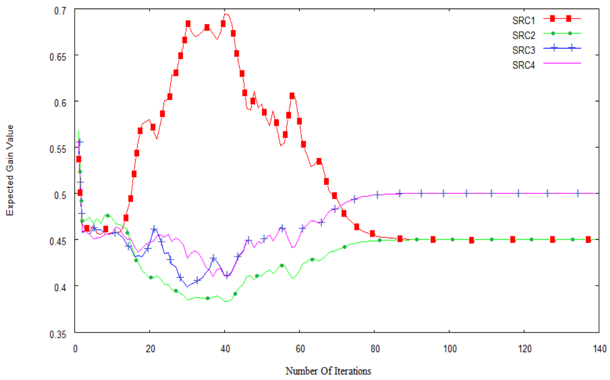

Figure 4: Variation of SRCs expected gain

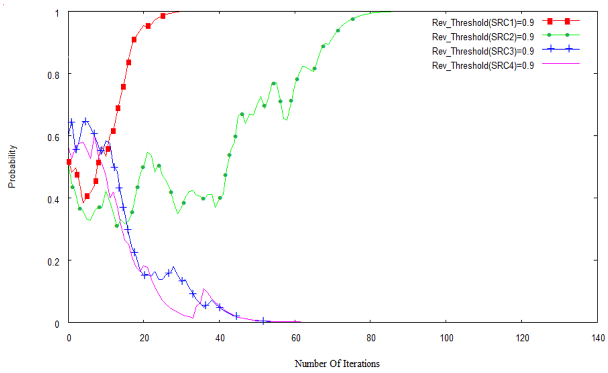

Figure 5: Variation of SRCs strategies probabilities

We will consider $T_2 = 300sec$, $\mu = 1$, $\delta = 0.1$, $\varepsilon = 0.1$, $\kappa = 0.1$, $\Psi_S = 0.5$ and $B_S = 20Mb/s$. Since a femto access could support only 8 communications, we will run simulations where $2 \leq N \leq 8$ keeping the same input presented above. We consider the following strategy notation $s_i = (Rev\_Th_i)$

5.2.1 Scenario with 4 SRCs and 1 SPC

For this scenario, we will as a first step analytically compute the game restricted to SRCs Nash equilibriums. To do so, we will consider the same steps from 2 to 5 presented in Figure 3 except that the SRC strategy profile is not generated with Algorithm A_{dist}: we will consider all the possible SRC strategy profiles and thus fill the SRCs payoff matrix with utilities corresponding to each SRC strategy profile. The SRCs payoff matrix highlights 6 pure Nash equilibriums and are the following:

$$\Pi^* = \{(0.9, 0.9, 1, 1), (0.9, 1, 0.9, 1), (0.9, 1, 1, 0.9), (1, 0.9, 0.9, 1),$$
$$(1, 0.9, 1, 0.9), (1, 1, 0.9, 0.9)\}.$$

Now, we will run the Algorithm A_{dist}. First, we will verify whether if it converges and then we will check whether if the point of convergence ,if any, is one among the pure Nash equilibriums detected analytically. To do so, we will consider $b = 0.1$ and 700 iterations. In Figure 4 and 5, each point represents the mean over 5 iterations of respectively SRCs expected gain value and SRCs strategy probability.

A_{dist} converges after 450 Iterations. Figure 4 shows that the SRCs expected gain stabilize as follows:

Expected gain(SRC_1)=Expected gain(SRC_2)=0.45 and Expected gain(SRC_3)=Expected gain(SRC_4)=0.5.

In Figure 5, we remark that the convergence point (point where each SRC has a pure strategy) $\Pi^* = (0.9, 0.9, 1, 1)$ matches with one of the pure Nash equilibriums computed analytically.

After 450 Iterations, the Algorithm A_{dist} converges. As a consequence the game restricted to SRCs stops and a signal is sent to the SPC to trigger the game restricted to SPCs. Since only once SPC is existing, so the game restricted to SPCs turns into an optimality problem where the SPC aims at finding its optimal bandwidth split maximizing its gain.

Now, we will focus on the economic aspects of this scenario. We will consider that the SPC is a Bill and that all SRCs are Aliens. We consider that a SRC should pay 1 Token per bandwidth unit denoted by bw_u and that a SPC gains 1 unit price denoted by $price_u$ per Token received by a SRC. We remind that BW_{Max} is the amount of bandwidth for which the utility of the SRC is at the top (equal to 1). If SRC_i receives BW_{Max} from the SPC, he will spend a number of tokens denoted by T_i^{Top} such as $T_i^{Top} = \frac{BW_{Max}}{bw_u}$.

At the convergence point of the Algorithm A_{Dist}, each SRC_i will spend a number of tokens denoted by T_i^{Spent} such as $T_i^{Spent} = T_i^{Top}.ExpectedGain(SRC_i)$.

The number of tokens that the SPC will receive by SRCs is denoted by $T^{receipt}$ such as $T^{receipt} = \sum_{i=1}^{N} T_i^{Spent}$

On one hand, each SRC_i is an Alien, so T_i^{Spent} correspond to POTs(Paying Own Tokens). On another hand, the SPC is a Bill, so $T^{receipt}$ correspond to PFTs(Paying Foreign Tokens).

Let's denote by $p (p \in [0, 1])$ the percentage of the whole SPC's gain the MO will get. Thus, the gain generated by the SPC after exchanging its received tokens against money is denoted by $Gain_{SPC}$ such that $Gain_{SPC} = (1 - p).T^{receipt}.price_u$.

The whole algorithm representing the sharing femto access problem and detailed in section 5.1 converges for this scenario after exactely 2 consecutive iterations of the SPC's optimality problem.

For a number of SRCs varying from 2 to 8 and $P = 1$, we have checked by simulations that pure Nash equilibrium in the game restricted to SRCs is reachable.

The table 2 summarizes the number of iterations necessary for A_{dist} to converge for N varying from 2 to 8 and $P = 1$. This result is true for SRCs with only two strategies.

Number of SRCs	2	3	4	5	6	7	8
Number of iterations	150	300	450	780	800	810	830

Table 2: Number of iterations for A_{dist} convergence

5.3 Simulation results

Simulations results have shown that the game restricted to SRCs using a distributed learning algorithm converges to a pure Nash equilibrium. We have checked that this result is available for a number of SRCs varying from 2 to 8 for SRCs with exactly 2 strategies or more than 2 strategies.

The distributed algorithm converges in a finite number of iterations. The distributed learning algorithm running in SRCs user equipments gives at convergence the minimum and the maximum amount of bandwidth to be requested by each SRC in Green and Yellow. The algorithm running in SPCs femto access gives the optimal bandwidth split to be fixed by each SPC in order to maximize its gain.

6. CONCLUSION AND PERSPECTIVES

In this article, we investigate the problem of sharing femto access taking a file transfer application as example. We have proved using potential games properties that the game restricted to SRCs is a potential game and thus admits at least one pure Nash equilibrium considering a game with N SRCs and P SPCs proposing only one type of connection. The simulations presented focus on examples with one SPC where SRCs objectives conflict: SRCs belonging to the same category of QoS sensitive SRCs. The Distributed Learning Algorithm always converges to a pure Nash equilibrium.

As a perspective, we will also focus on the convergence time of the algorithm A_{Dist} applied in a game with several SRCs and several SPCs and also on the optimzation of this time of convergence.

7. REFERENCES

[1] D.W. Stroock and SRS Varadhan. *Multidimensional Diffusion Processes.* Springer, 1979.

[2] R.W. Rosenthal. A class of games possessing pure-strategy Nash equilibria. *International Journal of Game Theory*, 2(1):65–67, 1973.

[3] M.A.L. Thathachar P.S. Sastry, V.V. Phansalkar. Decentralized learning of Nash equilibria in multi-person stochastic games with incomplete information. *IEEE transactions on system, man, and cybernetics*, 24(5), 1994.

[4] Nissan Nisan, Tim Roughgarden, Éva Tardos, and Vijay V. Vazirani. *Algorithmic Game Theory.* Cambridge University Press, 2007.

[5] Morris W. Hirsch, Stephen Smale, and Robert Devaney. *Differential Equations, Dynamical Systems, and an Introduction to Chaos.* Elsevier Academic Press, 2003.

[6] Jörgen W. Weibull. *Evolutionary Game Theory.* The MIT Press, 1995.

[7] Sahand Haji Ali Ahmad, Mingyan Liu, and Yunnan Wu. Congestion games with resource reuse and applications in spectrum sharing. *CoRR*, abs/0910.4214, 2009.

[8] H. J. Kushner. *Approximation and Weak Convergence Methods for Random Processes, with Applications to Stochastic Systems Theory.* Cambridge, MA: MIT Press, 1984.

[9] Niyaton Dusit and Ekram Hossain. *Microeconomic Models for Dynamic Spectrum Management in Cognitive Radio Networks*, chapter 14, pages 391–423. Springer, 2007.

[10] David Hausheer, Nicolas C. Liebau, Andreas Mauthe, Ralf Steinmetz, and Burkhard Stiller. Token-based accounting and distributed pricing to introduce market mechanisms in a peer-to-peer file sharing scenario. In *Proceedings of the 3rd International Conference on Peer-to-Peer Computing*, P2P '03, pages 200–, Washington, DC, USA, 2003. IEEE Computer Society.

[11] Mariem Krichen, Johanne Cohen, and Dominique Barth. File transfer application for sharing femto access : Game properties. Technical report, Université de Versailles, 2011.

[12] John F. Nash. Equilibrium points in n-person games. *Proceedings of the National Academy of Sciences of the United States of America*, 36:48–49, 1950.

[13] Nissan Nisan, Tim Roughgarden, Éva Tardos, and Vijay V. Vazirani. *Algorithmic Game Theory.* Cambridge University Press, 2007.

[14] M.A.L. Thathachar P.S. Sastry, V.V. Phansalkar. Decentralized learning of Nash equilibria in multi-person stochastic games with incomplete information. *IEEE transactions on system, man, and cybernetics*, 24(5), 1994.

[15] R.W. Rosenthal. A class of games possessing pure-strategy Nash equilibria. *International Journal of Game Theory*, 2(1):65–67, 1973.

[16] Takashi Ui. A shapley value representation of potential games. *Games and Economic Behavior*, 31(1):121–135, April 2000.

[17] Yiping Xing, Chetan N .Mathur, M. A Haleem, R. Chandramouli, and K. P Subbalakshmi. Real-time secondary spectrum sharing with qos provisioning. In *Consumer Communications and Networking Conference (CCNC)*, pages 630 – 634, 2006.

[18] J. Hofbauer and K. Sigmund. Evolutionary game dynamics. *Bulletin of the American Mathematical Society*, 4:479–519, 2003.