Optimal Base Station Placement: A Stochastic Method Using Interference Gradient In Downlink Case

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ABSTRACT

In this paper, we study the optimal placement and optimal number of base stations added to an existing wireless data network through the interference gradient method. This proposed method considers a sub-region of the existing wireless data network, hereafter called region of interest. In this region, the provider wants to increase the network coverage and the users throughput. In this aim, the provider needs to determine the optimal number of base stations to be added and their optimal placement.

The proposed approach is based on the Delaunay triangulation of the region of interest and the gradient descent method in each triangle to compute the minimum interference locations. We quantify the increase of coverage and throughput.

Categories and Subject Descriptors

C.2.1 [**Computer-Communication Networks**]: Network Architecture and Design—*wireless communication, network topology*; G.1.6 [**Numerical Analysis**]: Optimization—*gradient methods*

General Terms

Design

Keywords

Stochastic geometry, capacity, coverage, gradient methods

1. INTRODUCTION

Our objective is to increase the coverage and the capacity of a network, by adding new base stations. Therefore, we need to find the optimal number and optimal placement of additional base stations in an already deployed wireless data network. We consider a sub-region of the existing wireless data network, where the service provider wants to increase the coverage of the network and the throughput of the users.

The problem is to find the positions of K additional base stations (a variable number to be determined), within the region of interest in an existing wireless data network. Note that the addition of K new base stations to the network may impact the coverage and the capacity of the existing base stations which makes the problem very difficult. We show that the problem of finding the optimal location of a set of new base stations when there is a discrete number of mobile users is an NP-Hard problem. The problem in its more general form is much harder when we consider a random distribution of the mobile users, even for the uniform distribution of the users. We propose a related problem which is to find the points of minimum interference in an existing network. The optimality is then to find the minimum interference set of base stations.

We have studied the optimal placement and optimal number of base stations added to an existing wireless data network through the interference gradient method. Our approach is based on the Delaunay triangulation of the region of interest (where the triangulation completely covers the region of interest). We propose to take as the vertices of this Delaunay triangulation the positions of the existing base stations. This is justified by our proof that over each triangle of the considered triangulation there is only one global minimum (over the triangle) of the interference function. In consequence, there will be as many candidates of minima over the region of interest as triangles in the Delaunay triangulation. The problem is reduced to find over this set of candidates the minimum interference set of K base stations. We propose two heuristics to find this minimum interference points. Numerical simulations show that the coverage and the throughput through this method are highly incremented when our method is considered.

2. RELATED WORKS

Plastria [11] presented an overview of the research on locating one or more new facilities in an environment where other facilities already exist. The authors of [9] analyze the coverage and the capacity of a wireless network regularly distributed. Gabszewicz and Thisse [6] provided another general survey on the location problem. Buttazzo and Santambrogio [5] studied the location problem but their work does not consider the case when there is an existing wireless data network. Altman et al. [3] studied the case when there are two providers in the uplink scenario of a cellular network and the users are placed on a line segment. Prommak et al. [12] design a constraint satisfaction problem where they consider the transmitted power, frequency bands, and

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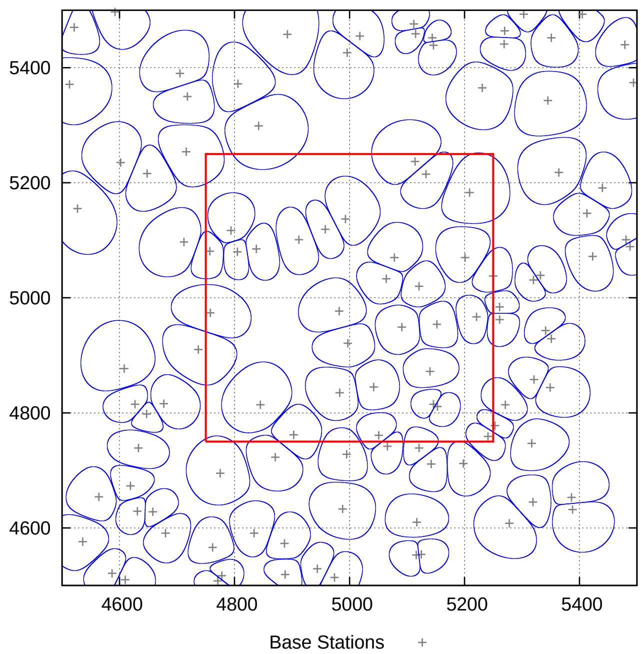

Figure 1: $\mathcal{A}_{\mathcal{R}}(\beta)$ of transmitters lying in the central square area of 1000×1000 square km. Region bounded by a square of side 500 km is the region of interest to place additional base stations. $\beta = 1$ and $\alpha = 4.0$.

location constraints. The authors of [10] studied a delay-constrained information coverage which targets an optimal base station placement with the objective of maximizing information collection within a constraint time, which is more suitable for wireless sensor networks.

3. THE MODEL

We consider an existing wireless data network. In particular, we put ourselves in the context of an LTE (OFDMA-based) system, where the relevant objective function is to find the optimal number of base stations and their optimal placement to maximize the throughput utility of the data flows.

We consider OFDMA systems supporting fractional frequency reuse (FFR) for interference mitigation similar to the one used on [13, 14]. This type of interference mitigation divide frequency and time resources into several resource sets. Fractional frequency reuse in the context of OFDMA systems has been discussed in cellular network standarization such as Third Generation Partnership Project (3GPP) and Third Generation Partnership Project 2 (3GPP2) [2, 1].

The model is given by the following:

- Let $\mathcal{S} = \{1, \ldots, S\}$ represent the set of base stations which are randomly distributed over the two-dimensional plane according to a Poisson point process (PPP) of intensity λ. We denote by $\mathbf{z}_i = (x_i, y_i)$[1] the position of base station $i \in \mathcal{S}$.
- and J sub-bands $j \in \mathcal{J} = \{1, \ldots, J\}$ where we denote W the bandwidth of each sub-band.
- Each sub-band consists of a fixed number c of sub-carriers.

[1]We denote the vectors by bold fonts.

Furthermore, time is divided into slots consisting of a number of OFDMA symbols and transmissions are scheduled to users by assigning a set of sub-carriers on specific slots. The time is slotted, so that transmissions within each cell are synchronized, and do not interfere with each other. To simplify the exposition we assume that the resource sets span the entire time period. Extension to more general resource sets is straightforward [13, 14]. A transmission in a cell, assigned to a sub-carrier in a sub-band, causes interference to only those users in other cells that are assigned to the same sub-carrier on the corresponding sub-band. We assume that there is no interference between sub-carriers.

- We denote by $\tau_{ij}(\mathbf{z}) \in [0, 1]$ the fraction of time an algorithm chooses the user located at position $\mathbf{z}$ in sub-carrier i in sub-band j.
- We denote by $R_{ij}(\mathbf{z}) \in [0, B]$, $B < \infty$ the transmission (nominal) rate in sub-carrier i in sub-band j, if the user located at position $\mathbf{z}$ is chosen.

Then the average rate a user located at position $\mathbf{z}$ actually receives is

$$\mathcal{C}(z) = \sum_j \sum_i \tau_{i,j}(\mathbf{z}) R_{ij}(\mathbf{z}). \tag{1}$$

Each base station may need to allocate a set of sub-carriers and average power $P_{ij}^{(k)}(\mathbf{z})$, if the user located at position $\mathbf{z}$ is to be assigned to sub-carrier i in sub-band j.

Let us denote by $G_{ij}^{(k)}(\mathbf{z})$ the propagation gain from cell k to a user located at position $\mathbf{z}$ in sub-carrier i in sub-band j.

Then, we consider the SINR (Signal to interference plus noise) function:

$$\mathrm{SINR}_{ij}(\mathbf{z}) = \frac{G_{ij}^{(k')}(\mathbf{z})P_{ij}^{(k')}(\mathbf{z})}{N_0 + \sum\limits_{k\in\mathcal{S},k\neq k'} G_{ij}^{(k)}(\mathbf{z})P_{ij}^{(k)}(\mathbf{z})}, \tag{2}$$

where N_0 is the thermal noise.

We use Shannon formula for the rate

$$\mathcal{C}(\mathbf{z}) := W\log_2(1+\mathrm{SINR}_{ij}(\mathbf{z})). \tag{3}$$

Let us consider that the propagation gain from cell k to a user located at position $\mathbf{z}$ associated to sub-carrier i in sub-band j is given by the channel gain

$$G_{ij}^{(k)}(\mathbf{z}) := \frac{\kappa}{(\sqrt{h^2+d_{ij}^2})^{\alpha}}, \tag{4}$$

where d_{ij} is the distance between the base station BS_i and the mobile terminal MT_j, h is the height of the base station, κ is a constant factor, and $\alpha > 2$ is the path loss exponent.

To simplify the exposition we consider the distance between base stations to be much greater that the height of the base station in eq. (4), and κ is constant. In the following we will drop the sub-indices and analyze just the interference between sub-carriers. This scenario is general and it may be applied to any interference problem.

The objective of this work is to find the optimal positions of K (variable) additional base stations, within a bounded region of interest in an existing wireless network, such that it maximizes the coverage and the capacity of the existing base stations and these K additional base stations. Note that the addition of K new base stations to the network may impact the coverage and the capacity of the existing base stations which makes the problem very difficult.

The coverage of a base station is given by the β-SINR threshold at which the basic service is provided. Considering that $P_{ij}^{(k)}$ is constant for all i,j,k, and assuming negligible thermal noise, we define the β-reception area of the base station $i\in\mathcal{S}$, denoted by $\mathcal{A_R}_i(\beta)$, which represents the area where the transmission from base station i is received with signal to interference plus noise ratio (SINR) greater or equal to β, i.e.,

$$\mathcal{A_R}_i(\beta) = \{\mathbf{z} \,:\, |\mathbf{z}-\mathbf{z}_i|^{-\alpha} \geq \beta\sum_{j\neq i}|\mathbf{z}-\mathbf{z}_j|^{-\alpha}\} \tag{5}$$

where $|\mathbf{z}|$ represents the Euclidean norm of the vector $\mathbf{z}=(x,y)$, i.e., $|\mathbf{z}|=\sqrt{x^2+y^2}$.

The coverage of the network is given by the sum over all the base stations of their β-reception areas

$$\mathrm{Coverage}(\beta) = \sum_{i\in\mathcal{S}}\mathcal{A_R}_i(\beta). \tag{6}$$

The β-reception area of the base station i, $\mathcal{A_R}_i(\beta)$, is computed using the gradient SINR methodology described in [8].

4. DESCRIPTION OF OUR SCHEME

4.1 Points of maximum capacity

Let us assume that the set of S existing base stations is given by

$$\mathcal{S} = \{1,\ldots,S\}. \tag{7}$$

Let us consider the case we want to incorporate a new base station $i\notin\mathcal{S}$. Then the set of $S+1$ base stations will be given by

$$\mathcal{S}' = \mathcal{S}\cup\{i\}, \tag{8}$$

The SINR function of a mobile terminal located at position $\mathbf{z}$ being served by base station BS_k with $k\in\mathcal{S}'$ located at position $\mathbf{z}_k$ is

$$\mathrm{SINR}_k(z) = \frac{|\mathbf{z}-\mathbf{z}_k|^{-\alpha}}{\sum\limits_{j\in\mathcal{S}',\,j\neq k}|\mathbf{z}-\mathbf{z}_j|^{-\alpha}} = \frac{f}{g}, \tag{9}$$

where $f=|\mathbf{z}-\mathbf{z}_i|^{-\alpha}$ and $g=\sum\limits_{j\neq i}|\mathbf{z}-\mathbf{z}_j|^{-\alpha}$

We want to compute the maximum of the capacity function obtained by adding a new base station depending on the location of the already existing base stations. The positions of the existing base stations are given, and we assume that the distribution of the mobile terminals is uniform, then the expected capacity function by adding the new base station is given by

$$\mathbf{E}(\mathcal{C}(\mathbf{z})) = \int_{\mathcal{A}}\mathcal{C}(\mathbf{z})\,d\mathbf{z}. \tag{10}$$

where

$$\mathcal{C}(\mathbf{z}) = \sum_{k\in S'} W\log_2(1+\mathrm{SINR}_k(\mathbf{z})). \tag{11}$$

The optima of this maximization problem are found at stationary points, where the first derivative or the gradient of the objective function is zero. An equation stating that the first derivative equals zero at an interior optimum is sometimes called a "first-order condition".

We want to study the behavior of the gradient of eq. (10):

$$\nabla\left(\int_{\mathcal{A}}\mathcal{C}(\mathbf{z})\,d\mathbf{z}\right). \tag{12}$$

We can rewrite eq. (11) as follows:

$$\sum_{k\in S'} W\log_2(1+\mathrm{SINR}_k(\mathbf{z})) = \tag{13}$$

$$\sum_{k\in S',k\neq i} W\log_2(1+\mathrm{SINR}_k(\mathbf{z})) + W\log_2(1+\mathrm{SINR_i}(\mathbf{z})) \tag{14}$$

Then in the case when there is a large number of base stations the term

$$\nabla\int_{\mathcal{A}}\sum_{k\in S',k\neq i} W\log_2(1+\mathrm{SINR}_k(\mathbf{z}))\,d\mathbf{z}\sim 0 \tag{15}$$

The relevant term in eq. (12) is then giving by the following

$$\nabla\int_{\mathcal{A}} W\log_2(1+\mathrm{SINR_i}(\mathbf{z}))\,d\mathbf{z} \tag{16}$$

In the low-SINR regime the problem of maximizing the existing capacity is equivalent to the problem of maximizing the SINR. In the high-SINR regime both quantities are related however we do not show a theoretical result. However, simulation results suggest that even in this scenario we obtain a good approximation. In this case, we consider the following function

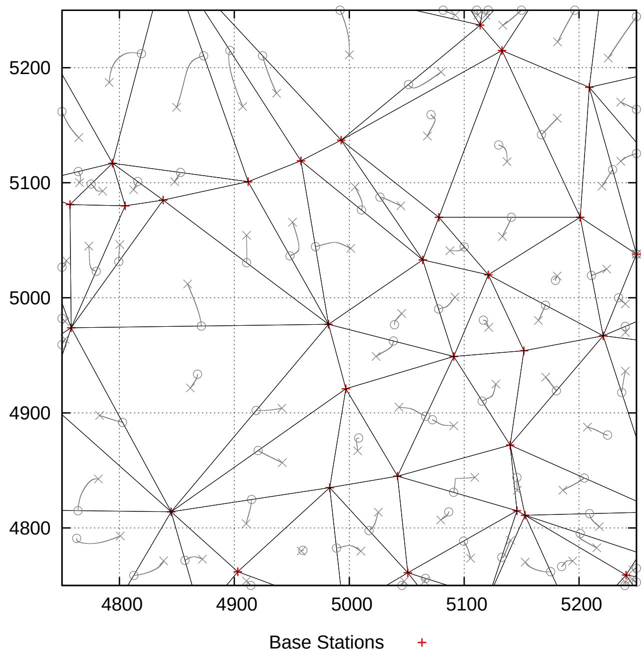

Figure 2: Finding points of local minimum interference in the central square area of 500×500 square km.

$$\nabla \int_{\mathcal{A}} \mathrm{SINR}(\mathbf{z})\, d\mathbf{z} \tag{17}$$

Notice that we can rewrite eq. (9)

$$\mathrm{SINR}_k(z) = \frac{|\mathbf{z} - \mathbf{z}_k|^{-\alpha}}{\sum\limits_{j \in \mathcal{S}', j \neq k} |\mathbf{z} - \mathbf{z}_j|^{-\alpha}} = \frac{f(\mathbf{z}_k, \mathbf{z})}{g(\mathbf{z})}, \tag{18}$$

$$\nabla \mathrm{SINR}(\mathbf{z}) = \nabla \left(\frac{f}{g}\right) = \frac{g \nabla f - f \nabla g}{g^2} \tag{19}$$

In general, this problem is NP-Hard (consider it as the extension of the coverage problem to the random distribution of users). Then we propose instead of considering the whole SINR function just to consider $F(\mathbf{z}_k) = \int_{\mathcal{A}_{\mathcal{R}}} f(\mathbf{z}_k, \mathbf{z})\, d\mathbf{z}$ to be a constant function, then in the previous equation we are interested on the behaviour of ∇g.

The problem of maximizing coverage in wireless data networks with K additional base stations can be reduced to the classical maximum coverage problem in computer science [7]. In maximum coverage problem, the inputs are several given sets and a number K. These sets may have some elements in common. The goal is to select at most K of these sets such that the maximum number of elements are covered, *i.e.*, the union of the selected sets has maximal size. Formally, the maximum coverage problem can be defined as:

Inputs: A number K and a collection of sets

$$S = S_1, S_2, ..., S_m \tag{20}$$

Objective: Find a subset $S' \subseteq S$ of sets, such that $|S'| \leq K$ and the number of covered elements $\left| \bigcup_{S_i \in S} S_i \right|$ is maximized.

The problem of maximum coverage with N given base stations and K additional base stations can be reduced to the above described maximum coverage problem. In case of wireless networks, the sets S_i represent base stations. The elements of each set may represent the users each base station may cover with given SINR threshold β. Note that these sets may overlap for $\beta \leq 1$. For $\beta \geq 1$, the zones covered by the base stations do not overlap [8]. The location of K additional base stations shall be chosen so that the coverage of users, in the network, by $N + K$ base stations is maximum. This is analogous to selecting K sets in maximum coverage problem which is an NP-hard problem.

4.2 Points of minimum interference

The interference or signal level received at any location z on the two-dimensional plane can be represented by the function

$$g(\mathbf{z}) = \sum_{i \in \mathcal{S}} |\mathrm{z} - \mathrm{z}_i|^{-\alpha} \tag{21}$$

There can be many local minima and to identify those points, we have used the following two-step approach.

1. We subdivide the region of interest in the network, by using the Delaunay triangulation scheme, with the location of the base stations.

2. We use the gradient descent method to locate the point of minimum interference in each triangle. Proof of the convexity of the interference function in each triangle is included in Proposition 4.2.

4.3 Delaunay Triangulation

A Delaunay triangulation for a set of points $\mathcal{S}$ in the two-dimensional plane, denoted by $\mathrm{DT}(\mathcal{S})$ is a triangulation (or subdivision of the two-dimensional plane into triangles) such that no point in $\mathcal{S}$ is inside the circumference of any triangle in $\mathrm{DT}(\mathcal{S})$.

The Delaunay triangulation of a discrete set of points $\mathcal{S}$ corresponds to the dual graph of the Voronoi tessellation for $\mathcal{S}$.

We choose the Delaunay triangulation because we have the following property.

PROPOSITION 4.1. *Any local minimum interference over the triangle $\mathcal{T} \setminus \{a_1, a_2, a_3\}$ is also a global minimum interference over the triangle.*

Proof. Consider the function defined in the two-dimensional plane except in $\mathbf{z}_i$:

$$g(\mathbf{z}, \mathbf{z}_i) := \frac{1}{|\mathbf{z} - \mathbf{z}_i|^{\alpha}} \tag{22}$$

Then the function of interference is given by

$$g(\mathbf{z}) = \sum_{i \in \mathcal{S}} g(\mathbf{z}, \mathbf{z}_i) \tag{23}$$

Let us consider the triangle of vertices a_1, a_2, a_3 where the three vertices are the position of three base stations:

$$\mathcal{T} = \{\mathbf{z} \,:\, \mathbf{z} = \sum_{i=1}^{3} \lambda_i a_i \,;\, 0 \leq \lambda_i \leq 1,\, i \in \{1,2,3\},\, \sum_{i=1}^{3} \lambda_i = 1\} \tag{24}$$

PROPOSITION 4.2. *The function $g(\mathbf{z}, \mathcal{A}_{\mathcal{R}}, \mathbf{z}_i)$ is convex in $\mathbf{z}$ over $\mathcal{T} \setminus \{a_1, a_2, a_3\}$.*

Proof. We just need to prove that $\frac{d^2 g}{d\mathbf{z}^2}$ is positive.
Let $r_i := |\mathbf{z} - \mathbf{z}_i|$. The gradient of $g(\mathbf{z}, \mathbf{z}_i)$ in $\mathbf{z} = (x, y)$ is:

$$\frac{dg}{d\mathbf{z}} = -\alpha |\mathbf{z} - \mathbf{z}_i|^{-\alpha - 1} \frac{\mathbf{z} - \mathbf{z}_i}{|\mathbf{z} - \mathbf{z}_i|} = -\alpha \frac{\mathbf{z} - \mathbf{z}_i}{|\mathbf{z} - \mathbf{z}_i|^{\alpha + 2}}. \tag{25}$$

This means that

$$\frac{dg}{dx} = -\alpha \frac{x - x_i}{r_i^{\alpha+2}} \quad \text{and} \quad \frac{dg}{dy} = -\alpha \frac{y - y_i}{r_i^{\alpha+2}} \tag{26}$$

$$\begin{aligned} \frac{d^2 g}{d\mathbf{z}^2} &= -\alpha \left(\frac{1}{|\mathbf{z} - \mathbf{z}_i|^{\alpha+2}} + (-\alpha - 2)|\mathbf{z} - \mathbf{z}_i|^{-\alpha-3} \frac{(\mathbf{z} - \mathbf{z}_i)^2}{|\mathbf{z} - \mathbf{z}_i|} \right) \\ &= -\alpha \left(\frac{1}{|\mathbf{z} - \mathbf{z}_i|^{\alpha+2}} - (\alpha + 2) \frac{(\mathbf{z} - \mathbf{z}_i)^2}{|\mathbf{z} - \mathbf{z}_i|^{\alpha+4}} \right) \\ &= \alpha(\alpha + 1) \frac{1}{|\mathbf{z} - \mathbf{z}_i|^{\alpha+2}} \end{aligned} \tag{27}$$

Since there is no element of $\mathcal{S}$ included on $\mathcal{T} \setminus \{a_1, a_2, a_3\}$, then the function g is convex on $\mathcal{T} \setminus \{a_1, a_2, a_3\}$

The sum of convex functions is convex, then over the domain $\mathcal{T} \setminus \{a_1, a_2, a_3\}$, the function

$$g(\mathbf{z}) = \sum_{i \in \mathcal{S}} g(\mathbf{z}, \mathbf{z}_i) \tag{28}$$

is convex over the domain $\mathcal{T} \setminus \{a_1, a_2, a_3\}$.

Any local minimum of a convex function is also a global minimum.

Important note. Notice that the fact that we are restricting our domain to be inside the triangle help us to determine that there is only one global minimum since the domain is a convex subset.

4.4 Gradient descent method

Gradient descent method [4] is based on the observation that if the real-valued function is defined and differentiable in a neighborhood of a point $\mathbf{z}^0$, then the function $g(\mathbf{z})$ decreases fastest if one goes from $\mathbf{z}^0$ in the direction of the negative gradient of g at $\mathbf{z}^0$, $-\nabla g(\mathbf{z}^0)$. Since our objective is to find the minimum of g, then we propose to use the gradient descent method. It follows that, if

$$\mathbf{z}^1 = \mathbf{z}^0 - \delta t \frac{\nabla g(\mathbf{z}^0)}{||\nabla g(\mathbf{z}^0)||} \tag{29}$$

and therefore:

$$\mathbf{z}^{n+1} = \mathbf{z}^n - \delta t \frac{\nabla g(\mathbf{z}^n)}{||\nabla g(\mathbf{z}^n)||} \tag{30}$$

where $\delta t > 0$ is the step size. Note that $g(\mathbf{z}^{n+1}) < g(\mathbf{z}^n)$. First approximate location of minimum interference point, $\mathbf{z}^0$, is the centroid of the triangle:

$$\mathbf{z}^0 = (x^0, y^0) = \left(\frac{x_{p_1} + x_{p_2} + x_{p_2}}{3}, \frac{y_{p_1} + y_{p_2} + y_{p_3}}{3} \right) \tag{31}$$

where p_1, p_2 and p_3 are the vertices of the triangle. In order to ensure that the points $\mathbf{z}^n$ $(n \geq 0)$ lie inside the triangle, we have used the method described in [15]

4.5 Heuristics

Here we propose two heuristics to add K additional base stations at the points of minimum interference identified by the gradient descent method [4]. Let $\mathcal{P}$ be the set of the location of these points of minimum interference.

1. *Heuristic 1:* The optimal position of an additional base station is found as follows.

   (a) Rank points in the set $\mathcal{P}$ in the ascending order of interference or signal level received from all base stations in the set $\mathcal{S}$.

   (b) Select the lowest ranked element p from the set $\mathcal{P}$. The location of p shall become the position of the additional base station.

   (c) Remove p from the set $\mathcal{P}$, *i.e.*, $\mathcal{P} := \mathcal{P} - p$.

   (d) For additional base stations, repeat from step (b).

2. *Heuristic 2:* The addition of a base station at minimum interference point in set $\mathcal{P}$ may increase the interference at other points of set $\mathcal{P}$ located in the immediate vicinity. Therefore, remaining points in set $\mathcal{P}$, $\{\mathcal{P} - p\}$, should be re-ranked. The Delaunay triangulation in this case will be provided by the Bowyer-Watson algorithm which gives another approach for incremental construction. It gives an alternative to edge flipping for computing the Delaunay triangles containing a newly inserted vertex.

   Steps (a) and (b) of *Heuristic 2* are the same as in *Heuristic 1*. After addition of a base station, the region of interest is re-triangulated to identify new points of minimum interference. In other words, the points of minimum interference in the region of interest shall be identified before the addition of all new base stations.

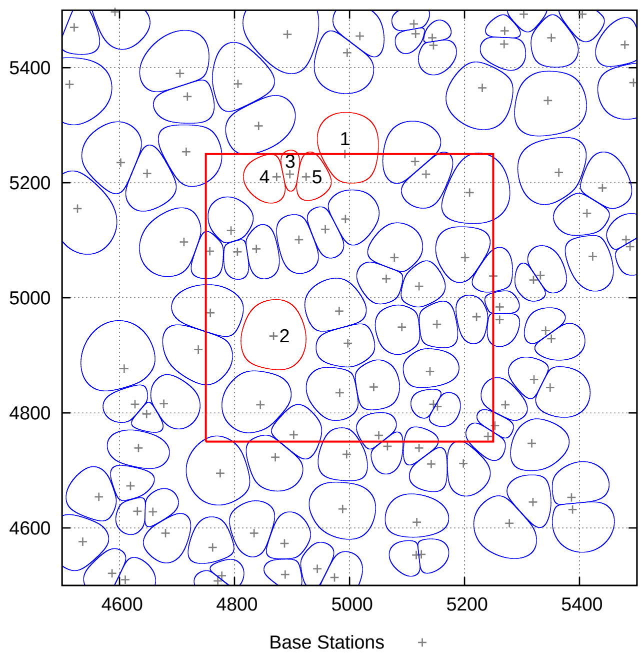

Figure 3: 5 base stations, denoted 1 to 5, are added in the region bounded by a square of side 500 km using *Heuristic 1*. The number indicates the order of addition to the network. $\beta = 1$ and $\alpha = 4.0$.

5. NUMERICAL SIMULATIONS

We consider a wireless data network over an area of 10000×10000 square km and base stations distributed according to a Poisson point process (PPP) of intensity $\lambda = 1$ base station per unit square mile. We consider the path loss exponent $\alpha = 4$.

In order to ignore edge effects, while computing minimum interference points, we assume that our region of interest lies in the center of this network and is a square region with each side of length 500 km.

Figure 1 shows the β-reception areas of the base stations distributed according to a Poisson point process of intensity $\lambda = 1$. It also shows the region of interest marked by a thick lines inside the network.

Figure 2 shows the process of identification of the points of minimum interference in this region of interest. Locations marked by 'x', show the first approximation of the points of minimum interference. Gradient descent method uses the first approximate position to arrive at the final location, marked by 'o', which is the true point of minimum interference in the region formed by the Delaunay triangle. Note that, in cases where a triangular region lies partially within the region of interest, we are interested in identifying the point of minimum interference lying within the region of interest.

Figures 3 and 4 shows the positioning of 5 additional base stations in the network by using the approaches of *Heuristic 1* and *Heuristic 2* respectively. In order to show the improvement achieved by the two heuristics, we have computed the total area covered by the base stations with SINR at least equal to β, *i.e.*, $\sum_{i \in \mathcal{S}'} \mathcal{A}_{\mathcal{R}i}(\beta)$ where $\mathcal{S}'$ is the set of base stations, also including the additional ones in case of *Heuristic 1* and *Heuristic 2*, lying in the region of interest.

Scenario 0 indicates the existing wireless data network to which we want to increase its coverage and its capacity.

	Scenario 0	*Heuristic 1*	*Heuristic 2*
Capacity	1.3559	1.5614	1.7007
Percentage Increase		15.15%	25.42%

Table 1: *Average capacity over the surface $[bits/s/Hz/km^2]$ over the network and percentage increase of the capacity under different heuristics*

	Scenario 0	*Heuristic 1*	*Heuristic 2*
Total coverage area	168776	190501	204628
Coverage percentage	67,51%	76,20%	81,85%
Percentage Increase		12.87%	21.25%

Table 2: *Coverage of the network (in square km) and percentage increase of the coverage under different heuristics*

Table 1 summarizes the results obtained by both heuristics. We compute the capacity over the network under three different scenarios: Scenario 0 which indicates the capacity on the existing wireless data network, and *Heuristic 1* and *Heuristic 2* explained above.

Table 2 summarizes the results obtained by both heuristics. We compute the coverage of the network under the three different scenarios described above.

6. CONCLUSIONS

We have studied the optimal placement and optimal number of base stations added to an existing wireless data network through the interference gradient method. The problem of finding the optimal location of a set of new base

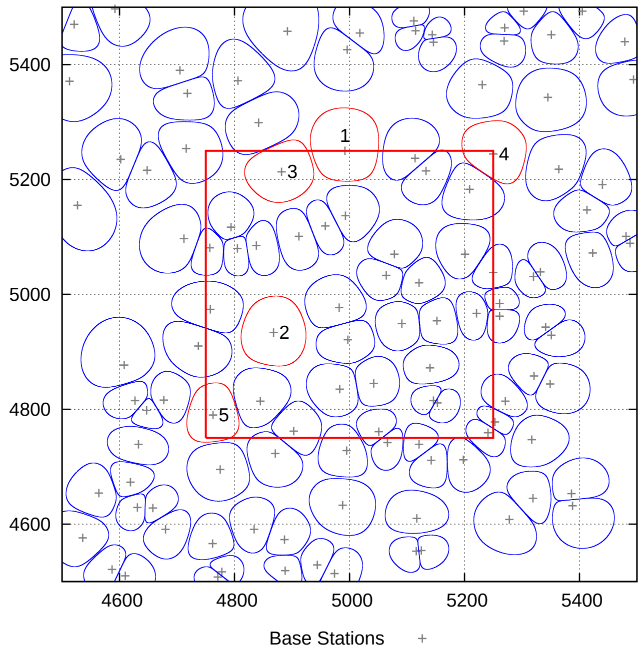

Figure 4: 5 base stations, shown in red, are added in the region bounded by a square of side 500 km using *Heuristic 2*. The number indicates the order of addition to the network. $\beta = 1$ and $\alpha = 4.0$.

stations when there is a discrete number of mobile users is NP-Hard. The problem considered is even harder when we consider a random distribution of the mobile users even for the uniform distribution. Our proposed method considers a sub-region of the existing wireless data network (hereafter called region of interest), where the service provider wants to increase the coverage and throughput of the users. Our approach is based on the Delaunay triangulation of the region of interest (this triangulation completely covers the region of interest), and it takes as the vertices of this triangulation the positions of the existing base stations. We prove that over each triangle there is only one global minimum of the interference function. In consequence, there will be as many candidates of minima over the region of interest as triangles. Then the problem is reduced to find over this set the minimum interference set. We propose two heuristics to find the minimum interference points. Numerical simulations show that the coverage and the throughput through this method are highly incremented.

7. PERSPECTIVES

There are many possible extensions to our work, but we think that the more interesting ones are the following:

- If we want to take into consideration a more realistic LTE (OFDMA) system, the mobile user may receive a set of sub-carriers.
- We have neglected the fact that the distribution of the users may be non-homogeneous over the region of interest.
- In a future work, the shadowing fluctuations over the network will be taken into account.

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