The Optimal User Scheduling for LTE-A Downlink with Heterogeneous Traffic Types

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***Abstract*—The current mobile broadband market experiences major growth in data demand and average revenue loss. To remain profitable from the perspective of a service provider (SP), one needs to maximize revenue as much as possible by making subscribers satisfied within the limited budget. On the other hand, traffic demands are moving toward supporting the wide range of heterogeneous services with different quality of service (QoS) requirements. In this paper, we consider *packet scheduling* problem in the 4th generation partnership project (3GPP) long term evolution-advanced (LTE-A) system to optimize the *long-term average revenue* of SPs subject to differential QoS constraints for *heterogeneous* traffic demands. The QoS-constrained control problem is first formulated as a constrained Markov decision process (CMDP) problem, of which the optimal control policy is achieved by utilizing the channel and queue information simultaneously. Subsequently, based on the proposed CMDP problem, we further formulated an optimization problem which stochastically grantees the QoS through a chance constraint. To make the proposed chance-constraint programming problem computationally tractable, we use *Bernstein approximation* technique to analytically approximate the chance constraint as a convex conservative constraint. Finally, the proposed scheduling framework and solution methods are validated via numerical simulation.**

***Index Terms*—resource scheduling; constrained Markov decision process; Bernstein approximation; LTE-A; heterogeneous delay requirements**

I. Introduction

As mobile broadband traffic demand shifts from voice-dominated to data-dominated traffic, the SP's revenue is not keeping pace with the dramatic increase in traffic volume [1]. In order to remain profitable, SPs are looking at ways to reduce their costs and improve their revenues. On the other hand, subscribers require SPs to ensure their QoS for the wide range of heterogeneous services. To provoke such a scheme to track the revenue rather than the demand, while fulfilling the stringent QoS guarantees, one requires an effective *resource scheduling* framework. The 3GPP LTE-A, as the fourth generation of cellular network mobile communication standard, promotes a flexible resource scheduling by allowing SP's desired algorithms to be developed. However, all the key parameters required to design a resource scheduler such as all signalling and users' QoS requirements are specified in details in the 3GPP LTE-A standard [2].

In LTE-A, the base station (eNodeB) schedules units of time-frequency resources known as *resource blocks* among LTE users. It is trivial to show that SP can maximize its revenue by allocating the resource blocks to the users which make the best profit based on channel condition. However, this resource allocation scheme may suffer from the violation of the 3GPP LTE-A scheduling constraints and QoS requirements as described in the following: First, although orthogonal frequency division multiple access (OFDMA) as the downlink radio access technology of the LTE-A system allows multiple resource blocks with different data rates to be assigned to a single user, 3GPP standard does not support multiple data rates for a single user in order to avoid excessive signaling overhead. Thus, to have a 3GPP standard-compliant resource scheduler we select a common modulation and coding scheme (MCS) over all resource blocks assigned to a user in our scheduling policy (refer to section 10.2 in [1]). This constraint is previously considered in multiple related works such as [3] and [4], whereas the formulated scheduling problems proved as NP-hard in these works. Authors in [5] achieved the optimal solution for the same scheduling problem by proving the total-unimodularity of the formulated problem and solved it as an standard linear programming problem. Second, the control policies which are only adaptive to channel variations can not guarantee the delay requirements for the real life applications. To fulfill the QoS requirement of the 3GPP LTE-A standard, the control policy should be designed based on both channel state information (CSI) and queue state information (QSI). By doing so, we can associate the users' traffic dynamics and channel variations with the SP's revenue. There are quite a number of works that considered the channel and queue information jointly and proposed a scheme for packet scheduling in OFDMA systems such as maximum-largest weighted delay (M-LWDF) [6], but most of them are not proper to use in the presence of the heterogeneous traffic since they do not provide bounded delay performance [7].

In this paper, an underlying information-theoretic principle is combined with a queuing-theoretic approach and attempts to achieve guaranteed QoS for users as well as maximum revenue for the SP. We consider a pricing structure that charges proportionally as the usage increases. So, the maximum achievable data rates by users are used as the revenue incentive for the SP. Two mathematical frameworks are proposed. Essentially, our proposed scheduling frameworks assign each resource block to the best user and select the best corresponding MCS for each user while maximizing the overall system performance (e.g., SP's revenue) and guaranteeing the QoS

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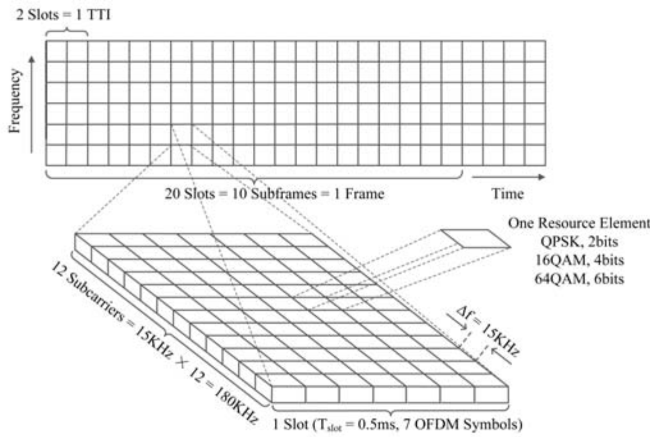

Fig. 1: Time-frequency resources in LTE-A

requirement for heterogeneous services. Our first mathematical framework is formed based on constrained Markov decision process (CMDP) which maximizes the long-term average SP's revenue subject to long-term average queue length constraint. The Lagrangian dynamic programming approach is used to convert the constrained MDP to unconstrained MDP. The optimal control policy for unconstrained MDP is obtained by solving the well-known *Bellman's equation* using value iteration method. As our second framework, we propose another formulation which maximizes the instantaneous revenue and provides the QoS provisioning for heterogeneous traffic using a set of stochastic constraints, e.g., chance constraints. To preserve the convexity and reduce the complexity of chance constraint programming, we use Bernstein approximation [8] to obtain a conservative and deterministic approximation of the affine chance constraints.

The rest of the paper is organized as follows. Section II gives an overview of main LTE-A features followed by the system model. The scheduling problems proposed CMDP framework are formulated in section III. We propose the method to solve the CMDP problem in section IV. In section V, the chance constrained revenue optimization problem is formulated and the solution to the stochastic optimization is proposed using Bernstein approximation. Section VI presents the performance of our scheduling scheme. Finally, section VI draws the conclusions.

II. Model

In this section, the OFDMA downlink system model and queueing model are outlined. The simplified architecture of the LTE-A downlink packet scheduler in the eNodeB of the LTE-A system is shown in Fig. 2. At the beginning of each decision epoch, eNodeB receives CSI from the users and captures QSI by observing the users' buffer. The packet scheduler in eNodeB makes a decision using this information and based on the scheduling policy and passes the resource allocation scheme to the radio access unit. The technology to access the radio spectrum in downlink is OFDMA, which divides the bandwidth into a series of flat fading narrow bands [2]. The radio resource divisions of the LTE-A SP in time-frequency domains are shown in Fig. 1. One resource block corresponds to 180 kHz in the frequency domain and one slot (*0.5 ms*) in the time domain. The minimum resource allocation unit is one scheduling block which is comprised of two consecutive resource blocks spanning a time duration of 1 ms known as transmit time interval (TTI). The update of the CSI, QSI and also the resource scheduling decision are carried out once every TTI.

A. Physical Layer

Consider a downlink OFDMA multiuser LTE-A system, let $\mathcal{U}$, $\mathcal{R}$, $\mathcal{M}$ and $\mathcal{P}$ be the sets of users, resource blocks, MCS schemes and price respectively. Define $U = |\mathcal{U}|$, $R = |\mathcal{R}|$, $M = |\mathcal{M}|$ and $P = |\mathcal{P}|$, where $|\cdot|$ represents the cardinality of a set. The channel between the eNodeB and any user $i \in \mathcal{U}$ is modeled as a frequency selective block fading channel, assuming that each resource block channel condition remains unchanged during a time interval of length TTI. At each TTI n, every user $i \in \mathcal{U}$ measures the signal to interference plus noise ratio (SINR) of the reference signals transmitted by the eNodeB over the channel, quantizes the SINR values and reports a channel quality indicator (CQI) vector $\mathbf{c}_i(n)$ to the eNodeB containing the $c_{ij}(n)$ values for all resource blocks $j \in \mathcal{R}$. Afterwards, eNodeB forms the CQI matrix $\mathbf{C}(n) = [\mathbf{c}_i(n)]$ and selects suitable set of MCS indexes corresponding to the $\mathbf{c}_i(n)$ to ensure a certain block error rate target (typically $< 10\%$) is met while achieving the highest transmit block size. Based on the 3GPP LTE-A standard, the scheduler should select a common MCS for each user over all resource blocks assigned to it at each TTI (refer to section 10.2 in [2]).

Denote $\mathbf{x}(n) = \{x_{ij}^m(n)\}$ as the resource block allocation strategy at TTI n, where $x_{ij}^m(n) = 1$ represents that resource block $j \in \mathcal{R}$ is assigned to user i with MCS m at TTI n. Accordingly, denote $r_{ij}^m(n)$ as achievable data rate when $x_{ij}^m(n) = 1$. Further denote $\mathbf{r}(n) = (r_1(n), \cdots, r_U(n))$ as the achievable data rate of the users at TTI n, where $r_i(n)$ is

$$r_i(n) = \sum_{j \in \mathcal{R}} \sum_{m \in \mathcal{M}} r_{ij}^m(n) x_{ij}^m(n). \tag{1}$$

Consider $\mathcal{P}(n) = \{p_{ij}(n)\}$ as the price set for all users $i \in \mathcal{U}$ over all resource blocks $j \in \mathcal{R}$. In this work, a pricing structure that assigns a rate per unit of usage and charges proportionately as the usage increases is used [9]. $p_{ij}(n)$ is associated with i-th user' maximum achievable data rate over resource block j, which can be expressed by

$$p_{ij}(n) = \alpha r_{ij}(n), \tag{2}$$

where α is the constant coefficient to charge the user per unit of data rate and r_{ij} is the maximum achievable data rate for user i over resource block j at TTI n. Basically, r_{ij} is used as the revenue incentive for the SP. Denote extra auxiliary MCS assignment strategy $d_i^m(n)$, where $d_i^m(n) = 1$ represents that user i chooses MCS m at TTI n based on the scheduling policy. We have the following widely-used assumption regarding the channel gains:

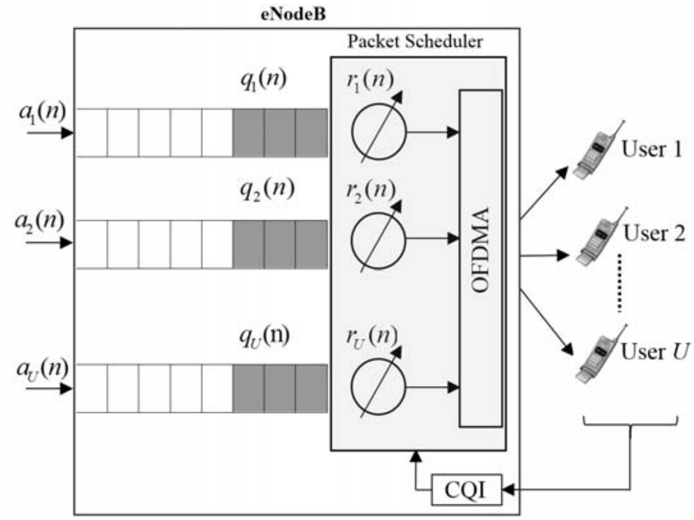

Fig. 2: Packet scheduling in the eNodeB of the LTE-A system

Assumption 1: The sequence of the fading channel variations follow an ergodic discrete time Markov chain [10]. It is also assumed that channel states are exactly known (or fully observed).

B. Source Model and Queue Dynamics

In this paper, we adopt a queuing model such that each user has a queue in the eNodeB (see Fig. 2). Denote $\mathbf{q}(n) = (q_1(n), ..., q_U(n))$ to be the queue lengths of the users, where $q_i(n)$ represents the number of bits in the i-th user's queue. Further denote $\mathbf{a}(n) = (a_1(n), ..., a_U(n))$ to be the incoming traffic within the n-th time interval, where $a_i(n)$ and $\overline{a}_i$ represent the number of arrival bits and the average arrival rate to the i-th user's queue, respectively. We put the following assumption for the incoming traffic:

Assumption 2: For all i and n, the random variables of the incoming traffic possess finite mean and finite variances, meanwhile, they are independent and identically distributed (i.i.d) over decision epochs.

In the n-th TTI, a batch of $\mathbf{a}(n)$ bits in the form of packets arrive, followed by the departure of $\mathbf{r}(n)$ bits. We assume the incoming traffic $\mathbf{a}(n)$ are captured after the packet scheduler's decision at time n. The value of $\mathbf{a}(n)$ is endogenous parameter, whereas $\mathbf{r}(n)$ is exogenous and affected by the SP's action. Hence, the evolution of the queues can be written as

$$\mathbf{q}(n+1) = [\mathbf{q}(n) - \mathbf{r}(n)]^+ + \mathbf{a}(n), \tag{3}$$

where $[\mathbf{x}]^+$ is componentwise operator defined as $\max\{0, x_i\}$.

C. Control Policy

Consider $\mathcal{S}(n) = \{\mathbf{C}(n), \mathbf{q}(n)\}$ to be the system state space, which composes of channel quality information and buffer state information. A policy is stationary if the decision rule is independent of the decision epochs. In our work, we assume a stationary and deterministic scheduling policy $\Omega_s = (\Omega_\mathcal{R}, \Omega_\mathcal{M})$ which is a mapping function from system state space $s \in \mathcal{S}$ to the set of resource blocks and MCSs allocation action spaces, which are given by $\Omega_\mathcal{R}(s) = \big\{x_{ij}^m \in \{0,1\}, \forall i \in \mathcal{U}, j \in \mathcal{R}, m \in \mathcal{M}\big\}$ and $\Omega_\mathcal{M}(s) = \{d_i^m \in \{0,1\}, \forall i \in \mathcal{U}, m \in \mathcal{M}\big\}$, respectively.

The policy $\Omega_s = (\Omega_\mathcal{R}, \Omega_\mathcal{M})$ should satisfy the practical constraints required by 3GPP LTE-A standard for all s. These constraints can be summarized as (i) each resource block can be assigned to only one user, (ii) each user can choose one MCS over all resource blocks assigned to it (iii) for each user the MCS is assigned only over those resource blocks assigned to it, otherwise the related MCS indicator is assigned to be zero (iv) the decision variables for resource blocks assignment and MCS assignment can take only zero or one values. With these physical constraints, the *per stage* constraint be mathematically modeled as

$$\textstyle\sum_{i\in\mathcal{U}} \sum_{m\in\mathcal{M}} x_{ij}^m(n) \le 1, \qquad \forall j, n \tag{4}$$

$$\textstyle\sum_{m\in\mathcal{M}} d_i^m(n) \le 1, \qquad \forall i, n \tag{5}$$

$$x_{ij}^m(n) \le d_i^m(n), \qquad \forall i, j, m, n \tag{6}$$

$$x_{ij}^m(n), d_i^m(n) \in \{0,1\}, \qquad \forall i, j, m, n. \tag{7}$$

We limit our policy space to unichain policies [11] and [12]. Given a unichain policy Ω_s, the induced Markov chain is ergodic and there exists a unique steady state distribution. Therefore, we have from the little's theorem that the average delay of the user i under policy Ω_s is according to:

$$\overline{D}_i^\Omega = \lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N-1} \frac{\mathbb{E}^\Omega[q_i(n)]}{\overline{a}_i}, \tag{8}$$

where $\mathbb{E}^\Omega$ is the expectation under the stationary policy Ω_s. Within the LTE-A network, the QoS requirements of heterogeneous services are classified to nine QoS class identifier (QCI) based on their tolerable packet delay budgets and packet error loss rates (refer to Table 2.1 in [2]). For example web-browsing can tolerate delay up to 300 *ms* with maximum 10^{-6} packet loss rate. To consider different delay requirements associated with different QCIs, heterogeneous queue thresholds $\beta = \{\beta_i, \forall i\}$ are assigned for users that uses different services. For example, for the user which uses a service with tighter delay budget, we assign smaller β. To guarantee the delay requirements of different types of traffic we have

$$\limsup_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N-1} E^\Omega[\mathbf{1}_{\{q_i(n)>\beta_i\}}] \le d_i, \ \forall i. \tag{9}$$

where d_i is the maximum bound for the expected average queue length of user i.

III. Proposed MDP Framework

In this section, we start to formally formulate the SP's revenue maximization problem. Based on the i.i.d traffic model and (3), the transition probabilities among the states can be

$$\mathrm{P}[s'|s,\Omega_s] = \mathrm{P}[s(n+1) \mid s(n)=\{\mathbf{C}(n),\mathbf{q}(n)\},\Omega_s(n)] = \mathrm{P}[\mathbf{C}(n+1) \mid \mathbf{C}(n)] \,.\, \mathrm{P}[\mathbf{q}(n+1)|\{\mathbf{C}(n),\mathbf{q}(n)\},\Omega_s(n)] \tag{10}$$

written as (10), which can be further compactly written as

$$\mathrm{P}[s'|s,\Omega_s] = \\ \mathrm{P}[\mathbf{C}(n+1)|\mathbf{C}(n)]\cdot\mathrm{P}\big[\mathbf{a}(n)=\mathbf{q}(n+1)-[\mathbf{q}(n)-\mathbf{r}(n)]^+\big]. \tag{11}$$

Due to the independence of incoming traffic among different users, the system state transition follows Markovian property. Therefore, the transition probability of the Markov chain can be further written as

$$\mathrm{P}[s'|s,\Omega_s] = \mathrm{P}[\mathbf{C}(n+1) \mid \mathbf{C}(n)]\cdot\prod_i \mathrm{P}[a_i(n)]. \tag{12}$$

In our work, we are interested in finding the optimal policy, denote by Ω_s^*, such that the long-term average reward over an infinite time horizon is maximized subject to all the per-stage resource allocation constraints (5) - (8) and the queue length constraint in (9). Mathematically, the problem is given by

$$\textbf{(P1):}\ \underset{\Omega}{\text{maximize}}\quad \limsup_{N\to\infty}\frac{1}{N}\sum_{n=0}^{N-1}E^{\Omega}[\sum_{i\in\mathcal{U}}\sum_{j\in\mathcal{R}}(p_{ij}(n)\sum_{m\in\mathcal{M}}x_{ij}^m(n))] \\ \text{subject to}\quad \text{constraint } (5)-(9)$$

The SP's revenue is affected by the queue threshold value of different users. When the delay requirement of a user is more stringent (lower β), more resource blocks are seized by the user regardless of its possible bad channel quality over the seized resource blocks. This reduces the chance of assigning those resource blocks to users with better channel condition and incur revenue loss for the SP.

Remark 1: Note that the optimal user scheduling problem with delay requirement **(P1)** is a constrained MDP in essence, which is widely used to deal with dynamical, multi-objective, decision problems. Without constraint (9), **(P1)** can be easily resolved using traditional value iteration or policy iteration method [13]. However, it becomes technically challenging since the delay requirement (9) may couple all the sequential decisions in addition to per-stage resource constraints. In the section IV, we shall introduce the concept of *Marginal Delay Cost* as an Lagrange Multiplier, which can be proved to be efficient in solving **(P1)** with optimality guarantee under some conditions.

IV. Marginal Delay Cost and the Optimal Scheduling Policy

A. Marginal Delay Cost and Optimal Condition

In this section the optimal solution for **(P1)** is studied. We define the marginal delay cost for user i, which is denoted by λ_i, as the Lagrange multiplier for the delay constraint (9). Consider the following problem:

$$\textbf{(P2):}\ \underset{\Omega}{\text{maximize}}\quad \limsup_{N\to\infty}\frac{1}{N}\sum_{n=0}^{N-1}E^{\Omega}\Big[\sum_{i\in\mathcal{U}}\sum_{j\in\mathcal{R}}\Big(p_{ij}(n)\sum_{m\in\mathcal{M}}x_{ij}^m(n) \\ -\lambda_i[\mathbf{1}_{\{q_i(n)>\beta_i\}}]\Big)\Big] \\ \text{subject to}\quad \text{constraints } (5)-(8),\ \text{and } \lambda_i\geq 0\forall i.$$

Let $R^{\Omega}(s(n),\Omega_{s(n)})$ be the per stage reward that SP can achieve by choosing resource block action under the policy Ω_s when the system state is s. Define the reward function at stage n as

$$R^{\Omega}(s(n),\Omega_{s(n)}) \\ = E^{\Omega}[\sum_{i\in\mathcal{U}}\sum_{j\in\mathcal{R}}p_{ij}(n)\sum_{m\in\mathcal{M}}x_{ij}^m(n)-\lambda_i[\mathbf{1}_{\{q_i(n)>\beta_i\}}]]. \tag{13}$$

Note that the optimal user scheduling problem **(P2)** with marginal delay cost λ_i is equivalent to the Lagrangian function of problem **(P1)** after incorporating the delay constraint (9) into the objective function. Furthermore, we have the following theorem to show the relationship between **(P1)** and **(P2)**. The detailed proof is omitted in this paper for brevity, we have a similar and detailed proof given in our previous work in [14].

Theorem 1: *Let Ω^* and λ_i be the optimal policy and the delay cost for problem **(P2)**. If the delay constraint* (9) *is strictly binding with policy Ω^*, then the policy Ω^* is also optimal for problem **(P1)** with λ_i serving as the corresponding optimal Lagrange multiplier.*

Note that if β_i is sufficiently large, such that the delay constraint (9) for user i is slack at the optimum of problem **(P1)**, the value of $\lambda_i = 0$. This can be trivially understood by that the user can tolerate large delay such that the SP does not need to consider any delay cost. When β_i is not sufficiently large, the value of λ_i can be determined by using the bisection method, which can iteratively reach the specified average delay β_i. Denote $\Omega_{\lambda_i}^*$ as the optimal scheduling policy for a given Lagrangian multiplier λ_i. The mariginal delay cost for the scheduling policy $\Omega_{\lambda_i}^*$ is given by

$$D^{\Omega_{\lambda_i}^*} = \limsup_{N\to\infty}\frac{1}{N}\sum_{n=0}^{N-1}E^{\Omega_{\lambda_i}^*}[\mathbf{1}_{\{q_i(n)>\beta_i\}}],\ \forall i. \tag{14}$$

It was proved in [15] that $D^{\Omega_{\lambda_i}^*}$ is a piecewise linear non-increasing function of λ_i. We can find the optimal Lagrangian multiplier λ_i^* through the following update:

$$\lambda_i^{l+1} = \max(\lambda_i^l + \gamma^l(D^{\Omega^*_{\lambda_i^l}} - d_i), 0), \tag{15}$$

where $\gamma_i^l = \frac{1}{l}$. The convergence to λ_i^* is ensured due to the fact that $D^{\Omega_{\lambda_i}^*}$ is a piecewise convex function of Lagrangian multiplier λ. Later on in this paper, we consider that the marginal delay cost is predetermined to reach a certain average delay requirement. By doing so, two advantages that facilitate our following performance analysis and the implementation of this scheduling policy in practical system are:

- The existence of an optimal policy that is both deterministic and stationary for Problem **(P2)** has already been shown in [11], in which significantly reduces the implementation complexity. Similar results also appeared in energy-efficiency and delay-constraint problems in Cognitive Radio networks [14].
- The marginal delay cost λ_i represents the delay sensitivity of the traffic of user i. By using the fixed λ_i, we can evaluate how the delay sensitivity of user traffic influences the optimal scheduling policy of the SP. Moreover, we can analyze the impact of multiple users with heterogeneous delay requirements on the optimal scheduling policy, which actually matches more to the real system.

B. Optimal Scheduling Policy

The structure of our MDP problem can be expressed as follows:

- State: $\mathcal{S} = \{\mathbf{C}, \mathbf{q}\}$
- Action: $\{x_{ij}^m \in \{0,1\}\}, \{d_i^m \in \{0,1\}\}$
- Reward: R^{Ω} is given in (13).
- Transition probability matrix: $\mathrm{P}[s'|s,\Omega_s]$ is given in (10).

All per stage constraints in (5) - (8) are formed the action space. In this problem, the channel condition variations across the users provides the opportunity for the SP to increase its revenue by using multiuser diversity gain. However, the SP should make sure that it satisfies the delay requirement of the traffic of different types. When the SP applies a stationary policy Ω, the induced Markov chain is recurrent and the optimal long-term average sum revenue is independent of the initial state. Under the unichain policy assumption, there exists an optimal control policy Ω^* for the problem given in (13), such that for any state $s \in \mathcal{S}$ the following *Bellman's equation* is satisfied [13].

$$V^*(s) = \max_{\Omega}\big\{R^{\Omega}(s,\Omega_s) + \sum_{s'\in\mathcal{S}} \mathrm{P}[s' \mid s,\Omega_s]V(s')\big\}, \forall s \in \mathcal{S} \tag{16}$$

where $V^*(s)$ is the optimal value function for state s and $R^{\Omega}(s,a)$ is the reward function defined in (13). With the stationarity assumption, time index n is eliminated. The *Bellman's equation* can be derived numerically through *Value Iteration* algorithm [13].

Note that by the traffic delay requirement in (9), the expected delay of each user is guaranteed to be exactly not larger than the corresponding tolerant, which is, however, somehow conservative for the SP since violation of the delay requirement with small probability may be acceptable for some users. Therefore, we will propose a novel revenue maximization framework with chance constraint in the next section.

V. Revenue Maximization Framework with Chance Constraint

In this section, we formulate the optimization problem of the SP's revenue based on stochastic chance constraint on queue length of the users. Since the queue state of the users vary in a slower time scale than the channel state of the users, we consider the slow fading channel, in which the source of randomness in the SINR value is from long-term channel variations i.e., path loss and shadowing effects. Consider the channel gain g_{ij} for user i at resource block j to follow a general probability density function (PDF) $f_{g_{ij}}(\xi)$. The achievable data rate $r_{ij}(n)$ for user i over resource block j at time n can be expressed as

$$r_{ij}(n) = W\log_2(1+\frac{pg_{ij}}{\Delta\sigma^2}) \tag{17}$$

where σ^2 is power of additive white Gaussian noise as background noise and Δ is the capacity gap for the bit error rate and MCS.

The stochastic QoS guarantee for users with heterogeneous service classes can be expressed as a chance constraint on queue length of the users as follows

$$\mathrm{P}\big\{[q_i(n) - \sum_{j\in\mathcal{R}}\sum_{m\in\mathcal{M}} r_{ij}^m(n)x_{ij}^m(n)]^+ + a_i(n) < \beta_i\big\} \leqslant \epsilon_i \ \forall i, \tag{18}$$

For the given current queue state information (18) grantees the overflow probability of the user i to be below a predefined threshold ϵ_i in the next time frame. Now, we can model the revenue maximization problem as follows

$$\begin{aligned} \textbf{(P3):}\ \underset{\mathbf{x},\mathbf{d}}{\text{maximize}} \quad & \sum_{i\in\mathcal{U}}\sum_{j\in\mathcal{R}}(p_{ij}(n)\sum_{m\in\mathcal{M}}x_{ij}^m(n)) \\ \text{subject to} \quad & \text{constraints } (5)-(8), \text{ and } (18) \end{aligned}$$

The chance constraint (18) in **(P3)** makes the optimization highly intractable due to difficulty of verifying its convexity. In this work, Bernstein approximation [8] is used to obtain a conservative convex approximation of the affine chance constraint in (18). Bernstein approximation is a recent advance in the field of chance constraint programming that provides a tractable conservative deterministic and convex approximation for the chance constraint [16].

Proposition 1: *The stochastic queue-constrained problem in* ***(P3)*** *can be approximated by the deterministic and convex optimization problem defined as*

$$\begin{aligned} \textbf{(P4):}\ \underset{\mathbf{x},\mathbf{d}}{\text{maximize}} \quad & \textstyle\sum_{i\in\mathcal{U}}\sum_{j\in\mathcal{R}}(p_{ij}(n)\sum_{m\in\mathcal{M}}x_{ij}^m(n)) \\ \text{subject to} \quad & \inf_{\varrho_i}\{\Psi_i(\mathbf{x},\zeta_\mathbf{i}) - \varrho_i\log\epsilon_i\} \le 0,\ \forall i \quad (19) \\ & \text{constraints } (5)-(8) \end{aligned}$$

where $\Psi_i(\mathbf{x},\zeta_\mathbf{i}) = m_i - q_i + \varrho_i\sum_{j\in\mathcal{R}}\sum_{m\in\mathcal{M}}\Lambda_{r_i}(\varrho_i^{-1}x_{ij}^m) + \varrho_i\Lambda_{a_i}(-\varrho_i^{-1})$. Note that we denote Λ_{r_i} and Λ_{a_i} as the cumulant (log-moment) generating function of the data rate and arrival process, respectively.

Proof: See Appendix A.

According to [8], the chance constraint in (18) holds if there exists a $\varrho_i > 0$ satisfying constraint (19) in **(P4)**. Note that Problem **(P4)** is a convex optimization problem which has a convex subproblem that requires to minimize function $\Psi_i(\mathbf{x},\zeta_\mathbf{i}) - \varrho_i\log\epsilon_i$ over ϱ_i. According to (23), $\Psi_i(\mathbf{x},\zeta_\mathbf{i})$ is convex and differentiable over ϱ_i. Therefore, it is always easy to obtain the minimum of function $\Psi_i(\mathbf{x},\zeta_\mathbf{i}) - \varrho_i\log\epsilon_i$ by setting the first derivative to be 0. However, it is far from trivial to solve the Problem **(P4)** directly by using standard solvers due to the calculation of the subproblem ”on-the-fly”. In this paper, the Problem **(P4)** is solved by the logarithmic barrier cutting plane method proposed in [16], we omit the details in this paper due to space limit.

VI. Performance Evaluation

In this section, the performance bound of the proposed scheduling policies are evaluated in terms of SP's revenue and delay for LTE-A downlink system with heterogeneous traffic types according to QCI Table in the 3GPP LTE-A standard (refer to Table 2.1 in [2]). We assume a multi-cell system with hexagonal grid of equidistantly-spaced eNodeB sites. The target cell is set to be the center cell with 6 neighbor cells, whereas users of the center cell do not leave it within the simulation. The proposed scheme is implemented in MATLAB. Without loss of generality, we assume the *frequency granularity* of the CQI measurement to be one RB and the period of the CQI reporting to one TTI. We set the decision epochs to be equal to the same time duration of the CQI reports, e.g. 1 TTI. Table I shows the list of parameters that are considered in our implementation.

TABLE I: Implementation Parameters

Parameter	**Value**
Antennas Configuration	7 hexagonal grid equidistantly -spaced eNodeB sites
Traffic Model	Backlogged traffic model
Total no. of RBs	6
CQI report period	1 ms
Frequency granuality for CQI	One RB
ARQ process	Zero transmission attempt
TTI	1 ms

A network with three users and two distinct classes of traffic in terms of arrival rate and queue threshold are used such that the arrival rates are 0.1packet/TTI and 0.2packet/TTI and queue thresholds are 120kb/TTI and 50kb/TTI for class 1 and class 2, respectively. A user from class 2 represents a less delay-sensitive user, while a user from class 2 represents a user with stringent delay requirements. A fixed packet size of 100 kb is considered for both classes of traffic. The incoming traffic rates and delay requirements are determined in a way that do not result in long queue threshold, therefore the system is stable and state space dimension does not increase due to large queue size. Let assume SP acquires one unit of currency (1 USD) for every 1 kb of data it transmits to each user at each TTI. The small numbers are considered for simplification, otherwise considering of large numbers do not changes the applied method.

In the first test, the SP's revenue is evaluated for different combinations of users of different classes. The results presented in Table II show that generally by increasing the number of users, e.g., compare row 1 against other rows, the system achieves higher revenue. However, the improvement in revenue is less when a users with more stringent delay constraint is added to the system, e.g. row 3 achieves less revenue than row

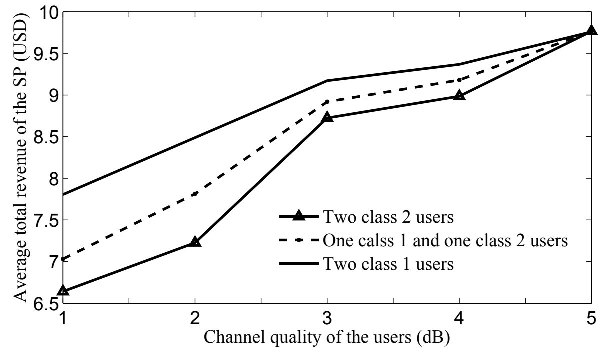

Fig. 3: Average total revenue of the SP for different channel qualities

2. We also compared the SP's revenue when there are three users of class 1 compare to the case that there are three users of class 2. As it is shown in rows 3 and 6 in Table II, SP's revenue is higher when users have more relaxed delay requirement in the system.

TABLE II: SP's Revenue for Different Number and Classes of the Users

	Number and class of users	**Average total service SP's revenue (USD)**
1	One class 1	5.1875
2	Two class 1	8.5417
3	Three class 1	10.3594
4	Two class 1 and one class 2	8.918
5	Two class 2	7.593
6	Three class 2	8.0218

The delay requirement violation for different classes of traffic is compared in Table III. The result in row 1 in Table III shows a zero average delay violation when there is a single class one user in the system. When the number of users with stringent delay requirement increases, it results in more delay violation as shown in rows 2 and 3.

TABLE III: Delay Constraint Violation for Different Number and Classes of the Users

	Number and class of users	**Delay requirement violation**
1	One class 1	0
2	Two class 1	0.3167
3	Two class 2	0.6073
4	Two class 1 and one class 2	0.6934

Next, we examine the revenue under different channel qualities for two classes of the users. The channel quality of the users over all the states $\mathcal{S}$ is quantized to 5 levels, in which channel quality 5 is the best. The Average total revenue over different channel quality levels is compared when there are *case1*: (1,1), *case2*: (2,0) and *case3*: (0,2) number of class 1 and class 2 users, respectively. The results in Fig. 3 shows that when the channel condition improves, SP's revenue increases as well. However, the improvement in the revenue is higher when delay requirement of the users is less stringent.

VII. Conclusion

In this paper, we proposed two scheduling policies to maximize the LTE-A SP's revenue subject to the heterogeneous QoS constraints of the users, as well as satisfying the resource scheduling constraints of the LTE-A system according to the 3GPP LTE-A standard. The mathematical frameworks were formed based on CMDP and chance constrained problems. The key contribution in chance constrained problem is to use Bernstein approximation to transform the chance constrain to a convex, deterministic and computationally tractable constraint.

Appendix A
Proof of Proposition 1

To apply the Bernstein approximation for the constraint in (18), the inequality $[q_i(n) - \sum_{j\in\mathcal{R}}\sum_{m\in\mathcal{M}} r_{ij}^m(n)x_{ij}^m(n)]^+ + a_i(n) < \beta_i$ can be equivalently expressed as

$$F_i(\mathbf{x}, \zeta_\mathbf{i}) \geq 0, \tag{20}$$

where $\zeta_\mathbf{i} = (\mathbf{r}_i, a_i)$ and

$$F_i(\mathbf{x}, \zeta_\mathbf{i}) \triangleq \beta_i - [q_i(n) - \sum_{j\in\mathcal{R}}\sum_{m\in\mathcal{M}} r_{ij}^m(n)x_{ij}^m(n)]^+ - a_i(n). \tag{21}$$

$F_i(\mathbf{x}, \zeta_\mathbf{i})$ is in the form of affine chance constraint which involves linear form of the random variables $\zeta_\mathbf{i} = (\mathbf{r}_i, a_i)$. Based on Bernstein approximation, constraint (18) can be approximated by

$$\inf_{\varrho_i}\{\Psi_i(\mathbf{x}, \zeta_\mathbf{i}) - \varrho_i \log \epsilon_i\} \leq 0, \forall i \tag{22}$$

where

$$\begin{aligned}\Psi_i(\mathbf{x}, \zeta_\mathbf{i}) &= \varrho_i \log \mathbb{E}[\exp(\varrho_i^{-1}(F_i(\mathbf{x}, \zeta_\mathbf{i}))] \qquad (23)\\ &= \beta_i - q_i + \varrho_i \sum_{j\in\mathcal{R}}\sum_{m\in\mathcal{M}} \Lambda_{r_i}(\varrho_i^{-1}x_{ij}^m) + \varrho_i \Lambda_{a_i}(-\varrho_i^{-1}),\end{aligned}$$

where Λ_{r_i} and Λ_{a_i} are the cumulant (log-moment) generating function of the data rate and arrival process respectively.

In the sequel, we derive the cumulant generating function of the random variable r_i, which is a function of the channel gain random variable. The moment generating function (MGF) of the instantaneous data rate $r(\gamma_{ij}) = \log(1 + \gamma_{ij})$, where $\gamma_{ij} = \frac{pg_{ij}}{\Delta\sigma^2}$, is given in [17] as follows

$$\begin{aligned}M_{r_{i.j}}(y) &= \mathbb{E}[\exp(-yr(\gamma_{ij}))]\\ &= 1 + \int_0^{+\infty} Q(\ln(2)y, \xi) M_{\gamma_{ij}}(\xi) \mathrm{d}\xi, \qquad (24)\end{aligned}$$

where $Q(a, u) = \Gamma(a, u)/\Gamma(a)$ is the regularized Gamma function[1] and $M_{\gamma_{ij}}(\cdot)$ is the MGF of γ_{ij} which is known in closed-form for many fading distributions [18]. Without loss of generality, consider the channel gain as an exponentially distributed random variable with PDF given by $f_{g_{ij}}(\xi) = \frac{1}{\mu_i}\exp(-\frac{\xi}{\mu_i})$, where μ_i is the long-term average channel gain for user i. Then, $M_{\gamma_{ij}}(\xi)$ can be computed as

$$M_{\gamma_{i.j}}(\xi) = \frac{\Delta N_o}{\Delta N_o - p\mu_i\xi} \tag{25}$$

[1] $\Gamma(a, u) = \int_u^{+\infty} t^{a-1}e^{-t}\mathrm{d}t$ and $\Gamma(a) = \int_0^{+\infty} t^{a-1}e^{-t}\mathrm{d}t$

By substituting (25) in (24), MGF for the data rate can be obtained. Hence, the cumulant generating function of the data rate can be achieved as $\Lambda_{r_i}(\cdot) = \log M_{r_{ij}}(\cdot)$.

To calculate the cumulant generating function of the arrival process, e.g., Λ_{a_i}, without loss of generality consider the arrival process for the traffic of the user i follows Poisson distribution with parameteras $f_{a_i}(k) = \frac{\bar{a}_i^k e^{-\bar{a}_i}}{k!}$, where $\bar{a}_i$ presents the average rate. Λ_{a_i} can be computed as

$$\Lambda_{a_i}(y) = \log \mathbb{E}[e^{ya_i}] = \log(e^{\lambda(e^y-1)}) = \frac{\lambda(e^y - 1)}{\ln 10}. \tag{26}$$

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