Joint Fair Resource Allocation for Opportunistic Spectrum Sharing in OFDM-based Cognitive Radio Networks

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ABSTRACT

This paper considers a cooperative Orthogonal Frequency Division Multiplexing (OFDM)-based cognitive radio network, where the primary system leases a fraction of its subcarriers to the secondary system in exchange for the secondary users (SUs) acting as decode-and-forward relays. Our aim is to determine an fair resource allocation strategy among the primary users and SUs as so to maximize the network capacity. To this end, a network utility maximization optimization problem of power, subcarrier allocation and relay selection is formulated based on a class of α-fair utility. This problem is solved by applying the lagrangian dual method and a joint fair resource allocation policy at the SUs is derived in a closed-form expression. Moreover, a novel stochastic algorithm is developed to approach the optimal policy by dynamically learning the intended wireless channels. Simulation results demonstrate that both primary and secondary systems can benefit from the proposed resource allocation policy.

Categories and Subject Descriptors

G.1.6 [**Optimization**]: Convex programming

General Terms

Algorithms

1. INTRODUCTION

The cognitive radio network (CRN) has been proposed as a method to solve the spectrum scarcity problem by allowing the secondary users (SUs) to dynamically access the licensed frequency bands or spectrum holes left by the primary users (PUs). In most of the works on dynamic spectrum access, SUs do not participate directly in the primary data transmission. And the secondary transmission is regarded as a harmful interference to the PUs. Recently, a new cooperation strategy between the primary system and the secondary system, named as spectrum leasing, was proposed in [1]. Therein, PUs lease their band to SUs for a fraction of time in exchange for SUs acting as relays to assist the primary transmission. This cooperative scheme can enhance the overall performance of both the primary and secondary systems. In this paper we focus on this cooperative communication scheme joint with spectrum leasing.

There has been a variety of research work dealing with topic. And quite a lot good solutions have been proposed. As a summary, those previous researches can be divided into two categories. As for the first category, the spectrum leasing problem in CRN was investigated by employing the widely-used economical concepts [2, 3]. For example, [2] proposed an auction framework in which an iterative and negotiation-based approach was suggested for spectrum leasing. The second category, the Lagrangian dual decomposition is adopted to solve such kind of spectrum leasing problem, in which the globally optimal resource allocation can be found [4, 5]. For example, by using Lagrangian dual decomposition theory, [5] aimed to determine the cooperative power allocation strategy among the primary and secondary systems so as to maximize the sum-rate of SUs while maintaining quality-of-service (QoS) requirements of PUs in multi-channel multi-user CRN.

As the extended works of spectrum leasing strategy via employing Lagrangian theory, in this paper we study the resource allocation in orthogonal frequency division multiplexing (OFDM)-based CRN. We aim to optimally allocate three types of wireless resources, power, subcarriers, relay nodes, among the primary and secondary systems while guaranteeing the fairness of resource allocation. To the best of our knowledge, such optimization has not been investigated in the literature and is crucial for achieving the best system performance. Specifically, based on a class of α-fair utility [6], a fair power, subcarrier allocation and relay selection policy is given. Besides, by taking into account the time-varying nature of fading channels without *a priori* knowledge of the cumulative distribution function (cdf), a stochastic resource allocation schemes is also put forward to learn the underlying channel distribution by employing the stochastic optimization tools [7].

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2. SYSTEM MODEL AND PROBLEM FORMULATION

2.1 System Model

We consider a cooperative CRN, which is composed of a pair of primary transmitter-receiver (PT-PR), and N pairs of secondary transmitters-receivers (STs-SRs). SUs are randomly distributed in a area wherein PU will choose a set of SUs acting as relays for cooperation when necessary. As reward, PU leases some of its subcarriers to the secondary system for a fraction of time.

Assume that both the primary and secondary signals are OFDM modulated. The wireless fading environment is be modelled as a frequency selective fading channel with severe Doppler spread effect. The licensed spectrum B is divided into K orthogonal narrow-band subcarriers, with each subbandwidth small enough for each subcarrier to experience flat fading. For $\forall k \in \{1, 2, ...K\}, n \in \{1, 2, ...N\}$, let $\gamma_{n,k,1}, \gamma_{n,k,2}, \gamma_{n,k,3}$ denote the channel power gains of the PT$\rightarrow$$n$-th ST link, n-th ST$\rightarrow$PR link and n-th ST$\rightarrow$ n-th SR link on subcarrier k, respectively. $\gamma_{k,0}$ denotes the channel power gain of the PT$\rightarrow$PR link over the k-th subcarrier. They all are assumed to remain invariant during a frame transmission and be independent for different n and k. A time-division based half duplex decode-and-forward (DF) protocol is utilized. In the first phase, PT transmits signals over all the subcarriers while SUs and PR listen. In the second phase, SUs decode the received signal, re-encode it and use a subset of K subcarriers to help forward the decoded primary signal to PR. Then the end-to-end transmission rate of the primary system at PR over two phases reads

$$R_{pu} = \min\{R_1, R_2\}, \tag{1}$$

where

$$R_1 = \frac{1}{2}\mathbb{E}\Big[\sum_{n=1}^{N}\sum_{k\in\Omega_{n,1}} \log(1+\gamma_{n,k,1}p_{p,k})\Big], \tag{2}$$

$$R_2 = \frac{1}{2}\mathbb{E}\Big[\sum_{n=1}^{N}\sum_{k\in\Omega_{n,p}} \log(1+\gamma_{k,0}p_{p,k}+\gamma_{n,k,2}p_{n,k,p}) + \sum_{k\in\overline{\Omega}_p}\log(1+\gamma_{k,0}p_{p,k})\Big], \tag{3}$$

where $\mathbb{E}[\cdot]$ is the expectation operator, $p_{n,k,p}$ is the power allocated to subcarrier k at the n-th ST used for relaying the data of the primary system, and $p_{p,k}$ denotes the PT's transmit power over the k-th subcarrier. $\Omega_{n,1}, \Omega_{n,p}$ are the sets of subcarriers assigned to the n-th SU in the link of PT $\rightarrow$ ST and ST $\rightarrow$ PR, respectively. $\Omega_p = \cup_{n=1}^{N}\Omega_{n,p}$. In the second phase, excepting relaying the primary data for PU, STs also use the remaining subcarriers $\overline{\Omega}_p$ to transmit their own data to SRs. The maximum average rate of the n-th secondary user reads

$$R_{su} = \mathbb{E}\Big[\frac{1}{2}\sum_{k\in\Omega_{n,s}} \log(1+\gamma_{n,k,3}p_{n,k,s})\Big], \tag{4}$$

where $p_{n,k,s}$ is the power allocated to subcarrier k at the n-th ST used for transmiting its own data, $\Omega_{n,s}$ is the set of subcarriers assigned to the link of n-th ST $\rightarrow$ n-th SR. Furthermore, to avoid interference subcarrier sets assigned to different SUs in the second phase over each link must be mutually exclusive.

2.2 Problem Formulation

In order to balance the total throughput and fairness among SUs, a class of α-fair utiltiy function is introduced [6]. α-fair utiltiy function refers to a family of functions parameterized by $\alpha \geq 0$, shown as

$$U_\alpha(\cdot) = \begin{cases} (1-\alpha)^{(-1)}(\cdot)^{(1-\alpha)} & \text{for } \alpha \neq 1 \\ \log(\cdot) & \text{for } \alpha = 1 \end{cases}, \tag{5}$$

which is a concave and increasing function. Larger α means more fairness. The notion of α-fairness includes max-min fairness (when $\alpha \rightarrow \infty$), proportional fairness (when $\alpha = 1$), and throughput maximization (when $\alpha = 0$).

With $U_\alpha(\cdot)$, this paper aims to maximize the network utility by determining optimally the transmit power at the SUs, the relay nodes and the subcarriers that used for relaying transmission or those leased to each SU. Based on the above definitions and analysis, the relay assignment and subcarrier allocation are represented by a collection of binary variable $\mathbf{\Psi} = \{\psi_{n,k,t}, t \in \{1, 2, p\}\}$, where $\psi_{n,k,1} = 1$ means that $k \in \Omega_{n,1}$, $\psi_{n,k,2} = 1$ means that $k \in \Omega_{n,s}$, $\psi_{n,k,p} = 1$ means that $k \in \Omega_{n,p}$, and $\psi_{n,k,t} = 0$ otherwise. Let P_s denote the power budget for all SUs, R_p denote the required minimum average data rate for the primary user, and R_n^s denote the achievable data rate of the n-th SU. Mathematically, the optimization problem can be formulated as

$$(\mathbf{P}) \quad \max_{\mathbf{\Psi},\boldsymbol{p}} \sum_{n=1}^{N} U_\alpha(R_n^s) \tag{6}$$

$$\text{s.t.} \quad R_n^s \leq \mathbb{E}\Big[\frac{1}{2}\sum_{k=1}^{K}\psi_{n,k,2}\log(1+\gamma_{n,k,3}p_{n,k,s})\Big], \forall n, \tag{7}$$

$$\frac{1}{2}\mathbb{E}\Big[\sum_{n=1}^{N}\sum_{k=1}^{K}\psi_{n,k,1}\log(1+\gamma_{n,k,1}p_{p,k})\Big] \geq R_p, \tag{8}$$

$$\frac{1}{2}\mathbb{E}\Big[\sum_{n=1}^{N}\sum_{k=1}^{K}\psi_{n,k,p}\log(1+\gamma_{k,0}p_{p,k}+\gamma_{n,k,2}p_{n,k,p}) + \sum_{k=1}^{K}\sum_{n=1}^{N}\psi_{n,k,2}\log(1+\gamma_{k,0}p_{p,k})\Big] \geq R_p, \tag{9}$$

$$\mathbb{E}\Big[\sum_{n=1}^{N}\sum_{k=1}^{K}\psi_{n,k,2}p_{n,k,s} + \sum_{n=1}^{N}\sum_{k=1}^{K}\psi_{n,k,p}p_{n,k,p}\Big] \leq P_s, \tag{10}$$

$$\sum_{n=1}^{N}\psi_{n,k,1} = 1, \forall k, \tag{11}$$

$$\sum_{n=1}^{N}(\psi_{n,k,2}+\psi_{n,k,p}) = 1, \forall k, \tag{12}$$

$$\psi_{n,k,t} \in \{0,1\}, t \in \{1,2,p\}, \forall n, k. \tag{13}$$

In this problem, $\boldsymbol{p} = \{p_{n,k,p}, p_{n,k,s}\}$ and $\mathbf{\Psi}$ are the sets of the optimization variables. (8) and (9) are the target rate constraints of the primary system, i.e. $\min\{R_1, R_2\} \geq R_p$. The last three constraints can guarantee that subcarrier sets are mutually exclusive.

3. OPTIMAL POWER AND SUBCARRIER ALLOCATION POLICY

The optimization problem $(\mathbf{P})$ in (6)-(13) is a (nonconvex) 0-1 integer programming problem, in which nonlinear constraints and integer variables are involved. Its computational complexity increases exponentially with the number of subcarriers, which makes it difficult to solve for even medium-size problems. However, it has been shown in [8] that under the condition that the number of subcarriers is sufficiently large, the duality gap of nonconvex resource optimization problems in multi-carrier systems is zero. In this section we shall apply the result from [8] joint with Lagrange dual decomposition approach [9] to solve the problem $(\mathbf{P})$.

3.1 Dual Decomposition

Let $\boldsymbol{\lambda}=[\lambda_1,...,\lambda_N]^T, \boldsymbol{\beta}=[\beta_1,...,\beta_N]^T,\ \xi_{R1},\ \xi_{R2}$ be the Lagrange multiplier variables associated with constraints (7)-(10), respectively. With convenient notations $\boldsymbol{X}:=\{\boldsymbol{p},\boldsymbol{\Psi}\}$ and $\boldsymbol{Y}:=\{\boldsymbol{\lambda},\boldsymbol{\beta},\xi_{R1},\xi_{R2}\}$, the lagrangian is given by (14) presented at the bottom of this page. And the Lagrange dual function can be expressed as

$$g(\boldsymbol{Y})=\max_{\boldsymbol{X}} L(\boldsymbol{X},\boldsymbol{Y}). \tag{15}$$

Computing the dual function $g(\boldsymbol{Y})$ involves determining the optimal $\boldsymbol{X}$ at a given $\boldsymbol{Y}$. To this end, we shall solve the following three decoupled subproblems, i.e.

$$\max_{R_n^s\geq 0}\sum_{n=1}^{N}\big[U_\alpha(R_n^s)-\lambda_n R_n^s\big], \tag{16}$$

$$\max_{\boldsymbol{p},\psi_{n,k,2},\psi_{n,k,p}}\sum_{k=1}^{K}\mathbb{E}\big[\vartheta_k\big], \tag{17}$$

$$\max_{\psi_{n,k,1}}\frac{\xi_{R1}}{2}\mathbb{E}\Bigg[\sum_{n=1}^{N}\sum_{k=1}^{K}\psi_{n,k,1}\log(1+\gamma_{n,k,1}p_{p,k})\Bigg], \tag{18}$$

where function ϑ_k is defined as

$$\vartheta_k=\sum_{n=1}^{N}\big[\psi_{n,k,2}\phi_{n,k,1}+\psi_{n,k,p}\phi_{n,k,2}\big],$$

$$\phi_{n,k,1}=\frac{\lambda_n}{2}\log(1+\gamma_{n,k,3}p_{n,k,s})+\frac{\xi_{R2}}{2}\log(1+\gamma_{k,0}p_{p,k}) -\beta_n p_{n,k,s},$$

$$\phi_{n,k,2}=\frac{\xi_{R2}}{2}\log(1+\gamma_{k,0}p_{p,k}+\gamma_{n,k,2}p_{n,k,p})-\beta_n p_{n,k,p}.$$

It is clear that the subproblem (16) is equivalent to

$$\max_{R_n^s\geq 0} U_\alpha(R_n^s)-\lambda_n R_n^s. \tag{19}$$

From (5), we can see that $U_\alpha(\cdot)$ is differentiable and its first derivative $U_\alpha'(\cdot)$ has a well defined inverse $U_\alpha'^{-1}(\cdot)$. Thus the solution of (19) is

$$R_n^{s\,*}=U_\alpha'^{-1}(\lambda_n),\quad \forall n. \tag{20}$$

It can proved that the maximization in (17) can be decoupled across different subcarrier and fading state. Thus, (17) can be reduced to the following optimization problem

$$\max_{\boldsymbol{p},\psi_{n,k,2},\psi_{n,k,p}}\sum_{n=1}^{N}\Big[\psi_{n,k,2}\phi_{n,k,1}+\psi_{n,k,p}\phi_{n,k,2}\Big],\forall k, \tag{21}$$

from which we can see that the power allocation policy is independent of the subcarrier allocation. For any given $\psi_{n,k,2}$ and $\psi_{n,k,p}$, the optimal $p_{n,k,s}^*$ is derived from maximizing $\phi_{n,k,1}$. And irrespective of the subcarrier allocation, $p_{n,k,p}^*$ maximizes $\phi_{n,k,2}$. Furthermore, the function $\phi_{n,k,1}$ and $\phi_{n,k,2}$ are concave function of $p_{n,k,s}$ and $p_{n,k,p}$, respectively. Then relying on the Karush-Kuhn-Tucker optimality conditions [9], we can easily obtain the optimal power allocation for given $\boldsymbol{Y}$, as shown

$$p_{n,k,s}=\left(\frac{\lambda_n}{2\beta_n}-\frac{1}{\gamma_{n,k,3}}\right)^+, \tag{22}$$

$$p_{n,k,p}=\left(\frac{\xi_{R2}}{2\beta_n}-\frac{1+\gamma_{k,0}p_{p,k}}{\gamma_{n,k,2}}\right)^+, \tag{23}$$

where $(x)^+=\max(0,x)$. With $p_{n,k,s}^*,p_{n,k,p}^*,\forall n,k$, it is clear that the optimal subcarrier assignment should solve

$$\max_{\psi_{n,k,2},\psi_{n,k,p}}\vartheta_k^*, \tag{24}$$

which is a linear optimization problem. With the subcarrier assignment constraints of $\psi_{n,k,t}\in\{0,1\},t\in\{2,p\}$ and $\sum_{n=1}^{N}(\psi_{n,k,2}+\psi_{n,k,p})\leq 1$, it is easy to see that the k-th subcarrier should be allocated to user n_k^*, shown as

$$n_k^*=\arg\max\{\phi_{1,k,1}^*,...,\phi_{N,k,1}^*,\phi_{1,k,2}^*,...,\phi_{N,k,2}^*\}. \tag{25}$$

Then, if $\phi_{n_k^*,k}=\max\{\phi_{1,k,1}^*,...,\phi_{N,k,1}^*,\phi_{1,k,2}^*,...,\phi_{N,k,2}^*\}$, the optimal subcarrier allocation policy can be described as

$$\begin{cases} k\in\Omega_{n_k^*,2}, & \text{if}\quad \phi_{n_k^*,k,1}=\phi_{n_k^*,k} \\ k\in\Omega_{n_k^*,p}, & \text{if}\quad \phi_{n_k^*,k,2}=\phi_{n_k^*,k}\end{cases}. \tag{26}$$

Similarly, via solving (18) we can obtain the subcarrier k belong to $\Omega_{n,1}$ if and only if

$$k=\arg\max\sum_{n=1}^{N}\log(1+\gamma_{n,k,1}p_{p,k}). \tag{27}$$

Note that this subcarrier allocation policy has nothing with lagrange variable ξ_{R_1}, which is ignored in the following below.

$$L(\boldsymbol{X},\boldsymbol{Y})=\sum_{n=1}^{N}U_\alpha(R_n^s)+\sum_{n=1}^{N}\lambda_n\Bigg\{\mathbb{E}\Bigg[\frac{1}{2}\sum_{k=1}^{K}\psi_{n,k,2}\log(1+\gamma_{n,k,3}p_{n,k,s})\Bigg]-R_n^s\Bigg\}+ \sum_{n=1}^{N}\beta_n\Bigg\{P_s-\mathbb{E}\Bigg[\sum_{k=1}^{K}\psi_{n,k,2}p_{n,k,s}+\sum_{k=1}^{K}\psi_{n,k,p}p_{n,k,p}\Bigg]\Bigg\}+\xi_{R1}\Bigg\{\frac{1}{2}\mathbb{E}\Bigg[\sum_{n=1}^{N}\sum_{k=1}^{K}\psi_{n,k,1}\log(1+\gamma_{n,k,1}p_{p,k})\Bigg]- R_p\Bigg\}+\xi_{R_2}\Bigg\{\frac{1}{2}\mathbb{E}\Bigg[\sum_{n=1}^{N}\sum_{k=1}^{K}\psi_{n,k,p}\log(1+\gamma_{k,0}p_{p,k}+\gamma_{n,k,2}p_{n,k,p})+\sum_{k=1}^{K}\sum_{n=1}^{N}\psi_{n,k,2}\log(1+\gamma_{k,0}p_{p,k})\Bigg]-R_p\Bigg\}. \tag{14}$$

3.2 Solving the dual problem

In order to obtain the optimal lagrange variables, next we focus on the the dual problem of $(\mathbf{P})$, which is written as

$$\begin{aligned} \min_{\boldsymbol{Y}} \quad & g(\boldsymbol{Y}) \\ \text{s.t.} \quad & \boldsymbol{Y} \succeq 0. \end{aligned} \tag{28}$$

Evidently, the dual function $g(\boldsymbol{Y})$ can be calculated with optimal power and subcarrier allocation policy. And it is convex, based on which a subgradient iteration algorithm with the expected values can be used to minimize $g(\boldsymbol{Y})$ by updating $\boldsymbol{Y}$ simultaneously along some appropriate search directions. Because of lack of space, we omit the the algorithm details.

3.3 Stochastic Resource Allocation

To solve the dual problem (28), we need the explicit knowledge of fading channel cdf to evaluate the expected values involved in the subgradient algorithm. But in some practical mobile environments, it is infeasible or impossible to obtain the cdf of the fading channels. Thus, the power and subcarrier allocation problem of operating without the knowledge of channel cdf should be solved urgently. As it turns out, this problem can be solved via employing the stochastic optimization theory [7]. Accordingly, a stochastic subgradient iteration algorithm based on per slot fading realization is put forward. The algorithm details is shown at the bottom of the page. Thereinto t is the iteration index, $s(t) > 0$ is a positive step-size. It only requires the fading state of the channels at the current iteration, which can be easily measured. Till now we have proposed an iterative algorithm to solve the dual problem (28) without knowledge of cdf.

4. NUMERICAL RESULTS

In this section, we provide some numberical results to evaluate the performance of the proposed scheme. Throughout our simulation, we consider a cognitive network including 4 pairs of SUs, and set the number of OFDM subcarriers be $K = 32$. The fading processes are generated from quasi-static frequency-selective Rayleigh fading channels with a 6-tap delay profile. Without loss of generality, we assume the fading of links associated with each SU (i.e. the links from each ST to PT, PR, and SR) follow the same fading channel model. But they are different from links associated with another SU. We set the average channel power gain for each SU is 1, 1.5, 2, 1, respectively. More detailed parameters will accompany with results figures to be shown.

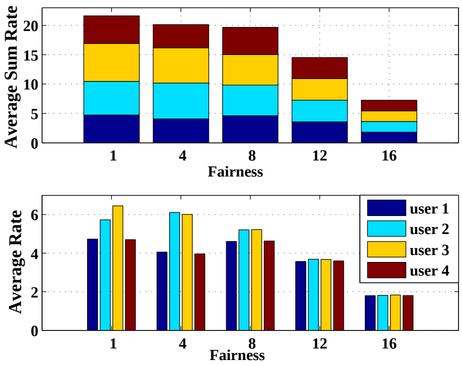

Figure 1: (Top) Average sum rate of all SUs, (Bottom) Average rate of each SUs.

We ran the proposed resource allocation algorithm with the constraints of $P_s = 10$ Watt, $R_p = 5$ Nats/s and $\alpha = \{1, 4, 8, 12, 16\}$. We assume that the PT transmits with a constant power over all subcarriers, and $p_{p,k} = 0.02$ Watt. Fig. 1 (top) shows the average sum rate of all SUs, and Fig. 1 (bottom) depicts the average rates of each SU. It is observed that, when $\alpha = 1$ the sum rate is maximal. However, this is achieved in an unfair manner, in which the average rates of SUs differ greatly. Especially the average rate of the third SU is much larger than the first and fourth one, which occurs because each SU suffers different wireless fading. With a larger α, it is shown that the sum rate decreases but fairness improves. For example, when $\alpha = 16$, all SUs have almost the same average rates, whereas total network throughput decreases 65 percent than the case with $\alpha = 1$. This demonstrates that we can employ α-fair utility function to trade off the cognitive network throughput and fairness.

To gauge the performance of the proposed algorithm, we compare it with other power and subcarrier allocation policies. With an equally divided power $p_{n,k,s} = p_{n,k,p} = P_s/K$ consumed per subcarrier, the first scheme is derived from the α-fair utility maximization problem. It is named constant power policy. The other, named fixed subcarrier allocation, equally divides the whole sub-carriers into four group and allocates one group to each SU. The corresponding power and subcarrier allocation policy is also derived from the α-fair utility maximization problem. Numerical results are shown in Fig. 2 when $\alpha = 1$. The proposed policy in Fig. 2 is the op-

$$\begin{aligned}
&\lambda_n[t+1] = \lambda_n[t] + s[t]\left(R_n^s - \frac{1}{2}\sum_{k=1}^{K}\psi_{n,k,2}(\boldsymbol{Y}[t],\boldsymbol{\gamma}[t])\log(1+\gamma_{n,k,3}[t]p_{n,k,s}(\boldsymbol{Y}[t],\boldsymbol{\gamma}[t]))\right)^{+},\\
&\beta_n[t+1] = \beta_n[t+1] - s[t]\left(P_s - \sum_{k=1}^{K}\psi_{n,k,2}(\boldsymbol{Y}[t],\boldsymbol{\gamma}[t])p_{n,k,s}(\boldsymbol{Y}[t],\boldsymbol{\gamma}[t]) + \sum_{k=1}^{K}\psi_{n,k,p}(\boldsymbol{Y}[t],\boldsymbol{\gamma}[t])p_{n,k,p}(\boldsymbol{Y}[t],\boldsymbol{\gamma}[t])\right)^{+},\\
&\xi_{R_2}[t+1] = \xi_{R_2}[t] + \frac{s[t]}{2}\left(R_p - \sum_{n=1}^{N}\sum_{k=1}^{K}\psi_{n,k,p}(\boldsymbol{Y}[t],\boldsymbol{\gamma}[t])\log(1+\gamma_{k,0}p_{p,k}+\gamma_{n,k,2}(\boldsymbol{Y}[t],\boldsymbol{\gamma}[t])p_{n,k,p}(\boldsymbol{Y}[t],\boldsymbol{\gamma}[t]))\right.\\
&\left.+\sum_{k=1}^{K}\sum_{n=1}^{N}\psi_{n,k,2}(\boldsymbol{Y}[t],\boldsymbol{\gamma}[t])\log(1+\gamma_{k,0}p_{p,k})\right)^{+}.
\end{aligned} \tag{24}$$

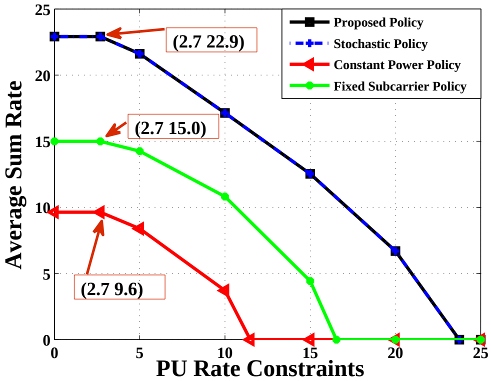

Figure 2: Average sum rate (Nats/s) versus the PU rate constraints (Nats/s). $\alpha = 1, P_s = 10$ **Watt.**

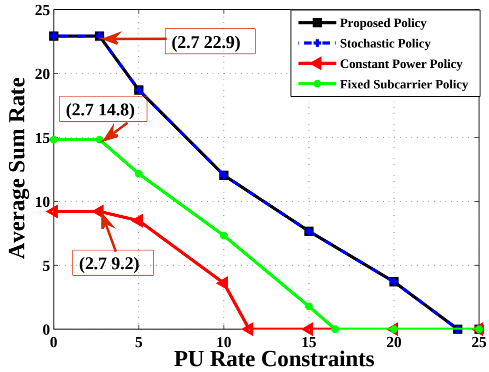

Figure 3: Average sum rate (Nats/s) versus the PU rate constraints (Nats/s). $\alpha = 5, P_s = 10$ **Watt.**

timal power and subcarrier policy put forward in this paper, whose subgradient iterative is based on the expected values. We can see that the proposed policy demonstrates the same performance with the stochastic policy. This verifies that the proposed stochastic scheme can learn the channel fading knowledge on the fly and can approach the optimal policy. Also it is expected that under different PU rate constraints the proposed policy outperforms the other two policies obviously in improving the network throughput of the secondary system. In Fig. 2, the value of 2.7 is the maximum rate of PU without relaying. Note that when R_p is smaller than 2.7, with increase of R_p the average sum rate of SUs stays the same. This is because in this case even there is no relay serving for the primary system, PU's average rate is still larger than R_p. Thus the secondary system is granted full access to the licensed primary in the second phase. It is also seen that the average sum rate of SUs decreases with the increase of R_p, when R_p is larger than 2.7. In this case, some SUs are selected as relays which forward the primary data using a fraction of the secondary system's power. The secondary system uses its remaining power and the leased subcarrier to transmit its own data. Note that when R_p increases, the secondary system will have less chances to access the spectrum. When R_p is larger than some value, the average sum rate of the secondary system reduces to zero. That is, the secondary system serves as a pure DF relay for the primary system by devoting all its power to relay the primary signal. Results when the secondary system is of different fairness value of α are also given, as shown in Fig. 3. The curves in Fig. 3 are similar to those of Fig. 2. Similar conclusions can be drawn as those from Fig. 2.

5. CONCLUSION

We study the optimal resource allocation among the primary and secondary systems for DF-based CRN over fading OFDM channels. A joint optimization problem of power, subcarrier allocation and relay selection is present for maximizing a α-fair utility function of average rates while fulfilling the QoS requirement of the primary system. By using the dual decomposition method, we efficiently solved the optimization problem in an asymptotically optimal manner. Furthermore, we have presented a stochastic resource allocation scheme that can learn the statistics of the fading channels and adaptively approach the optimal strategy on the fly. Moreover, the numerical results demonstrates that our proposed schemes exhibit excellent performance in improving network throughput compared with constant power policy and fixed subcarrier allocation for CRN.

6. ACKNOWLEDGMENTS

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7. REFERENCES

[1] O. Simeone, I. Stanojev, S. Savazzi, Y. Bar-Ness, U. Spagnolini, and R. Pickholtz, "Spectrum leasing to cooperating secondary ad hoc networks," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 1, pp. 203-213, Jan. 2008.

[2] S. M. M. Toroujeni, S. M. Sadough, S. A. Ghorashi,"Spectrum Leasing for OFDM-Based Cognitive Radio Networks," *IEEE Trans. Vehi. Technology*, vol. 62, no. 5, pp. 2131-2139, June 2013.

[3] L. Duan, L. Gao, J. Huang, "Cooperative Spectrum Sharing: A Contract-Based Approach," *IEEE Trans. Mobi. Comput.*, vol. 13, no. 1, pp. 174-187, Jan. 2014.

[4] W. Lu, Y. Gong, S. Ting, X. Wu, N. Zhang, "Cooperative OFDM Relaying for Opportunistic Spectrum Sharing: Protocol Design and Resource Allocation," *IEEE Trans. Wireless Commun.*, vol. 11, no. 6, pp. 2126-2135, June 2012.

[5] M. Tao, Y. Liu,"Spectrum Leasing and Cooperative Resource Allocation in Cognitive OFDMA Networks," *Journal of Commun. and networks*, vol. 15, no.1, pp. 102-110, Feb. 2013.

[6] J. Mo, J. Walrand, "Fair end-to-end window-based congestion control," *IEEE/ACM Trans. Netw.*, vol. 8, no. 5, pp. 556-567, Oct. 2000.

[7] P. Kall, S. W. Wallace, *Stochastic Programming.* Wiley, 1994.

[8] W. Yu, R. Lui, "Dual methods for nonconvex spectrum optimization of multicarrier systems," *IEEE Trans. Commun.*, vol. 54, no. 7, pp. 1310-1322, July 2006.

[9] Stephen Boyd and Lieven Vandenberghe, *Convex Optimization.* Cambridge University Press, 2004.