Maximum Service Rate of Two Interacting Queues with Delay Constraint

(Invited Paper)

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***Abstract*—When two queues interact with each other, the delay of one queue depends on the behavior of the other queue. More specifically, the delay of one user depends on the access probability of the other user, since this probability controls the level of mutual interference in the network. In this sense, the delay of a given user can be limited by appropriately adjusting the access probability of the other user in the network, as investigated by several authors. In this paper we investigate the effects of spectral efficiency of the communication links on this optimization problem related to the access probability.**

I. Introduction

In the last few years the performance of interacting queues has received a lot of attention, motivated by the increasing interest in wireless networks operating under opportunistic channel access techniques, such as cognitive radio networks (see, for instance [1], [2], and references therein). The channel sharing nature of the operation of such networks naturally leads to interacting queues, that must be appropriately investigated.

In this paper we consider two users, namely user 1 and user 2, sharing the communication channel, where user 1 has priority to access the channel. If user 1 is transmitting, user 2 is allowed to transmit with a given access probability, what certainly will cause mutual interference and will increase the delay of user 1. Hu, Yang and Hanzo [1] studied this problem to determine the maximum allowed access probability of user 2 that keeps the delay of user 1 below a given threshold. In this paper we revisit this problem, to investigate the effects of the communication link spectral efficiency on the access probability of user 2 and, therefore, its service rate. We show that if high spectral efficiency modulation is employed, in the limit user 2 may be restricted to transmit when the channel is idle only, in order to guarantee that the delay of user 1 does not exceed the desired target. On the other hand, when more robust modulation scheme is used, users 1 and 2 may share the channel more often.

II. System Model

In this work we basically follow the models assumed in several works related to interfering queues, such as [3], [4], and many others. We consider two users, namely user 1, represented by the pair (S_1, D_1), and user 2, represented by (S_2, D_2). These users share the same channel, such that they may interfere to each other, as illustrated in Figure 1. Each

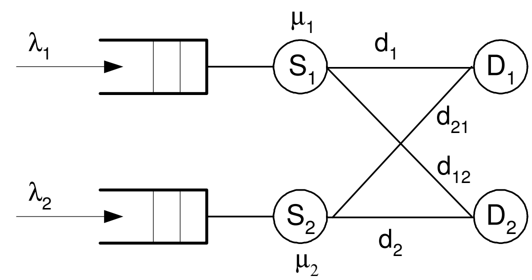

Fig. 1. Two interfering queues with bursty arrival of packets.

source S_i, $i = 1, 2$, is modeled by a queue of infinite length, with the packet arrival processes modeled as independent and identically distributed Bernoulli processes with mean arrival rate λ_i. We assume that time is slotted, and that transmissions from both users are synchronized on a slot basis. A packet requires one time-slot to be transmitted, and we assume that acknowledge messages are transmitted instantaneously and error-free.

The transmission channel has bandwidth $B = 1$ Hz (normalized), and M-ary quadrature modulation (M-QAM) is employed. The transmission rate R_i depends on the modulation order assumed, and is evaluated as $R_i = \log_2 M_i$. We assume that all packets consist of S modulation symbols, such that the number of bits per packet depends on the modulation order.

The propagation channel model assumed here includes deterministic path loss, with path loss exponent η, additive noise with power W, and a block fading model, with Rayleigh distribution. The transmit power levels are $P_{\text{tx},i}$, $i = 1, 2$, and d_i and d_{ij} are the transmitter-receiver separation distances and the interferer-receiver separation distances, respectively (see Figure 1).

As far as the channel access policy is concerned, we assume that user 1 transmits a packet when its queue is non-empty, regardless of the channel status (busy or idle). On the other hand, user 2 transmits its packets with probability one if the channel is idle, and with probability p when the channel is busy. Therefore, user 1 has priority to access the channel, but

This work was partially supported by CNPq Grant No. 312146/2012-4

CROWNCOM 2014, June 02-04, Oulu, Finland

DOI 10.4108/icst.crowncom.2014.255670

due to the possibly nonzero access probability p of user 2, interference may occur between transmissions from both users. This channel access model is frequently used to model the interaction between primary and secondary users in cognitive radio networks, with perfect spectrum sensing [2].

A. Successful Transmission Probability

A key parameter in the context of channel sharing is the success probability of a transmission. In our model, a transmission from user i may find the channel free or busy due to a transmission from user j. We assume here that a transmission is successful if the the signal-to-interference-plus-noise ratio (SINR) of the received signal is above a given threshold. Formally, the successful transmission probability for user i when a set of users $\mathcal{M}$ is transmitting simultaneously is defined as

$$q_{i,\mathcal{M}} = \Pr\{\gamma_{i|\mathcal{M}} \geq \gamma_{\text{min}}\}, \quad (1)$$

where $\gamma_{i|\mathcal{M}}$ is the SINR at the receiver, and γ_{min} is the required SINR for successful transmission,

Recalling that the channel model assumed here includes Rayleigh fading and deterministic path loss, we can show that [5]

$$q_{i,|\{i,j\}} = \exp\left(\frac{-\gamma_{\text{min}} W}{\overline{P}_i}\right)\left(\frac{\gamma_{\text{min}}\overline{I}_j}{\overline{P}_i} + 1\right)^{-1}, \quad (2)$$

where W is the noise power, $\overline{P}_i$ is the average received power and $\overline{I}_j$ is the average interference power, caused by user j, which are defined as

$$\overline{P}_i = P_{\text{tx},i}\, d_i^{-\eta}, \quad (3)$$

$$\overline{I}_j = P_{\text{tx},j}\, d_{ij}^{-\eta}. \quad (4)$$

Clearly, $q_{i,|\{i,j\}}$ is a decreasing function of γ_{min}. We assume in this work that both users transmit at the same power, and expression (2) becomes

$$q_{i,|\{i,j\}} = \exp\left(\frac{-\gamma_{\text{min}} W}{\overline{P}_i}\right)\left[\gamma_{\text{min}}\left(\frac{d_i}{d_{ij}}\right)^{\eta} + 1\right]^{-1}. \quad (5)$$

If we additionally assume, as usual, that $d_i < d_{ij}$ and the additive noise is negligible (i.e., $W = 0$), then $q_{i,|\{i,j\}}$ will be an increasing function of the path loss exponent η.

The minimum required SINR γ_{min} depends on several parameters, such as modulation order, target packet error rate, packet length, etc. For M-QAM modulation, the symbol error probability is approximately given by [6]

$$P_s = 4\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3}{M-1}\gamma}\right). \quad (6)$$

Recalling that each packet has S M-QAM symbols, then the packet error rate P_P can be written as

$$P_P = 1 - (1 - P_s)^S. \quad (7)$$

Therefore, for target packet error rate $P_{P,\text{max}}$, the minimum acceptable SINR γ_{min} for successful transmission is

$$\gamma_{\text{min}} = \frac{M-1}{3}\left[Q^{-1}\left(\frac{1-(1-P_{P,\text{max}})^{1/S}}{4(1-1/\sqrt{M})}\right)\right]^2. \quad (8)$$

TABLE I. SYSTEM MODEL AND PARAMETER SETTING

Parameter	Value
Modulation M-QAM	$M_1 = M_2 = 2, 4, 8, 16$
Primary T-R separation distance	$d_1 = 100$ m
Secondary T-R separation distance	$d_2 = 100$ m
Interferer-receiver separation distances	$d_{12} = d_{21} = 200$ m
Channel model	Rayleigh, noise-free, deterministic path loss
Target Packet error rate	$P_{P,\text{max}} = 10^{-3}$

III. Maximum Secondary Throughput

According to the channel access model assumed, if user 1 is transmitting, user 2 will transmit concurrently with probability p. Clearly, the access probability p controls the interference level caused by user 2, which in turn affects the performance of both users. We are particularly interested in determining the access probability of user 2 that maximizes its average service rate, while guaranteeing the delay of user 1 is kept below a given threshold. The analysis presented here is similar to the ones found in [1] and [7], but it is extended by taking into account the spectral efficiency of the communication links.

The average service rates μ_1 and μ_2 of user 1 and 2 are given by

$$\begin{aligned}\mu_1 \;=\;& q_{1|\{1\}}\Pr\{Q_2 = 0\} + \\ &+ q_{1|\{1\}}\Pr\{Q_2 \neq 0\}(1-p) + \\ &+ q_{1|\{1,2\}}\Pr\{Q_2 \neq 0\}\, p, \end{aligned} \quad (9)$$

and

$$\begin{aligned}\mu_2 \;=\;& q_{2|\{2\}}\Pr\{Q_1 = 0\} + \\ &+ q_{2|\{1,2\}}\Pr\{Q_1 \neq 0\}\, p. \end{aligned} \quad (10)$$

Expressions (9) and (10) show that the queues of users 1 and 2 are coupled, due to the interference between the communication links, making the analysis of the performance of both queues very difficult. Using the *Principle of Stochastic Dominance*, introduced by Rao and Ephremides in [8] in the context of interacting queues, we build a *Dominant System* where user 2 transmits dummy packets when it does not have packets in its queue. Therefore, $\Pr\{Q_2 = 0\} = 0$, and μ_1 is now written as

$$\begin{aligned}\mu_1 &= q_{1|\{1\}}(1-p) + q_{1|\{1,2\}}\, p \\ &= q_{1|\{1\}} - Q_1\, p, \end{aligned} \quad (11)$$

where $Q_1 = q_{1|\{1\}} - q_{1|\{1,2\}}$. Note that $Q_1 \geq 0$, and $Q_1 = 1$ for collision channels, while $Q_1 = 0$ for orthogonal channels. Recalling that $\Pr\{Q_1 = 0\} = 1 - \lambda_1/\mu_1$, the average service rate of user 2 is now given by

$$\mu_2 = q_{2|\{2\}} + \left(q_{2|\{1,2\}}p - q_{2|\{2\}}\right)\frac{\lambda_1}{\mu_1}, \quad (12)$$

where μ_1 is given by (11).

Using Little's Law, the average delay D_1 of user 1, measured in number of time-slots, is

$$\begin{aligned}D_1 &= \frac{1}{\mu_1 - \lambda_1} \\ &= \frac{1}{q_{1|\{1\}} + Q_1 p - \lambda_1}. \end{aligned} \quad (13)$$

As expected, the delay D_1 of user 1 increases as the access probability p of user 2 increases. On the other hand, the service rate μ_2 of user 2 is clearly a function of p, but, as we will shortly see, μ_2 may increase or decrease with p. We are here interested in maximizing μ_2 by adjusting the access probability p, while keeping the delay of user 1 below a given maximum acceptable delay D_0.

We begin by rewriting μ_2 as

$$\begin{aligned} \mu_2(p) &= q_{2|\{2\}} + \frac{q_{2|\{1,2\}}p - q_{2|\{2\}}}{q_{1|\{1\}} + Q_1 p}\lambda_1, \\ &= q_{2|\{2\}} + H(p)\lambda_1, \end{aligned} \tag{14}$$

where

$$H(p) = \frac{q_{2|\{1,2\}}p - q_{2|\{2\}}}{q_{1|\{1\}} + Q_1 p}. \tag{15}$$

We can check whether $\mu_2(p)$ is a decreasing or an increasing function of p by analyzing the sign of the first derivative of $H(p)$:

$$\frac{dH}{dp} = \frac{q_{1|\{1\}}q_{2|\{1,2\}} - Q_1 q_{2|\{2\}}}{\left(q_{1|\{1\}} + Q_1 p\right)^2}. \tag{16}$$

Therefore, the behavior (increasing or decreasing) of μ_2 does not depend on p, but on the values of successful transmission probabilities. More specifically, if $q_{1|\{1\}}q_{2|\{1,2\}} - Q_1 q_{2|\{2\}} > 0$, then μ_2 is an increasing function of p, and vice-versa. By using simple variable manipulation, we can redefine this condition as:

$$\begin{aligned} &\frac{q_{1|\{1,2\}}}{q_{1|\{1\}}} + \frac{q_{2|\{1,2\}}}{q_{2|\{2\}}} > 1 \quad \Rightarrow \quad \mu_2 \text{ increases with } p \\ &\frac{q_{1|\{1,2\}}}{q_{1|\{1\}}} + \frac{q_{2|\{1,2\}}}{q_{2|\{2\}}} < 1 \quad \Rightarrow \quad \mu_2 \text{ decreases with } p \end{aligned} \tag{17}$$

The inequalities in (17) have appeared in [4], where the authors investigated the stability region of the two-user interference channel. It was shown in [4] that the condition (17) can be use to determine whether the stability region[1], for $p = 1$, is concave or convex.

Based on condition (17), we can determine the access probability of user 2 that maximizes its average service rate, under a constraint on the delay of user 1, as follows.

Let D_0 be the target delay of user 1, with $D_0 \geq D_{1,\text{min}}$, where $D_{1,\text{min}}$ is the minimum possible delay at arrival rate λ_1, given by

$$D_{1,\text{min}} = \frac{1}{q_{1|\{1\}} - \lambda_1}. \tag{18}$$

The access probability p^* that maximizes the average service rate μ_2 of user 2, while keeping $D_1 < D_0$, is given by

$$p^* = \begin{cases} 0 & \text{if } G < 1, \\ \min\left\{1, p_0\right\} & \text{if } G > 1. \end{cases} \tag{19}$$

where G and p_0 are given by

$$G = \frac{q_{1|\{1,2\}}}{q_{1|\{1\}}} + \frac{q_{2|\{1,2\}}}{q_{2|\{2\}}}, \tag{20}$$

and

$$p_0 = \frac{D_0(q_{1|\{1\}} - \lambda_1) - 1}{D_0 Q_1}. \tag{21}$$

Proof of expression (19): We begin by noting the transmission of user 2 can only increase the delay of user 1, and, therefore, the target delay D_0 of user 1, at input rate λ_1, cannot be smaller than the delay observed when user 1 is accessing the channel alone, i.e., we must have $D_0 \geq D_{1,\text{min}}$. Let us first consider the case when $G < 1$, *i.e.*, when μ_2 is a decreasing function of p. Since D_1 is always an increasing function of p, then μ_2 is maximized by setting $p^* = 0$. Now, let us consider the case when $G > 1$, which means that μ_2 is an increasing function of p and, therefore, we would like to set p to its maximum possible value. Recalling that D_1 is an increasing function of p, and we want to guarantee that $D_1 < D_0$, then, using (13), the largest possible value of p is determined by solving for p_0 the inequality

$$\frac{1}{q_{1|\{1\}} + Q_1 p_0 - \lambda_1} < D_0. \tag{22}$$

Clearly, if $D_0 > D_1|_{p=1}$, then the value of p_0 that solves (22) is larger than unity, and $p^* = \min\left\{1, p_0\right\}$. This concludes the proof. ■

Using expressions (19) and (14), the maximum secondary service rate that does not violate the constraint $D_1 < D_0$ is then given by

$$\mu_{2,\text{max}} = q_{2|\{2\}} + \frac{q_{2|\{1,2\}}p^* - q_{2|\{2\}}}{q_{1|\{1\}} + Q_1 p^*}\lambda_1. \tag{23}$$

The service rate μ_2 is measured in number of packets per time-slots. Recalling that the number of modulation symbols S per packet is fixed with respect to the modulation order, then the maximum throughput of user 2 in bits per packet can be measured by

$$T_2 = \log_2(M_2) \times \mu_{2,\text{max}}. \tag{24}$$

Throughput T_2 will be used in the numerical analysis (in the sequel) to investigate the effects of spectral efficiency on the access probability.

A. Numerical Analysis

In this section we investigate the effects of some parameters on the maximum average service rate of user 2, under some constraints on the delay of user 1, using the expressions derived in the previous section. Particularly, we are interested in evaluating the effects of spectral efficiency on the maximum throughput of user 2.

We assume in this analysis that both users employ the same modulation order. Table II shows the values of $q_{i|\mathcal{M}}$ for the modulation orders considered in this analysis, and for $\eta = 5.0$. Note that, for $\eta = 5.0$ and the parameter setting shown in Table I, we have $G < 1$ for modulation formats 8-QAM and 16-QAM, and therefore we can expect $p^* = 0$ for any value of λ_1 for these two modulation orders.

Figure 2 shows the required p^* for $D_0 = 8$ time-slots (TS) and $\eta = 5.0$. As expected, for modulation formats 8-QAM and 16-QAM we must have $p^* = 0$, since $G < 1$. This means that, with $M = 8$ and 16, user 2 cannot transmit when user 1

[1] Stability region is set of vectors (λ_1, λ_2) for which queues of users 1 and 2 are stable.

TABLE II. SUCCESSFUL TRANSMISSION PROBABILITY, FOR $\eta = 5.0$.

$q_{i\|\mathcal{M}}$	$M_1 = 2$	$M_1 = 4$	$M_1 = 8$	$M_1 = 16$
$q_{1\|\{1\}}$	1.00	1.00	1.00	1.00
$q_{1\|\{1,2\}}$	0.81	0.58	0.37	0.21
$q_{2\|\{2\}}$	1.00	1.00	1.00	1.00
$q_{2\|\{1,2\}}$	0.81	0.58	0.37	0.21
G	1.63	1.16	0.74	0.42

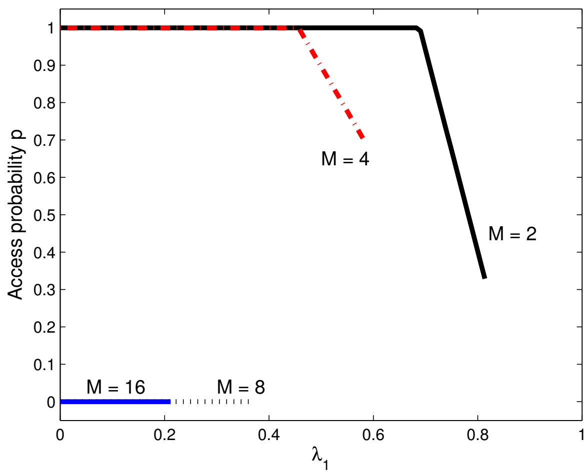

Fig. 2. Access probability p^* that guarantees $D_1 < 8$ TS, and maximizes the secondary service rate, for path loss exponent $\eta = 5$, and different modulation orders.

is already transmitting, if the constraint $D_1 < 8$ TS is to be guaranteed. On the other hand, for modulation formats 2-QAM and 4-QAM the access probability can be set to $p^* = 1$ for small values of primary input rate λ_1, indicating that $D_0 > D_1|_{p=1}$ in these cases. For larger values of λ_1, p^* must be reduced as λ_1 increases, in order to control the interference caused by user 2 to user 1, and guarantee $D_1 < D_0$. Note that in Figure 2 we have used $\lambda_1 < q_{1|\{1,2\}}$ for each modulation order, in order to guarantee that the queue of user 1 is stable in the worst scenario case, that is, when $p = 1$ (see expression (11)).

Figure 3 shows the maximum secondary service rate for the access probabilities shown in Figure 2. Note that, for $M_1 = M_2 = 2$ and 4 (*i.e.*, for the cases where $p^* > 0$ is possible), the maximum secondary service rate decays more slowly as λ_1 increases. In other words, with lower modulation order M, $\mu_{2,\max}$ is less sensitive to λ_1.

Figure 4 shows the maximum throughput T_2, given by (24), for the scenarios investigated. We can see that, as we increase the spectral efficiency of the communication links (recall that we are assuming $M_1 = M_2$), more *secondary bits* can be sent without violating the delay constraint $D_1 < D_0$. However, as already noted, when high spectral efficiency modulation is employed, the maximum throughput T_2 decays more quickly with increasing arrival rate λ_1.

Next, we consider path loss exponent $\eta = 3$, which corresponds to a propagation environment with stronger interference. Figure 5 shows that the maximum service rate $\mu_{2,\max}$ becomes more and more sensitive to the arrival rate λ_1.

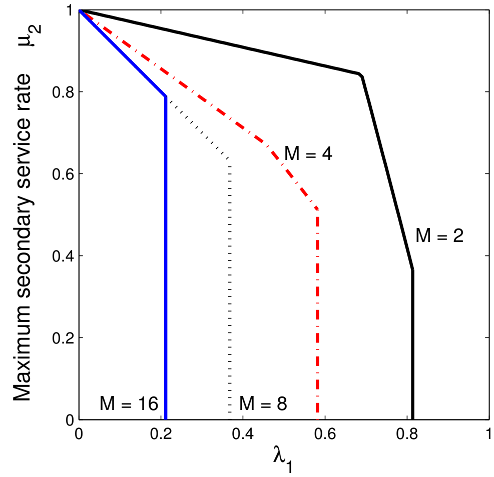

Fig. 3. Maximum secondary service rate $\mu_{2,\max}$ that guarantees $D_1 < 8$ TS, for path loss exponent $\eta = 5$, and different modulation orders.

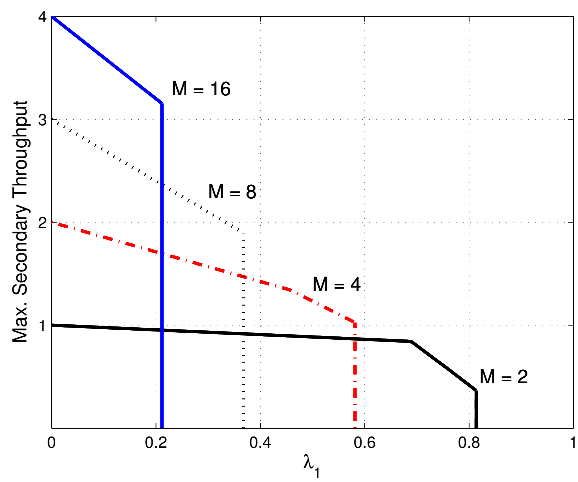

Fig. 4. Maximum secondary throughput T_2 in bits per packet that guarantees $D_1 < D_0 = 8$ TS, for path loss exponent $\eta = 5$, and different modulation orders.

This conclusion is corroborated by the curves shown in Figure 6, where the maximum throughput for $\eta = 3$ and $\eta = 5$ are plotted together for ease of comparison. Note that, with robust modulation scheme (M = 2, in our study) and strong signal attenuation (high η), the maximum service rate $\mu_{2,\max}$ is almost insensitive to the arrival rate λ_1, for small values of λ_1.

IV. CONCLUSIONS

We studied the performance of two interacting queues, where one of the users (user 1) has priority to access the channel over the other user (user 2). In our model, user 2 transmits with probability one when the channel is idle, and with access probability p when the channel is busy. As a consequence, the delay of user 1 is affected by transmissions of user 2, due to the interference caused by the latter to the former.

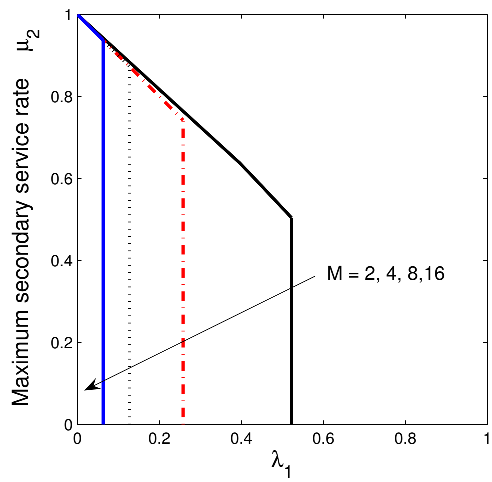

Fig. 5. Maximum secondary service rate $\mu_{2,\max}$ that guarantees $D_1 < 8$ TS, for path loss exponent $\eta = 3$, and different modulation orders.

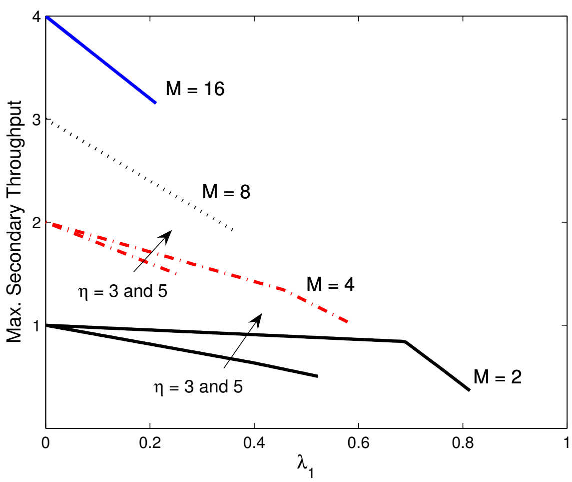

Fig. 6. Maximum secondary throughput T_2 in bits per packets that guarantees $D_1 < D_0 = 8$ TS, for path loss exponents $\eta = 3$ and $\eta = 5$, and different modulation orders. Note that, for $M = 8$ and 16, the curves for $\eta = 3$ and for $\eta = 5$ coincide.

Particularly, we investigated the effects of the spectral efficiency of user 2 on its throughput, when its transmissions are controlled, by means of the access probability p, in order to keep the delay of user 1 below some threshold. We have seen that when higher modulation order is employed, we may need to adjust the access probability to zero, which means that user 2 can only transmit when the channel is idle. On the other hand, by using a more robust and less spectrally efficient modulation, user 2 is allowed to transmit more often.

REFERENCES

[1] J. Hu, L.-L. Yang, and L. Hanzo, "Maximum average service rate and optimal queue scheduling of delay-constrained hybrid cognitive radio in nakagami fading channels," *Vehicular Technology, IEEE Transactions on*, vol. 62, no. 5, pp. 2220–2229, Jun 2013.

[2] J. Jeon, M. Codreanu, M. Latva-aho, and A. Ephremides, "The stability property of cognitive radio systems with imperfect sensing," *Selected Areas in Communications, IEEE Journal on*, vol. 32, no. 3, pp. 628–640, March 2014.

[3] V. Naware, G. Mergen, and L. Tong, "Stability and delay of finite-user slotted aloha with multipacket reception," *Information Theory, IEEE Transactions on*, vol. 51, no. 7, pp. 2636–2656, July 2005.

[4] N. Pappas, M. Kountouris, and A. Ephremides, "The stability region of the two-user interference channel," in *Information Theory Workshop (ITW), 2013 IEEE*, Sept 2013, pp. 1–5.

[5] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*. New York, NY, USA: Cambridge University Press, 2005.

[6] J. G. Proakis, *Digital Communications*. McGraw-Hill, 2000.

[7] A. El Shafie and A. Sultan, "Optimal random access and random spectrum sensing for an energy harvesting cognitive radio," in *Wireless and Mobile Computing, Networking and Communications (WiMob), 2012 IEEE 8th International Conference on*. IEEE, 2012, pp. 403–410.

[8] R. Rao and A. Ephremides, "On the stability of interacting queues in a multiple-access system," *Information Theory, IEEE Transactions on*, vol. 34, no. 5, pp. 918–930, Sep 1988.