

A Distributed Transmit Antenna Selection Scheme for Fixed-Gain Multi-Antenna AF Relaying Systems

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Abstract—Antenna selection has arisen as an attractive strategy in multi-antenna systems due to its reduced complexity and cost. In this paper, the outage performance of a distributed transmit antenna selection scheme for dual-hop fixed-gain amplify-and-forward relaying systems is investigated. Our analysis considers a multi-antenna source, a multi-antenna relay, and a single-antenna destination, in which selection combining and maximal-ratio combining are employed at the relay and destination receptions, respectively. Lower and upper bounds for the outage probability of the proposed scheme are derived as the sum of two terms, one term given in closed form and the other term given in single integral form. Moreover, asymptotic expressions for these bounds at high signal-to-noise ratio are obtained in closed form. The results show that our scheme presents full diversity order. More importantly, the underlying distributed strategy is shown to perform closely to the costly optimal centralized antenna selection.

I. INTRODUCTION

Multi-antenna relaying systems are expected to be widely utilized in future wireless communication networks, since the advantages offered by multiple-input multiple-output (MIMO) and relaying techniques can be fully exploited. However, a potential drawback of the use of multiple antennas is their complex hardware and increased cost [1]. To alleviate this, transmit antenna selection (TAS) has arisen as an alternative low-cost low-complexity strategy to capture the advantages of MIMO systems. Nevertheless, in order to provide the knowledge necessary for the transmitter to select the best antenna, the implementation of TAS schemes usually requires feedback channels for acquiring the channel state information (CSI) of all links. The required bits of feedback information vary depending on the number of antennas at the nodes [1]. In this case, when the number of antennas increases, the delay and complexity involved in the selection mechanism may rapidly become prohibitive in practice.

Several works have shown and explored the advantages of TAS schemes, but a substantial amount of CSI feedback has been observed in most of them. For instance, the optimal TAS strategy was studied in [2] for a dual-hop MIMO amplify-and-forward (AF) relaying system, in which an exhaustive search for the best antenna at the source and relay was required. A different optimal TAS strategy was examined in [3] assuming a variable-gain AF relay and Nakagami- m fading. Two suboptimal TAS strategies were also analyzed

in [3], in which the feedback overhead was reduced at the expense of system performance. More recently, a distributed TAS scheme was proposed in [4] assuming a single-antenna fixed-gain AF relay.

In this work, we generalize the scheme proposed in [4] by considering a fixed-gain AF relay with multiple antennas in both transmission and reception. Our scheme provides a low and constant delay/feedback overhead, despite of the number of transmit antennas. We derive lower and upper bounds for the outage probability of the proposed scheme as the sum of two terms, one term given in closed form and the other term given in single integral form. Moreover, an asymptotic expression for each of these bounds at high signal-to-noise ratio (SNR) is obtained in closed form. It is noteworthy that such analysis is considerably more intricate than that of the scheme in [4]. Our results show that the proposed scheme presents full diversity order. More importantly, its underlying distributed strategy is shown to perform closely to the costly optimal centralized TAS.

Throughout this paper, $f_Z(\cdot)$ denotes the probability density function (PDF) of a generic random variable Z , $E[\cdot]$ denotes expectation, and $\Pr(\cdot)$ denotes probability.

II. SYSTEM MODEL AND ANTENNA SELECTION SCHEME

A. System Model

Consider a dual-hop relaying system composed of one source S with N_t antennas, one single-antenna destination D , and one half-duplex fixed-gain AF relay R with N_{rr} receive antennas and N_{rt} transmit antennas. A time-division multiple access scheme is assumed and all the channels undergo flat Rayleigh fading.

Before data transmission, the transmit antenna at S that maximizes the end-to-end instantaneous SNR is chosen according to the procedure to be described in the next subsection. Afterwards, a conventional cooperative transmission takes place in two time slots. In the first time slot, the signals received by the multiple antennas at R are combined using a selection combining (SC) scheme, and, in the second time slot, the signals coming from the direct and relaying links¹

¹The relay employs a TAS scheme to forward to the destination the signal received from the source. This will be described next.

are combined at D using a maximal-ratio combining (MRC) scheme. The end-to-end SNR from the i th antenna at S to D (through the k th receive antenna and j th transmit antenna at R) can be written as

$$\gamma_{i,k,j} = \gamma_{S_i D} + \frac{\gamma_{S_i R_k} \gamma_{R_j D}}{\gamma_{R_j D} + C}, \quad (1)$$

where $\gamma_{S_i D} \triangleq \frac{P_S}{N_0} |h_{S_i D}|^2$, $\gamma_{S_i R_k} \triangleq \frac{P_S}{N_0} |h_{S_i R_k}|^2$, $\gamma_{R_j D} \triangleq \frac{P_R}{N_0} |h_{R_j D}|^2$, and $C = 1 + \bar{\gamma}_{S_i R_k}$, with $\bar{\gamma}_{S_i R_k} = E[\gamma_{S_i R_k}]$. In these expressions, $|h_{S_i D}|^2$, $|h_{S_i R_k}|^2$, and $|h_{R_j D}|^2$ denote the channel power gains of the links from the i th antenna at S to D , from the i th antenna at S to the k th antenna at R , and from the j th antenna at R to D , respectively; P_S and P_R stand for the transmit powers at S and R , respectively; and N_0 denotes the additive white Gaussian noise (AWGN) at each receiving terminal. We assume a homogeneous network, in which $E[\gamma_{S_i R_k}] = \bar{\gamma}_{SR}$, $E[\gamma_{S_i D}] = \bar{\gamma}_{SD}$, and $E[\gamma_{R_j D}] = \bar{\gamma}_{RD}$, for any $i = 1, \dots, N_t$, any $k = 1, \dots, N_{rr}$, and any $j = 1, \dots, N_{rt}$. The fixed-gain relaying factor is adjusted according to [5] as $G^2 = E\left[\frac{P_R}{P_S |h_{S_i R_k}|^2 + N_0}\right]$.

B. Antenna Selection Strategy

The optimal selection criterion chooses the best transmit antennas at S and R that maximize the end-to-end SNR, i.e.,

$$(i^*, k^*, j^*) = \arg \max_{i,k,j} [\gamma_{i,k,j}]. \quad (2)$$

Note that the optimal scheme entails a large amount of delay and feedback overhead due to the full system CSI required for decision. To alleviate this, relying on the idea pioneered in [4], a suboptimal (yet efficient) distributed antenna selection (DAS) scheme is presented next. In this scheme, the local CSI available at S is exploited to its furthest extent in order to assist the decision, incurring a negligible delay and feedback overhead.

The DAS concept is motivated and supported by the following inequality involving the end-to-end SNR:

$$\gamma_{i,k,j} < \gamma_{S_i D} + \gamma_{S_i R_k} \min\left[\frac{\gamma_{R_j D}}{C}, 1\right] \triangleq \tilde{\gamma}_i. \quad (3)$$

The DAS scheme is performed in two time slots [4]. In the first time slot, D sends to R and S a 1-bit reverse pilot signaling. Then, R and S use this bit to estimate their respective local CSIs $\gamma_{R_j D}$ and $\gamma_{S_i D}$. At this time, based on $\gamma_{R_j D}$, the relay selects its optimal antenna to transmit data, by applying the selection rule $j^* = \arg \max_j [\gamma_{R_j D}]$. In the second time slot, R compares its local CSI $\gamma_{R_{j^*} D}$ with C , yielding two cases:

- Case I: If $\gamma_{R_{j^*} D} \geq C$, R sends to S a 1-bit message “1”. In this case, by using (3), $\gamma_{S_i D}$ and $\gamma_{S_i R_k}$, which are available at S , are sufficient to apply the selection rule $\max\{\tilde{\gamma}_{i,k,j^*}\}$.

- Case II: If $\gamma_{R_{j^*} D} < C$, R sends to S a 1-bit message “0”. In this case, by using (3), the application of $\max\{\tilde{\gamma}_{i,k,j^*}\}$ would require the additional knowledge of $\gamma_{R_{j^*} D}$, which is unavailable at S . Therefore, somewhat arbitrarily, a suboptimal decision will be employed that depends solely on the maximization of $\gamma_{S_i D}$, which is available at S . All in all, the TAS

rule of the proposed DAS scheme is summarized as follows:

$$i^* = \begin{cases} \bar{i} = \arg \max_i \left[\gamma_{S_i D} + \max_k [\gamma_{S_i R_k}] \right], & \text{if } \gamma_{R_{j^*} D} \geq C \\ \underline{i} = \arg \max_i [\gamma_{S_i D}], & \text{if } \gamma_{R_{j^*} D} < C \end{cases} \quad (4)$$

It is noteworthy that the major advantage of DAS over other antenna selection schemes is its reduced delay/feedback overhead, requiring only a 2-bit pilot/feedback signaling, at the expense of some additional hardware complexity at the source.

III. PERFORMANCE ANALYSIS

In this section, the outage probability for the proposed scheme will be analyzed. Such a metric can be defined as the probability that the end-to-end SNR falls below a given threshold, commonly written in terms of a target spectral efficiency \mathfrak{R}_0 , being mathematically formulated as

$$P_{out}^{DAS} = \underbrace{\Pr\left(\gamma_{R_{j^*} D} \geq C, \left(\gamma_{S_{\bar{i}} D} + \frac{\gamma_{S_{\bar{i}} R_{k^*}} \gamma_{R_{j^*} D}}{\gamma_{R_{j^*} D} + C}\right) < z \triangleq 2^{2\mathfrak{R}_0} - 1\right)}_{P_1} + \underbrace{\Pr\left(\gamma_{R_{j^*} D} < C, \left(\gamma_{S_{\underline{i}} D} + \frac{\gamma_{S_{\underline{i}} R_{k^*}} \gamma_{R_{j^*} D}}{\gamma_{R_{j^*} D} + C}\right) < z\right)}_{P_2}. \quad (5)$$

Since an exact closed-form expression for the above expression is mathematically intractable, lower and upper bounds for the outage probability will be derived instead, based on the following paramount relationships:

$$\frac{\gamma_{S_i R_k}}{2} \min\left[\frac{\gamma_{R_j D}}{C}, 1\right] \leq \frac{\gamma_{S_i R_k} \gamma_{R_j D}}{\gamma_{R_j D} + C} \leq \gamma_{S_i R_k} \min\left[\frac{\gamma_{R_j D}}{C}, 1\right]. \quad (6)$$

In order to evaluate the terms P_1 and P_2 , we first derive a lower bound P_1^{LB} for P_1 , using (6), as follows:

$$\begin{aligned} P_1 &> \Pr\left(\gamma_{R_{j^*} D} \geq C, \left(\gamma_{S_{\bar{i}} D} + \gamma_{S_{\bar{i}} R_{k^*}} \min\left[\frac{\gamma_{R_{j^*} D}}{C}, 1\right]\right) < z\right) \\ &\stackrel{(a)}{=} \Pr\left(\gamma_{R_{j^*} D} \geq C, \max_i \left(\gamma_{S_i D} + \max_k (\gamma_{S_i R_k})\right) < z\right) \triangleq P_1^{\text{LB}} \\ &= \Pr(\gamma_{R_{j^*} D} \geq C) \Pr\left(\left(\gamma_{S_i D} + \max_k (\gamma_{S_i R_k})\right) < z\right)^{N_t} \\ &= \Pr(\gamma_{R_{j^*} D} \geq C) \\ &\times \left(\int_0^z f_{\gamma_{SD}}(x) \Pr\left(\left(x + \max_k (\gamma_{S_i R_k})\right) < z\right) dx\right)^{N_t} \\ &= \Pr(\gamma_{R_{j^*} D} \geq C) \\ &\times \underbrace{\left(\int_0^z \frac{1}{\bar{\gamma}_{SD}} e^{-\frac{x}{\bar{\gamma}_{SD}}} \left(1 - e^{-\frac{z-x}{\bar{\gamma}_{SR}}}\right)^{N_{rr}} dx\right)^{N_t}}_{\alpha}, \end{aligned} \quad (7)$$

where in step (a) we applied the DAS rule given in (4) for $\gamma_{R_{j^*} D} \geq C$. The term α can be rewritten using the binomial

theorem [6, Eq. (1.111)] as

$$\begin{aligned}\alpha &= \int_0^z \frac{1}{\bar{\gamma}_{SD}} e^{-\frac{x}{\bar{\gamma}_{SD}}} \sum_{j=0}^{N_{rr}} \binom{N_{rr}}{j} (-1)^j e^{-j\frac{z-x}{\bar{\gamma}_{SR}}} dx \\ &= \sum_{j=0}^{N_{rr}} \binom{N_{rr}}{j} (-1)^j e^{-j\frac{z}{\bar{\gamma}_{SR}}} \frac{1}{\bar{\gamma}_{SD}} \int_0^z e^{j\frac{x}{\bar{\gamma}_{SR}} - \frac{x}{\bar{\gamma}_{SD}}} dx \quad (8) \\ &= \sum_{j=0}^{N_{rr}} \binom{N_{rr}}{j} (-1)^j e^{-j\frac{z}{\bar{\gamma}_{SR}}} \left(\frac{(1 - e^{-\frac{z}{\bar{\gamma}_{SD}} + \frac{jz}{\bar{\gamma}_{SR}}}) \bar{\gamma}_{SR}}{j\bar{\gamma}_{SD} - \bar{\gamma}_{SR}} \right).\end{aligned}$$

Then, by substituting (8) in (7), a lower bound for P_1 can be derived in closed form as

$$\begin{aligned}P_1^{\text{LB}} &= \left(1 - \left(1 - e^{-\frac{C}{\bar{\gamma}_{RD}}}\right)^{N_{rt}}\right) \\ &\quad \times \left(\sum_{j=0}^{N_{rr}} \binom{N_{rr}}{j} (-1)^j \left(\frac{(e^{-j\frac{z}{\bar{\gamma}_{SR}}} - e^{-\frac{z}{\bar{\gamma}_{SD}}}) \bar{\gamma}_{SR}}{j\bar{\gamma}_{SD} - \bar{\gamma}_{SR}} \right) \right)^{N_t}.\end{aligned} \quad (9)$$

In order to assess the asymptotic behavior of P_1^{LB} , a high-SNR expression for the term α can be derived from its definition as

$$\begin{aligned}\alpha &= \int_0^z \frac{1}{\bar{\gamma}_{SD}} e^{-\frac{z-y}{\bar{\gamma}_{SD}}} \left(1 - e^{-\frac{y}{\bar{\gamma}_{SR}}}\right)^{N_{rr}} dy \\ &\simeq \left(\frac{1}{\bar{\gamma}_{SR}}\right)^{N_{rr}} \frac{1}{\bar{\gamma}_{SD}} \frac{z^{N_{rr}+1}}{(N_{rr}+1)},\end{aligned} \quad (10)$$

which, when replaced in (7), yields a corresponding high-SNR asymptotic lower bound for P_1 as

$$P_1^{\text{LB}} \simeq \left(1 - \left(1 - e^{-\frac{C}{\bar{\gamma}_{RD}}}\right)^{N_{rt}}\right) \left(\left(\frac{1}{\bar{\gamma}_{SR}}\right)^{N_{rr}} \frac{1}{\bar{\gamma}_{SD}} \frac{z^{N_{rr}+1}}{(N_{rr}+1)} \right)^{N_t}. \quad (11)$$

Using (6), it can be shown that an upper bound P_1^{UB} for P_1 and its asymptote can be directly attained by replacing z with $2z$ into (9) and (11), respectively.

Similarly, a lower bound P_2^{LB} for P_2 can be formulated as

$$\begin{aligned}P_2 &> \Pr\left(\gamma_{R_j^* D} < C, \left(\gamma_{S_i D} + \gamma_{S_i R_k^*} \min\left[\frac{\gamma_{R_j^* D}}{C}, 1\right]\right) < z\right) \\ &= \Pr\left(\gamma_{R_j^* D} < C, \left(\gamma_{S_i D} + \frac{\gamma_{S_i R_k^*} \gamma_{R_j^* D}}{C}\right) < z\right) = P_2^{\text{LB}} \\ &= \Pr\left(\gamma_{R_j^* D} < C, \left(\max_m [\gamma_{S_m D}] + \max_k [\gamma_{S_i R_k}] \frac{\gamma_{R_j^* D}}{C}\right) < z\right).\end{aligned} \quad (12)$$

By using the principles of probability theory, P_2^{LB} can be elaborated as

$$\begin{aligned}P_2^{\text{LB}} &= \int_0^C f_{\gamma_{RD}}(x) \Pr\left(\left(\max_m [\gamma_{S_m D}] + \max_k [\gamma_{S_i R_k}] \frac{x}{C}\right) < z\right) dx \\ &= \int_0^1 C f_{\gamma_{RD}}(Cu) \Pr\left(\left(\max_m [\gamma_{S_m D}] + \max_k [\gamma_{S_i R_k}] u\right) < z\right) du \\ &= \int_0^1 C f_{\gamma_{RD}}(Cu) \int_0^z f_{\gamma_{SR}}(y) \Pr\left(\left(\max_m [\gamma_{S_m D}] + yu\right) < z\right) dy du \\ &= \int_0^1 C f_{\gamma_{RD}}(Cu) \int_0^z \frac{1}{u} f_{\gamma_{SR}}\left(\frac{v}{u}\right) \Pr\left(\max_m [\gamma_{S_m D}] < z - v\right) dv du \\ &= \int_0^1 \frac{CN_{rt}}{\bar{\gamma}_{RD}} e^{-\frac{Cu}{\bar{\gamma}_{RD}}} \left(1 - e^{-\frac{Cu}{\bar{\gamma}_{RD}}}\right)^{N_{rt}-1} \frac{N_{rr}}{u\bar{\gamma}_{SR}} \\ &\quad \times \underbrace{\int_0^z e^{-\frac{v}{u\bar{\gamma}_{SR}}} \left(1 - e^{-\frac{v}{u\bar{\gamma}_{SR}}}\right)^{N_{rr}-1} \left(1 - e^{-\frac{z-v}{\bar{\gamma}_{SD}}}\right)^{N_t}}_{\varphi} dv du,\end{aligned} \quad (13)$$

where the term φ can be calculated using the binomial theorem [6, Eq. (1.111)] as

$$\begin{aligned}\varphi &= \sum_{m=0}^{N_t} \sum_{j=0}^{N_{rr}} \binom{N_t}{m} \binom{N_{rr}-1}{j} (-1)^m (-1)^j \\ &\quad \times \frac{\left(e^{-\frac{(1+j)z}{u\bar{\gamma}_{SR}}} - e^{-\frac{mz}{\bar{\gamma}_{SD}}}\right) u\bar{\gamma}_{SR}\bar{\gamma}_{SD}}{mu\bar{\gamma}_{SR} - (1+j)\bar{\gamma}_{SD}}.\end{aligned} \quad (14)$$

Then, by substituting (14) into (13), a lower bound for P_2 can be derived in single integral form as

$$\begin{aligned}P_2^{\text{LB}} &= \int_0^1 \frac{CN_{rt}}{\bar{\gamma}_{RD}} e^{-\frac{Cu}{\bar{\gamma}_{RD}}} \left(1 - e^{-\frac{Cu}{\bar{\gamma}_{RD}}}\right)^{N_{rt}-1} \frac{N_{rr}}{u} \\ &\quad \times \sum_{m=0}^{N_t} \sum_{j=0}^{N_{rr}} \binom{N_t}{m} \binom{N_{rr}-1}{j} (-1)^m (-1)^j \\ &\quad \times \frac{\left(e^{-\frac{(1+j)z}{u\bar{\gamma}_{SR}}} - e^{-\frac{mz}{\bar{\gamma}_{SD}}}\right) u\bar{\gamma}_{SD}}{mu\bar{\gamma}_{SR} - (1+j)\bar{\gamma}_{SD}} du.\end{aligned} \quad (15)$$

In order to assess the asymptotic behavior of P_2^{LB} , we simplify the term φ defined in (13) by using the binomial theorem [6, Eq. (1.111)] and the MacLaurin series of exponential functions [6, Eq. (1.211.1)], obtaining

$$\begin{aligned}\varphi &= \sum_{i=1}^{N_{rr}} \binom{N_{rr}-1}{i-1} (-1)^{i-1} \int_0^z e^{-\frac{iv}{u\bar{\gamma}_{SR}}} \left(\frac{z-v}{\bar{\gamma}_{SD}}\right)^{N_t} dv \\ &= \sum_{i=1}^{N_{rr}} \binom{N_{rr}-1}{i-1} (-1)^{i-1} e^{-\frac{iz}{u\bar{\gamma}_{SR}}} \left(\frac{1}{\bar{\gamma}_{SD}}\right)^{N_t} \\ &\quad \times \sum_{n=0}^{\infty} \binom{i}{u\bar{\gamma}_{SR}}^n \frac{z^{N_t+n+1}}{n!(N_t+n+1)}.\end{aligned} \quad (16)$$

By substituting (16) into (13) and using again the binomial

theorem, P_2^{LB} can be rewritten as

$$P_2^{\text{LB}} = N_{rt} \frac{N_{rr}}{\bar{\gamma}_{SR}} \left(\frac{1}{\bar{\gamma}_{SD}} \right)^{N_t} \sum_{i=1}^{N_{rr}} \binom{N_{rr}-1}{i-1} (-1)^{i-1} \times \sum_{j=0}^{N_{rt}} \binom{N_{rt}-1}{j-1} (-1)^{j-1} \sum_{n=0}^{\infty} \underbrace{\frac{z^{N_t+n+1} i^n \bar{\gamma}_{SR}^{-n}}{n! (N_t+n+1)} \int_0^1 \frac{C}{\bar{\gamma}_{RD}} u^{-n-1} e^{-\frac{iz}{u\bar{\gamma}_{SR}}} e^{-\frac{jCu}{\bar{\gamma}_{RD}}} du}_{\Phi_n}. \quad (17)$$

The behavior of Φ_n in the high-SNR regime is characterized in Apps. A and B. In App. A, Φ_n is determined for $n = 0$, yielding

$$\Phi_0 \simeq \begin{cases} \frac{z^{N_t+1}}{(N_t+1)} \frac{\left(\frac{jCiz}{\bar{\gamma}_{SR}\bar{\gamma}_{RD}} \right)^{n_1}}{n_1!^2 \mu_2} \left(-\ln \left(\frac{jiz}{\bar{\gamma}_{RD}} \right) + \psi(n_1+1) - (-1)^{n_1} \Gamma \left(-n_1, \frac{jC}{\bar{\gamma}_{RD}} \right) \right), & \text{if } N_{rr} = N_{rt} \\ \frac{z^{N_t+1}}{(N_t+1)} \frac{\left(\frac{jCiz}{\bar{\gamma}_{SR}\bar{\gamma}_{RD}} \right)^{n_1}}{n_1!^2 \mu_2} \left(-\ln \left(\frac{jiz}{\bar{\gamma}_{RD}} \right) - (-1)^{n_1} \Gamma \left(-n_1, \frac{jC}{\bar{\gamma}_{RD}} \right) \right), & \text{if } N_{rr} < N_{rt} \\ \frac{z^{N_t+1}}{(N_t+1)} \frac{\left(\frac{jCiz}{\bar{\gamma}_{SR}\bar{\gamma}_{RD}} \right)^{n_1}}{n_1!^2 \mu_2} \left(-\ln \left(\frac{jiz}{\bar{\gamma}_{RD}} \right) \right), & \text{if } N_{rr} > N_{rt}, \end{cases} \quad (18)$$

where $\mu_2 \triangleq \bar{\gamma}_{RD}/\bar{\gamma}_{SR}$ and $n_1 \triangleq \min(N_{rr}, N_{rt}) - 1$. In App. B, Φ_n is determined for $n > 0$, in which two intervals for n are established ($1 \leq n \leq n_1$ and $n > n_1$), yielding

$$\Phi_{n|n_1} \simeq \begin{cases} \frac{z^{N_t+n_1+1}}{n!(N_t+n_1+1)\mu_2} \frac{\left(\frac{jCi}{\bar{\gamma}_{SR}\bar{\gamma}_{RD}} \right)^{n_1}}{\left((-1)^{n_1+1} \frac{1}{(n_1-n)!n_1!} \ln \left(\frac{jiz}{\bar{\gamma}_{RD}} \right) - \frac{(-1)^{n_1-n}}{(n_1-n)!} \Gamma \left(-n_1, \frac{jC}{\bar{\gamma}_{RD}} \right) \right)}, & \text{if } N_{rr} \leq N_{rt} \\ \frac{z^{N_t+n_1+1}}{n!(N_t+n_1+1)\mu_2} \frac{\left(\frac{jCi}{\bar{\gamma}_{SR}\bar{\gamma}_{RD}} \right)^{n_1}}{\left((-1)^{n_1+1} \frac{1}{(n_1-n)!n_1!} \ln \left(\frac{jiz}{\bar{\gamma}_{RD}} \right) \right)}, & \text{if } N_{rr} > N_{rt} \end{cases} \quad (19)$$

$$\Phi_{n|n_1+1} \simeq \begin{cases} \frac{z^{N_t+n_1+1}}{n!(N_t+n_1+1)\mu_2} \frac{\left(\frac{jCi}{\bar{\gamma}_{SR}\bar{\gamma}_{RD}} \right)^{n_1}}{\frac{(n-n_1-1)!}{n_1!} (-1)^{n_1}}, & \text{if } N_{rr} = N_{rt} \\ 0, & \text{otherwise.} \end{cases} \quad (20)$$

As done before for the term P_1 , using (6), it can be shown that an upper bound P_2^{UB} for P_2 and its asymptote can be directly attained by replacing z with $2z$ into (15) and (17).

Finally, by adding as in (5) the asymptotic bounds derived for P_1 and P_2 , corresponding asymptotic lower and upper bounds for P_{out} are obtained, respectively, as

$$P_{out}^{\text{DAS,LB}} \simeq \begin{cases} P_1^{\text{LB}} + P_2^{\text{LB}}, & \text{if } N_t = 1 \text{ and } N_{rr} \leq N_{rt} \\ P_2^{\text{LB}}, & \text{otherwise} \end{cases} \quad (21)$$

$$P_{out}^{\text{DAS,UB}} \simeq \begin{cases} P_1^{\text{UB}} + P_2^{\text{UB}}, & \text{if } N_t = 1 \text{ and } N_{rr} \leq N_{rt} \\ P_2^{\text{UB}}, & \text{otherwise.} \end{cases} \quad (22)$$

Remarks:

1. Using (21), (22), and the asymptotic bounds for P_1 and P_2 , it can be shown that the proposed DAS scheme achieves full diversity order, being equal to $N_t + \min(N_{rr}, N_{rt})$.

2. As shown in (17), both lower and upper asymptotic bounds for P_2 are given in terms of an infinite series. In order to observe the convergence of this series, two cases are analyzed: (i) when $N_{rr} \neq N_{rt}$, only the first $n_1 + 1$ terms are taken into consideration, since the series is zero for terms greater than n_1 ; (ii) when $N_{rr} = N_{rt}$, similarly to [4], the convergence is proved using the convergence test of [6, Eq. (0.223)].

IV. NUMERICAL RESULTS AND DISCUSSIONS

In this section, representative numerical results are presented and Monte Carlo simulations are run to support the derived analytical bounds. In our plots, as a sample case, we assume that the target spectral efficiency is $\mathfrak{R}_0 = 1$ bit/s/Hz and that the path loss exponent is $\beta = 4$. We also assume that the channel mean power is proportional to $d^{-\beta}$, with d being the distance between the transceivers. The distance between S and D is normalized to unity, as in [4]². Figs. 1 and 2 show the outage probability for two major system configurations, namely $\{N_t = 2, d_{SR} = 0.7\}$ and $\{N_t = 3, d_{SR} = 0.8\}$, respectively. It is noteworthy that, in each case, the relay has been placed at the position d_{SR} that provides the best outage performance, and such a position was previously established through simulations. These prior simulation results have not been presented here due to space limitations. From the figures, note that the performed analysis for the exact and asymptotic bounds is validated, and that the diversity order, determined as $N_t + \min(N_{rr}, N_{rt})$, is verified. For instance, using the system configuration $\{N_t = 2, N_{rr} = 2, N_{rt} = 2\}$ the diversity order equals 4, which agrees with the asymptotes's slope in Fig. 2. In addition, both figures compares the performance of the proposed DAS and optimal TAS schemes, which can be seen to be very similar.

V. APPENDIX A

HIGH-SNR EXPRESSION FOR THE TERM Φ_0 IN (17)

Here we consider Φ_n for $n = 0$, which is defined in (17) as

$$\Phi_0 = \frac{z^{N_t+1}}{(N_t+1)\bar{\gamma}_{RD}} \underbrace{\int_0^1 u^{-1} e^{-\frac{iz}{u\bar{\gamma}_{SR}}} e^{-\frac{jCu}{\bar{\gamma}_{RD}}} du}_{\rho_0}. \quad (23)$$

The term ρ_0 can be rewritten through the help of [6, Eq. (3.471.9)], [6, Eq. (1.211.1)], and [6, Eq. (8.446)] as (24), given at the top of the next page, in which $K_n(\cdot)$ denotes

²Again, as in [4], we assume a linear network topology, in which S and R transmit with the same SNR P , and $d_{SD} = d_{SR} + d_{RD}$, where d_{SD} , d_{SR} , and d_{RD} represent the distance of the links $S \rightarrow D$, $S \rightarrow R$, and $R \rightarrow D$, respectively. The corresponding average link SNRs can be formulated as $\bar{\gamma}_{SD} = Pd_{SD}^{-\beta}$, $\bar{\gamma}_{SR} = Pd_{SR}^{-\beta}$ and $\bar{\gamma}_{RD} = Pd_{RD}^{-\beta}$.

$$\rho_0 = 2K_0 \left(2\sqrt{\frac{jCiz}{\bar{\gamma}_{SR}\bar{\gamma}_{RD}}} - \int_1^\infty u^{-1} e^{-\frac{jCu}{\bar{\gamma}_{RD}}} \sum_{l=0}^\infty \frac{\left(-\frac{iz}{u\bar{\gamma}_{SR}}\right)^l}{l!} du \right) = -\ln \frac{jCiz}{\bar{\gamma}_{SR}\bar{\gamma}_{RD}} \sum_{k_1=0}^\infty \frac{\left(\sqrt{\frac{jCiz}{\bar{\gamma}_{SR}\bar{\gamma}_{RD}}}\right)^{2k_1}}{k_1!^2} + \sum_{k_2=0}^\infty \frac{\left(\sqrt{\frac{jCiz}{\bar{\gamma}_{SR}\bar{\gamma}_{RD}}}\right)^{2k_2}}{k_2!^2} \times 2\psi(k_2+1) - \sum_{l=0}^\infty \frac{\left(-\frac{iz}{\bar{\gamma}_{SR}}\right)^l}{l!} \left(\frac{jC}{\bar{\gamma}_{RD}}\right)^l \Gamma\left(-l, \frac{jC}{\bar{\gamma}_{RD}}\right). \quad (24)$$

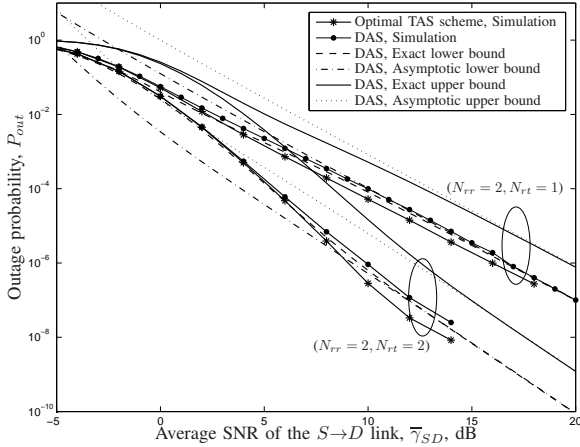


Fig. 1. Outage probability versus average SNR of the $S \rightarrow D$ link for different AS schemes ($d_{SR} = 0.7$, $N_t = 2$).

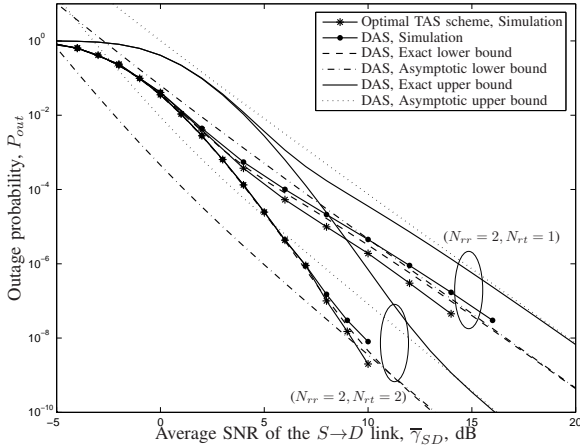


Fig. 2. Outage probability versus average SNR of the $S \rightarrow D$ link for different AS schemes ($d_{SR} = 0.8$, $N_t = 3$).

the n th-order modified Bessel function of the second kind [6, Eq.(8.446)], $\psi(\cdot)$ represents the Euler psi function [6, Eq. (8.360)] and $\Gamma(\cdot)$ is the gamma function [6, Eq. (8.310.1)]. It is important to mention that in order to conduct the high-SNR analysis of the inner expression that defines Φ_0 in (17), the outer summations in i and j must be considered. To this end,

we define the variable λ_0 as

$$\lambda_0 = \sum_{i=1}^{N_{rr}} \binom{N_{rr}-1}{i-1} (-1)^{i-1} \sum_{j=1}^{N_{rt}} \binom{N_{rt}-1}{j-1} \times (-1)^{j-1} \Phi_0. \quad (25)$$

Plugging (24) into (23) and this into (25), λ_0 can be rewritten as

$$\lambda_0 = \frac{z^{N_t+1}}{(N_t+1)} \frac{C}{\bar{\gamma}_{RD}} \left(-\sum_{k_1=0}^\infty \frac{\left(\frac{Cz}{\bar{\gamma}_{SR}\bar{\gamma}_{RD}}\right)^{k_1}}{k_1!^2} \sum_{i=1}^{N_{rr}} \binom{N_{rr}-1}{i-1} i^{k_1} \times (-1)^{i-1} \sum_{j=1}^{N_{rt}} \binom{N_{rt}-1}{j-1} (-1)^{j-1} j^{k_1} \left(\ln\left(\frac{j}{\bar{\gamma}_{RD}}\right) + \ln(iz) \right) + \sum_{k_2=0}^\infty \frac{\left(\frac{Cz}{\bar{\gamma}_{SR}\bar{\gamma}_{RD}}\right)^{k_2}}{k_2!^2} \psi(k_2+1) \sum_{i=1}^{N_{rr}} \binom{N_{rr}-1}{i-1} (-1)^{i-1} i^{k_2} \times \sum_{j=1}^{N_{rt}} \binom{N_{rt}-1}{j-1} (-1)^{j-1} j^{k_2} - \sum_{l=0}^\infty \frac{\left(-\frac{Cz}{\bar{\gamma}_{SR}\bar{\gamma}_{RD}}\right)^l}{l!^2} \sum_{i=1}^{N_{rr}} \binom{N_{rr}-1}{i-1} \times (-1)^{i-1} i^l \sum_{j=1}^{N_{rt}} \binom{N_{rt}-1}{j-1} (-1)^{j-1} j^l \Gamma\left(-l, \frac{jC}{\bar{\gamma}_{RD}}\right) \right). \quad (26)$$

Observe that λ_0 is composed by the sum of three terms, corresponding to the summations in k_1 , k_2 , and l . Using the fact that $\sum_{a=1}^b \binom{b-1}{a-1} (-1)^{a-1} a^c$ is null for $c = 0, 1, 2, \dots, b-2$, after a careful inspection it was attested that each term presents a different behavior at high-SNR regime, and this will be described next. The first term is non-null for values of k_1 greater than $\min(N_{rr}, N_{rt}) - 1$, the second term is non-null for values of k_2 greater than $\max(N_{rr}, N_{rt}) - 1$, and the third term is non-null for values of l greater than $N_{rr} - 1$. Using this and preserving only the lowest-order terms so as to derive a high-SNR expression, we arrive at three cases: (i) if $N_{rr} = N_{rt}$, the three terms must be considered; (ii) if $N_{rr} < N_{rt}$, the first and third terms must be considered; and (iii) if $N_{rr} > N_{rt}$, only the first term must be considered. In all the cases, the diversity order of λ_0 is observed to be given by $n_1 \triangleq \min(N_{rr}, N_{rt}) - 1$. By incorporating these results into the definition of Φ_0 given by (23) and (24), we finally arrive at the high-SNR expression for Φ_0 as given in (18).

$$\begin{aligned}
\rho_n &\stackrel{(b)}{=} 2\bar{\gamma}_{SR}^{-\frac{n}{2}} \left(\frac{iz\bar{\gamma}_{RD}}{jC}\right)^{-\frac{n}{2}} K_n \left(2\sqrt{\frac{jCiz}{\bar{\gamma}_{SR}\bar{\gamma}_{RD}}}\right) - \sum_{l=0}^{\infty} \frac{\bar{\gamma}_{SR}^{-n} \left(-\frac{iz}{\bar{\gamma}_{SR}}\right)^l}{l!} \int_1^{\infty} u^{-n-l-1} e^{-\frac{jCu}{\bar{\gamma}_{RD}}} du \\
&\stackrel{(c)}{=} 2\bar{\gamma}_{SR}^{-\frac{n}{2}} \left(\frac{iz\bar{\gamma}_{RD}}{jC}\right)^{\frac{n}{2}} \sum_{m_1=0}^{n-1} \frac{(-1)^{m_1} (n-m_1-1)}{m_1!} \left(\frac{jCiz}{\bar{\gamma}_{SR}\bar{\gamma}_{RD}}\right)^{m_1-\frac{n}{2}} + 2\bar{\gamma}_{SR}^{-n} \left(\frac{iz\bar{\gamma}_{RD}}{jC}\right)^{-\frac{n}{2}} (-1)^{n+1} \\
&\times \sum_{m_2=0}^{\infty} \frac{\sqrt{\frac{jCiz}{\bar{\gamma}_{SR}\bar{\gamma}_{RD}}}}{m!(n+m_2)!} \left(\ln \sqrt{\frac{jCiz}{\bar{\gamma}_{SR}\bar{\gamma}_{RD}}} - \frac{1}{2}\psi(m_2+1) - \frac{1}{2}\psi(n+m_2+1)\right) - \sum_{l=0}^{\infty} \frac{\bar{\gamma}_{SR}^{-l-n} (iz)^l}{l!} \left(\frac{jC}{\bar{\gamma}_{RD}}\right)^l \Gamma\left(-n-l, \frac{jC}{\bar{\gamma}_{RD}}\right) \\
&= \sum_{m_1=0}^{n-1} (-1)^{m_1} \frac{(n-m_1-1)}{m_1!} z^{m_1-n} \bar{\gamma}_{SR}^{-m_1} i^{m_1} \left(\frac{jC}{\bar{\gamma}_{RD}}\right)^{m_1} + 2 \sum_{m_2=0}^{\infty} \frac{1}{m_2!(n+m_2)!} z^{m_2} \left(\frac{jC}{\bar{\gamma}_{RD}}\right)^{m_2+n} \left(\frac{i}{\bar{\gamma}_{SR}}\right)^{n+m_2} \\
&\times \left(\ln \sqrt{\frac{jCiz}{\bar{\gamma}_{SR}\bar{\gamma}_{RD}}} - \frac{1}{2}\psi(m_2+1) - \frac{1}{2}\psi(n+m_2+1)\right) - \sum_{l=0}^{\infty} \frac{\bar{\gamma}_{SR}^{-l-n} i^{n+l} z^l}{l!} \left(\frac{jC}{\bar{\gamma}_{RD}}\right)^{l+n} \Gamma\left(-n-l, \frac{jC}{\bar{\gamma}_{RD}}\right). \quad (28)
\end{aligned}$$

$$\begin{aligned}
\lambda_n &= \frac{z^{N_t+n+1}}{n!(N_t+n+1)} \frac{C}{\bar{\gamma}_{RD}} \left(\sum_{m_1=0}^{n-1} (-1)^{m_1} \frac{(n-m_1-1)}{m_1!} \bar{\gamma}_{SR}^{-m_1} z^{m_1} \left(\frac{C}{\bar{\gamma}_{RD}}\right)^{m_1} \sum_{j=1}^{N_{rt}} \binom{N_{rt}-1}{j-1} (-1)^{j-1} j^{m_1} \sum_{i=1}^{N_{rr}} \binom{N_{rr}-1}{i-1}\right) \\
&\times (-1)^{i-1} i^{m_1} + 2 \sum_{m_2=0}^{\infty} \frac{\left(\frac{C}{\bar{\gamma}_{RD}}\right)^{n+m_2} \left(\frac{i}{\bar{\gamma}_{SR}}\right)^{n+m_2}}{m_2!(n+m_2)!} \sum_{j=1}^{N_{rt}} \binom{N_{rt}-1}{j-1} (-1)^{j-1} j^{n+m_2} \sum_{i=1}^{N_{rr}} \binom{N_{rr}-1}{i-1} (-1)^{i-1} i^{n+m_2} \\
&\times \left(\ln \sqrt{\frac{jCiz}{\bar{\gamma}_{SR}\bar{\gamma}_{RD}}} - \frac{1}{2}\psi(m_2+1) - \frac{1}{2}\psi(n+m_2+1)\right) - \sum_{l=0}^{\infty} \frac{\bar{\gamma}_{SR}^{-l-n} (-z)^l}{l!} \left(\frac{C}{\bar{\gamma}_{RD}}\right)^l \sum_{j=1}^{N_{rt}} \binom{N_{rt}-1}{j-1} (-1)^{j-1} \\
&\times j^{n+l} \sum_{i=1}^{N_{rr}} \binom{N_{rr}-1}{i-1} (-1)^{i-1} i^{n+l} \Gamma\left(-n-l, \frac{jC}{\bar{\gamma}_{RD}}\right). \quad (30)
\end{aligned}$$

HIGH-SNR EXPRESSION FOR THE TERM Φ_n IN (17), $n > 0$

We now consider Φ_n for $n > 0$, which is defined in (17) as

$$\Phi_n = \frac{z^{N_t+n+1}}{n!(N_t+n+1)} \underbrace{\bar{\gamma}_{SR}^{-n} i^n \int_0^1 \frac{C}{\bar{\gamma}_{RD}} u^{-n-1} e^{-\frac{iz}{u\bar{\gamma}_{SR}}} e^{-\frac{jCu}{\bar{\gamma}_{RD}}} du}_{\rho_n}. \quad (27)$$

The term ρ_n can be rewritten as (28), given at the top of this page, in which we have used [6, Eq. (3.471.9)] and [6, Eqs. (8.446) and (3.381.3)] in steps (c) and (d), respectively. The analysis that follows is similar to that in App. A. To this end, we define the variable λ_n as

$$\lambda_n = \sum_{i=1}^{N_{rr}} \binom{N_{rr}-1}{i-1} (-1)^{i-1} \sum_{j=1}^{N_{rt}} \binom{N_{rt}-1}{j-1} (-1)^{j-1} \Phi_n. \quad (29)$$

Plugging (28) into (27) and this into (29), λ_n can be rewritten as (30), given at the top of this page. Observe that, similarly to λ_0 , λ_n is composed by the sum of three terms, corresponding to the summations in m_1 , m_2 , and l . The first term is non-null for values of m_1 greater than $\max(N_{rr}, N_{rt}) - 1$; the second term is non-null for values of m_2 greater than $\min(N_{rr}, N_{rt}) - 1$; and the third term is non-null for values of l greater than $N_{rr} - 1$. Using this and preserving only the

lowest-order terms so as to derive a high-SNR expression, we arrive at three cases: (a) if $N_{rr} = N_{rt}$, the three terms must be considered; (b) if $N_{rr} < N_{rt}$, the second and third terms must be considered; and (c) if $N_{rr} > N_{rt}$, only the second term must be considered. In all the cases, the diversity order of λ_n is observed to be given by $n_1 \triangleq \min(N_{rr}, N_{rt}) - 1$. By incorporating these results into the definition of Φ_n given by (27) and (28), we finally arrive at the high-SNR expression for Φ_n as given in (19) and (20).

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