Performance of Transmit Beamforming for Interference Mitigation with Random Codebooks

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***Abstract*—In this paper, we study the interference mitigation capability of Transmit Beamforming (TBF) when used to combat cross-layer interference in a two-tier Heterogeneous Network (Het-Net) scenario. Since generic practical codebook designs are not known when TBF is applied for interference mitigation purposes, a randomly generated codebook is used as a simple way to provide a lower bound performance for any efficient (deterministic) codebook design. Closed-form expressions for the rate performance are derived when *altruistic* TBF is applied to mitigate interference with different number of transmit antennas and feedback bit resolutions. Our analysis reveals that the use of additional feedback bits has potential to provide more performance gain at lower outage probability regimes (when compared to higher outage probability ones). In addition, for random codebooks with a fixed number of feedback bits, the number of transmit antennas does not have a notable effect on the interference mitigation capability of an altruistic TBF scheme.**

I. Introduction

Heterogeneous Networks (Het-Nets) have been introduced to alleviate the problems that arise when the ever-growing demand of higher data rates needs to be tackled. Heterogeneous Networks exploit the fact that data rate demands in a mobile network is not uniform over the whole coverage region, and its name describes the nature of a network deployment that contains a mixture of different types of low-power nodes (i.e., micro, pico, and femto base stations) which operate under the overlaying umbrella coverage of a macrocell system [1]. Research in Het-Net technology started from femtocells, which are low-power and low-cost Base Stations (BSs) designed to serve limited indoor areas (like a home or an office). Due to femto BSs operate on licensed spectrum, a main area of interest has been focused on the management of the interference that femto BSs generate in the wireless environment [2].

The problem of inter-layer and co-layer interference in a co-channel Het-Net deployment has been widely addressed in academic research [3]. Precisely, investigations have strongly focused on two-tier networks, where interference between macrocells and femtocells is seen as a key limiting factor. This problem becomes especially serious when the so-called Closed Subscriber Group (CSG) configuration is applied, and a handover operations (towards a femto BS) of a macro Mobile Station (MS) that is not identified as member of the CSG is prevented. As a consequence, the macro MS is expected to face very strong interference when both femtocells and macrocells operate on the same frequency band (this can be interpreted as the creation of a coverage hole in the macrocell layer).

Since macrocells serve a large number of users, they are usually considered as resource constrained cells. On the other hand, due to femtocells are envisioned to serve only few users in its coverage area, they are expected to provide high data rates to their femto MSs, even when the Signal-to-Interference plus Noise power Ratio (SINR) is low. In such commonly faced scenario, femtocells can be viewed as secondary (priority) cells in nature, and the network may demand an altruistic behavior when a femto BS needs to define the best way to use its communication resources. Macrocells, on the contrary, can be considered as primary (priority) cells, and an egoistic behavior from their side is expected to be observed when defining the way to serve its macro MSs.

The use of multi-antenna techniques for interference mitigation purposes is considered in [4], where the authors present the concept of *altruistic* Transmit Beamforming (TBF) to manage co-layer interference in a femtocellular scenario. Basically, this work proposes that a victim femto MS establishes a low-rate signaling connection to the interfering femto BS, and informs the beamforming vector that should be applied in transmission to minimize the interfering signal power. Based on this simplified interference mitigation approach, the performance of altruistic TBF has been characterized analytically in [5] for a dominant co-layer interfering source, and in [6] for multiple dominant cross-layer interfering sources. In [7], the performance of a scheme that combines altruistic TBF with channel-aware scheduling is studied for the single interfering source case. Nevertheless, in all these papers, the codebook designs to implement altruistic TBF are generalized versions of practical UTRA-based transmit-diversity methods (originally designed for scenarios with 2 transmit antennas), which cannot be easily extended when a larger number of transmit antennas is used for interference mitigation purposes.

In this paper, we study the downlink achievable data rate in a simple two-tier system composed by single macrocell and femtocell (from now on, primary cell and secondary cell, respectively). For this purpose, we consider that the primary (macro) BS always applies *egoistic* TBF to serve its associated primary (macro) MS, while the secondary (femto) BS always implement *altruistic* TBF to mitigate interference in the primary (macro) cell. Simple practical codebook designs are known for egoistic TBF in presence of an arbitrary number of transmit antennas and feedback bits [8]. Nevertheless, the definition of practical codebook designs for altruistic TBF in generic system scenarios (i.e., for any number of transmit antennas and feedback bits) is a topic that still needs to be tackled. For this purpose, in this initial paper we study the interference mitigation capabilities of Random Vector Quantization (RVQ) beamforming as a baseline approach [9]. Due to the codebook elements of a RVQ beamforming scheme

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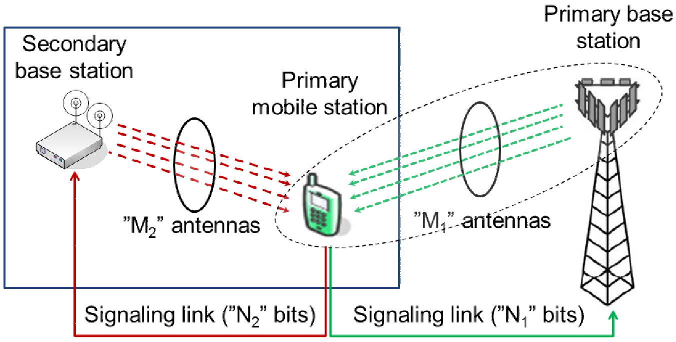


Fig. 1: The downlink system model with co-channel deployment comprises a primary (secondary) BS with M_1 (M_2) transmit antennas, and a primary MS equipped with a single receive antenna. A dedicated low-rate signaling link exists between primary MS and the primary (secondary) BS to convey N_1 (N_2) feedback bits. Primary BS implements egoistic TBF with deterministic codebook design to serve its associated user. Secondary BS mitigate interference at the primary MS using altruistic TBF with randomly generated codebook.

are randomly selected every time the channel changes, it provides a performance lower bound characterization for any deterministic beamforming codebook that is designed off-line [10]. Note that since both femto and macro BSs are not expected to have tight time synchronization in practical deployments (and no ideal backhaul will likely exist between them), advanced Interference Alignment (IA) schemes are not good candidates to control interference effectively in this situation [11]. However, the lack of synchronization between femtocells and macrocells does not affect the performance of an altruistic TBF scheme, since its goal is the mitigation of individual interference sources rather than the alignment of them (in reception) at the desired destination.

The rest of the paper is organized as follows: Section II introduces the system model and the adopted assumptions. Section III discusses the TBF techniques, and explains the analyzed codebook designs for implementing egoistic and altruistic TBF according to the priority of the cell. Section IV derives the closed-form formulas that are used to characterize the interference mitigation capabilities of the proposed approach, while Section V presents the performance results and validates the analysis through numerical simulation comparisons. Finally, conclusions are drawn in Section VI.

II. System Model

This section presents the system model and the assumptions that are used to model the two-cell heterogeneous scenario.

A. Inter-cell interference scenario

The general layout of our downlink system model is shown in Fig. 1. The network contains two fixed network elements, known as primary and secondary BSs, equipped with multiple transmit antennas. Single-antenna MSs are also deployed in the coverage area of the network, and are also classified as primary or secondary according to the type of the BS to which they are associated. A co-channel deployment scenario is considered, with both primary (wide area) and secondary (local area) cells operating in the same frequency band (to guarantee an efficient use of scarce spectrum resources) [12]. The coverage region of the secondary cell is assumed to overlap (at least partially) with the coverage region of the primary cell. As a consequence, a primary MS may eventually suffer strong downlink interference when visiting the secondary cell if no interference mitigation mechanism is implemented. In this paper, we focus on the use of TBF techniques for interference mitigation purposes; in case of a Frequency Division Duplex (FDD) air interface, this requires the establishment of a signaling link under request between victim MS and interfering BS.

In absence of strong downlink interference between cells, a primary (secondary) MS informs to its serving primary (secondary) BS the egoistic TBF vector $\widehat{\mathbf{w}}_1$ ($\widehat{\mathbf{w}}_2$) that should be applied in transmission to maximize the received SINR of its associated user. However, when a primary MS visits the coverage region of the secondary cell, a better performance is obtained if the secondary BS replaces its egoistic TBF vector (i.e., $\widehat{\mathbf{w}}_2$) by the altruistic TBF vector (i.e., denoted as $\check{\mathbf{w}}_2$) that minimize the interference power at the victim primary MS. To carry out this action, it is assumed that a primary MS can establish a signaling link over-the-air to the secondary BS every time that it visits the secondary cell. The interference mitigation capability of TBF was studied in [5] when a BS makes use of practical TBF codebooks, originally designed for 2 transmit antennas. Nevertheless, the extension of this study when a BS relies on more advanced TBF codebooks, designed to mitigate interference in presence of a larger number of antennas and/or feedback links, has not been yet addressed in the literature to our best understanding.

B. Adopted assumptions

In our system model, the following assumptions are made:

(A1) We focus on a two cell scenario, where the primary MS experiences strong downlink co-channel interference from the secondary BS. Transmission power in both BSs is constant, and handover operations between cells are not allowed. Nevertheless, this limitation does not preclude the possibility of establishing an *ad hoc* signaling link between the primary MS and the secondary BS (whenever the primary MS visits the secondary cell).

(A2) There are M_1 (M_2) transmit antennas at the primary (secondary) BS, and a single receive antenna at each MS. Mobile Stations can perfectly estimate the channel gains of each individual transmit antenna of the primary (secondary) BS using common pilot signals. A primary MS can send N_1-bit (N_2-bit) feedback messages to the primary (secondary) BS to implement an egoistic (altruistic) TBF scheme. The impacts of feedback delay and the effect of signaling errors are ignored.

(A3) Channel gains related to the different antennas of the same BS are modeled as independent and identically distributed (i.i.d.) zero-mean circularly symmetric complex Gaussian Random Variables (RVs). Average path loss and shadow fading components are identical for all the antennas of the same BS; however, fast fading components for each individual antenna of the same BS are considered uncorrelated.

III. Transmit Beamforming Techniques

To implement TBF in the downlink of a FDD system, a BS makes use of quantized Channel State Information (CSI) that a target MS encodes into a feedback message and reports over a reverse signaling link. The basic idea behind this approach is to adapt the transmitted signal to the instantaneous channel conditions, and enhance the quality of the received signal. In an interference-free case, the received signal at the desired destination attains the form

$$r = \mathbf{h} \cdot \mathbf{x} + n = (\mathbf{h} \cdot \mathbf{w})s + n, \tag{1}$$

where $\mathbf{x} \in \mathbb{C}^{1\times M}$ is the transmitted signal vector from the M BS antennas, $\mathbf{h} \in \mathbb{C}^{1\times M}$ is the channel gain vector (with zero-mean complex Gaussian coefficients), and n refers to Additive White Gaussian Noise (AWGN) with power P_N. Transmit vector $\mathbf{x}$ is related to the information symbol s via linear beamforming, where $\mathbf{w} \in \mathbb{C}^{1\times M}$ is a beamforming vector that verifies $\|\mathbf{w}\| = 1$. We assume that beamforming vector $\mathbf{w}$ belongs to a common beamforming codebook $\mathcal{W}$, which is known at both extremes of the link beforehand.

The selection of a the best beamforming vector is done in the receiver to optimize a given objective function. In what follows, two different approaches are analyzed: egoistic TBF and altruistic TBF [7].

A. Egoistic and Altruistic Transmit Beamforming

In egoistic TBF, the goal is to find the beamforming vector $\widehat{\mathbf{w}}$ that maximize the power of the received signal. In other words, the MS should first determine

$$\widehat{\mathbf{w}} = \arg \max_{\mathbf{w}\in\mathcal{W}} |\mathbf{h} \cdot \mathbf{w}|^2, \tag{2}$$

and then inform the index of the corresponding beamforming vector to the serving BS via a dedicated/shared signaling link. Various codebooks that are compatible with egoistic TBF have been proposed in literature for arbitrary transmit antenna numbers and feedback resolutions; see e.g. [8], where simplified beamforming codebook structures are analyzed. With no loss of generality, in this paper we use the so-called Quantized Cophasing (QCP) scheme.

In altruistic TBF, on the other hand, the goal is to find the beamforming vector $\check{\mathbf{w}}$ that minimize the power of a received signal (originated in an interfering source of information). In this case, the MS needs to first determine

$$\check{\mathbf{w}} = \arg \min_{\mathbf{w}\in\mathcal{W}} |\mathbf{h} \cdot \mathbf{w}|^2, \tag{3}$$

and then inform the corresponding index to the interfering BS via an *ad hoc* signaling link that can be established on request. Previous works have studied the mitigation capability of altruistic TBF in 2 transmit antenna systems, using beamforming codebooks originally designed for egoistic TBF, see e.g. [5]. However, it is not clear yet how to construct (and model) beamforming codebooks for altruistic TBF in presence of an arbitrary larger number of transmit antennas (i.e., $\forall M$ greater than 2). In the following pages, we provide a performance lower-bound characterization for any efficient beamforming codebook design for altruistic TBF. For this purpose, we use RVQ beamforming approach to minimize the interference power that the secondary BS generates in the primary MS.

B. Quantized Cophasing (egoistic TBF scheme)

When using a beamforming codebook for QCP, the power per transmit antenna element is kept constant, and the phases of antenna signals are adjusted with respect to a reference antenna, such that signals combine constructively at the receiver side [8]. Since phases of circularly symmetric complex Gaussian RVs are uniformly distributed on interval $(-\pi, \pi)$, a uniform phase quantizer is used to define the elements of the beamforming codebook in this approach.

When a BS is equipped with M transmit antenna elements, the components of a beamforming vectors $\mathbf{w} = (w_1, \ldots, w_M)$ used in a QCP scheme are of the form

$$w_m = \begin{cases} 1 & m = 1 \\ v_m & m = 2, \ldots, M \end{cases}, \tag{4}$$

where

$$v_m = e^{-j\frac{(2n-1)\pi}{2^{N_p}}} \qquad n = 1, \ldots, 2^{N_p}, \tag{5}$$

and N_p is the number of feedback bits that are used to adjust the relative phase per individual transmit antenna. Note that when $M = 2$, the beamforming codebook for QCP is

$$\mathcal{W}_{\text{qcp}} = \left\{ \frac{1}{\sqrt{2}} \left(1, e^{-j\frac{(2n-1)\pi}{2^{N_p}}}\right) : n = 1, \ldots, 2^{N_p} \right\}. \tag{6}$$

Exact distribution for RV

$$\rho_{\max} = |\mathbf{h} \cdot \widehat{\mathbf{w}}|^2 \tag{7}$$

is difficult to obtain in closed-form when implementing QCP, even for the $M = 2$ case. So, we make use of the Chi-Square (χ^2) distribution approximation presented in [13], and approximate the Cumulative Distribution Function (CDF) of RV $\rho_{\max}$ as

$$F_{\rho_{\max}}(x) = 1 - \left(1 + \frac{2\,x}{\mathcal{G}\,\overline{\gamma}}\right) e^{-\frac{2\,x}{\mathcal{G}\,\overline{\gamma}}} \qquad x \geq 0, \tag{8}$$

where

$$\mathcal{G} = \mathbb{E}\left\{|\mathbf{h} \cdot \widehat{\mathbf{w}}|^2\right\} \tag{9}$$

is the so-called Signal-to-Noise power Ratio (SNR) gain of the QCP scheme, and $\overline{\gamma}$ is the mean received power from each individual transmit antenna. The SNR gain for QCP depends on N_p [8], and its value is given in closed-form by the following formula:

$$\mathcal{G}_{\text{qcp}} = 1 + \frac{\pi}{4}\,a_N, \qquad a_N = \frac{2^N}{\pi} \sin\left(\frac{\pi}{2^N}\right). \tag{10}$$

C. Random Vector Quantization (altruistic TBF scheme)

In RVQ beamforming, the beamforming vector that is applied in transmission is chosen from a randomly generated codebook $\mathcal{W}_{\text{rvq}}$, which is made available at both transmitter and receiver sides beforehand [9]. The elements of the RVQ beamforming codebook are taken from a uniform distribution on the complex unit-norm sphere, and are updated every time the channel changes. In this paper, we use RVQ beamforming to characterize the performance lower bound for any efficient beamforming codebook, designed for altruistic TBF with a given number of transmit antennas M and feedback bits N. Intuitively, it is clear that any deterministic beamforming codebook that is designed off-line (to optimize a pre-defined performance measure) should lead to better performance than RVQ beamforming (since in RVQ beamforming codebook elements are randomly selected). In addition, it is important to highlight that, based on asymptotic results presented in [10], the performance gap that exists between RVQ and any other advanced codebook design is expected to vanish as the number of feedback bits and/or transmit antennas grows large.

Following the similar reasoning presented in [14], it is possible to show that the Probability Distribution Function (PDF) of the minimum normalized inner product

$$\nu_{\min} = \min_{\mathbf{w}\in\mathcal{W}_{\text{rvq}}} \frac{|\mathbf{h}\cdot\mathbf{w}|^2}{\|\mathbf{h}\|^2} \tag{11}$$

is given by

$$f_{\nu_{\min}}(x) = N_D(N-1)(1-x)^{N_D(M-1)-1} \qquad 0 \le x \le 1, \tag{12}$$

where N_D is the number of beamforming vectors in a randomly generated beamforming codebook $\mathcal{W}_{\text{rvq}}$. In case of spatially uncorrelated Rayleigh fading channels with identical mean received power $\overline{\gamma}$, RV $\|\mathbf{h}\|^2$ is χ^2 distributed with $2M$ degrees of freedom, i.e.,

$$f_{\|\mathbf{h}\|^2}(x) = \frac{1}{\Gamma(M)\,\overline{\gamma}} \left(\frac{x}{\overline{\gamma}}\right)^{M-1} e^{-\frac{x}{\overline{\gamma}}} \qquad x \ge 0, \tag{13}$$

where $\Gamma(x)$ is the (complete) gamma function. To find the PDF of the minimum interference signal power

$$\rho_{\min} = \nu_{\min}\|\mathbf{h}\|^2, \tag{14}$$

we use the well known formula for the product $Z = X\,Y$ of two continuous RVs [15]

$$f_Z(z) = \int_{-\infty}^{\infty} f_{X,Y}\left(x, \frac{z}{x}\right) \frac{1}{|x|} dx. \tag{15}$$

Since RVs ν and $\|\mathbf{h}\|^2$ are independent [14], we have $f_{X,Y}(x,y) = f_X(x)\,f_Y(y)$ in (15) and distributions (12), (13) can be used to compute PDF of $\rho_{\min}$. After tedious but rather simple computations, we obtain

$$\begin{aligned} f_{\rho_{\min}}(y) = & \frac{N_D}{\Gamma(M-1)} \Bigg\{ \sum_{k=0}^{M-2} \frac{(-1)^k}{\overline{\gamma}} \left(\frac{y}{\overline{\gamma}}\right)^k \\ & \times \binom{N_D(M-1)-1}{k} \Gamma\left(M-k-1, \frac{y}{\overline{\gamma}}\right) \\ & + \sum_{k=M-1}^{N_D(M-1)-1} \frac{(-1)^k}{\overline{\gamma}} \left(\frac{y}{\overline{\gamma}}\right)^{M-1} \\ & \times \binom{N_D(M-1)-1}{k} E_{-M+k+2}\left(\frac{y}{\overline{\gamma}}\right) \Bigg\} \quad y \ge 0, \end{aligned} \tag{16}$$

where

$$\binom{n}{k} = \frac{n!}{k\,(n-k)!} \tag{17}$$

is the binomial coefficient with indices n and k,

$$\Gamma(a,x) = \int_x^{\infty} e^{-t}\, t^{a-1} dt \tag{18}$$

is the upper incomplete gamma function, and

$$E_n(x) = \int_1^{\infty} \frac{e^{-x\,t}}{t^n} dt \qquad n = 0, 1, \ldots \tag{19}$$

is the generalized exponential integral function of order n [16].

IV. Rate performance analysis

In this section, we use the CDF of the received SINR at the primary MS as performance measure. Later on, the study is extended to cover the achievable rate. Note that the procedure that is used to derive the closed-form formulas for the CDF is similar to the one presented in [5]. The main difference lies in the PDF distribution that is used to model the interfering signal that is originated in the secondary BS, as explained in Section III-C.

A. Cumulative Distribution Function for received SINR

To carry out the computation of the received SINR at the primary MS, we consider the RV

$$Z = \frac{X}{1+Y}, \tag{20}$$

assuming that RVs X and Y are independent. It is shown in [17] that if (20) holds, the CDF of RV Z is of the form

$$F_Z(z) = \int_0^{\infty} F_X\big[z(t+1)\big]\, f_Y(t)\, dt, \tag{21}$$

where $f_Y(y)$ is the PDF of RV Y, while $F_X(x)$ represents the CDF of RV X.

Then, since our goal is to derive the stochastic behavior of the received SINR at the primary MS, i.e.,

$$\Upsilon_{(1)} = \frac{\overline{\gamma}_{1,1}|\mathbf{h}_{1,1}\cdot\widehat{\mathbf{w}}_1|}{1+\overline{\gamma}_{2,1}|\mathbf{h}_{2,1}\cdot\check{\mathbf{w}}_2|}, \tag{22}$$

where

$$\overline{\gamma}_{k,l} = \frac{P_k}{\overline{L}_{k,l}P_N} \tag{23}$$

is the mean received SNR for the link between BS k and MS l and $\overline{L}_{k,l}$ is the path loss attenuation of the corresponding link (i.e., combined effect of both average path loss and shadow fading components), we need to compute (21) assuming

$$X = \overline{\gamma}_{1,1}|\mathbf{h}_{1,1}\cdot\widehat{\mathbf{w}}_1|, \qquad Y = \overline{\gamma}_{2,1}|\mathbf{h}_{2,1}\cdot\check{\mathbf{w}}_2|. \tag{24}$$

When primary BS applies egoistic TBF with a QCP codebook designed for $M_1 = 2$ transmit antennas and N_1 feedback bits (i.e., when $\widehat{\mathbf{w}}_1 \in \mathcal{W}_{\text{qcp}}$), the CDF of RV X can be approximated by the formula presented in (8). Similarly, when the secondary BS applies altruistic TBF with a RVQ beamforming codebook designed for M_2 transmit antennas and N_2 feedback bits (i.e., when $\check{\mathbf{w}}_2 \in \mathcal{W}_{\text{rvq}}$), the PDF of RV Y is given by (16). Note that the use of N_2 feedback bits in RVQ beamforming enables the identification of up to $N_D = 2^{N_2}$ element in the randomly generated beamforming codebook.

Finally, carrying out many tedious but relative simple manipulations after combining (8) and (16) with (21), the closed-form CDF expression presented in (25) is obtained. The definite integrals that appear in (25) are derived with the aid of definite integral formulas and relations of [16], [18] (details are omitted for the sake of brevity):

$$\begin{aligned} \int_0^{\infty} \Gamma(a, \beta x)\, x^k e^{-\mu x} dx = & (a-1) \int_0^{\infty} \Gamma(a-1, \beta x)\, x^k e^{-\mu x} dx \\ & + \beta^{a-1} \frac{\Gamma(k+a)}{(\mu+\beta)^{k+a}} \quad a = 1, 2, \ldots, \end{aligned} \tag{26}$$

$$F_Z(z) = 1-\frac{N_D}{\Gamma(M_2-1)}\left\{\sum_{k=0}^{M_2-2}\frac{(-1)^k}{(\overline{\gamma}_{2,1})^{k+1}}\binom{N_D(M_2-1)-1}{k}e^{-\frac{2z}{\mathcal{G}\overline{\gamma}_{1,1}}}\left[\left(\frac{2z}{\mathcal{G}\overline{\gamma}_{1,1}}\right)\int_0^\infty\Gamma\left(M_2-k-1,\frac{t}{\overline{\gamma}_{2,1}}\right)t^{k+1}e^{-\frac{2z}{\mathcal{G}\overline{\gamma}_{11}}t}dt\right.\right.$$
$$+\left.\left(1+\frac{2z}{\mathcal{G}\overline{\gamma}_{1,1}}\right)\int_0^\infty\Gamma\left(M_2-k-1,\frac{t}{\overline{\gamma}_{2,1}}\right)t^{k}e^{-\frac{2z}{\mathcal{G}\overline{\gamma}_{11}}t}dt\right]+\sum_{k=M_2-1}^{N_D(M_2-1)-1}\frac{(-1)^k}{(\overline{\gamma}_{2,1})^{M_2}}\binom{N_D(M_2-1)-1}{k}e^{-\frac{2z}{\mathcal{G}\overline{\gamma}_{1,1}}}$$
$$\left.\times\left[\left(\frac{2z}{\mathcal{G}\overline{\gamma}_{1,1}}\right)\int_0^\infty E_{-M_2+k+2}\left(\frac{t}{\overline{\gamma}_{2,1}}\right)t^{M_2}e^{-\frac{2z}{\mathcal{G}\overline{\gamma}_{11}}t}dt+\left(1+\frac{2z}{\mathcal{G}\overline{\gamma}_{1,1}}\right)\int_0^\infty E_{-M_2+k+2}\left(\frac{t}{\overline{\gamma}_{2,1}}\right)t^{M_2-1}e^{-\frac{2z}{\mathcal{G}\overline{\gamma}_{11}}t}dt\right]\right\} \quad (25)$$

$$\int_0^\infty E_1(\beta x)\,x^k e^{-\mu x}dx = \frac{\Gamma(k+1)}{\mu^{k+1}}\left[\log_e\left(1+\frac{\mu}{\beta}\right) - \sum_{m=1}^{k}\frac{1}{m}\left(1+\frac{\mu}{\beta}\right)^{-m}\left(\frac{\mu}{\beta}\right)^{m}\right], \quad (27)$$

and

$$\int_0^\infty E_n(\beta x)\,x^k e^{-\mu x}dx = \frac{1}{(n-1)}\left[\frac{\Gamma(k+1)}{(\mu+\beta)^{k+1}} - \beta\int_0^\infty E_{n-1}(\beta x)\,x^{k+1}e^{-\mu x}dx\right]. \quad (28)$$

B. Cumulative Distribution Function for achievable rate

Once the stochastic behavior of the received SINR Υ is known, an approximation for the achievable data rate can be obtained using the modified Shannon formula

$$R = A\,W\log_2(1+B\Upsilon) := g(\Upsilon), \quad (29)$$

where W is the communication bandwidth, while A and B are parameters that model the bandwidth efficiency and the SINR efficiency of the system, respectively. In practice, both parameters should be properly selected to fit the achievable rates of the different adaptive modulation and coding combinations that are implemented. Then, from (29), we find that the CDF for the achievable rate is

$$F_R(r) = F_\Upsilon[g^{-1}(r)]. \quad (30)$$

V. Performance Results

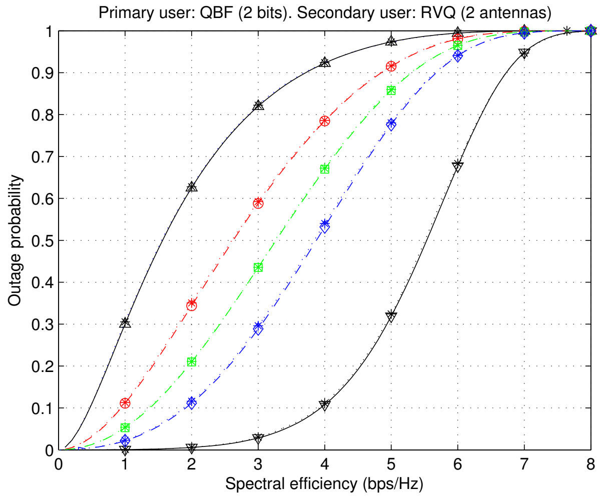
In this section we analyze the rate performance of different codebook designs when the primary BS applies egoistic TBF with QCP codebook (to serve the primary MS), and the secondary BS implements altruistic TBF with RVQ codebook (to mitigate the interference that it generates towards the primary cell). The evaluation is carried out considering that both desired signal (from the primary BS) and interfering signal (from the secondary BS) are received with an equally good strength at the primary MS (i.e., $\overline{\gamma}_{1,1} = \overline{\gamma}_{2,1} = 15$ dB). Since the goal of this paper is to quantify the interference mitigation capability of altruistic TBF, the number of transmit antennas and the number of feedback bits are kept fixed in the primary cell (i.e., $M_1 = 2$ and $N_1 = 2$ for all cases). Then, we study the effect that the number of transmit antennas M_2 and the number of feedback bits $N_2 = \log_2(N_D)$ have in the interference mitigation capability of RVQ beamforming, when applied in the secondary BS.

Figures 2a and 2b show the spectral efficiencies that can be achieved when an altruistic TBF scheme with RVQ codebook and different feedback resolutions is applied in the secondary BS, in presence of 2 and 4 transmit antennas, respectively. Solid lines with up-pointing triangles represents the achievable spectral efficiency when the secondary BS applies egoistic TBF to serve its associated secondary MS (performance lower bound), while the solid lines with down-pointing triangles identifies the maximum spectral efficiency that is obtained when the secondary BS is silent (performance upper bound). In addition, the spectral efficiencies that are achievable when the secondary BS implements an altruistic TBF scheme with a RVQ codebook are represented with dashed curves for different feedback bit resolutions: 2 bits (circles), 3 bits (squares), and 4 bits (diamonds). In all cases, simulated values are denoted with stars ($*$), and are included as a simple way to validate the closed-form formulas derived in Section IV-A.

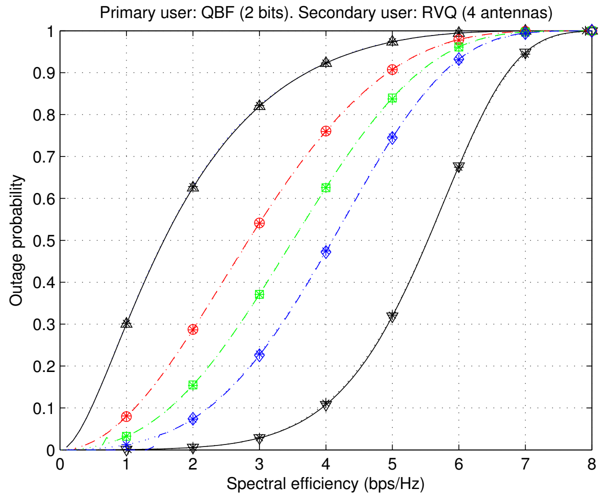
Based on the fact that RVQ beamforming provides a performance lower bound for any well-designed codebook, it is possible to claim that the use of additional feedback bits provides higher *relative* spectral efficiency gains at low outage probability regimes (e.g., at 10-th percentile outage rates). Therefore, lower *relative* rate performance gains are expected to be observed at higher outage probability regimes (e.g., at 50-th percentile outage rates). In addition, for a fixed number of feedback bits, the number of transmit antennas does not seem to have a notable effect in the interference mitigation capabilities of an altruistic TBF scheme. Nevertheless, this does not preclude that the design of a practical beamforming codebook for interference mitigation purposes could be simplified as the number of transmit antenna grows (e.g., even simple transmitter antenna selection scheme could provide excellent interference mitigation capabilities if the number of transmit antennas is allowed to grow large).

VI. Conclusions

The deployment of secondary priority femtocells under the overlaying umbrella coverage of primary priority macrocells may generate serious cross-layer interference problems to macro MSs, particularly in presence of the so-called CSG configuration. Trying to alleviate this problem, altruistic TBF can be used in a secondary (priority) BS to mitigate the downlink co-channel interference that a primary (priority) MS may experience, when it visits the coverage area of the secondary cell. Due to it is still not clear how to design efficient beamforming codebooks for interference mitigation



(a) $M_2 = 2$ antennas



(b) $M_2 = 4$ antennas

Fig. 2: Rate Cumulative Distribution Function for the primary MS when primary BS applies *egoistic* TBF with Quantized Cophasing codebook with $N_1 = 2$ bits, and secondary BS applies *egoistic/altruistic* TBF with Random Vector Quantization codebook of variable size ($\overline{\gamma}_{1,1} = 15$ dB, $\overline{\gamma}_{2,1} = 15$ dB). Solid lines: secondary BS egoistic (up-pointing triangles), and secondary BS silent (down-pointing triangles). Dashed lines: $N_2 = 2$ bits (circles), $N_2 = 3$ bits (squares), and $N_2 = 4$ bits (diamonds). Simulated values denoted by stars ($*$).

purposes, the use of randomly generated codebook elements was used in this initial paper as a simple way to provide a lower bound performance for any efficient (deterministic) beamforming codebook design. Based on this baseline approach, it was observed that the use of extra feedback bits has potential to provide additional performance gain at lower outage probability regimes, when compared to higher outage probability ones. In addition, for a random codebook with a fixed number of feedback bits, the number of transmit antennas does not seem to have a notable effect on the interference mitigation capabilities of an altruistic TBF scheme. The design of practical (and efficient) beamforming codebook to mitigate interference for an arbitrary number of transmit antennas and feedback bits felt out of the scope of this paper. Nevertheless, based on the results reported in this paper, it represents an interesting topic that calls for further research.

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