# A Practical Precoding Approach for Radar/Communications Spectrum Sharing

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***Abstract*—In this paper, we consider spectrum sharing between multiple-input multiple-output (MIMO) radar system and a communication system modeled as MIMO interference channel. We derive a zero-forcing precoder for radar transmitter which completely eliminates the radar interference to communication users. Obtaining the precoder requires the knowledge of an effective interference channel matrix composed of the channel matrices to all of the communication receivers and the post-processing matrices employed by them. We propose a channel estimation phase in which all of the communication receivers coordinate in their choice of training symbols and power transmission and the radar transmitter can estimate the effective interference channel. We investigate the effect of radar precoder and channel estimation error on the performance of radar and interference to communication receivers. Our results indicate that while the precoder null steers the radar interference to communication users it degrades the radar performance by introducing correlation to the probing signals. We show that this performance loss can be compensated for by increasing the number of radar antennas.**

## I. Introduction

Radar systems, traditionally operating in the S band (2-4 GHz), are one of the major consumers of radio spectrum. Up until recently, the general view has been that such spectrum can not be shared with communication systems due to very different nature of the uses [1]. New advances in communication technology and the increasing demand for radio spectrum has led to multiple efforts to reexamine the sharing options. According to the national broadband plan released in the US in 2010, 500 MHz of bandwidth must be freed until 2020 for mobile broadband application [2]. One candidate is the 3550-3650 MHz band used by military radars. While radar receiver performance is inherently robust against interference, radar interference mitigation is important for successful spectrum sharing between radar and communication systems.

Most of the previous work for radar/communications spectrum sharing are based on geographic separation. In United Kingdom (UK), ofcom, the independent regulator and competition authority for the UK communications industries, has investigated the feasibility of spectrum sharing between maritime radars and commercial wireless networks like WiMAX and LTE in 2.6 GHz band. The proposed recommendations include ensuring appropriate separation between radar and communication users in addition to improved filtering, receiver sensitivity and also coordination between radar and communication operators [3]. In [4], coexistence of a rotating radar with a secondary network has been considered by taking into account the radar's beam direction. A secondary device is allowed to transmit when the radar's directional antenna is pointing to a direction different from its own transmission. In [5], a projection based approach, inspired by previous work on the null space based coexistence in underlay cognitive radio [6, 7], is proposed and the effect on the radar target localization is investigated. However, the proposed approach assumes a single communication receiver only. The performance when the number of communication receivers increases is not studied. In addition, the effect of post-processing at the communication receiver which changes the effective interference channel is not included. This is particularly important when multiple communication transmitter-receivers pairs operate simultaneously in the form of an interference channel (IFC). The authors also assume the perfect knowledge of interference channel through cooperation between radar and communication system and using a cognitive pilot channel. The effect of inaccurate channel knowledge in system performance is not included.

In this paper, we consider MIMO radar, i.e. a multiple antenna radar system capable of transmitting arbitrary waveforms from each antenna element. MIMO radar allows more degrees of freedom for radar signal design and enables the spatial coding of probing signals. We propose a precoder for radar which completely eliminates the interference to communication receivers. The precoder requires the knowledge of an effective interference channel matrix. To estimate this effective channel, we propose a channel estimation phase in which all of the communication receivers coordinate in their choice of training symbols. We investigate the effect of radar precoder and channel estimation error on the performance of radar and interference to communication receivers.

***Notations:*** We represent the vectors and matrices by lowercase and uppercase boldface letters, respectively (e.g., $\mathbf{h}$ and $\mathbf{H}$). The rank, null space, transpose, and Hermitian transpose of $\mathbf{H}$ are denoted by rank$\{\mathbf{H}\}$, $\mathcal{N}\{\mathbf{H}\}$, $\mathbf{H}^T$ and $\mathbf{H}^*$, respectively. The subspace spanned by a set of vectors $S$ is denoted by Span$\{S\}$. The $n$ by $n$ identity matrix is denoted by $\mathbf{I}_n$, and the function $(x)^*$ is equal to $\max\{0, x\}$.

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## II. System Model

We consider a communication system with $K$ transmitter-receiver pairs, each equipped with multiple antennas. The signal transmitted by any transmitter is desired by its corresponding receiver only and will act as interference for all other $K-1$ receivers. This gives rise to a $K$-user MIMO interference channel. This architecture can be useful to model a distributed, e.g. an ad-hoc, wireless network with $K$ simultaneous unicast communications or a cellular network with $K$ base stations (BSs) where only one user equipment (UE) is active in any cell at any given time. The number of antenna elements at transmitter and receiver $k$ are $M_k$ and $N_k$ respectively ( $1 \leq k \leq K$). The channel between transmitter $j$ and receiver $i$ at time $n$ is denoted by $\mathbf{H}_{ij}(n) \in \mathbb{C}^{N_i \times M_j}$.

The communication system coexists and shares the spectrum with a monostatic MIMO radar system. The radar system consists of two collocated arrays of $M_R$ transmit antenna elements and $N_R$ receive antenna elements respectively (See Fig. 1 for system model and Subsection II-A for more information about target localization using MIMO radar). The channel between radar transmitter and receiver at time $n$ is denoted by $\mathbf{H}_R(n)$ and the channel between radar transmitter and communication receiver $i$ at time $n$ is denoted by $\mathbf{H}_{iR}(n) \in \mathbb{C}^{N_i \times M_R}$. We assume block fading and that channel matrices remain quasi-static for $L$ channel uses and change independently from one block to another.

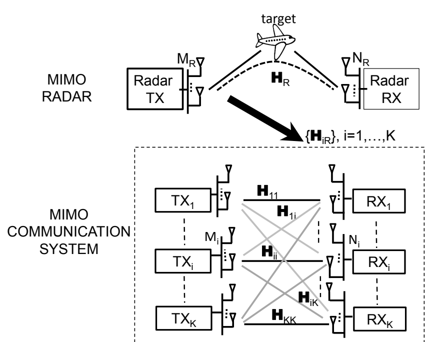


Fig. 1: System model.

Let $\mathcal{K} = \{1, 2, \cdots, K\}$ denote the indices of communication users. The $N_k$-dimensional signal vector received at the $k^{th}$ receiver ($k \in \mathcal{K}$) at time $n$ ($n \in \mathbb{N}$) can be written as

$$\mathbf{y}_k(n) = \sum_{i \in \mathcal{K}} \mathbf{H}_{ki}(n)\mathbf{x}_i(n) + \mathbf{H}_{kR}(n)\mathbf{x}_R(n) + \mathbf{z}_k(n) \quad (1)$$

where $\mathbf{x}_i(n)$ is the signal vector transmitted by communication user $i$, $\mathbf{x}_R(n)$ is the signal vector transmitted by MIMO radar transmitter and $\mathbf{z}_k(n) \in \mathbb{C}^{N_k \times 1}$ is the circularly symmetric additive white Gaussian noise (AWGN) at the $k^{th}$ receiver whose elements are independent and identically distributed (i.i.d.) and drawn from a complex Gaussian distribution with zero mean and variance $\sigma^2$. The elements of $\mathbf{H}_{ij}(n)$ and $\mathbf{H}_{iR}(n)$ are assumed to be i.i.d. and drawn from a continuous distribution. From random matrix theory, the matrices $\mathbf{H}_{ij}(n)$ and $\mathbf{H}_{iR}(n)$ have, almost surely, ranks equal to the minimum of the number of rows and columns [9]. For ease of exposition, we hereafter omit the channel use index $n$.

For the communication system and considering the transmitter-receiver pair $k$ with $M_k$ transmit and $N_k$ receive antennas, the degrees of freedom (DoF) is defined to be the number of signalling dimensions, where one signaling dimension corresponds to one interference-free information stream [10]. Denoting the DoF for user $k$ as $d_k$, clearly we must have $d_k \leq \min(M_k, N_k)$, i.e., the number of interference-free information streams can not be larger than the minimum of the numbers of transmitter and receiver antennas. At transmitter $k$, the $d_k$-dimensional information message is denoted by $\tilde{\mathbf{x}}_k$. The information message $\tilde{\mathbf{x}}_k$ is mapped onto the $M_k$-dimensional transmitted signal $\mathbf{x}_k$ using a $M_k \times d_k$ precoder $\mathbf{P}_k$ and we have $\mathbf{x}_k = \mathbf{P}_k\tilde{\mathbf{x}}_k$. At the receiver side, the $N_K$-dimensional received signal, $\mathbf{y}_k$, is mapped onto a $d_k$-dimensional signal, $\tilde{\mathbf{y}}_k$, using a post-processor, $\mathbf{F}_k$, with dimension $N_k \times d_k$ (See Fig. 2) and we have $\tilde{\mathbf{y}}_k = \mathbf{F}_k\mathbf{y}_k$.

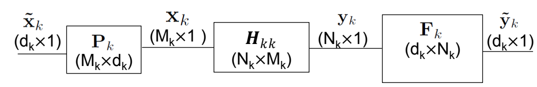


Fig. 2: Precoding and post-processing for user $k$.

The MIMO interference channel has been considered by many researchers [11]. While the exact capacity of these channels for a general case remains unknown, recent research has provided insight about the capacity in high signal to noise ratio (SINR) regimes. It is shown that in a wireless network, every user can obtain half of its interference-free capacity regardless of the number of users. The approach to achieve this capacity is known as interference alignment [12]. With interference alignment and using appropriate precoding matrices, the transmitted signals are constructed such that the desired signal and the total interference are received at orthogonal subspaces at each receiver. Obtaining the closed-form transmit precoding matrices for interference alignment requires global channel knowledge which can be overwhelming in practice [12, 13]. In [13], assuming local channel knowledge, precoding at the transmitters and post-processing (interference suppression) at the receiver, an iterative algorithms is proposed for distributed interference alignment.

In low SNR regimes, the interference received at each receiver is overwhelmed by the noise power. In such scenarios, the optimal rate maximizing approach for precoder and post-processing matrices of user $k$ are simply the $d_k$ dominant right and left singular vectors obtained from the singular value decomposition (SVD) of direct link $\mathbf{H}_{kk}$ which leads to diagonalization of $\mathbf{H}_{kk}$ [11].

Note that while we assume each communication receiver includes a post-processing block, our proposed approach is independent from the types of precoding and post-processing matrices that are employed by the communication users. However, for the rest of this paper, we assume that the precoding and post-processing matrices of communication users are obtained from SVD of direct communication links. The SVD of $\mathbf{H}_{kk}$ is $\mathbf{H}_{kk} = \mathbf{U}_k \mathbf{S}_k \mathbf{V}_k^*$ from which $\mathbf{P}_k$ and $\mathbf{F}_k$ are obtained from the columns of $\mathbf{V}_k$ and $\mathbf{U}_k$ corresponding to largest $d_k$ singular values, respectively [14].

### *A. MIMO RADAR*

MIMO radars, radars with multiple antenna elements at both transmitter and receiver sides, is an emerging technology which allows array processing at both transmit and receive modes and enables spatial coding of probing signals [15]. In this paper, we consider a collocated MIMO radar with $M_R$ transmit and $N_R$ receive antenna elements. Denoting the samples of baseband equivalent of $M_R$-dimensional transmitted radar signals as $\{\mathbf{s}(n)\}_{n=1}^{M}$, the coherence matrix (i.e., the spatial covariance matrix) is [16]

$$\mathbf{R}_s = \frac{1}{M}\sum_{n=1}^{M} \mathbf{s}(n)\mathbf{s}^*(n) = \begin{bmatrix} 1 & \beta_{12} & \cdots & \beta_{1M_R} \\ \beta_{21} & 1 & \cdots & \beta_{2M_R} \\ \vdots & \vdots & \vdots & \vdots \\ \beta_{M_R 1} & \beta_{M_R 2} & \cdots & 1 \end{bmatrix}$$

where $n$ is the time index and $\beta_{ij}$ is the correlation coefficient between $i$th and $j$th signals ($1 \le i, j \le M_R$). The phases of $\{\beta_{ij}\}$ control beam direction and for $\beta_{ij} = 0, i \neq j$, we have $\mathbf{R}_s = \mathbf{I}_{M_R}$ corresponding to omni-directional transmission.

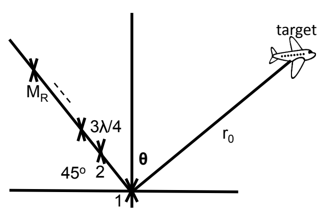


Fig. 3: Array configuration.

For a single target at direction $\theta$, the received signal by $m$th receive element is

$$y_m(n) = \alpha \sum_{i=1}^{M_R} A_{im} s_i(n) + w_m(n), \quad m = 1, \cdots, N_R \quad (2)$$

where $\tau_{im}(\theta) = \tau_{t,i}(\theta) + \tau_{r,m}(\theta)$ is the total delay of the signal transmitted by the $i$th transmit element and received by the $m$th receive element and $A_{im} = \exp(-j\omega_c \tau_{im}(\theta))$ is the corresponding phase delay, $\tau_{t,i}(\theta)$ and $\tau_{r,m}(\theta)$ are the delays between $i$th transmit element and target and target and $m$th receive element, respectively, $\alpha$ is the complex path loss and $w_m(n)$ is the additive noise received at the $m$th receive element. $A_{im}(\theta)$ can be decomposed to

$$A_{im}(\theta) = \exp\left(-j\omega_c(\tau_{t,i}(\theta) + \tau_{r,m}(\theta))\right) = a_{t,i}(\theta) a_{r,m}(\theta)$$

where $a_{t,i}(\theta) = \exp(-j\omega_c \tau_{t,i}(\theta))$ and $a_{r,m}(\theta) = \exp(-j\omega_c \tau_{r,m}(\theta))$. Defining

$$\mathbf{a}_t(\theta) = [a_{t,1}(\theta)\ a_{t,2}(\theta)\ \cdots\ a_{t,M_R}(\theta)]^T,$$
$$\mathbf{a}_r(\theta) = [a_{r,1}(\theta)\ a_{r,2}(\theta)\ \cdots\ a_{r,N_R}(\theta)]^T,$$

and the transmit-receive steering matrix $\mathbf{A}(\theta) = \mathbf{a}_t(\theta)\mathbf{a}_r^T(\theta)$, (2) can be written as $\mathbf{y}(n) = \alpha \mathbf{A}(\theta)\mathbf{s}(n) + \mathbf{w}(n)$. Using this model, the Cramer Rao bound (CRB) for target direction estimation is obtained in closed form in [15, 16] as follows:

$$\text{CRB}(\theta) = \frac{1}{2\text{SNR}}\Bigg( N_R \dot{\mathbf{a}}_t^*(\theta)\mathbf{R}_s^T \dot{\mathbf{a}}_t(\theta) - \mathbf{a}_t^*(\theta)\mathbf{R}_s^T \mathbf{a}_t(\theta) ||\dot{\mathbf{a}}_r(\theta)||^2 - \frac{N_R |\mathbf{a}_t^*(\theta)\mathbf{R}_s^T \dot{\mathbf{a}}(\theta)|^2}{\mathbf{a}_t^*(\theta)\mathbf{R}_s^T \mathbf{a}_t(\theta)} \Bigg)^{-1} \quad (3)$$

where $\dot{\mathbf{a}}_t(\theta) = \frac{d\mathbf{a}_t(\theta)}{d\theta}$ and $\dot{\mathbf{a}}_r(\theta) = \frac{d\mathbf{a}_r(\theta)}{d\theta}$. To obtain $\mathbf{a}_t(\theta)$, $\mathbf{a}_r(\theta)$, $\dot{\mathbf{a}}_t(\theta)$ and $\dot{\mathbf{a}}_r(\theta)$, we need to know the delays as a function of $\theta$ and therefore the radar array configuration must be known (See Fig. 3). From (3), CRB($\theta$) depends, among other factors, on the coherence matrix of radar transmitted signals. It is shown in [15, 16] that, when all other parameters are fixed, the performance of target direction estimation in terms of maximum likelihood or Cramer Rao bound is optimal for $\mathbf{R}_s = \mathbf{I}_{M_R}$ (i.e., for orthogonal probing signals).

## III. Problem Description and Proposed Solution

In this paper, our goal is to design a precoder for radar ($\mathbf{P}_R$) which completely eliminates the interference generated at all of the communication receivers. We assume that the interference among communication users is accounted for either because the communication users are operating in low SNR regime and therefore interference can be ignored compared to the noise (See Section II) or because the channels used by the communication users are orthogonal using an appropriate medium access control algorithm. We therefore focus on the interference from radar only. With above assumption, after post-processing of the received signal $\mathbf{y}_k$ at communication receiver $k$, we have

$$\tilde{\mathbf{y}}_k = \mathbf{F}_k \mathbf{y}_k = \mathbf{F}_k \mathbf{H}_{kk} \mathbf{x}_k + \mathbf{F}_k \mathbf{H}_{kR} \mathbf{x}_R + \mathbf{F}_k \mathbf{z}_k.$$

The interference from radar to communication receiver $k$ is $\mathbf{F}_k \mathbf{H}_{kR} \mathbf{x}_R$. The goal is to construct the radar signal $\mathbf{x}_R$ such that its interference at all of the communication receivers is zero-forced.

### *A. Zero-forcing of Radar Interference*

We propose using a zero-forcing precoder at radar transmitter. After applying this precoder, the transmitted radar signal $\mathbf{x}_R$ is constructed as $\mathbf{x}_R = \mathbf{P}_R \tilde{\mathbf{x}}_R$ where $\tilde{\mathbf{x}}_R$ is the radar probing signal. Using this precoder the interference at communication receiver $k$ will be $\mathbf{F}_k \mathbf{H}_{kR} \mathbf{P}_R \tilde{\mathbf{x}}_R$. To completely eliminate the radar interfeence at all of the communication receivers, we must choose $\mathbf{P}_R$ such that

$$\mathbf{F}_k \mathbf{H}_{kR} \mathbf{P}_R \tilde{\mathbf{x}}_R = \mathbf{0}, \quad \forall k \in \mathcal{K}$$

which leads to the requirement,

$$\mathbf{P}_R \tilde{\mathbf{x}}_R \in \mathcal{N}(\mathbf{F}_k \mathbf{H}_{kR}), \quad \forall k \in \mathcal{K}$$

In other words, the radar transmitted signal must lie in the null space of $\mathbf{F}_k \mathbf{H}_{kR}$ for all $k$. Equivalently, we must have

$$\mathbf{P}_R \tilde{\mathbf{x}}_R \in \mathcal{N}(\mathbf{F}_1 \mathbf{H}_{1R}) \cap \mathcal{N}(\mathbf{F}_2 \mathbf{H}_{2R}) \cdots \mathcal{N}(\mathbf{F}_K \mathbf{H}_{KR})$$

Using the equality $\mathcal{N}(\mathbf{A}) \cap \mathcal{N}(\mathbf{B}) = \mathcal{N}(\mathbf{C})$ where $\mathbf{C} = [\mathbf{A}^* \ \mathbf{B}^*]^*$, we conclude that the precoder must satisfy the condition $\mathbf{P}_R \tilde{\mathbf{x}}_R \in \mathcal{N}(\tilde{\mathbf{H}})$ where

$$\tilde{\mathbf{H}} = [(\mathbf{F}_1 \mathbf{H}_{1R})^* \ (\mathbf{F}_2 \mathbf{H}_{2R})^* \cdots (\mathbf{F}_K \mathbf{H}_{KR})^*]^* \tag{4}$$

### B. Radar Precoder Design

In the previous subsection, we found that the radar precoder must be chosen such that the radar transmitted signal belongs to the null space of the matrix $\tilde{\mathbf{H}}$. To find the null space of $\tilde{\mathbf{H}}$, we first obtain the SVD of $\tilde{\mathbf{H}}$: $\tilde{\mathbf{H}} = \tilde{\mathbf{U}}\tilde{\mathbf{S}}\tilde{\mathbf{V}}^*$. Denoting the columns of $\tilde{\mathbf{V}}^*$ corresponding to zero singular values of $\tilde{\mathbf{H}}$ as $\bar{\mathbf{V}}$, null space of $\tilde{\mathbf{H}}$ will be Span$\{\bar{\mathbf{V}}\}$. To satisfy the condition $\mathbf{P}_R \tilde{\mathbf{x}}_R \in \mathcal{N}(\tilde{\mathbf{H}})$, the precoder $\mathbf{P}_R$ must be the projection matrix into Span$\{\bar{\mathbf{V}}\}$, i.e.

$$\mathbf{P}_R = \bar{\mathbf{V}} \left(\bar{\mathbf{V}}^* \bar{\mathbf{V}}\right)^{-1} \bar{\mathbf{V}}^* \tag{5}$$

The use of radar precoder will eliminate the effect of radar interference on communication users. On the other hand, radar precoder will change the spatial correlation of radar probing signals and impacts on their coherence matrix. For precoded radar signals, we will have $\mathbf{R}_s = \mathbf{P}_R \mathbf{P}_R^*$. Note that target localization performance is optimal for $\mathbf{R}_s = \mathbf{I}_{M_R}$ [15, 16].

### C. Feasibility of Radar Precoder Design

It was shown in the previous subsection that a radar precoder which eliminates its interference to the communication users reduces to the projection matrix into null space of an effective interference channel ($\mathcal{N}(\tilde{\mathbf{H}})$). In following proposition, we state the necessary condition for a non-trivial precoder ($\mathbf{P}_R \neq \mathbf{0}$) to exist.

**Proposition 1.** *MIMO radar can operate without creating interference at any of the communication receivers if number of radar transmit antennas is greater than sum of the requested degrees of freedom of all of the communication users.*

*Proof:* Dimension of the matrix $\mathbf{F}_k \mathbf{H}_{kR}$ is $d_k \times M_R$ and dimension of $\tilde{\mathbf{H}}$ is $\sum_k d_k \times M_R$. Consider the $l$th row of $\tilde{\mathbf{H}}$ and let $l = \sum_{i=1}^{D-1} d_i + b$ for some $D$ and $b$. The $l$th row of $\tilde{\mathbf{H}}$ is a linear combination of elements of $\mathbf{H}_{DR}$ weighted by the $b$th row of $\mathbf{F}_D$. Since the elements of channel matrices are assumed to be drawn from a continuous distribution, the rows of $\tilde{\mathbf{H}}$ are linearly independent [9]. The matrix $\tilde{\mathbf{H}}$ is therefore full rank. The nullity (dimension of null space) of $\tilde{\mathbf{H}}$ is therefore $(M_R - \sum_k d_k)^+$. To have a non-zero nullity for $\tilde{\mathbf{H}}$, and hence a non-zero precoder, we must have $M_R > \sum_k d_k$. ■

### D. Estimation of $\tilde{\mathbf{H}}$

We consider a fixed period of duration $L_t$ channel uses at the beginning of each block of $L$ channel uses where the channel matrices, $\mathbf{H}_{iR}, \ i \in \mathcal{K}$, remain constant and use this time period to estimate $\tilde{\mathbf{H}}$. During this estimation phase, communication receivers send training symbols and the radar transmitter uses the received signal to estimate $\tilde{\mathbf{H}}$ and find $\mathbf{P}_R$. We assume that communication receivers can cooperate in this phase in their choice of training symbols and transmission power. We consider channel reciprocity and therefore the channels from communication receiver $k$ to the radar transmitter is $\mathbf{H}_{kR}^*$. The communication receiver $k$ uses the Hermitian transpose of its post-processing matrix (i.e., $\mathbf{F}_k^*$) as a precoder (See Fig. 4). The effective channel from communication receiver $k$ to the radar transmitter will therefore be $\mathbf{H}_{kR}^* \mathbf{F}_k^*$. The composite channel from all of the communication receivers to the radar transmitter in this estimation phase will be $\bar{\mathbf{H}} = [\mathbf{H}_{1R}^* \mathbf{F}_1^* \ \cdots \ \mathbf{H}_{kR}^* \mathbf{F}_k^* \ \cdots \ \mathbf{H}_{KR}^* \mathbf{F}_K^*]$ which has a dimension of $M_R \times \sum_k d_k$. Clearly, $\tilde{\mathbf{H}} = \bar{\mathbf{H}}^*$.

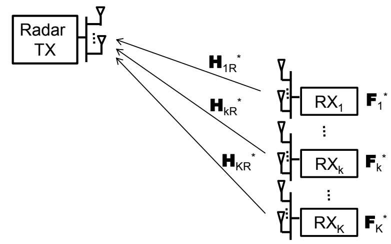

Fig. 4: Estimation of $\tilde{\mathbf{H}}$.

By assuming coordination among communication receivers, $\bar{\mathbf{H}}$ reduces to a standard MIMO channel with $\sum_k d_k$ inputs and $M_R$ outputs and we can estimate it using standard MIMO channel estimation algorithms. Let $\mathbf{S}$ denote the $\sum_k d_k \times L_t$ matrix of training symbols, $\mathbf{S} = [\mathbf{s}_1 \ \mathbf{s}_2 \ \cdots \ \mathbf{s}_{L_t}]$ where $\mathbf{s}_i, \ 1 \le i \le L_t$ is a $\sum_k d_k$-dimensional vector which contains the concatenation of training symbols sent by all of the communication receivers at time $i$. The received signal vector by radar transmitter at time $i$, assuming the average SNR at each receiving antenna is $\rho$, will be

$$\mathbf{y}_i = \sqrt{\frac{\rho}{\sum_k d_k}} \bar{\mathbf{H}} \mathbf{s}_i + \mathbf{w}_i, \ 1 \le i \le L_t \tag{6}$$

where $\mathbf{y}_i$ is the $M_R$-dimensional received signals at time $i$ and $\mathbf{w}_i$ is the noise vector at time $i$. Let $\mathbf{Y} = [\mathbf{y}_1 \ \mathbf{y}_2 \ \cdots \ \mathbf{y}_{L_t}]$ and $\mathbf{W} = [\mathbf{w}_1 \ \mathbf{w}_2 \ \cdots \ \mathbf{w}_{L_t}]$. The maximum likelihood (ML) estimation of $\bar{\mathbf{H}}$ is found in [17, 18] as

$$\hat{\bar{\mathbf{H}}}_{\text{ML}} = \sqrt{\frac{\sum_k d_k}{\rho}} \mathbf{Y} \mathbf{S}^* (\mathbf{S}\mathbf{S}^*)^{-1} \tag{7}$$

and the optimal training symbols which minimizes the mean square error is chosen such that $\mathbf{S}\mathbf{S}^* = L_t \mathbf{I}_{\sum d_k}$. Note that to choose this optimal training sequence it is necessary for communication receivers to cooperate. Consequently, $\tilde{\mathbf{H}}$ can be estimated as $\hat{\tilde{\mathbf{H}}} = \hat{\bar{\mathbf{H}}}_{\text{ML}}^*$.

## IV. Numerical and Simulation Results

In this section, we compare the performance of MIMO radar in terms of CRB for target direction estimation with and without radar precoder and as a function of degrees of freedom requested by communication users, number of communication receivers, and number of radar antennas. We also investigate the effect of null space estimation error on target direction estimation and radar interference to communication users.

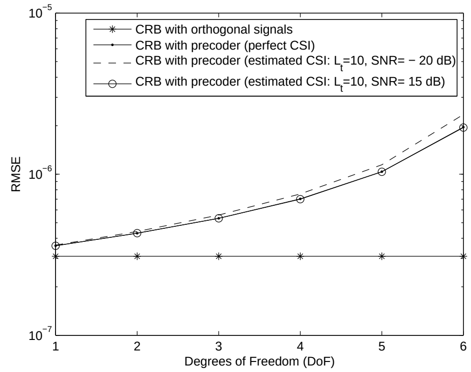


Fig. 5: CRB($\theta$) vs. DoF ($K = 1$, $M_R = N_R = 8$, $M_1 = N_1 = 6$).

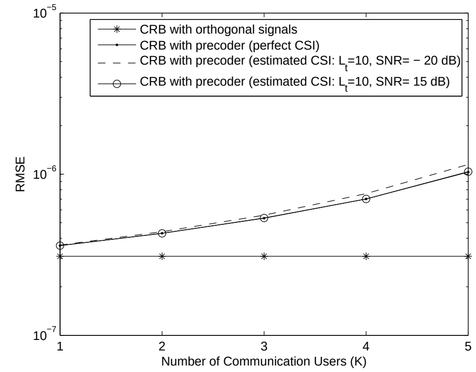


Fig. 7: CRB($\theta$) vs. $K$ ($d_k = 1$, $M_R = N_R = 8$, $M_1 = N_1 = 6$).

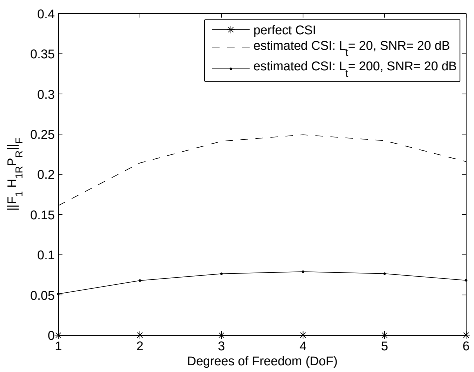


Fig. 6: $||\mathbf{F}_1\mathbf{H}_{1R}\mathbf{P}_R||_F$ vs. DoF ($K = 1$, $M_R = N_R = 8$, $M_1 = N_1 = 6$).

As we mentioned earlier (See Section III-B), the effect of radar precoder on radar probing signals is to change their coherence matrix ($\mathbf{R}_s$). On the other hand, as seen in equation (3), $\mathbf{R}_s$ impacts on the performance of target direction estimation. We compare the radar performance with orthogonal radar signals (i.e., $\mathbf{R}_s = \mathbf{I}_{M_R}$) which is known to lead to the optimal performance [15, 16] and precoded radar signals (i.e., $\mathbf{R}_s = \mathbf{P}_R\mathbf{P}_R^*$). The distance of target to radar array is assumed to be $r_0 = 5000\ m$, The radar inter-element spacing is assumed to be $3\lambda/4$ and frequency of operation is 3.5 GHz. We assume that the SNR of received radar signal is 20 dB. The target direction is assumed to be at $\theta = 0^o$.

In Fig. 5, the radar CRB performance is shown for orthogonal radar signals as well as precoded radar signals with perfect channel state information (CSI) (i.e., $\tilde{\mathbf{H}}$) knowledge at radar transmitter and with estimated CSI. For CSI estimation, we consider a training period of $L_t = 10$ channel uses and SNR values (i.e., $\rho$ in (7)) of -20 dB and 10 dB. In this figure, we consider a single communication receiver (i.e., $K = 1$) with $M_1 = N_1 = 6$ antennas and varying degrees of freedom ($1 \leq d_1 \leq 6$). The number of radar transmit and receive antennas is assumed $M_R = N_R = 8$ for this figure. As we found in Section III-C, the nullity of $\tilde{\mathbf{H}}$ is $M_R - \sum_{k=1}^{K} d_k$ and therefore as the requested DoF of communication user increases, null space of $\tilde{\mathbf{H}}$ shrinks. This will impact on the choice of radar precoder and deteriorate the target localization performance. The CSI estimation error further degrades the radar performance, however performance loss due to CSI estimation error, even in low SNR, is marginal. In Fig. 6, using the same parameters, we have shown the effect of CSI estimation error on the interference perceived at the communication receiver. To see the effect of interference, we have used $||\mathbf{F}_1\mathbf{H}_{1R}\mathbf{P}_R||_F$ as a metric where $||.||_F$ denotes the Frobenius norm [19]. As shown in this figure, with perfect knowledge of $\tilde{\mathbf{H}}$, interference can be completely eliminated at the communication receiver. Assuming SNR=20 dB, the channel estimation error and the effect of interference at communication user becomes less significant as $L_t$ increases.

In Fig. 7, the radar CRB performance is shown as a function of number of communication receivers ($K$). We set the requested DoF of every user equal to 1, $M_k = N_k = 6$ and $M_R = N_R = 8$. Note that we must have $M_R > \sum d_k$ so that radar can choose a non-zero precoder (See Proposition 1). In this figure we assume $K \leq 5$ to make sure nullity of $\tilde{\mathbf{H}}$ is non-zero. With increasing $K$ null space of $\tilde{\mathbf{H}}$ shrinks which impacts on the choice of precoder and worsens the radar CRB performance. The effect of $\tilde{\mathbf{H}}$ estimation error on radar performance is again marginal. In Fig. 8, we consider the interference performance in terms of the metric $\sum_i ||\mathbf{F}_i\mathbf{H}_{iR}\mathbf{P}_R||_F$. The effect of interference becomes less significant as the duration of training phase increases while it will be completely eliminated with perfect knowledge of $\tilde{\mathbf{H}}$.

In Fig. 9 we consider a single communication user with $M_1 = N_1 = 6$ and DoF $d_1 = 2$ and investigate the effect of number of radar antennas ($M_R = N_R$) on the CRB performance of radar. Note from (3) that CRB explicitly depends on $N_R$ and improves when $N_R$ increases. For precoded radar signals, increasing $M_R$ results in increasing the nullity of $\tilde{\mathbf{H}}$ which impacts the choice of $\mathbf{P}_R$ and will lead to better CRB performance. The CSI estimation error slightly worsens the performance of radar with precoder. Results in Fig. 9 indicate that increasing the number of radar antennas can compensate the performance loss in target direction estimation due to correlation in precoded radar signals. In other words, for a

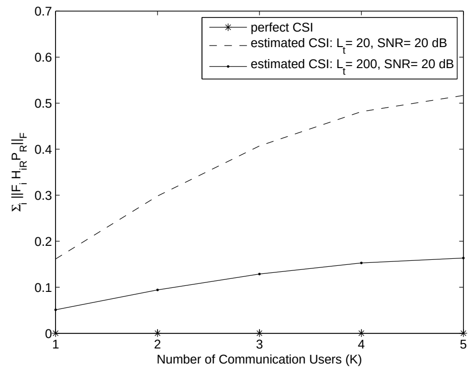


Fig. 8: $\sum_i ||\mathbf{F}_i\mathbf{H}_{iR}\mathbf{P}_R||_F$ vs. $K$ ($d_k = 1$, $M_R = N_R = 8$, $M_1 = N_1 = 6$).

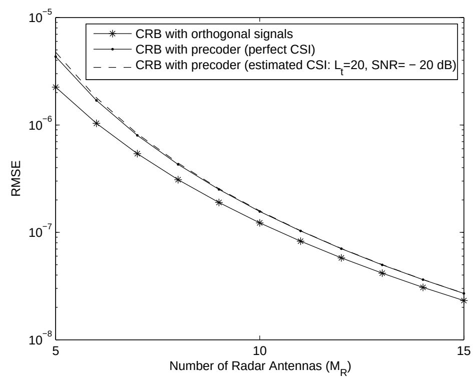


Fig. 9: CRB($\theta$) vs. $M_R$ ($d_1 = 2$, $K = 1$, $M_1 = N_1 = 6$).

given desired RMSE, number of radar antennas must increase when radar employs a precoder to zero force its interference at the communication receivers. As shown in Fig. 10, the effect of radar interference (measured in terms of $||\mathbf{F}_1\mathbf{H}_{1R}\mathbf{P}_R||_F$) becomes less significant as $L_t$ increases.

## V. Conclusion

Radar systems with high resolution have traditionally been one of the major consumers of radio spectrum. Spectrum sharing between radar and communication systems has been considered due to high spectrum demand from commercial data-intensive communication systems. In this paper, we propose a null-steering precoder for a MIMO radar system to avoid interference to a communication system sharing the spectrum with radar. The proposed approach requires the estimation of an effective interference channel at radar. An estimation phase is proposed in which communication receivers coordinate and the radar transmitter estimates the effective interference channel and obtains the precoder. We investigate the effect of degrees of freedom requested by communication users, number of communication user and number of radar antennas and the channel estimation error on the radar performance and interference at the communication users. Our results show that precoder introduces correlation to radar probing signals which incurs some performance loss for target direction estimation. We show that this loss can be compensated by increasing the number of radar antennas.

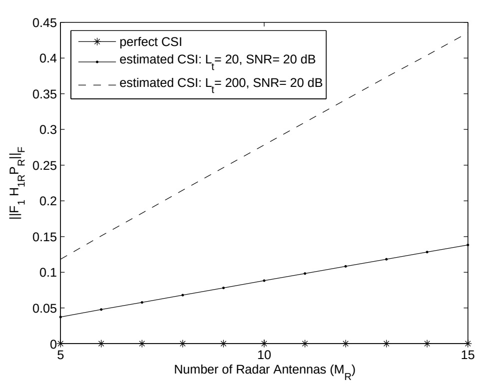


Fig. 10: $||\mathbf{F}_1\mathbf{H}_{1R}\mathbf{P}_R||_F$ vs. $M_R$ ($d_1 = 2$, $K = 1$, $M_1 = N_1 = 6$).

## References

[1] M. J. Marcus, "Thoughts on Radar/Communications Spectrum Sharing," in Proceedings of ISART 2011, Boulder, CO, USA, 2011.

[2] Connecting America: The National Broadband Plan, editted by Barry Leonard, available at www.broadband.gov.

[3] Communication Signals in the 2.6 GHz Band and Maritime Radar, Technical Assessment of Interference, Ofcom Technica Report, 2011.

[4] R. Saruthirathanaworakun, J. M. Peha and L. M. Correia, "Opportunistic Primary-Secondary Spectrum Sharing with a Rotating Radar," in *Proc. of ICNC 2012*.

[5] S. Sodagarai, A. Khawar, T. C. Clancy and R. McGwier, "A Projection Based Approach for Radar and Telecommunication Systems Coexistence," in *Proc. of Globecom 2012*.

[6] H. Yi, H. Hu, Y. Rui, K. Guo, and J. Zhang, "Null Space-Based Precoding Scheme for Secondary Transmission in a Cognitive Radio MIMO System Using Second-Order Statistics," in *Proc. of ICC 2009*.

[7] H. Yi, "Nullspace-Based Secondary Joint Transceiver Scheme for Cognitive Radio MIMO Networks Using Second-Order Statistics," in *Proc. of ICC 2010*

[8] A. M. Tulino and S. Verdu, "Random Matrix Theory and Wireless Communication," *Foundations and Trends in Communications Information Theory*, vol. 1, no. 1, 2004.

[9] S. A. Jafar, "Interference Alignment: A New Look at Signal Dimensions in a Communication Network", *Foundations and Trends in Communications and Information Theory*, vol. 7, no. 1, 2011.

[10] F. Negro, S. P. Shenoy, I. Ghauri and D. T. M. Slock, "On the MIMO Interference Channel," in *Proc. of ITA 2010*.

[11] V. R. Cadambe and S. A. jafar, "Interference Alignment and Degrees of Freedom of the K-User Interference Channel," *IEEE Trans. Inf. Theory*, vol. 54, no. 8, pp. 3425-3441, Aug. 2008

[12] K. Gomadam, V. R. Cadambe, S. A. Jafar, "Approaching the Capacity of Wireless Networks through Distributed Interference Alignment," Submitted to *IEEE Trans. Inf. Theory*.

[13] D. Tse and P. Viswanath, Fundamentals of Wireless Communications, Cambridge University Press, 2005.

[14] J. Li and P. Stoica, MIMO Radar Signal Processing. Wiley-IEEE Press, 2009.

[15] I. Bekkerman and J. Tabrikian, "Target Detection and Localization Using MIMO Radars and Sonars," *IEEE Trans. Signal Pocess.*, vol. 54, no. 10, Oct. 2006.

[16] T. L. Marzetta, "BLAST Training: Estimating Channel Characteristics for High Capacity Space-Time Wireless," in *Proc. 37th Annual Allerton Conference on Communications, Control, and Computing*, 1999.

[17] Q. Sun, D. C. Cox, H. C. Huang, and A. Lozano, "Estimation of Continuous Flat Fading MIMO Channels," *IEEE Tans. Wireless Commun.*, vol. 1, no.4, Oct. 2002.

[18] G. H. Golub and C. F. Van Loan, Matrix Computations, 3rd ed. Johns Hopkins Studies in Mathematical Sciences, 1996.