# Cross-Layer Performance Analysis for Cognitive Radio Network with a Random Transmission Protocol in Presence of Sensing Errors

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***Abstract*—In this paper, we develop a queueing analytic model that incorporates imperfect sensing in order to measure different performance parameters e.g., collision probability, packet loss probability and queueing delay. This analytic model is useful for call admission control (CAC) decision in cognitive radio network (CRN) when there is a certain sensing error as well as certain quality of service (QoS) requirements for both primary and cognitive radio (CR) users. Using our developed model, we compare the performance of a random transmission protocol with that of the traditional deterministic transmission protocol. Selected numerical results show that for given QoS requirements, the random transmission protocol can support higher number of CR users than the traditional deterministic transmission protocol when there is a certain probability of sensing error.**

***Index Terms*—Cognitive radio, call admission control, sensing error, cross-layer performance, random transmission, Markov chain.**

## I. Introduction

The inefficient usage of radio spectrum demands dynamic spectrum access (DSA) as opposed to the conventional fixed spectrum allocation policy. Cognitive radio (CR) technology can facilitate DSA [1]. CR is a mechanism that senses the spectrum of primary users (PUs) and based on sensing outcome, it adapts the transmission parameters. In the literature CR users are also referred to as secondary users (SUs). Spectrum sensing is one of the most challenging tasks for DSA using CR technology [2]. A number of spectrum sensing mechanisms has been proposed in the literature [2], [3]. However, most of the sensing mechanisms have a certain probability of sensing error. In particular, there are two types of sensing errors i.e., miss-detection and false alarm [4]. Due to a miss-detection, transmission from cognitive radio network (CRN) leads to a collision with the transmission of PUs.

In a CRN, queueing analysis enables to measure different cross-layer performance parameters e.g., packet loss probability, queueing delay and throughput [5]. Authors of [5], [6] developed a queueing model to analyze packet-level performances assuming perfect channel sensing in their models. Recently, Wang *et. al* [7] also developed a Markov model for queueing analysis with sensing errors. However, they considered the traditional deterministic transmission protocol[1] in their Markov model. In this paper, we develop a queuing analytic model for CRN that considers sensing errors as well as a random transmission protocol. According to the random transmission protocol the CRN accesses the channel with a certain probability based on sensing outcome. In particular, we develop a Markov chain model and analyze it as a quasi birth and death (QBD) process. With the help of our analytic model, a CRN can easily measure different packet level performance parameters for given number of SUs, probability of sensing errors, transmission probabilities of CRN and other system parameters. With the developed model, the collision probability to the PUs can also be measured. As such our analysis is useful in making call admission control (CAC) decision in CRN for a given collision probability to the PUs while maintaining target quality of service (QoS) requirements for SUs. For example, the SUs have certain QoS requirements in terms of delay and packet loss probability. On the other hand, the PUs can tolerate collisions with a certain collision probability. Using our model one can determine the number of SUs that can be supported while maintaining these QoS requirements for given false alarm and miss-detection probabilities. We also compare the performance of the random transmission protocol with the performance of the traditional deterministic transmission protocol.

Presented numerical results demonstrate the significance of our analytic model in terms of CAC decision. Numerical results also show that CRN can support higher number of SUs using random transmission protocol than the deterministic transmission protocol by properly choosing transmission probabilities of the random transmission protocol.

## II. The System model

### A. Network Model

We consider a single channel infrastructure based CRN where a CR transmitter (Tx) transmits information to $K$ number of SUs opportunistically. In particular, in a given time slot the CRN senses the channel. Based on the channel

[1] By deterministic transmission protocol we mean that CRN access the channel when it is sensed as idle and it does not access the channel when the channel is sensed as busy.

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sensing outcome, it makes transmission decision and assigns the channel to a SU as described in later section.

### B. Primary User Activity

We consider so called ON-OFF behavior (see e.g., [5]) for the PUs' activity. According to this model, the channel occupancy state of a PU can be represented by a two state time homogeneous first order Markov process. Let us use $O$ to denote the actual channel occupancy state. In a given time slot, if the channel is occupied i.e., busy, $O$ takes value 0. On the other hand, $O$ takes value 1 if the channel is not occupied i.e., idle. $P_{\mathrm{o}}$ denotes the transition probability matrix of the PU actual channel occupancy state, $O$ which is defined as follows:

$$\mathbf{P}_{\mathrm{o}} = \begin{bmatrix} O_{0\to 0} & O_{0\to 1} \\ O_{1\to 0} & O_{1\to 1} \end{bmatrix}, \quad (1)$$

where $O_{i\to j}$ denotes the transition probability from state $i$ to state $j$ and $i, j \in \{0, 1\}$. PU activity, $\rho$ can be obtained from the steady state probability of eq. (1).

The CRN senses the channel at the beginning of each time slot. Let us use random variable $\hat{O}$ to denote the estimated channel occupancy where $\hat{O} = 1$ represents the fact that the channel is sensed as idle and vice versa. Due to the sensing errors, in a given time slot $n$, the value of $\hat{O}_n$ may not be same as the value of $O_n$. In particular, when there is a miss-detection, $\hat{O}_n = 1$ given $O_n = 0$. On the other hand, due to the false alarm, $\hat{O}_n = 0$ given $O_n = 1$.

### C. Channel Model and Rate Adaptation

We consider that the channel between CR Tx and SUs is time varying and independent from one user to another user. The time varying fading gain between the CR Tx and a SU can be modeled as a finite state Markov chain (see e.g., [5]). We also consider that each channel fading amplitude can be modeled as the Nakagami-$m$ distribution. The possible channel states are denoted by a set $\mathcal{C} = \{0, 1, ...., Z-1\}$ with total $Z$ states. Packet transmission rate at state $x$ is, $\gamma_x = rx$ where $x \in \mathcal{C}$ i.e., $0 \le x \le (Z-1)$ and $r$ is an integer depends on slot duration, modulation and coding parameters.

### D. Opportunistic Spectrum Scheduling

We assume an opportunistic spectrum scheduling scheme which utilizes multiuser diversity so that CRN can maximize its throughput. According to this scheme, CR Tx assigns the channel to a SU who has the highest transmission rate. If more than one SU has the highest transmission rate, the channel is assigned randomly to a SU among the SUs who have the highest transmission rate.

### E. Packet Arrival

Batch Bernoulli random process is considered for packet arrival for the SUs as it can capture different bursty traffic arrival process (see e.g., [5]). Batch Bernoulli model can be described by $\beta = [\beta_0, \beta_1, ...., \beta_N]$, where $N$ is the maximum number of packets that can arrive at a time slot. $\beta_j$ $(0 \le j \le N)$ is the probability of $j$ packet(s) arrival at a particular time slot. Average packet arrival rate can be calculated as, $\alpha = \sum_{j=0}^{N} j\beta_j$. We assume that packet(s) arriving during current time slot will be served in the next time slot at the earliest.

### F. Joint State Considering Channel State and Opportunistic Scheduling

We consider a homogenous system where all the SUs have identical and independent packet arrival and channel fading process. Therefore, we analyze the performance of a particular user referred to as tagged user. Let us use random variable $m_n \in$\{0,1\} to denote whether the channel is assigned to the tagged user or not at time slot $n$. $m_n = 1$ represents that the channel is assigned to the tagged user at time slot $n$ and vice versa. $c_n \in \mathcal{C}$ denotes the channel state at time slot $n$. The joint state at time slot $n$ is denoted as $(m_n, c_n)$ and the transition probability can be written as follows:

$$\mathbf{J} = \begin{bmatrix} \mathbf{J}_{0\to 0} & \mathbf{J}_{0\to 1} \\ \mathbf{J}_{1\to 0} & \mathbf{J}_{1\to 1} \end{bmatrix}, \quad (2)$$

where $\mathbf{J}_{i\to j}(p, q)$ denotes the transition probability from state $i$ to state $j$, $i, j \in \{0, 1\}$ and channel state goes from state $p$ to state $q$, $0 \le p, q \le (Z-1)$. $\mathbf{J}$ depends on number of SUs, $K$. One can obtain the joint transition probability, $\mathbf{J}$ using the procedure mentioned in [5]. Let us construct a new state variable, $a_n = m_n c_n$ and state space, $\mathbf{\Theta} = \{a_n | 0 \le a_n \le Z-1\}$. From $a_n$ one can obtain transmission rate at a particular time slot $n$, $f_n = ra_n$ and corresponding transition probability matrix can be expressed as follows:

$$\mathbf{A}_{l\to\hat{l}} = \frac{\sum_{x\in l}\sum_{\hat{x}\in\hat{l}} \tau_x \mathbf{J}_{x\to\hat{x}}}{\sum_{x\in l}\tau_x}, \quad (3)$$

where $0 \le f_n \le Y$ and $Y$ is the maximum transmission rate given by $Y = r(Z-1)$. $\mathbf{A}_{l\to\hat{l}}$ denotes the transition probability from state $l$ to state $\hat{l}$ and $l, \hat{l} \in \{0, 1, ..., Y\}$ and $\tau_{\mathbf{x}}$ denotes the steady state probability vector for the state space $\mathbf{\Theta}$ at state $x$.

## III. Random Transmission Protocol

In our analysis, we consider a random transmission protocol that takes sensing errors into account. It is important to mention that in previous works [5]-[7], random transmission protocol is not considered. Let us use $T$ to denote the transmission decision of CRN where $T$ $\in$\{0,1\}. $T$ takes value 1 when CRN decides to transmit otherwise $T$ takes value 0. According to the random transmission protocol, CR Tx transmits probabilistically based on estimated channel occupancy, $\hat{O}_n$. The details of the protocol is described below:

- *Step 1*: At a time slot $n$, CRN estimates PUs' channel occupancy, $\hat{O}_n$.
- *Step 2*: Based on the estimated channel occupancy, $\hat{O}_n$ CR Tx makes transmission decision as follows: If $\hat{O}_n = 0$, CR Tx decides to transmit with probability, $P_1$. On the other hand if $\hat{O}_n = 1$, CRN decides to transmit with probability, $P_2$.[2]

[2]The actual number of packets that will be transmitted depends on the buffer as well as channel fading state.

With the traditional deterministic transmission protocol [5]-[7], CR Tx transmits with probability 1 if the channel is sensed as idle. On the other hand, if the channel is sensed as busy, CR Tx does not attempt to transmit. It is obvious that the traditional deterministic protocol is a special case of the random transmission protocol when $P_1 = 0$ and $P_2 = 1$.

After a transmission is made by the CR Tx, the transmitted packet(s) can be received successfully by the SUs or they can collide with PUs' transmission based on the actual channel occupancy. If CR Tx receives an acknowledgement from its receiver within a certain time duration, it discards the transmitted packet(s) from the buffer. Otherwise, it retransmits the packet(s) in the next transmission opportunity. So, no packet of SUs is lost due to the collision with PUs' transmission. Packets are lost only due to the buffer overflow. The significance of the random transmission protocol is two fold. When there is a false alarm, by increasing $P_1$, CRN may improve SUs' transmission rate. On the other hand, decreasing $P_2$ CRN can reduce collision probability in presence of miss-detection. As shown in Section VI, by adjusting $P_1$ and $P_2$ CRN can improve QoS parameters of SUs, while keeping the collision probability below the threshold specified by the PUs.

## IV. Queuing Analytic Model

### A. Joint State of Primary User Activity and Transmission Decision

According to the random transmission protocol, the transmission decision of CR Tx depends on PUs' channel occupancy. As such they are not independent random variable and we need to find joint transition probability of these variables. Let us define the joint transition probability, $\mathbf{R}$ as follows:

$$\mathbf{R} = \begin{bmatrix} (T_{0\to0}, O_{0\to0}) & (T_{0\to0}, O_{0\to1}) & (T_{0\to1}, O_{0\to0}) & (T_{0\to1}, O_{0\to1}) \\ (T_{0\to0}, O_{1\to0}) & (T_{0\to0}, O_{1\to1}) & (T_{0\to1}, O_{1\to0}) & (T_{0\to1}, O_{1\to1}) \\ (T_{1\to0}, O_{0\to0}) & (T_{1\to0}, O_{0\to1}) & (T_{1\to1}, O_{0\to0}) & (T_{1\to1}, O_{0\to1}) \\ (T_{1\to0}, O_{1\to0}) & (T_{1\to0}, O_{1\to1}) & (T_{1\to1}, O_{1\to0}) & (T_{1\to1}, O_{1\to1}) \end{bmatrix}, \tag{4}$$

where $T_{m\to k}$ denotes the transition probability of transmission decision from state $m$ to state $k$ and $O_{a\to b}$ represents the transition probability of actual occupancy from state $a$ to state $b$ where $(m, k, a, b) \in \{0, 1\}$. Mathematically, $(T_{m\to k}, O_{a\to b})$ can be written as:

$$\begin{aligned} & (T_{m\to k}, O_{a\to b}) \\ = \; & [(1-P_1)^{(2-m-k)} P_1^{(m+k)} (1-P_d)^{(2-a-b)} P_f^{(a+b)} \\ & +(1-P_1)^{(1-m)}(1-P_2)^{(1-k)} P_1^{(m)} P_2^{(k)} \\ & (1-P_d)^{(1-a)} P_d^{(1-b)} (1-P_f)^{(b)} P_f^{(a)} \\ & +(1-P_2)^{(2-m-k)} P_2^{(m+k)} P_d^{(2-a-b)} (1-P_f)^{(a+b)} \\ & +(1-P_1)^{(1-k)}(1-P_2)^{(1-m)} P_1^{(k)} P_2^{(m)} \\ & (1-P_d)^{(1-b)} P_d^{(1-a)} (1-P_f)^{(a)} P_f^{(b)}] O_{a\to b}, \end{aligned} \tag{5}$$

where $P_f$, denotes the false alarm probability and $P_d$, denotes the miss-detection probability.

### B. Markov Chain Analysis

We consider that state transition occurs at the slot boundary and state variables can take only a discrete value. So, we can model our system as a discrete time Markov chain (DTMC). State space for finite buffer size of our system can be expressed as: $\mathbf{\Phi} \equiv \{b_n, f_n, T_n, O_n | 0 \le b_n \le Q; 0 \le f_n \le Y; 0 \le T_n \le 1; 0 \le O_n \le 1\}$, where $b_n$ is the queue length or number of packets in the buffer at a time slot $n$ and $Q$ in the maximum buffer size. After obtaining the transition probabilities of eq. (4), the state transition matrix for the DTMC can be written as in eq. (6) for infinite buffer space. However, finite buffer space is more realistic. We consider a finite buffer size $Q$ and buffer level $X$ where $X = \lfloor \frac{Q}{Y} \rfloor$. By making block of sub matrices of eq. (6) as a QBD process, eq. (6) for the finite buffer case can be expressed as follows:

$$\mathbf{P} = \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ \vdots \\ X-1 \\ X \end{array} \begin{bmatrix} B & C & & & & & \\ D & E & F_0 & & & & \\ & F_2 & F_1 & F_0 & & & \\ & & F_2 & F_1 & F_0 & & \\ & & & \ddots & \ddots & \ddots & \\ & & & & F_2 & F_1 & F_0' \\ & & & & & F_2' & F_1' \end{bmatrix}. \tag{7}$$

Let us consider a set of sub matrices $\mathbf{U}^{(x)}(0 \le x \le Y)$ and $\mathbf{S}^{(r)}(1 \le r \le 4)$. $\mathbf{U}^{(x)}(0 \le x \le Y)$ can be written as follows:

$$\mathbf{U}_{i,j}^{(x)} = \begin{cases} \mathbf{A}_{i,j} & \text{if} \quad i = x \\ 0 & \text{if} \quad i \ne x \end{cases}, \tag{8}$$

where $0 \le i, j \le Y$ and $\mathbf{A}_{i,j}$ denotes the transition probability of transmission rate from state $i$ to state $j$. Similarly, $\mathbf{S}^{(r)}(1 \le r \le 4)$ can be written as follows:

$$\mathbf{S}_{i,j}^{(r)} = \begin{cases} \mathbf{R}_{i,j} & \text{if} \quad i = r \\ 0 & \text{if} \quad i \ne r \end{cases}, \tag{9}$$

where $1 \le i, j \le 4$ and $\mathbf{R}_{i,j}$ represents the transition probability of eq. (4) from state $i$ to state $j$. Using eq. (8) and eq. (9) we can define the inner sub matrices of eq. (6) as follows:

$$\mathbf{H}_0^{(0)} = \beta_0 \mathbf{AR}; \qquad \mathbf{H}_{d+}^{(0)} = \beta_{d+} \mathbf{AR}, \quad 1 \le d \le N, \tag{10}$$

$$\mathbf{H}_0^{(e)} = \begin{cases} \beta_0 \mathbf{U}^{(0)}\mathbf{R} + \beta_0 \sum_{q=1}^{Y} \mathbf{U}^{(q)} \sum_{r=1}^{3} \mathbf{S}^{(r)} \\ + \sum_{1 \le p \le e} \beta_p \mathbf{U}^{(p)} \mathbf{S}^{(4)} \\ + \sum_{(e+1) \le x \le Y} \beta_e \mathbf{U}^{(x)} \mathbf{S}^{(4)}, \text{ for } 1 \le e \le N, \\ \beta_0 \mathbf{U}^{(0)}\mathbf{R} + \beta_0 \sum_{1 \le q \le Y} \mathbf{U}^{(q)} \sum_{r=1}^{3} \mathbf{S}^{(r)} \\ + \sum_{(1 \le p \le N)} \beta_p \mathbf{U}^{(p)} \mathbf{S}^{(4)}, \text{ for } N < e \le Y, \end{cases} \tag{11}$$

$$\mathbf{H}_{d-}^{(e)} = \begin{cases} \beta_0 \sum_{e \le q \le Y} \mathbf{U}^{(q)} \mathbf{S}^{(4)}, \quad \text{if} \quad d = e \\ \sum_{0 \le q \le e-d-1} \beta_q \mathbf{U}^{(q+d)} \mathbf{S}^{(4)} \\ + \beta_{e-d} \sum_{e \le q \le Y} \mathbf{U}^{(q)} \mathbf{S}^{(4)}, \\ \text{for } e - N \le d \le e - 1, \\ \sum_{0 \le p \le N} \beta_p \mathbf{U}^{(p+d)} \mathbf{S}^{(4)}, \\ \text{for } 1 \le d \le e - N, 1 \le e \le Y - 1, \end{cases} \tag{12}$$

$$
\mathbf{P} = \left[ \begin{array}{c|ccccc|ccccc|ccc}
\mathbf{H}_0^{(0)} & \mathbf{H}_{1+}^{(0)} & \cdots & \mathbf{H}_{N+}^{(0)} & & & & & & & & & & \\
\hline
\mathbf{H}_{1-}^{(1)} & \mathbf{H}_0^{(1)} & \mathbf{H}_{1+}^{(1)} & \cdots & \mathbf{H}_{N+}^{(1)} & & & & & & & & & \\
\vdots & \vdots & & & \ddots & & & & & & & & & \\
\mathbf{H}^{(Y-N+1)}_{(Y-N+1)-} & \mathbf{H}^{(Y-N+1)}_{(Y-N)-} & \cdots & \mathbf{H}_0 & \mathbf{H}^{(Y-N+1)}_{(1)+} & \mathbf{H}^{(Y-N+1)}_{(N-1)+} & \mathbf{H}^{N+} & & & & & & & \\
\vdots & \vdots & & & & \vdots & \vdots & \ddots & & & & & & \\
\mathbf{H}^{(Y)}_{Y-} & \mathbf{H}^{(Y)}_{(Y-1)-} & \cdots & \cdots & \mathbf{H}^{(Y)}_{(1)-} & \mathbf{H}_0 & \mathbf{H}_{1+} & \cdots & \mathbf{H}_{N+} & & & & & \\
\hline
 & \mathbf{H}_{Y-} & \cdots & \cdots & \cdots & \mathbf{H}_{1-} & \mathbf{H}_0 & \mathbf{H}_{1+} & \cdots & \mathbf{H}_{N+} & & & & \\
 & & \ddots & & & \vdots & \vdots & & & \ddots & & & & \\
 & & & \mathbf{H}_{Y-} & & \mathbf{H}_{(Y-N+1)-} & \mathbf{H}_{(Y-N)-} & \cdots & \cdots & \cdots & \mathbf{H}_{(N-1)+} & \mathbf{H}_{N+} & & \\
 & & & & \ddots & \vdots & \vdots & & & \ddots & \vdots & \vdots & \ddots & \\
 & & & & & \mathbf{H}_{Y-} & \mathbf{H}_{(Y-1)-} & \cdots & \cdots & \mathbf{H}_{1-} & \mathbf{H}_0 & \mathbf{H}_{1+} & \cdots & \mathbf{H}_{N+} \\
\hline
 & & & & \ddots & & & & & \ddots & & & & \ddots
\end{array} \right] \quad (6)
$$

$$
\begin{aligned}
\mathbf{H}_{d+} \quad &= \quad \beta_d \mathbf{U}^{(0)}\mathbf{R} + \beta_d \sum_{q=1}^{Y} \mathbf{U}^{(q)} \sum_{r=1}^{3} \mathbf{S}^{(r)} \\
&+ \sum_{x=d+1}^{N} \beta_x \mathbf{U}^{(x-d)} \mathbf{S}^{(4)},\ 1 \leq d \leq N,
\end{aligned} \quad (13)
$$

$$
\mathbf{H}_{d-} = \sum_{0 \leq p \leq N} \beta_p \mathbf{U}^{(p+d)} \mathbf{S}^{(4)},\ 1 \leq d \leq Y, \quad (14)
$$

$$
\begin{aligned}
\mathbf{H}_{d+}^{(e)} \quad &= \quad \beta_d \mathbf{U}^{(0)}\mathbf{R} + \beta_d \sum_{q=1}^{Y} \mathbf{U}^{(q)} \sum_{r=1}^{3} \mathbf{S}^{(r)} \\
&+ \beta_{(e+d)} \sum_{e \leq q \leq Y} \mathbf{U}^{(q)} \mathbf{S}^{(4)} \\
&+ \sum_{d+1 \leq p \leq d+e-1} \beta_p \mathbf{U}^{(p-d)} \mathbf{S}^{(4)},
\end{aligned} \quad (15)
$$

$$
\begin{aligned}
\mathbf{H}_0 \quad &= \quad \beta_0 \mathbf{U}^{(0)}\mathbf{R} + \beta_0 \sum_{q=1}^{Y} \mathbf{U}^{(q)} \sum_{r=1}^{3} \mathbf{S}^{(r)} \\
&+ \sum_{p=1}^{N} \beta_p \mathbf{U}^{(p)} \mathbf{S}^{(4)},
\end{aligned} \quad (16)
$$

$$
\begin{aligned}
\mathbf{H}'_0 \quad &= \quad \sum_{p=0}^{N} \beta_p \mathbf{U}^{(0)}\mathbf{R} + \sum_{p=0}^{N} \beta_p \sum_{q=1}^{Y} \mathbf{U}^{(q)} \sum_{r=1}^{3} \mathbf{S}^{(r)} \\
&+ \sum_{1 \leq j \leq N} \sum_{1 \leq h \leq j} \beta_j \mathbf{U}^{(h)} \mathbf{S}^{(4)},
\end{aligned} \quad (17)
$$

$$
\mathbf{F}'_2 = \left[ \begin{array}{c} \mathbf{F_2} \\ \mathbf{0}_{(4 \times X'Z) \times (4 \times YZ)} \end{array} \right], \quad (18)
$$

$$
\mathbf{F}'_0 = \left[ \begin{array}{cc} \mathbf{F_0} & \mathbf{0}_{(4 \times YZ) \times (4 \times X'Z)} \end{array} \right], \quad (19)
$$

$$
\begin{aligned}
\mathbf{H}'_{k+} \quad &= \quad \sum_{p=k}^{N} \beta_p \mathbf{U}^{(0)}\mathbf{R} + \sum_{p=k}^{N} \beta_p \sum_{q=1}^{Y} \mathbf{U}^{(q)} \sum_{r=1}^{3} \mathbf{S}^{(r)} \\
&+ \sum_{k+1 \leq j \leq N} \sum_{1 \leq h \leq j-k} \beta_j \mathbf{U}^{(h)} \mathbf{S}^{(4)},
\end{aligned} \quad (20)
$$

$$
\mathbf{F}'_1 = \left[ \begin{array}{cccccc}
\mathbf{H}_0 & \mathbf{H}_{1+} & \cdots & \mathbf{H}_{N+} & & \\
\mathbf{H}_{1-} & \mathbf{H}_0 & \mathbf{H}_{1+} & \cdots & \mathbf{H}_{N+} & \\
\vdots & & \ddots & & \ddots & \\
\mathbf{H}_{(Y-N+X')-} & \cdots & \cdots & \cdots & & \mathbf{H}'_{(N-1)+} \\
\vdots & & \ddots & & & \vdots \\
\mathbf{H}_{Y-} & \cdots & \cdots & \cdots & \cdots & \mathbf{H}'_{(X'-1)+} \\
\ddots & & \ddots & & & \vdots \\
 & & \mathbf{H}_{Y-} & \cdots & \cdots & \mathbf{H}'_{(0)}
\end{array} \right], \quad (21)
$$

where

$$
X' = Q - \lfloor \frac{Q}{Y} \rfloor \times Y. \quad (22)
$$

Let us use $\vec{\boldsymbol{\pi}}_l$ to denote the steady state solution associated with probabilities in level $l$ where $l = 0, 1, ..., Q$. One can calculate the steady state probability vector $\boldsymbol{\pi}$ according to the procedure described in [8].

## V. Performance Analysis

Once the steady state probability of eq. (7) is obtained, one can measure different packet-level performance parameters. We are mainly interested to measure collision probability, packet loss probability due to buffer overflow and average queueing delay for given values of $P_1$ and $P_2$ as well as other operating parameters.

### A. Collision Probability

In a given time slot $n$, collision happens for a particular user if the tagged SU transmits one or more packets but the channel is actually occupied by the PU i.e., $O_n = 0$. CR Tx makes a transmission decision, $T_n = 1$, if buffer state of the tagged SU, $b_n > 0$ and $f_n > 0$. Therefore, the collision probability corresponds to the summation of the steady state probabilities associated with the states having $T_n = 1$, $b_n > 0$, $f_n > 0$ and $O_n = 0$. Mathematically, the collision probability for a tagged SU can be calculated from the steady state probability as follows:

$$P_{\text{tag,col}} = \sum_{b=1}^{Q} \sum_{f=1}^{Y} \boldsymbol{\pi}(b, f, 1, 0). \quad (23)$$

For a homogeneous system the collision probability due to the transmission of each SU will be identical to eq. (23). So total collision probability can be calculated as follows:

$$P_{\text{col}} = P_{\text{tag,col}} \times K. \quad (24)$$

### B. Packet Loss Probability

A packet is lost if it finds the buffer of the tagged SU is full upon arrival. Considering steady state probability of those states that can lead to buffer overflow and associated packet arrival probability, we can finally express the packet loss probability of the tagged SU as follows:

$$P_{loss} = \sum_{b=(Q-N+1)}^{Q} \boldsymbol{\pi}(b, f, T, O)\mathbf{1} \times \sum_{i=(Q-b+1)}^{N} \beta_i, \quad (25)$$

where $\mathbf{1}$ is a column vector with length $4Z$.

### C. Average Queueing Delay

Using the well known Little's law [4], the average queuing delay of a packet for the tagged SU can be written as follows:

$$D_{avg} = \frac{\sum_{b=1}^{Q} p(b) \times b}{\alpha \times (1 - P_{loss})} \quad (26)$$

where $\alpha$ is the average arrival rate and $P_{loss}$ is the packet loss probability of the tagged SU which can be calculated using eq. (25). $p(b)$ is the marginal probability that corresponds to the probability of having $b$ packets in the buffer of the tagged SU. Marginal probability, $p(b)$ can be obtained as follows:

$$p(b) = \boldsymbol{\pi}(b, f, T, O)\mathbf{1}, \qquad 1 \leq b \leq Q. \quad (27)$$

## VI. Numerical Results and Application

In this section, we present selected numerical examples to demonstrate the significance of our developed queueing analytic model. As a potential application of our developed queueing analytic model in CAC decision in CRN, we provide an example. We also compare the performance of the random transmission protocol with the deterministic protocol. All the presented numerical results are validated via computer simulation. In our numerical results, we assume that the maximum buffer size of each SU, $Q$=20, the number of channel states, $Z$=4 and primary user activity, $\rho$=0.6. We also assume that the average packet arrival rate of each SU, $\alpha$= 0.3, false alarm probability, $P_f$=0.3, miss-detection probability, $P_d$=0.3 and Doppler frequency of 30 Hz.

### A. Numerical Results

In Fig. 1, we have plotted collision probability for different number of SUs in the CRN. As the number of SUs increases, the collision probability increases. This can be explained as follows: higher number of SUs, increases the overall transmission probability due to multiuser diversity [5]. But due to the miss-detection, increased transmission probability leads to a higher collision probability. Fig. 1 also shows that the effect of $P_1$ and $P_2$ of random transmission protocol on the collision probability. According to the random transmission protocol, the CR Tx transmits with probability, $P_1$ if the channel is sensed as busy. Therefore, decreasing the value of $P_1$ leads to a less aggressive transmission. As such lowering the value of $P_1$, decreases the collision probability. Similarly, when there is a certain miss-detection, decreasing the value of $P_2$ leads to a lower collision probability. So, a CRN can decrease the collision probability by reducing the values of $P_1$ and $P_2$.

Fig. 2 shows the effect of SUs on packet loss probability of the tagged SU. As the number of SUs increases, the performance of each SU's degrades. But the packet loss probability can be decreased by increasing the value of $P_1$ and $P_2$. In Fig. 3, we have plotted the effect of SUs on average queueing delay of the tagged SU which shows similar behaviour as the packet loss probability in Fig. 2.

TABLE I
Transmission Probabilities $(P_1, P_2)$ Vs number of SUs for given QoS requirements ($p_{t,col} = 0.07$, $p_{t,ploss} = 0.14$ and $D_{t,avg} = 77$ (time slots))

| $(P_1, P_2)$ | $K_{p_{t,ploss}}$ | $K_{D_{t,avg}}$ | $K_{p_{t,col}}$ | $K_s$ |
|---|---|---|---|---|
| (0,0.9) | 6 | 9 | 15 | 6 |
| (0,1) | 7 | 11 | 13 | 7 |
| (0.1,1) | 8 | 12 | 8 | 8 |

### B. Application of Our Developed Analytic Model

In what follows, we explain an application of our queueing analytical model. In general, PUs can tolerate a certain probability of collision determined by the PUs' QoS requirement [9]. Let us assume that the PU system has specified a collision probability, $p_{t,col}$ of 0.07 as a threshold limit. Let us consider that each SU has a target packet loss probability, $p_{t,ploss}$ of 0.14 and average queuing delay, $D_{t,avg}$ of 77 (time slots). Given other system parameters, in order to maintain QoS requirements, the CRN can support a certain number of SUs for given values of $P_1$ and $P_2$. Let us denote the maximum number of SUs that satisfies $p_{t,col}$ is $K_{p_{t,col}}$ for particular values of $(P_1, P_2)$. From Fig. 1 one can obtain the value of $K_{p_{t,col}}$ for given values of $(P_1, P_2)$. Similarly, let us denote, the maximum number of SUs that satisfies $p_{t,ploss}$ and $D_{t,avg}$ is $K_{p_{t,ploss}}$ and $K_{D_{t,avg}}$ respectively for particular values of $(P_1, P_2)$. The values of $K_{p_{t,ploss}}$ and $K_{D_{t,avg}}$ can be obtained from Figs. 2 and 3 respectively. For example, different values of $P_1$, $P_2$ and corresponding $K_{p_{t,ploss}}$, $K_{D_{t,avg}}$, $K_{p_{t,col}}$ are listed in Table I. For the given pair of values of $P_1$ and $P_2$, the maximum number of SUs that can be supported in order to satisfy all three QoS requirements is equal to the minimum of $K_{p_{t,ploss}}$, $K_{D_{t,avg}}$ and $K_{p_{t,col}}$ i.e., $K_s = min(K_{p_{t,ploss}}, K_{D_{t,avg}}, K_{p_{t,col}})$ which is listed in the last column of Table I.

From Table I, it is obvious that CRN should choose transmission probability of $P_1 = 0.1$ and $P_2 = 1$ as it can support maximum number of SUs for the given QoS requirements and other operating parameters. From the column $5$ of Table I it is clear that CRN can admit at most $8$ SUs. It is important to note that the deterministic transmission protocol (i.e., $P_1 = 0$ and $P_2 = 1$) can support maximum $7$ SUs. Interestingly, the random transmission protocol with probability $P_1 = 0.1$ and $P_2 = 1$ can support one more SU that the deterministic protocol for the given setup.

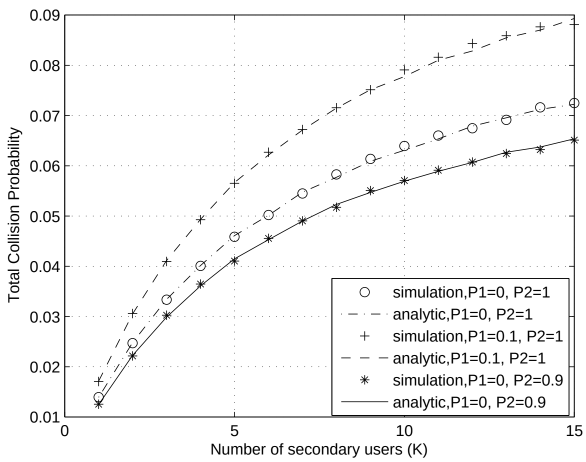


Fig. 1. Effect of the values of $P_1$, $P_2$ and secondary users $(K)$ on the collision probability.

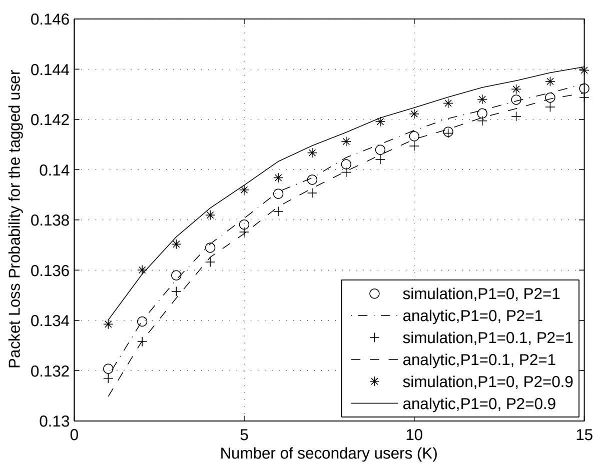


Fig. 2. Effect of the values of $P_1$, $P_2$ and secondary users $(K)$ on the packet loss probability.

## VII. Conclusion

In this paper, we developed a queueing analytic model for a random transmission protocol that incorporates sensing error into account in order to measure packet-level performance parameters as well as collision probability with PUs' transmission. The analytic model is useful in CAC decision in CRN for specific QoS requirements. We also compare

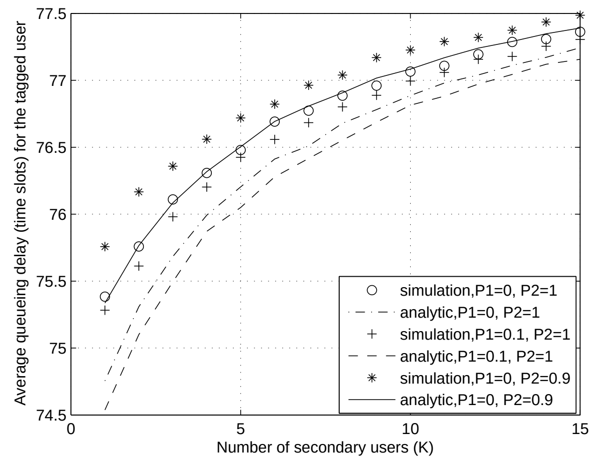


Fig. 3. Effect of the values of $P_1$,$P_2$ and secondary users $(K)$ on average queueing delay.

the performance of the random transmission protocol with the deterministic protocol. The presented numerical results show that by properly selecting the probabilities of random transmission, CRN can support higher number of SUs than the deterministic transmission protocol while satisfying specified QoS requirements.

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