A joint approach for PAPR reduction and predistortion by adding signal in Cognitive Radio

O. Abel Gouba and Yves Louët
IETR/SUPELEC labs, Rennes campus,
Avenue de la Boulaie - CS 47601 - F-35576 Cesson-Sévigné CEDEX, France,
Email: {Abel.Gouba, Yves.Louet}@supelec.fr.

Abstract—**Multi-carrier and multi-standard systems are prone to high Peak-to-Average Power Ratio (PAPR). Due to the non-linearity of the high Power Amplifier (PA), this results in interferences and/or low power efficiency. In this paper, we propose to use a joint approach for PAPR reduction and memoryless predistortion by adding signal in order to improve PA linearity and efficiency performances depending on the radio environment. First, the radio environment is sensed and informations such as signal's PAPR, channel estimation, Signal-to-Noise Ratio (SNR) and battery level are collected. Accordingly, a decision engine updates additional signals for PAPR reduction and predistortion in order to meet targeted linearity and power efficiency requirements. Ideally suited for Cognitive Radio (CR) systems, this dynamic joint approach by adding signal is simulated and validated through two scenarios represented on two examples of radio environment. The PAPR reduction performance is evaluated by the Complementary Cumulative Density Function (CCDF) and the PA linearity by Error Vector Magnitude (EVM) criteria.**

I. Introduction

Power Amplifiers (PA) with perfect linearity and high power efficiency are more and more required for wireless transmitters especially when multi-carrier signals are used. However, PA is an intrinsic non-linear component and multi-carrier signals have high Peak-to-Average Power Ratio (PAPR). The amplification of these signals causes spectral leakage, warping and clustering of the constellation. Thus, the overall consequence of this is Bit Error Rate (BER) degradation at the receiver and co-channel interferences. This situation is aggravated in Cognitive Radio (CR) context [1] where signals' PAPR vary in real time due to the usage of multi-standard signals and a dynamic management of the spectrum [2].

Solutions proposed in literature [3], [4] for PA linearity and efficiency are grouped in two categories including linearization and PAPR reduction respectively. They are complementary solutions [8], [9]. In addition, they have mutual effects such that when the PA is perfectly linearized, its efficiency is implicitly and slightly improved as the Input Back-Off (IBO) is reduced. Conversely, when a PAPR reduction technique with no additional distortions is used, the dynamic range of the signal is reduced. By amplifying near the saturation region, this increases the PA efficiency and for a given PA IBO, this slightly improves the linearity as well [6], [7]. However, linearization and PAPR reduction techniques have been initially designed separately and applied independently in conventional systems. A joint approach is then proposed to enhance interoperability.

A joint approach consists in a synergistic combination of PAPR reduction and linearization with information exchange. It takes into consideration mutual effects in order to improve the efficiency and the linearity of the PA. Several works have been done on joint approach [10]–[13]. In [11] for example, a way to jointly optimize PAPR reduction by Tone Reservation scheme and predistortion is investigated considering IEEE 802.11 standard. The authors propose to tune the targeted clipping level of the PAPR reduction block according to the updated saturation information of the combined AM/AM response of the predistorter followed by the PA. For the same purpose, authors of [12] propose to adjust the polynomial degree of the predistorter characteristic according to the PAPR reduction performance. [10] noticed that by bypassing the PAPR reduction block during the characterization step and by applying it concurrently with the Digital predistortion in the linearization step, the performance of the PA is enhanced. The idea of our paper is to perform dynamically a joint approach by adding signal depending on the radio environment of the transmitter. For this, predistortion as a linearization technique is formulated as an adding signal technique thanks to Bussgang theorem [14]. Then, we propose to collect information such as channel estimation, Signal-to-Noise Ratio (SNR), battery level and signal's PAPR in order to update the additional signals for PAPR reduction and predistortion.

The paper is organized as follows. Section II first introduces Bussgang theorem and then proves that as PAPR reduction techniques based on non-linear functions, memoryless predistortion can be formulated too as an adding signal technique. Accordingly, this allows a common vision of PAPR reduction and predistortion. Section III describes CR context and the proposed methodology of joint approach by adding signal. In Section IV, two scenarios are presented and the performance of the proposed methodology is simulated. Conclusions are given in Section V.

II. From a separate to a common vision of PAPR reduction and predistortion

Since PAPR reduction and predistortion are complementary for PA efficiency and linearity, the methodology of designers focuses on their association [8], [9]. The problem of such association is that the two operations, PAPR reduction and

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predistortion, have been initially designed and optimized separately. In this section, we start by recall the Bussgang theorem and the principle of PAPR reduction by adding signal. Then, we prove that as a non-linear function, predistortion can be also formulated as an adding signal technique thanks to the Bussgang theorem. This makes possible a common vision of PAPR reduction and predistortion and finally a joint approach by adding signal.

A. Recall of the Bussgang theorem and the principle of PAPR reduction by adding signal

Bussgang theorem states that when a Gaussian stationary process passes through a non-memory non-linear device, the cross-correlation function of input and output is proportional to the auto-correlation function of input [14]. Let $f(.)$ be a non-linear function and let us take $x[n]$ and $y[n]$ as its input and output signals respectively. The theorem can be written as follows:

$$\begin{aligned} y\left[n\right] &= f\left(x[n]\right) \\ &= \alpha x[n] + d[n] \; where \; \alpha = \frac{R_{yx}(0)}{R_{xx}(0)}. \end{aligned} \quad (1)$$

$R_{yx}(\tau)$ and $R_{xx}(\tau)$ are the cross-correlation and the auto-correlation functions of the input signal and the output signal respectively. It is shown that the distortion term $d[n]$ is uncorrelated to the input signal $x[n]$ i.e $R_{xx}(\tau) = E\left\{d[n+\tau]x^{*}[n]\right\} = 0$ [14].

Considering that multi-carrier signal can be approached by a complex Gaussian process [17] and using Bussgang decomposition presented in (1), authors of [15] proved that all PAPR reduction methods based on a non-linear function (like Clipping, Companding, etc.) can be formulated as adding signal techniques and the uncorrelated term $d[n]$ is the useful signal for PAPR reduction. Generally speaking, the principle of adding signal techniques supposes a peak cancellation signal $c^{papr}[n]$ that, added to the input multi-carrier signal $s[n]$, reduces its PAPR. Then, the peak reduced signal $\hat{s}[n]$ can be expressed as follows:

$$\hat{s}[n] = s[n] + c^{papr}[n]. \quad (2)$$

By using null or reserved sub-carriers of the multi-carrier standards to generate the peak cancellation signal $c^{papr}[n]$, [15] also proved that PAPR can be reduced without BER degradation. Let N_u be the set of useful data sub-carriers and N_r that of reserved sub-carriers. The total set of sub-carriers is given by $N = N_u \cup N_r$ and $N_u \cap N_r = \varnothing$. Accordingly, peak cancellation signal can be generated in order to keep BER unchanged. Many well known and effective PAPR reduction methods in the literature are based on adding signal techniques. Some examples are Tone Reservation (TR) [16], Active Constellation Extension (ACE) or Tone Injection [3]. TR and ACE have been recently adopted in the Digital Video Broadcasting Terrestrial (DVB-T2) standard for their performance.

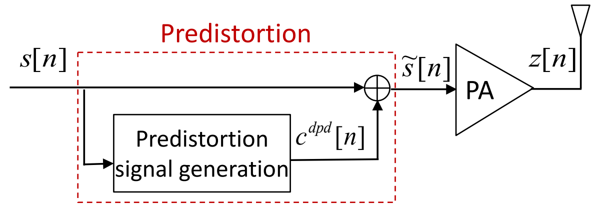

Fig. 1. Principle of predistortion by adding signal

B. Memoryless predistortion formulated as an adding signal technique

In this sub-section, we propose a new vision of predistortion as an adding signal technique. Traditional predistortion consists in applying to an input signal, a non-linear function $P(.)$ which is the inverse of the PA characteristic so that the concatenation of this function and the PA will ideally be equivalent to a linear function. In addition, if the memory effects of the PA are negligible, predistortion is therefore based on non-linear and non-memory function. As a consequence, it can be formulated as an adding signal technique thanks to Bussgang theorem considering as previously that multi-carrier signal converges on a complex Gaussian process. Let $s[n]$ denote the input signal and $c^{dpd}[n]$ be the additional signal for predistortion. This predistortion signal $c^{dpd}[n]$ is in fact the signal for PA non-linearity compensation. The output of the predistortion $\tilde{s}[n]$ is then expressed as follows:

$$\tilde{s}[n] = s[n] + c^{dpd}[n]. \quad (3)$$

The principle of predistortion by adding signal is illustrated in Fig.1. From (1) and (3), the predistortion signal is expressed as:

$$c^{dpd}[n] = (\alpha - 1)s[n] + d[n]. \quad (4)$$

We notice from (4) that, the predistortion signal depends on the correlation factor α and the uncorrelated signal $d[n]$ resulting from the non-linear process of the input signal.

In the following, we propose to derive the correlation factor α. To do so, we use a memoryless polynomial model of predistortion [4]. Let us consider a m degree polynomial with coefficients $\{d_1, d_3, \ldots, d_{2m-1}\}$. Considering that multi-carrier signal converges to complex Gaussian distribution [17], the correlation factor can be expressed as follows after some maths:

$$\begin{aligned} \alpha = &\sum_{k=0}^{m-1} d_{2k+1} {P_x}^k \left(\gamma\left(k+2, IBO\right)\right) \\ &+ \sum_{k=0}^{m-1} d_{2k+1} {P_x}^k \left(IBO^{\frac{2k+1}{2}} \Gamma\left(\frac{3}{2}, IBO\right)\right), \end{aligned} \quad (5)$$

where $\gamma\left(a,z\right)$ and $\Gamma\left(a,z\right)$ are "lower" and "upper" incomplete Gamma functions given by (6) and (7) respectively. IBO represents the PA Input Back-Off and P_x the input signal

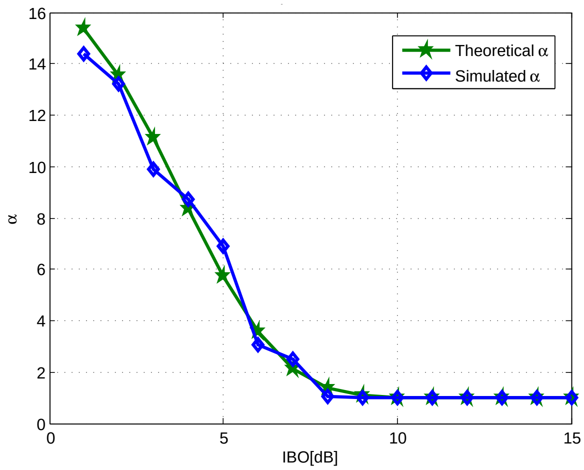

Fig. 2. The correlation factor α as a function of the input back-off (IBO)

power, $P_x = E\left\{|s[n]|^2\right\}$.

$$\gamma(a, z) = \int_0^z t^{a-1} e^{-t} dt \tag{6}$$

$$\Gamma(a, z) = \int_z^{+\infty} t^{a-1} e^{-t} dt. \tag{7}$$

To illustrate, α is plotted against IBO in Fig.2. Each simulation point considers 10^4 randomly generated Orthogonal Frequency Division Multiplexing (OFDM) symbols with 64 sub-carriers 16-QAM modulated. We can notice that α is a decreasing function of the IBO and converges to 1 for IBO$\geq$ 7dB. So, if IBO is sufficiently large so that $\alpha \approx 1$, the predistortion signal $c^{dpd}[n]$ is approximatively equal to the uncorrelated distortion term $d[n]$ of (1):

$$\begin{aligned} c^{dpd}[n] &= (\alpha - 1)\, s[n] + d[n], \\ &\approx d[n]. \end{aligned} \tag{8}$$

From (8), it can be concluded that when IBO is large enough (7dB under the aforementioned parameters), the PA compensation signal is approximatively equivalent to the uncorrelated distortion term resulting from the Bussgang decomposition of the predistortion function.

C. PAPR reduction and predistortion by adding signal

In addition to PAPR reduction by adding signal [15], we have proved that predistortion can also be formulated as an adding signal technique thanks to Bussgang theorem. The possibility of performing PAPR reduction and predistortion by adding signal as shown in (2) and (3) respectively, allows a common vision of theses two operations. Let us consider $s[n]$ as an input multi-carrier signal, this supposes that it can be generated a peak reducing signal $c^{papr}[n]$ and a predistortion signal $c^{dpd}[n]$. Then, the output signal is expressed as follows:

$$\breve{s}[n] = s[n] + c^{papr}[n] + c^{dpd}[n], \tag{9}$$

Intuitively from (9), a new additional signal can be defined:

$$c[n] = c^{papr}[n] + c^{dpd}[n], \tag{10}$$

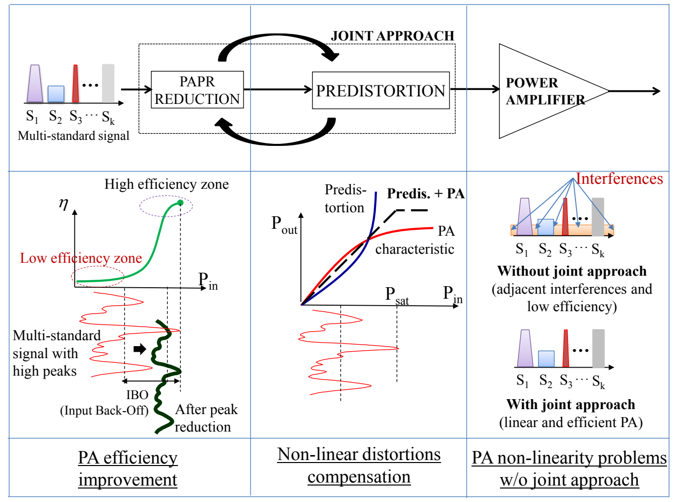

Fig. 3. Amplification effects on SWR signal with/without joint approach of PAPR reduction and predistortion

for PAPR reduction and predistortion simultaneously.

From (10), it can be concluded that a unique additional signal can be generated to perform at the same time PAPR reduction and predistortion. This joint approach by adding signal is particularly interesting for systems where a dynamic adaptation of PAPR and predistortion is necessary in real time. This is the target of CR. Thanks to the well known performance of adding signal techniques [16], a joint approach by adding signal will allow an better compromise between linearity and PA efficiency [6], [7].

III. Joint approach by adding signal in Cognitive Radio context

Introduced by J. Mitola in [1], the idea of CR is to define a radio that is autonomous and has some intelligence to observe, analyze, decide and adapt its parameters according to the local radio environment. So, CR implies a flexible technology also called Software Radio (SWR). SWR permits to deal with the growing number of telecommunications' protocols, services and standards dynamically and autonomously. A SWR signal is a multi-standard signal where each standard has its own system parameters. This combination of standards has consequently a high PAPR value. With the non-linearity of the PA, amplification of a high PAPR signal causes many problems. In this section, we briefly describe the effects of PA on SWR signals and then we present the proposed joint approach by adding signal.

A. PA non-linearity problem in CR context

PA non-linearity and efficiency problems in the case of SWR signal is summarized in Fig.3. $S_1,\ S_2,\ S_3, \ldots S_k$ represent any standards combined in the input SWR signal. The amplification of this high peak signal results in interferences and low efficiency. With the joint approach of PAPR reduction and predistortion, the linearity and the PA efficiency are jointly improved.

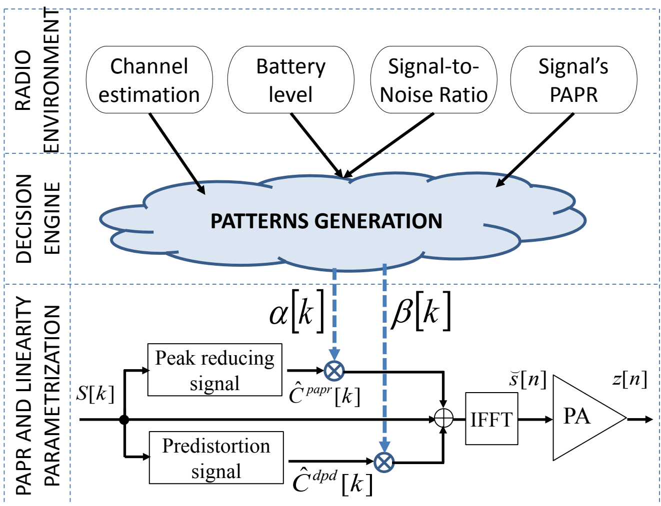

Fig. 4. Principle of patterns generation for dynamic joint approach by adding signal.

The particularity of CR context (based on SWR) is that the PAPR of the signal varies in real time (due to the dynamic access of the spectrum) and consequently, PAPR reduction and predistortion must be updated dynamically and efficiently. Thanks to the common vision described in Section II, we propose a dynamic joint approach by adding signal. The additional signal for PAPR reduction and that for predistortion are generated and updated depending on the radio environment.

B. Dynamic joint approach by adding signal

The proposed dynamic joint approach by adding signal consists in updating additional signals for PAPR reduction and predistortion in accordance to the radio environment. It is based on the common vision described in Section II; the PAPR reduction technique and the predistortion are formulated as adding signal techniques as presented in sub-section II-B. In order to avoid BER degradation, PAPR reduction is based on Tone Reservation (TR) and its gain is directly proportional to the number of reserved sub-carriers.

In CR context, information on radio environment is collected from a set of sensors. So, the optimality of decisions made by the decision engine depends directly on the number and the quality of the sensors. The informations collected are all useful for the radio to adapt its parameters for a given service in a given environment. Many sensors have been proposed, they are classified according to the OSI layers [2]. Our proposed approach is based on sensors like channel estimation, Signal-to-Noise Ratio (SNR), battery level and signal's PAPR.

The principle of dynamic joint approach by adding signal is illustrated in Fig.4. It can be explained in three items:

- First, information on radio equipment's environment is collected and analyzed.
- Accordingly, a decision engine generates two patterns, namely α and β, to update PAPR reduction and predistortion respectively.

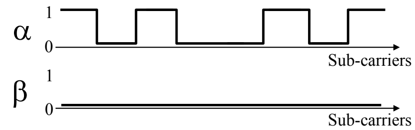

Fig. 5. Example of patterns generated for PAPR reduction only

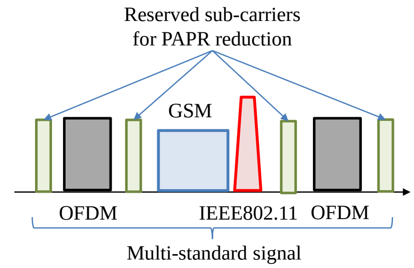

Fig. 6. Simulation model of SWR signal

- Finally, The additional signal for PAPR reduction and predistortion is generated and expressed as:

$$C\left[k\right] = \alpha\left[k\right]\hat{C}^{papr}\left[k\right] + \beta\left[k\right]\hat{C}^{dpd}\left[k\right], \qquad (11)$$

where $\hat{C}^{papr}[k]$ and $\hat{C}^{dpd}[k]$ represent the frequency response of the peak cancellation signal $c^{papr}[n]$ and the predistortion signal $c^{dpd}[n]$ respectively.

Patterns generation is supposed to be done in accordance to the information collected on the radio environment. To keep our study generic enough, the algorithm of the decision engine is considered out of the scope of this work and is not treated here. Patterns are vectors of 0 or 1 whose length corresponds to the number of sub-carriers of the SWR signal. A pattern of ones means that the corresponding sub-carriers are turned on and zero means turned off. The number of 1's in α corresponds to the number of peak reducing sub-carriers and that of β represents the number of carriers used for predistortion. Accordingly, PAPR reduction and predistortion are adjusted jointly by deciding the number and position of 1 or 0 in each pattern i.e α and β. An example of patterns is illustrated in Fig.5 representing a context where only PAPR reduction is performed on dedicated sub-carriers and predistortion is turned off. As shown, α corresponds to a vector with 1 at the positions of peak cancellation sub-carriers and 0 in the others. β is zero.

IV. Scenarios and Simulation Results

The objective in this section is to illustrate the principle of the dynamic joint approach by adding signal through two examples of radio environment. For this purpose, two scenarios are considered. As mentioned above, the algorithm of the decision engine is not treated in this paper so each scenario corresponds to values of α and β chosen empirically depending on the radio context to be analyzed.

In the first scenario, we consider that at a given instant, the radio equipment is close to the base station and the users intend to increase the data rate (streaming data communication for example). Information from radio environment (from sensors)

TABLE I
SCENARIOS EVALUATED

Radio context	need of linearity	need of efficiency
Scenario 1	low	low
Scenario 2	high	high

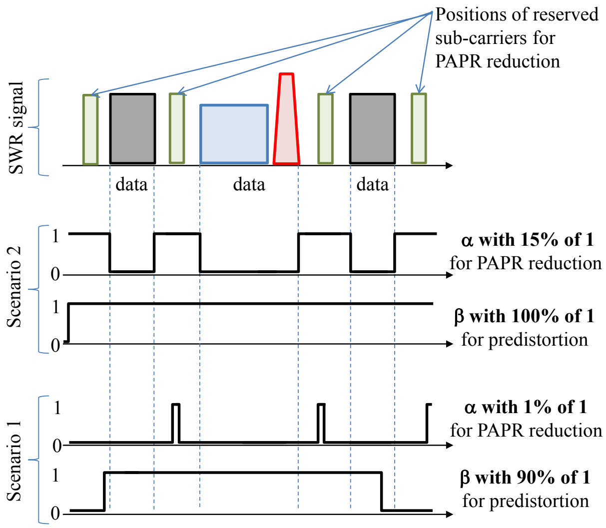

Fig. 7. Patterns representation for each scenario

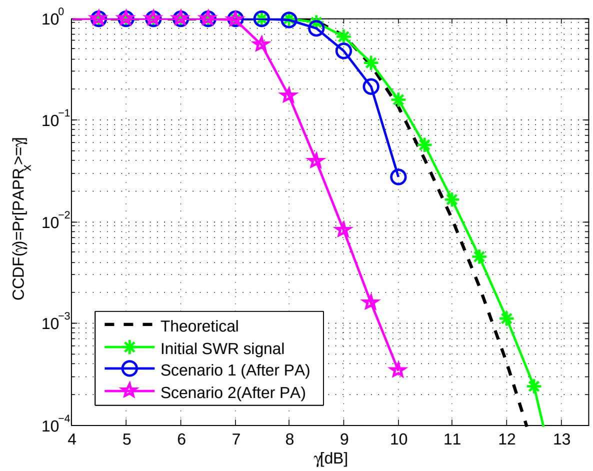

Fig. 8. CCDF of the PAPR distribution of amplified SWR signal with $IBO = 10\text{dB}$

exhibits good channel propagation conditions, high SNR and high battery level. In this situation, PAPR reduction is less needed, so the radio equipment can reduce the number of reserved sub-carriers for PAPR reduction. In addition, a powerful PA linearization is not necessary too but predistortion must be kept in order to avoid co-channel interferences. Accordingly, the number of reserved sub-carriers for PAPR reduction is set for example to 1% of the total sub-carriers so a corresponding vector α is generated with 1 at the position of the reserved sub-carriers and 0 otherwise. Likewise 90% of sub-carriers are used for predistortion signal and the corresponding vector β is generated as well.

The second scenario considers a situation where the radio equipment has a low battery level while a high data rate is not requested (voice communication for example). A good channel propagation is observed with an acceptable SNR value. This implies an important need to increase the power efficiency and therefore a high PAPR reduction gain. Consequently, the number of reserved sub-carriers for PAPR reduction has to be increased. We set it to 15% of total sub-carriers with a corresponding vector α. β is set to 1 for all sub-carriers in order to keep a maximal performance of predistortion and preserve transmitted data quality.

The two scenarios evaluated are summarized in table I. The corresponding patterns are illustrated in Fig. 7.

The simulation model is based on [19] with a multi-standard system presented in Fig. 6. Considered standards are Global System for Mobile communications (GSM) and IEEE802.11. The number of reserved sub-carriers varies according to the scenario. The performance of dynamic joint approach by adding signal is evaluated after the PA by simulating Complementary Cumulative Density Function (CCDF) of the PAPR and the Error Vector Magnitude (EVM). Indeed, power efficiency is inversely proportional to the PAPR of the signal after PA [7]. The PA considered is a memoryless Solid State Power Amplifier (SSPA) represented by a Rapp model [4].

Fig.8 shows the CCDF of the PAPR of the SWR signal after PA in comparison to its initial and theoretical values. Simulations are done over 5.10^4 randomly generated data symbols with a PA IBO set to 10dB. We can notice that the initial value of PAPR is very high about 11dB at CCDF of 10^{-2}. Thanks to joint approach by adding signal, particularly peak reducing signal, the PAPR is reduced before passing through the PA. Even if PA non-linearities are compensated by predistortion signal, PA saturation can not be compensated and it will contribute to reduce the input signal's PAPR; this is noticeable in scenario 1 where the PAPR reduction gain is low so after PA, the measured PAPR is 10dB. In scenario 2, the PAPR reduction is higher and after PA, PAPR is 9dB.

In Fig.9, the linearity of the PA is measured through EVM criteria according to the PA IBO. EVM in both scenarios decreases proportionally to the PA IBO and after IBO=10dB, the EVM is equal to zero, which means that the PA non-linearities are perfectly mitigated. We can noticed that in scenario 1 the EVM value is higher compared to that in scenario 2. This proved that by managing dynamically additional signals for PAPR reduction and predistortion, linearity through EVM criteria can be improved (as in scenario 2) or degraded (as in scenario 1).

From both figures, we can conclude that depending on the number of sub-carriers used for the peak reducing signal (1% in scenario 1 and 15% in scenario 2) and that for the predistortion signal (90% in scenario 1 and 100% in scenario 2), the linearity and PA efficiency are impacted.

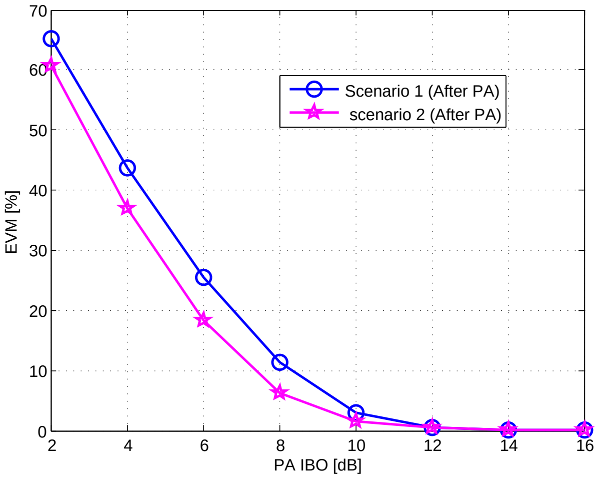

Fig. 9. EVM performance of SWR signals after PA

V. Conclusion

In this paper, we first proved that as PAPR reduction by adding signal, memoryless predistortion can be modeled as an adding signal technique thanks to Bussgang theorem. This provides a common vision of PAPR reduction and predistortion and finally a joint approach by adding signal. Then, we show that in multi-standard systems with dynamic spectrum management (i.e CR) where the PAPR varies in real time, a dynamic and joint approach by adding signal can be performed by taking into account the radio environment. The performance of this proposed method is simulated based on a multi-standard signal. Two different scenarios of radio environment are presented and analyzed in terms of needs of PA linearity and efficiency, then they are translated into predistortion and PAPR reduction respectively. For each scenario, simulation of CCDF and EVM validates the proposed methodology. It appears that by generating additional signals for PAPR reduction and predistortion depending on the radio environment analyzed, it is possible to tune the PA's linearity and efficiency and consequently increase its performance.

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