An Analysis of Competition among Autonomous Devices in Multichannel Cognitive Radios Networks

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***Abstract*—We investigate an autonomous opportunistic spectrum access (OSA) strategy for a multichannel cognitive radio (CR) network, where two or more autonomous CRs sense the channels sequentially in some sensing order to find a free channel to transmit on, if one exists. We evaluate the performance of the OSA strategy in terms of probability of success (probability that a given CR finds a channel free), mean time of success and throughput achievable by a CR. Surprisingly, we find that, when the number of potentially available channels is not large as compared to the number of CRs in the network, then having more free channels beyond a certain number actually hurts the CRs. This counter-intuitive result stems from an increased number of collisions among the autonomous CRs. To validate the closed form expressions that are derived in this paper, we compare our results to those obtained via simulations.**

I. Introduction

Cognitive radio networks are envisioned to utilize the licensed frequency spectrum more efficiently through opportunistic access to (temporarily) unused spectrum bands. Among different opportunistic spectrum access (OSA) schemes, sensing-based OSA is widely investigated because it does not require the licensed (primary) users to alter their existing hardware or behavior [1]. In sensing-based OSA, cognitive radios (CRs) monitor the environment to reliably detect the primary user signals and operate whenever the band is empty. In practice, detection of primary users may rely on a combination of sensing and the use of geolocation spectrum occupancy databases [2].

When multiple potential frequency bands (channels) are available, time slotted multiple access is widely considered [3]–[8]. The first portion of each time slot is used by CRs for spectrum sensing, and the second portion is used to access the free channel, if one is found. The CRs may sense only a single channel in any given time slot [6] or they may sense the potential channels sequentially in some order until they find a free channel to transmit on, if one exists [4]. However, if two or more uncoordinated CRs simultaneously decide to sense the same channel, find it free from primary user activity, and decide to transmit on the channel, this may lead to frame collisions. In this context, the channel sensing order $\mathbb{P}$, i.e., the order in which radios competing for the channels visit those channels, will affect their probability of successful access.

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This paper investigates autonomous OSA for a time-slotted distributed CR network, where in a given time slot two or more distributed CRs sense the potential channels sequentially in some order $\mathbb{P}$ until they find a free channel to transmit on, if one exists. We are particularly interested in the case where CRs autonomously choose the order $\mathbb{P}$ in which they visit channels, without coordination from a centralized entity. The channel availability statistics, i.e., the probabilities of the primary user being absent or present in each channel, are assumed to be unknown to the CRs.

We analyze an OSA strategy based on *random permutation sensing*, where in a given time slot the indices of N potential channels, i.e., $\{1,2,...,N\}$, are randomly permuted by each CR independently and then channels are sensed by each CR according to its own random permutation. We evaluate the performance of the *random permutation sensing* strategy in terms of the probability of success (the probability that a given CR finds a channel free from the primary user and other CR activity within the time slot), and mean time of success (if successful, the mean time to find a channel). By determining the probability of success and mean time of success, we evaluate the throughput achievable by a CR. Surprisingly, we find that, when the number of potential channels is not large as compared to the number of CRs in the network, then having more free channels beyond a certain number actually hurts the CRs. This counter-intuitive result stems from increased collisions among the distributed uncoordinated CRs. We also derive closed-form expressions for throughput and probability of success for CRs competing for opportunistic use of channels.

II. Related Work

The problem of multichannel sensing and allocation for CRs has recently attracted strong interest. Fan [4] investigates the problem for a two-user CR network in the presence of a coordinator. While the use of a coordinator simplifies the problem, for a network comprising a large number of CRs it may create significant signaling overhead to coordinate successful channel utilization. Also, in some practical scenarios, the CRs may be associated with different service providers, requiring an OSA strategy that does not rely on a common coordinator. Jiang

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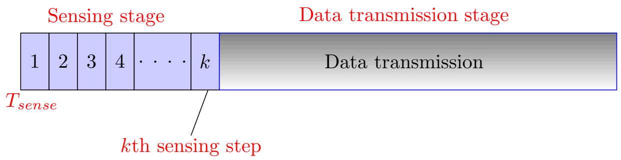

Fig. 1. Time slot structure with sensing and data transmission stages. The length of the data transmission stage depends on how many steps it takes for the CR to find a free channel.

[9] investigates the optimal selection of a channel sensing order for a single cognitive radio. In contrast to that, our work considers competition for channels among multiple CRs. Cheng [5] proposes a channel sensing order for distributed CRs, in which CRs are assumed to have knowledge of the gains for each channel. Based on this information, [5] proposes that each CR should sense channels in a descending order of their achievable rates and should transmit in the first channel that is sensed free. Unlike [5], our work does not assume knowledge of the channel gains. Our work in [10] investigates how CRs can autonomously select channel sensing orders so as to minimize the likelihood of collisions with other CRs also searching for channels to be utilized opportunistically. In particular, [10] is interested in the scenario where devices with false alarms autonomously select the sensing orders in which they visit channels, without coordination from a centralized entity. An adaptive persistent sensing order selection strategy that allows autonomous adaptations to collision-free sensing orders is proposed and evaluated. It is shown that the proposed strategy converges and maximizes cognitive network throughput compared to a random selection of sensing orders. Moreover, [10] investigates OSA strategies under independent and identically distributed (i.i.d.) PU channel occupancy model and also under the PU channel occupancy model that considers correlation in channel occupancy by a PU in consecutive time slots. Unlike [10], our work investigates random selection of sensing orders under the K-free PU channel occupancy model, as further explained in Section III. Moreover, unlike [10], we also analyze the mean time of success (if successful, mean time to find a channel) and throughput achievable by a CR.

The radio rendezvous problem studied by us in [11] is, in a sense, the dual of the problem we study here. While [11] proposes the use of non-orthogonal sequences to increase the probability of rendezvous, in the present work we investigate the OSA scenario in which the aim of the CRs is to disperse (avoid one another).

III. System Model

We consider a distributed cognitive radio (CR) network of M cognitive transmitter/receiver pairs and a set $\mathbf{N} = \{1,2,...,N\}$ of channels. A CR is allowed to make use of one of these channels when it is not occupied by a primary user. The primary users and CRs are both assumed to use a time slotted system with perfect time-synchronization, and each primary user is either present for the entire time slot, or absent for the entire time slot [3], [6], [12]. The probability of a primary user (PU) being present and the probability of a PU being absent in a given channel are assumed to be unknown to the CRs. In each time slot, we denote the number of channels free from PU activity by K, where $1 \leq K \leq N$, and we assume that the K free channels are randomly scattered over the set $\mathbf{N}$. Also, the random scattering of K free channels in each time slot is independent from the other slots. The status of the ith channel, $i \in \mathbf{N}$, in a given slot is a binary variable, $B_i \in \{0,1\}$, where 0 means the channel is free and 1 means the channel is occupied by a primary user. Sensing observations of each CR are assumed to be perfect, as in other works in the literature [6], [12]. (Our work in [10] considers the effects of sensing inaccuracies.) Due to hardware constraints, at any given time each CR can either sense or transmit, but not both. Also, each CR can sense only one channel at a time.

As illustrated in Fig. 1, the CRs use the beginning of each slot to sense the potential channels sequentially in some order $\mathbb{P}$ (based on their OSA strategy as explained in Section IV) to find a channel that is free of PU (or other CR) activity. We refer to this as the sensing stage. The CR then accesses the first vacant channel it finds. The sensing stage in each slot is divided into a number of *sensing steps*. Each sensing step is used by a CR to sense a channel. If a CR finds a channel free in its kth sensing step, it transmits in that channel until the end of the slot. We refer to this as the data transmission stage. The durations of the sensing stage and data transmission stage are kT_{sense} and $T - kT_{sense}$, respectively, where T_{sense} is the time required to sense each channel, T is the total duration of each slot and $T >> T_{sense}$. When the autonomous CRs search multiple potential channels for spectrum opportunities, then from an individual CR perspective one of the following three events will happen in each sensing step: i) A CR visits a channel that is currently occupied by either a PU or another CR (which may have found the channel free in an earlier step and started transmitting); ii) Two or more CRs visit the same given channel which is currently free; iii) A CR is the only one to visit a given channel that is currently free.

The CRs will employ one of the following three strategies (depending on the events): i) If in the kth step a CR visits a channel that is currently occupied by either a PU or another CR, then the CR continues looking for a channel in the $(k+1)$th step; ii) If in the kth step two or more CRs visit the same given channel and find it free, all of these CRs will start transmitting in that channel until the end of the time slot and a collision will occur. In this case none of these CRs can transmit successfully due to the collision; iii) If in the kth step a CR is the only one to visit a given channel and find it free, it will successfully transmit in that channel for the remainder of the time slot. Figure 2 illustrates examples of different scenarios for sequential channel sensing using sensing orders.

Let X be a random variable representing the number of sensing steps within a time slot until a CR is successful in finding a channel free from PU and other CR activity (given that the CR is successful). Note that, with a constant time slot of duration T, the duration of successful data transmission in each slot is a function of X. At the end of each slot, the

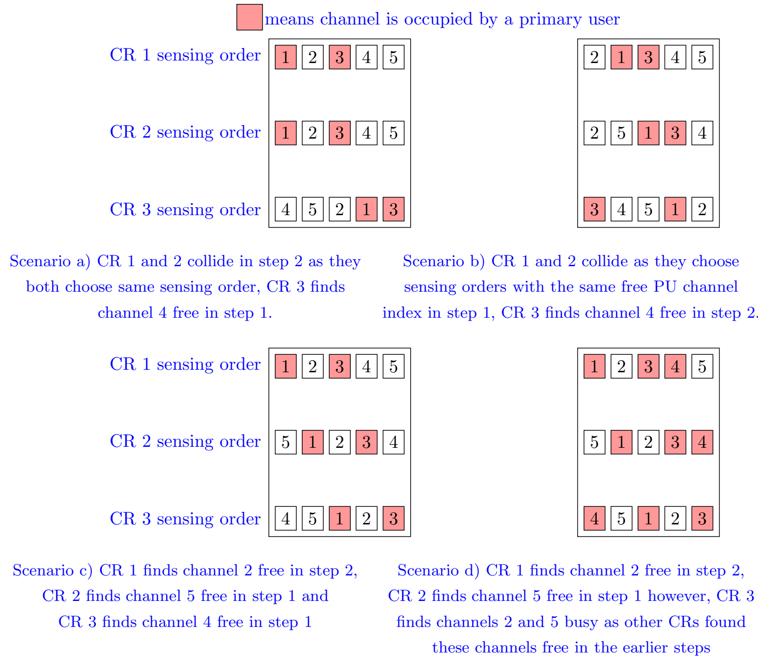

Fig. 2. Different scenarios for sequential channel sensing using sensing orders. In scenarios a, b and c for $M=3$ CRs there are $N=5$ potential channels with $K=3$ channels free from PU activity. In scenario d there are $K=2$ free channels.

instantaneous throughput $C(\mathrm{X})$ achieved by a CR is given by

$$C(\mathrm{X}) = \begin{cases} (1-\frac{\mathrm{X}T_{sense}}{T})R, & \text{if successful} \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

where R represents the transmission rate of each CR to its receiver when the channel is available. The average throughput achieved by a CR is given by

$$E[C] = P_s \sum_{k=1}^{N} (1-\frac{kT_{sense}}{T}) P[\mathrm{X}=k \mid s] R, \quad (2)$$

where P_s represents the probability that a CR finds a channel free from both primary and other CR transmissions in a given slot and $P[\mathrm{X}=k \mid s]$ represents the probability that the CR is successful in the kth sensing step (given that the CR is successful). Note that if N is larger than T/T_{sense}, then the CR does not have time to visit all channels within a time slot. In that case, $P[X=k \mid s]=0$ for $k > T/T_{sense}$, and the equation still holds. However, for simplicity and also for practical reasons, we assume throughout the paper that $N < T/T_{sense}$.

We next consider the performance of a *random permutation sensing* strategy.

IV. Random Permutation Sensing Strategy

We consider the OSA strategy based on random permutation sensing (RPS), where in a given slot the indices of N potential channels, i.e., $\{1,2,\ldots,N\}$, are randomly permuted by each CR independently and then channels are sensed by each CR according to its own random permutation. A permutation $\sigma^{(i)} = \left(\sigma_1^{(i)}, \sigma_2^{(i)}, \ldots, \sigma_N^{(i)}\right)$ of the N channels describes the sensing order in which the ith CR visits channels in a given slot, and the matrix of all possible permutations is denoted as Σ. For instance, Σ for $N=3$ channels is given as:

$$\Sigma = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \\ 2 & 1 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \\ 3 & 2 & 1 \end{pmatrix}$$

To evaluate the performance of the OSA strategy based on RPS in terms of average throughput, given by (2), we first investigate the probability of success for this strategy, i.e., the probability that a given CR finds a channel free from the primary user and other CR activity within the time slot. Under this assumption an exact closed-form expression for the probability of success can be derived for any N and $1 \leq K \leq N$ when $M=2$. For $M>2$, we are able to obtain an exact closed-form expression when $K=1$ or $K=N$. For large M and large N, obtaining an exact closed-form expression for the probability of success is challenging due to the combinatorial explosion in the number of ways that M CRs can find channels free from PUs and other CRs. To simplify the analysis we provide an approximation for any N and $2 \leq K \leq N$, for $2 < M \leq K$.

A. P_s for $K=1$ and any N and M

For one free channel ($K=1$) and any N and M, a closed-form expression for the probability of success for an individual CR can be derived as in (3) (at the top of the page 4).

B. P_s for $K=N$ and any M

For $K=N$ free channels and any M, the problem is reduced to a traditional multichannel access problem, and a closed-

$$P_s\{N,K=1,M\} = P(\text{CR } i \text{ visited the PU-free channel AND no other CR visited the same channel in the 1st step}) + P(\text{CR } i \text{ visited a channel occupied by a PU in the 1st step AND CR } i \text{ visited the PU-free channel in the 2nd step AND no other CR visited the PU-free channel in the 1st and 2nd steps}) + \cdots\cdots + P(\text{CR } i \text{ visited channels occupied by PUs in the first } (N-1) \text{ steps AND CR } i \text{ visited the PU-free channel in the } N\text{th step AND no other CR visited the PU-free channel in any of the } N \text{ steps})$$

$$= \frac{1}{N}(1-\frac{1}{N})^{M-1} + \frac{1}{N}(1-\frac{2}{N})^{M-1} + ... + \frac{1}{N}(1-\frac{N-1}{N})^{M-1}$$

$$= \sum_{k=1}^{N} \frac{1}{N}(1-\frac{k}{N})^{M-1} \quad (3)$$

$$P_s\{N,K=N,M\} = P(\text{ CR } i \text{ visited a PU-free channel AND no other CR visited the same channel in the 1st step}) = (1-\frac{1}{N})^{M-1} \quad (4)$$

$$P_s\{N,K,M=2\} = \sum_{k=1}^{N} [P(\text{In the first } (k-1) \text{ steps, the channels visited by CR } i \text{ are occupied by a PU AND in the } k\text{th step CR } i \text{ visits a free channel and does not collide with competing CR } j) + P(\text{In the first } (k-1) \text{ steps, the channels visited by CR } i \text{ are occupied by a PU and one channel is occupied by the competing CR } j\text{, which has already found a free channel and started transmitting, AND in the } k\text{th step CR } i \text{ finds a free channel})]$$

$$= \sum_{k=1}^{N} \left[p_k(K,N)\left(1-\frac{1}{K}\left(\sum_{l=1}^{k} p_l(K,N)\right)\right) + \sum_{l=2}^{k-1} \left\{ (\frac{1}{K})p_l(K,N)\left(\sum_{m=1}^{l-1} p_m(K,N)\right) p_u(K-1,N-l) \right\} \right] \quad (5)$$

where $p_1(x,y) = \frac{x}{y}$, $p_v(x,y) = \left(1-\sum_{n=1}^{v-1} p_n(x,y)\right)\left(\frac{x}{y-v+1}\right)$, $v > 1$ and $u = k-l$.

form expression for the probability of success for an individual CR is given in (4) (at the top of the page 4).

For $K = N$ free channels, P_s is the probability that CR i visits a channel free from PU and other CR activity in the first step. Note that with perfect sensing and radios employing no backoff mechanism, CRs can either transmit successfully or collide in the first step.

C. P_s for $M = 2$ CRs and any K and N

For $M = 2$ CRs and for any K and N, an exact closed-form expression for the probability of success for an individual CR can be obtained as in (5) (at the top of the page 4). The derivation of equation (5) is given in [13].

D. P_s for $M > 2$ CRs, $2 \le K \le N$ and $2 < M \le K$

For $M > 2$ CRs, the difficulty in deriving an exact closed-form expression is that, in any step k, the number of other competing CRs depends on how many CRs were successful in the previous steps (which in turn determines how many channels are available) and how many collided. Hence, instead of presenting an exact closed-form expression we present an approximation for the probability of success for an individual CR when $2 \le K \le N$ and $2 < M \le K$. The approximation is given as

$$P_s\{N, 2 \le K \le N, 2 < M \le K\} \approx \frac{K}{N}\left(1-\frac{1}{N}\right)^{M-1} + \frac{(N-K)}{N}\left(1-\frac{(N-K)!}{N!}\right)^{M-1} \quad (6)$$

The derivation of equation (6) is given in [13].

V. Analytical and Simulation Results

In Figure 3, we plot the probability of success P_s of an individual CR as a function of the number of free channels K, for $M = 2$ and $N = 10$ and 100. We compare the results given by the closed-form expression we derived in (5) and the estimated probability of success from a Monte Carlo simulation; some of the results are also tabulated in Table I. Observe that the probabilities estimated from Monte-Carlo simulations are within $\pm 1\%$ of those obtained from equation (5).

Table II lists the probability of success for several combinations of N, K and M. We tabulate both the probability estimated from Monte Carlo simulations and obtained from the approximation in (6). Observe that the probabilities estimated from Monte-Carlo simulations are within $\pm 8\%$ of those obtained from (6).

In Figs. 4, 5 and 6 using Monte Carlo simulations, we show the performance of the OSA strategy based on RPS in terms

TABLE I

ANALYTICAL AND SIMULATION RESULTS FOR THE PROBABILITY OF SUCCESS, P_s, IN A TWO CR SCENARIO $M = 2$.

	$N=2$		$N=4$			$N=8$			
	$K=1$	$K=2$	$K=1$	$K=2$	$K=4$	$K=1$	$K=2$	$K=4$	$K=8$
P_s									
simulated	0.2504	0.4977	0.3738	0.8087	0.7476	0.4380	0.9078	0.9102	0.8733
analytical	0.2500	0.5000	0.3750	0.8056	0.7500	0.4375	0.9107	0.9111	0.8750

	$N=16$					$N=32$				
	$K=1$	$K=2$	$K=4$	$K=8$	$K=16$	$K=1$	$K=2$	$K=4$	$K=8$	$K=16$
P_s										
simulated	0.4689	0.9569	0.9612	0.9583	0.9385	0.4843	0.9786	0.9814	0.9815	0.9781
analytical	0.4688	0.9569	0.9605	0.9571	0.9375	0.4844	0.9788	0.9813	0.9813	0.9789

TABLE II

ANALYTICAL AND SIMULATION RESULTS FOR THE PROBABILITY OF SUCCESS, P_s, IN M CR SCENARIOS.

	$N=20, K=12$			$N=40, K=15$			$N=40, K=30$		
	$M=4$	$M=8$	$M=12$	$M=4$	$M=8$	$M=12$	$M=8$	$M=16$	$M=24$
P_s									
simulated	0.8965	0.7822	0.6859	0.9517	0.8977	0.8482	0.8691	0.7430	0.6490
approximation	0.9144	0.8190	0.7413	0.9726	0.9391	0.9088	0.8782	0.7630	0.6690

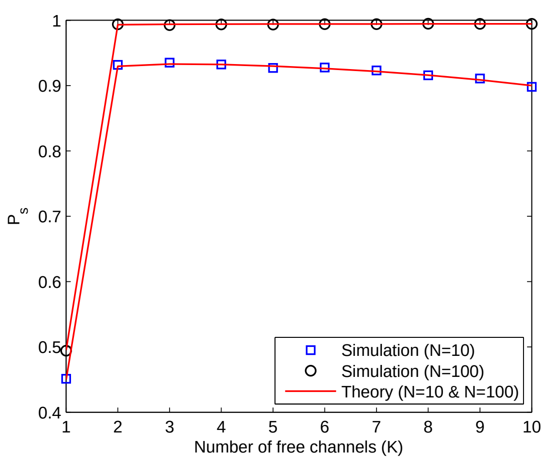

Fig. 3. Analytical and simulation results for the probability of success of an individual CR as a function of the number of free channels, K, with $M = 2$ CRs.

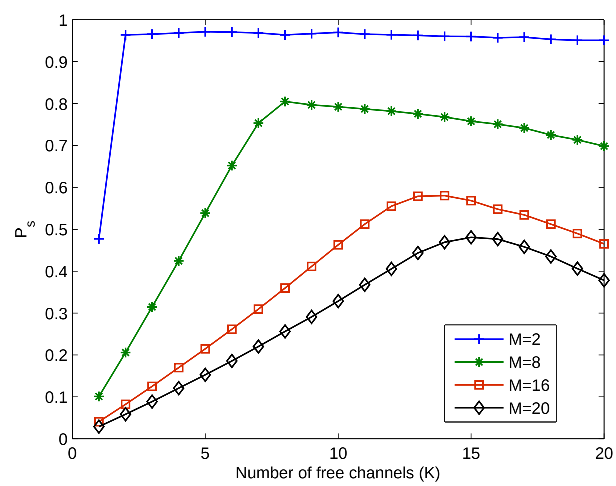

Fig. 4. Probability of success of an individual CR as a function of the number of free channels, K, with $N = 20$ potential channels.

of P_s, mean time of success (if successful, mean time to find a channel) and average throughput of an individual CR in the presence of $M-1$ other CRs, when the number of potential channels $N = 20$ and K free channels varies between 1 and 20. It can be seen from the three figures (Figs. 4, 5 and 6) that, for a given N and M, the P_s, mean time of success and average throughput of an individual CR are not monotonic in the number of free channels K. Surprisingly, when the number of potential channels is not large as compared to the number of CRs in the network, then having more free channels beyond a certain number actually hurts the CRs. This counter-intuitive result stems from increased collisions among the distributed uncoordinated CRs. Moreover for low values of K, as K increases it becomes more likely for an individual CR to find free channels in the later sensing steps. However, when K is increased beyond a certain number, then an individual CR can either find a free channel in its initial sensing steps or else it may collide with the other CRs and fail to find a free channel during that time slot. It can also be seen in Fig. 5 that for a given N, when the number of CRs M is increased the mean time to find a free channel decreases. This counter-intuitive result is because as M is increased the likelihood of collisions

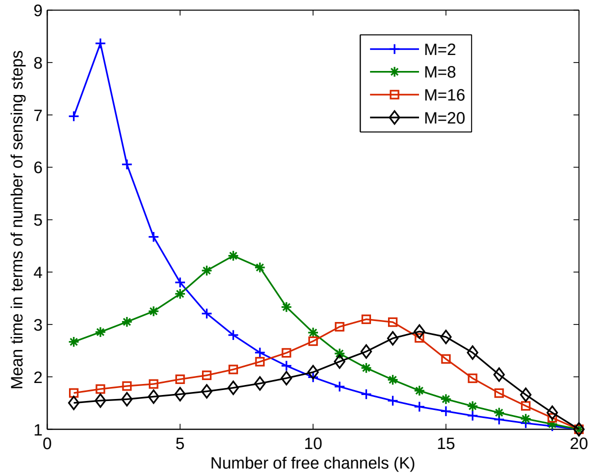

Fig. 5. Average number of steps required for a CR to find a free channel in each slot, as a function of the number of free channels, K, with $N = 20$.

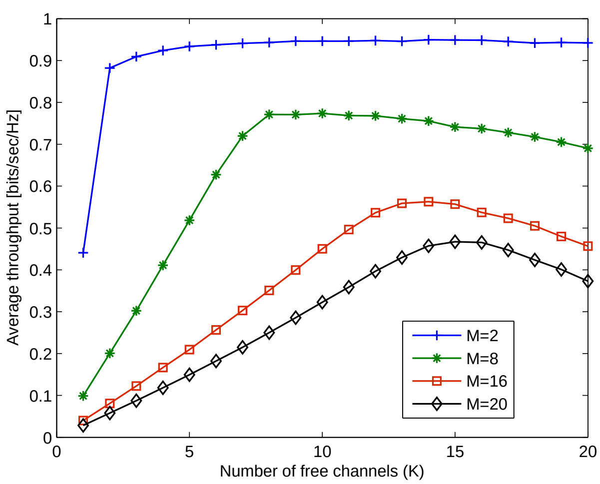

Fig. 6. Average throughput (bits/sec/Hz) of an individual CR as a function of the number of free channels, K, with $T_{sense} = 1$ ms, $T = 100$ ms and $R = 1$ (bit/sec/Hz) (see Section III and Eq. 2).

is also increased and an individual CR can either find a free channel in its initial sensing steps or else it may collide with the other CRs and fail to find a free channel at all.

VI. Conclusions

In this research we investigate autonomous *random permutation sensing* strategy for a distributed cognitive radio (CR) network. We find that the performance of the *random permutation sensing* strategy is limited by the collisions among the autonomous CRs. We derive closed-form expressions for throughput and probability of success for CRs competing for opportunistic use of channels. We validate the closed form expressions that are derived in this paper by comparing our results to those obtained via simulations. We show that for a given number of channels N and number of CRs M the probability of success (P_s), mean time of success and average throughput of an individual CR are not monotonic in the number of free channels K. It is also observed that for a given N, when the number of CRs M is increased the mean time to find a free channel decreases.

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