Bayesian Approach to Spectrum Sensing for Cognitive Radio Applications

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***Abstract*—In this paper, we address the spectrum sensing task of cognitive radio from Bayesian detection (BD) perspective. We first show that BD essentially simplifies to classical energy detection (ED) under Gaussian signal assumption but the threshold setting has more degrees of freedom to optimize the sensing performance, e.g., against given spectrum utilization. Then we propose a novel BD based algorithm where the sample energy is calculated iteratively, and the odds ratio is used to quantify the measurement reliability. Depending on the reliability, either a hard decision is forced or the algorithm progresses to accumulate more sample energy. When working under unknown SNRs, this allows the detector to reach reliable sensing decisions by using adaptive sample window, thus providing advantage over classical ED where fixed threshold is used regardless of channel conditions. Extensive computer simulations are provided to illustrate the performance advantages against classical ED in terms of e.g. sensing time.**

I. Introduction

The radio frequency (RF) spectrum as a very limited natural resource has inspired the concept of cognitive radio (CR) [1] that aims to utilize this resource more efficiently. In the past decade, there has been intensive research on spectrum sensing which is an essential ingredient in CR. There are recent surveys, e.g. [1] and [3], summarizing most of the proposed and applied spectrum sensing methods. Among these, perhaps energy detector (ED), cyclostationary based detection, and eigenvalue or covariance based detection are the most well known ones. Though these methods differ from each other in terms of complexity and the required information (e.g. noise or signal variance), none of them has the capability to utilize any prior information of the primary user spectrum utilization rate, if available, in the final sensing decisions.

On the other hand, Bayesian approach has the inherent capability to use such statistical primary user channel utilization information. The application of Bayesian methods to spectrum sensing problem has recently started to attract more attention. In [4], the Bayesian philosophy is used to establish a rigorous mathematical foundation for cognitive radios. In [5], optimal and suboptimal Bayesian detectors are studied for a primary user that is assumed to use a BPSK signal. A more interesting case is studied in [6] where the primary user is assumed to be a general Gaussian signal impaired by a Rayleigh fading channel. A generalized likelihood ratio test is used for the detection under the assumption of multiple sensors and observations. One drawback of Bayesian approach which is also encountered in this paper is the difficulty in the computation of posterior probabilities or distributions of interest when there are unknown parameters such as signal and noise power [2], [7]. In [7], a generalized likelihood ratio-test (GLRT) is presented that studies optimal solutions under different combinations of known/unknown noise power and signal covariance matrix.

In this paper, we apply the Bayesian approach to the spectrum sensing problem in a different way than [7], building on the assumption that the primary signal is complex Gaussian process distorted by an independent additive white Gaussian noise (AWGN) process. Thus as a whole, the received composite signal is also complex Gaussian; this is elaborated more in Section II. Since we will be comparing the Bayes detector (BD) against classical ED, an overview of ED is given first in the same section. Then, the optimum detection rule for Bayesian approach, with arbitrary PU utilization and Bayes penalties, are derived in Section III. Following that in Section IV, we propose a novel iterative algorithm that builds on the derived general Bayesian framework in Section III and aims to perform as well as the classical ED in terms of false alarm probability (P_{FA}) and detection probability (P_D) but requiring fewer samples (i.e. shorter observation window) on average. In Section V, computer simulation results are provided illustrating the equality of the performances for proposed iterative BD and classical ED as well as a significant decrease in the required number of samples and hence sensing time and arithmetic complexity. Finally Section VI draws the conclusion.

II. Principal Signal Model and Energy Detection

Based on above assumptions of Gaussian PU and Gaussian noise, the composite received signal is always Gaussian. To be precise, we assume that the real and imaginary parts of the received complex signal samples y_i are modeled as i.i.d. Normal variables having variance v ,

$$y_i^{re} \mid v \sim \mathcal{N}(0, v), \quad y_i^{im} \mid v \sim \mathcal{N}(0, v) \tag{1}$$

Above, and also in the continuation, $a \mid b$ denotes that the random variable a is conditioned on b . The parameter θ that indicates the absence ($\theta = 0$) or presence ($\theta = 1$) of PU signal determines the variance of the total received signal. In the absence of the signal the real and imaginary parts of the signal both have Gaussian distribution with variance v_0 whereas in the presence of the signal this variance is v_1 , i.e., $v_0 = E[|y_i|^2 \mid \theta = 0]$ and $v_1 = E[|y_i|^2 \mid \theta = 1]$. The signal to noise ratio (SNR) is denoted by ρ and is defined as,

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$$\rho = \frac{E[|y_i|^2 \mid \theta = 1] - E[|y_i|^2 \mid \theta = 0]}{E[|y_i|^2 \mid \theta = 0]} = \frac{v_1}{v_0} - 1 \quad (2)$$

An energy detector calculates the sample energy out of N received samples by

$$s = \frac{1}{2N}\sum_{i=1}^{N}[(y_i^{re})^2 + (y_i^{im})^2] \quad (3)$$

and compares it to a chosen threshold γ and decides on $\theta = 1$ if and only if $s > \gamma$. The performance measures, namely the probability of false alarm P_{FA} and probability of detection P_D, are defined as

$$P_{FA} \coloneqq \Pr(s > \gamma \mid \theta = 0) \cong Q\left(\frac{\gamma - v_0}{v_0 / \sqrt{N}}\right) \quad (4)$$

$$P_D \coloneqq \Pr(s > \gamma | \theta = 1) \cong Q\left(\frac{\gamma - v_1}{v_1 / \sqrt{N}}\right) \quad (5)$$

where $Q(.)$ is the classical Q-function and the approximation is tight for large N.

A. Classical Energy Detection: Unknown SNR

In the context of CR, typically only noise power is available at the receiver and there is no knowledge on the possible signal power and hence no knowledge on SNR. In such case, the threshold design is purely based on a target P_{FA} and number of samples N. As in well known from the literature, the threshold in that case, guaranteeing a given P_{FA} for known noise variance v_0 is

$$\gamma_{ED} = \frac{Q^{-1}(P_{FA})v_0}{\sqrt{N}} + v_0 \quad (6)$$

B. Classical Energy Detection: Known SNR

In a hypothetical case where SNR is assumed to be known, then the threshold could be chosen based on either a target P_D or a target P_{FA}. Such design for given target P_D would have the threshold

$$\gamma_{ED} = \frac{Q^{-1}(P_D)v_0(1+\rho)}{\sqrt{N}} + v_0(1+\rho) \quad (7)$$

as is easy to show. Another design strategy would be to set both a target P_D and P_{FA} and determine the N that satisfy both. The needed number of samples can be easily derived as $N = [(Q^{-1}(P_{FA}) - Q^{-1}(P_D)(1+\rho))/\rho]^2$.

Obviously in all the design strategies described above, one of the underlying assumptions is that there is perfect knowledge of the noise variance. When there is noise uncertainty, it has been shown that there is serious degradation in the performance, and the choice of threshold and sample-size N is more complicated [8].

III. Bayesian Approach to Signal Detection

In this Section, we will develop a framework of Bayesian philosophy applied to signal detection in CR which is also partially discussed in the existing literature. Starting from the so-called Bayesian inference that is the posterior likelihood or probability of θ, the form of the detector and the corresponding detection rules will be presented. This framework will then be deployed in Section IV to develop iterative Bayes detector for spectrum sensing.

A. Bayesian Inference

The overall received vector is the concatenation of real and imaginary parts, written here as $\mathbf{y} = [y_1^{re}, \cdots, y_N^{re}, y_1^{im}, \cdots, y_N^{im}]$. The pdf of $\mathbf{y}$ is in the form of standard multivariate normal distribution of the form $p(\mathbf{y} \mid v) \propto v^{-N} e^{-N\frac{s}{v}}$. The proportionality sign is used to omit the scaling constant and such notation will be used for most of the upcoming probability distributions. This is because constant scaling does not have any influence on the final results. In the case where the variance is known perfectly once θ is provided, the conditional distribution of v is $p(v \mid \theta) = \delta(v - v_\theta)$. Furthermore, the prior of θ is denoted in the continuation as $\pi_1 \coloneqq \Pr(\theta = 1)$ and $\pi_0 \coloneqq \Pr(\theta = 0)$.

Now, using the Bayes rule, the joint posterior distribution of θ and v is given as

$$p(\theta, v \mid \mathbf{y}) \propto p(\mathbf{y} \mid v)p(v \mid \theta)p(\theta) \propto v^{-N} e^{-N\frac{s}{v}} p(v \mid \theta)\pi_\theta$$

Marginalising out v and then inserting $p(v \mid \theta) = \delta(v - v_\theta)$ yields

$$\begin{aligned} p(\theta \mid \mathbf{y}) &\propto \pi_\theta \int v^{-n} e^{-N\frac{s}{v}} p(v \mid \theta) dv \propto p(\theta \mid \mathbf{y}) \\ &\propto \pi_\theta v_\theta^{-N} e^{-N\frac{s}{v_\theta}} \end{aligned} \quad (8)$$

B. Detection Rule

For notational purposes, we "encode" the alternatives for decision of presence and absence as $a = 1$ and $a = 0$. Then we introduce the loss or penalty function $L(a, \theta)$ whose values can be summarized by the following table as

	$\theta = 0$	$\theta = 1$
$a = 0$	0	L_{MD}
$a = 1$	L_{FA}	0

where L_{MD} and L_{FA} indicate the penalties for missed detection (MD) and false alarm (FA), respectively. Given the measurements (i.e., received signal), the expected losses are then

$$\begin{aligned} E[L(0, \theta \mid \mathbf{y})] &= L_{MD} p(\theta = 1 \mid \mathbf{y}) \\ E[L(1, \theta \mid \mathbf{y})] &= L_{FA} p(\theta = 0 \mid \mathbf{y}) \end{aligned} \quad (9)$$

Then the best decision that minimizes the expected loss is to decide $a = 1$, if $E[L(1, \theta \mid \mathbf{y})] < E[L(0, \theta \mid \mathbf{y})]$ which can be written in form of odds ratio as,

$$\Phi = \frac{p(\theta = 1 \mid \mathbf{y})}{p(\theta = 0 \mid \mathbf{y})} = \frac{\pi_1}{\pi_0}(1+\rho)^{N(\frac{s}{\overline{v}} - 1)} > \frac{L_{FA}}{L_{MD}} \quad (10)$$

where $\overline{v} = v_1 v_0 \ln(v_1 / v_0)/(v_1 - v_0)$. Notice that in [2], [7], the ratio of interest is $p(\mathbf{y} \mid \theta = 1)/p(\mathbf{y} \mid \theta = 0)$ as opposed to (10) and hence the channel utilization information π_1 and

π_0 are not part of the solution. After some manipulations, (10) can be shown to be equivalent to

$$s > \gamma_{BD} = \overline{v} + \frac{1}{N} \cdot \frac{v_1 v_0}{(v_1 - v_0)} . \ln\left(L_r \frac{\pi_0}{\pi_1} \right) \qquad (11)$$

where $L_r = L_{FA} / L_{MD}$ denotes the loss ratio. The right hand side of (11) gives the optimum threshold that minimizes the expected value of the posterior loss. The fact that the ratio of posterior odds is simplified to comparing the sample energy against a threshold is a similar result as developed also in Section IV.A of [7] when both noise and signal are white Gaussians and the noise variance is known.

Note that the threshold value in (11) requires the knowledge of v_1 (the variance of the signal) which is not necessarily directly available in a SU device. However, one interesting observation is that if we plug in an arbitrary value for the SNR in (10) and then set the loss ratio as

$$L_r^{ED} = \frac{L_{FA}}{L_{MD}} = \frac{\pi_1}{\pi_0} e^{N(\frac{\rho\gamma_{ED}}{v_1} - \log(1+\rho))} \qquad (12)$$

this will ensure the equality for the thresholds of BD and the ED, i.e., $\gamma_{BD} = \gamma_{ED} = Q^{-1}(P_{FA}) v_0 / \sqrt{N} + v_0$. The interpretation of the above action in some practical detection scenario can be explained via an example. For instance, if we have say -15 dB for the SNR and calculate the odds ratio, then the calculated value of odds tells what is the chance that there is signal of -15 dB against pure noise with that particular measured sample energy s. This gives a reliability value after which the detector might give a hard decision by using L_r^{ED} that guarantees certain target P_{FA} or might take some further actions such as waiting for more samples to calculate an improved estimate of sample energy which in turn will give sensing result that has higher reliability. This will be the main ideology in deriving the iterative BD-based sensing scheme in the forthcoming Section IV.

In a more general case where the loss ratio L_r is chosen arbitrarily, the corresponding P_{FA} and P_D are plotted in Fig. 1 and Fig. 2. The performance of "classical" ED with designs based on target P_{FA} and P_D are plotted as well. When threshold is based on a target P_{FA} then corresponding P_D is monotonically increasing for ED which is the natural consequence of equation (5). There the argument of the Q-function is decreasing by increasing SNR causing an increase in the value of the function. When the design is based on a target P_D and hence on a threshold given by (7), then it can be easily shown that the argument of the Q-function in the expression of P_{FA} is again monotonically increasing. Consequently this results in a monotonic decrease in the value of P_{FA} by increasing SNR. However, in case of Bayes detector (BD), the threshold in (11) is a more complicated function of SNR. This can result in scenarios where either P_{FA} or P_D (or both) have a non-monotonic behavior vs. SNR. Such increasing-decreasing behavior is visible in Fig. 1 and Fig. 2 for e.g. the P_{FA} when $L_r = 1$ and $\pi_0 = 0.75$ and for the P_D when $L_r = 0.2$ and $\pi_0 = 0.5$ or $\pi_0 = 0.75$. A general conclusion is that when the channel utilization rate and SNR are known, then for fixed N the BD-based threshold selection in (11) provides an interesting and useful trade-off between P_{FA} and P_D which can be controlled via the loss-ratio. For instance, if we have a reasonably challenging SNR level of say -15 dB and $\pi_0 = 0.75$, then Fig. 1 shows that the P_D performances of BD with $L_r = 1$ and $L_r = 0.2$ are very close to that of ED with target $P_D = 0.99$. At the same time, the corresponding P_{FA} performances for BD are somewhat better than those of ED.

IV. Iterative Bayes Detector

In the previous Section, we have developed the tools for Bayesian inference and the corresponding detection rules. It was shown by equation (11) that the Bayes detector has the same form as ED where the sample energy s is compared against a certain threshold. It was mentioned that when the loss ratio L_r is chosen as in (12), then the same threshold of an ED given in (6) which is based on a target P_{FA} is obtained. Then the false alarm and detection performance of more general case for arbitrary loss ratio L_r was also discussed through some example cases. However, in these general cases the receiver needs the knowledge of SNR which may not always be possible. In this section, we will present a novel iterative algorithm that uses the Bayesian philosophy but without explicit SNR knowledge, to have sensing performance as good as the classical ED but using fewer samples on average. As will be presented in Section V, the required sensing time can be half of an ED under reasonable assumptions. This is already a noteworthy decrease in the computational complexity and hence in the sensing time especially for large number of samples.

A. Motivation

For a cognitive radio that is operating for a considerably long time, it is reasonable to assume that the SNR values (if PU signal is present) are changing. In other words, it is reasonable to assume an interval of SNR values that the CR device can experience during its life span. In classical ED where threshold is designed by a target P_{FA} and used sample size N, the detector is unable to take advantage of good SNR situations where a sensing decision could have been reached with much smaller number of samples. Obviously a high SNR situation is reflected to the sample energy s and with even smaller number of samples than the maximum available, denoted here by N, a smarter detector can "judge" a clear sign for the existence of the signal.

The smartness of the detector is determined by its ability to understand how reliable the current calculated sample energy s is. The classical ED doesn't have such tools whereas the odds ratio given in (10) allows a Bayes detector to assign a reliability value to the decision for that particular calculated s and the used sample size. In the following sub-section, we will give the details of the mentioned novel algorithm which starts by calculating the sample energy with small sample window and iteratively accumulates more accurate sample energy estimate until a required confidence level is reached and final sensing decision is forced.

B. Iterative Algorithm

The starting point and objective for the algorithm derivation is to develop a sensing solution which always performs (at least) as good as the classical ED in terms of P_{FA} and P_D but with an additional target of reaching the performance with shorter sensing time on average, when the primary user signal variance is unknown and also varies from sensing realization

to another. We first start by noting that when the target P_{FA} and the allowed number of samples N are specified for an ED, then it is possible to find the SNR value for the ED where successful detection with probability of say $P_D = 0.99$ is possible. That value will be referred to as the SNR at which almost sure (a.s.) detection is feasible and it is given by the following formula

$$\rho_{0.99} = \frac{\gamma_{ED} - 1 - Q^{-1}(0.99)\frac{v_0}{\sqrt{N}}}{Q^{-1}(0.99)\frac{v_0}{\sqrt{N}} + v_0} \tag{13}$$

where γ_{ED} is given by (6). As noted already above, one of the objectives of our algorithm development is not to accept any inferior P_D performance compared to that of ED. Therefore we have to set a rule such that with the s that is calculated out of fewer samples than N above, we should still be guaranteeing a.s. detection for $\rho_{0.99}$ given by (13). This rule is described below:

Rule 1:

i) Set a high value for target odds, for e.g. $\Phi_1^{tgt} = 99$

ii) For $n < N$ *, calculate odds for* $\rho_{0.99}$ *and compare it against* Φ_1^{tgt} *.*

iii) If $\Phi > \Phi_1^{tgt}$ *, decide* $\theta = 1$ *; else increase the number of samples* n *and go back to ii) until you reach the maximum allowable number of samples specified for ED.*

Now assume that an ED has a target P_{FA} that is e.g. $P_{FA} = 0.01$ and the provided N is 10^4 and consequently $\rho_{0.99} = -13.5$ dB. In other words, an ED whose threshold is given by (6) achieves $P_D = 0.99$ at -13.5 dB. When the above rule is applied, a sample energy s is first calculated with $n < N$ for e.g. $n = 10^3$. Then the odds ratio is calculated for that particular (s, n) pair which translates to "the odds of a signal with SNR of $\rho_{0.99}$ against pure noise for the pair (s, n)". Note that for a fixed SNR value such as $\rho_{0.99}$, the odds given by (10) is a monotonically increasing function with increasing s. Hence the larger s is, the larger the odds and higher the confidence for θ being equal to 1. The target odds, Φ_1^{tgt}, then tells when the pair (s, n) produce a result that is highly reliable to force a decision. The pessimistic looking number of 99 is chosen as an illustrative example, in order not to cause any inferior P_{FA} performance. Hence a decision is given only when $\theta = 1$ is at least 99 times more likely than $\theta = 0$. Also note that by algebraic manipulations we can turn the comparison $\Phi > \Phi_1^{tgt}$ in the third step of *Rule 1* to the form $s > \gamma_{1,n}$ where $\gamma_{1,n}$ is given as

$$\gamma_{1,n} = \overline{v}_{0.99} + \frac{1}{n}\frac{\overline{v}_{0.99}}{\log(1+\rho_{0.99})}\log(\Phi_1^{tgt}\frac{\pi_0}{\pi_1})$$

where $\overline{v}_{0.99} = v_0(1+\rho_{0.99})\log(1+\rho_{0.99})/\rho_{0.99}$.

By similar philosophy we can also set a rule for deciding on $\theta = 0$ at an early stage. This rule is described below,

Rule 2:

i) Set a low value for target odds, for e.g. $\Phi_0^{tgt} = 1/99$

ii) For $n < N$ *, calculate odds for* $\rho_{0.99}$ *and compare it against* Φ_0^{tgt} *.*

iii) If $\Phi < \Phi_0^{tgt}$ *, decide* $\theta = 0$ *; else increase the number of samples* n *and go back to ii) until you reach the maximum allowable number of samples specified for ED.*

Similar to *Rule 1*, a decision of $\theta = 0$ is given prior to reaching maximum allowable samples N only if the odds favor 0 at least 99 times more than it favors 1. Again it is possible to turn the comparison $\Phi < \Phi_0^{tgt}$ into the form $s < \gamma_{0,n}$ where $\gamma_{0,n}$ is given as

$$\gamma_{1,n} = \overline{v}_{0.99} + \frac{1}{n}\frac{\overline{v}_{0.99}}{\log(1+\rho_{0.99})}\log(\Phi_0^{tgt}\frac{\pi_0}{\pi_1})$$

Now by *Rule 1* and *Rule 2,* we have given the rules for deciding on $\theta = 1$ or $\theta = 0$ when s is calculated out of n samples where $n < N$. In practice a predetermined set of values can be used in the iterative step such as $n = [n_1\ n_2 \cdots n_M]$ where $n_1 < n_2 < \cdots < n_M = N$. Hence, there is at most M iterations if the intermediate value of s is not sufficient to produce a result that is enough to make a decision. In that case, when n reaches $n_M = N$, then the standard ED is used to guarantee the same P_{FA} and P_D performance offered by it.

A pseudo-code combining *Rule 1* and *Rule 2* with the standard ED at the final iteration can be given as follows:

ALGORITHM I

```
set n = [n_1 n_2 ... n_M], Φ_0^tgt and Φ_1^tgt, i=1, endDetection=0
for j = 1 : M calculate γ_{1,n_j} and γ_{0,n_j}

while endDetection = 0
    calculate s out of n_i samples
    if i=M
        if s > γ_ED,
            decide θ = 1
        else,
            decide θ = 0
        end
        set endDetection=1
    else
        if s > γ_1,n_i
            decide θ = 1, set endDetection=1
        elseif s < γ_0,n_i
            decide θ = 0, set endDetection=1
        else
            i=i+1
        end
    end
end
```

Note that in the above algorithm, we are still not doing anything more complicated than the comparison of the sample energy against certain thresholds. Thus complexity-wise, the proposed solution is very simple. The point that differentiates this algorithm from classical ED is that certainty ($s < \gamma_{0,n_i}$ and $s > \gamma_{1,n_i}$) and uncertainty ($\gamma_{0,n_i} < s < \gamma_{1,n_i}$) regions are derived based on odds ratio. If during any iteration an uncertainty region is encountered, the algorithm goes to the next iteration where s is calculated out of n_{i+1} samples. Obviously in such case, we will use the value of sum from the previous iteration and use only $n_{i+1} - n_i$ operations to calculate

the new sum (accumulate more sample energy). To elaborate this, we first denote

$$s_{n_i} = \frac{1}{2n_i}\sum_{j=1}^{n_i}[(y_j^{re})^2 + (y_j^{im})^2]$$

Then it is straightforward to see that

$$s_{n_{i+1}} = \frac{1}{2n_{i+1}}(2n_i s_{n_i} + \sum_{j=n_i+1}^{n_{i+1}}[(y_j^{re})^2 + (y_j^{im})^2]$$

Hence even if the algorithm cannot make a prior decision, it still requires only a similar number of arithmetic operations compared to classical ED (assuming that the extra multiplication and summation coming from $2n_i s_{ni}$ is negligible). Finally we define the average complexity as the sum of used number of samples at each attempt of detection divided by the total number of detection attempts which can be written as,

$$N_{av} = \frac{1}{N_r}\sum_{j=1}^{Nr} N_j \qquad (14)$$

In the next section we will provide simulation results where the algorithm described above is compared against a classical ED in terms of the required average complexity given in (14) to achieve the same performance of P_{FA} and P_D.

V. Simulations and Discussions

In our simulations, we set 5 different values for N_{ED} which is the number of samples that ED is allowed to use. Then for each N_{ED}, $N_r = 10^5$ independent Monte-Carlo realizations are done where in each realization, θ is chosen randomly as 0 or 1 according to π_1 and π_0. If $\theta = 1$ then SNR for single realization is randomly chosen from the interval from -30 to 0 dB assuming a uniform distribution, and an i.i.d complex Gaussian signal with variance v_θ is generated accordingly. The noise power is set to $v_0 = 1$. For the target odds ratios, we use $\Phi_0^{tgt} = 1/99$ and $\Phi_1^{tgt} = 99$. The vector of samples that are allowed to be used in different iterations for a given N_{ED} is set as $n = [10^3\ (1{:}5)M]$ where $M = N_{ED}/5$. Hence there are at most 6 iterations starting with the use of 10^3 samples and if the intermediate steps are not satisfactory then the introduced algorithm ends up using N_{ED} samples at most.

In Fig. 3, it is seen that the iterative BD can achieve the same detection probability performance as ED under different channel utilization values of $\pi_0 = 0.5$ and $\pi_0 = 0.75$. The P_{FA} plots are omitted since they are constant at $P_{FA} = 0.01$ for both ED and iterative BD. On the other hand, in Fig. 4 it is clearly seen that iterative BD achieves this identical performance by using clearly less samples on average. For instance when $N_{ED} = 10^6$, the N_{av} for iterative BD is around 5×10^5 and hence saving considerably in sensing time and arithmetic operations. Finally in Fig. 5, the relative frequency (number of times n_i is used divided by N_r) of the used samples are shown for $N_{ED} = 5 \times 10^5$. As can be seen, the maximum number of samples is used in less than 20 percent of the realizations whereas considerably lower numbers of samples such as 10^3, 10^4 and 2×10^4 are used much more often.

VI. Conclusions

In this paper, we have presented the general Bayesian framework to the Gaussian signal detection problem in the context of cognitive radio. It was first shown that the Bayesian spectrum detector simplifies to energy detector where sample energy is compared against certain threshold. The form of the threshold that minimizes the posterior loss functions was derived explicitly as a function of Bayes penalties and prior channel utilization statistics. Thus the channel utilization information is built-in to the threshold expression which is then giving more degrees of freedom in the design compared to classical energy detector which makes no use of such information. One practical complication in the derived threshold is that it depends on the received SNR which in turn may not be directly available in most of the applications. Following that, and stemming from the general developed Bayesian framework, we also presented a novel iterative algorithm that is capable of assigning reliability measures to existence/absence of the primary user signal with adaptive sample energy measurement window. Thus, when the SNR in practice is unknown, this approach automatically tunes the sensing time such that desired reliability is achieved under the prevailing received signal conditions. More specifically, the exact value of the SNR under operation is not needed in the developed iterative approach but the SNR value at which a classical ED achieves almost sure detection is used instead. Computer simulations showed that such iterative BD can be built to perform essentially as well as the classical ED concerning the P_{FA} and P_D performance, but using on average considerably shorter sensing time which also depends on the values of channel utilization. Compared to classical energy detection, this approach thus saves considerably in the required sensing time and thereby also in required number of arithmetic operations to yield a reliable sensing decision.

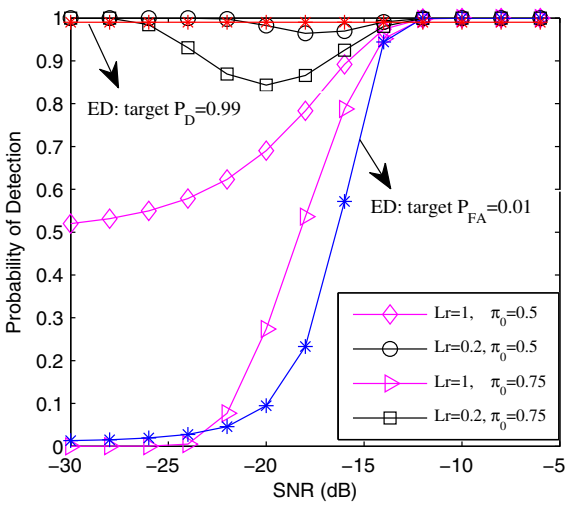

Fig. 1. P_D vs. SNR with $N = 10^4$ for classical energy detector (ED) and Bayes detector (BD) with two loss ratios and prior probabilities.

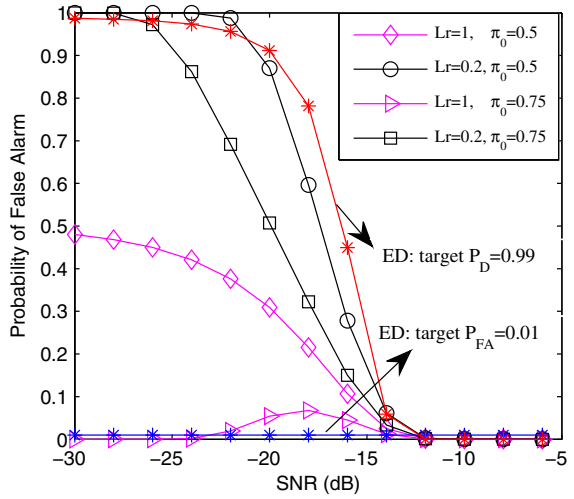

Fig. 2. P_{FA} vs. SNR with $N = 10^4$ for classical energy detector (ED) and Bayes detector (BD) with two loss ratios and prior probabilities.

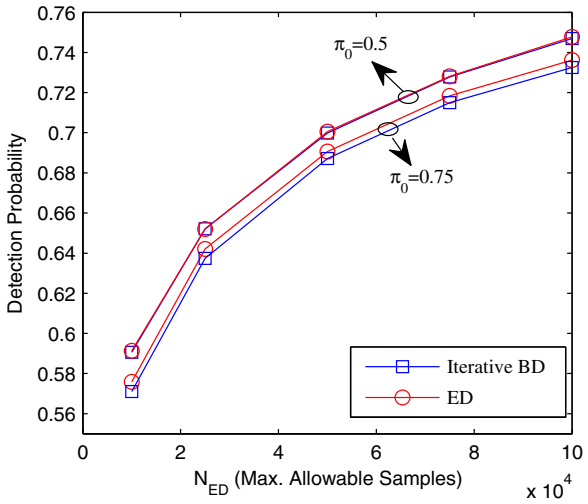

Fig. 3. P_D vs. N_{ED} under fixed $P_{FA} = 0.01$ for classical energy detector and iterative Bayes detector. In the realizations, the received SNR is uniformly distributed from -30 dB to 0 dB.

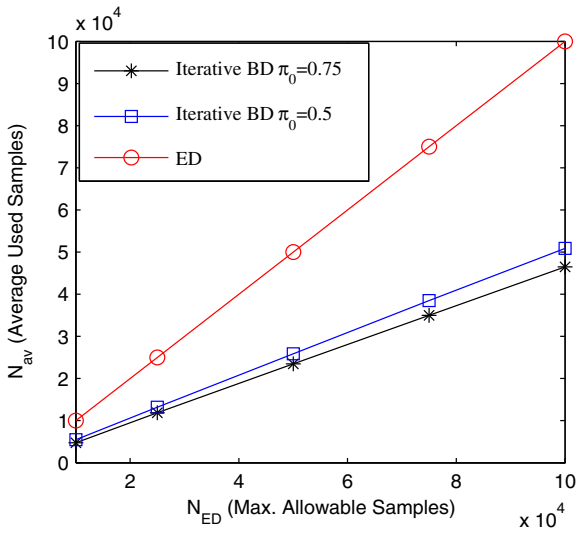

Fig. 4. N_{av} vs. N_{ED} under fixed $P_{FA} = 0.01$ for classical energy detector and iterative Bayes detector. In the realizations, the received SNR is uniformly distributed from -30 dB to 0 dB.

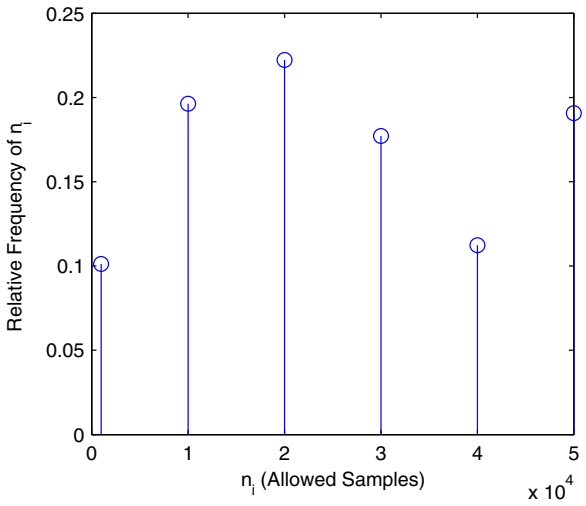

Fig. 5. Relative frequency of needed sample-size n_i for the iterative Bayes detector with fixed $P_{FA} = 0.01$ and maximum sample-size $N_{ED} = 5 \times 10^4$. In the realizations, the received SNR is uniformly distributed from -30dB to 0 dB.

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