

# Per-Node Throughput Performance of Overlapping Cognitive Radio Networks

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**Abstract**—Multiple Cognitive Radio (CR) networks may exist in the same spatial domain in many cases. In this paper we consider two uncoordinated and geographically overlapping CR networks coexisting together with a primary network. We specifically study the achievable per-node throughput performance of the CR networks. Firstly, an interference model is specified which models the situation. By using this model we derive the per-node throughput for overlapping CR networks. Furthermore, the upper bound for the probability of false alarm, which is required to achieve a certain throughput, is deduced. The results of this paper illustrate how the different CR network parameters, such as network density, transmission probability, and sensing performance, affect the achievable per-node throughput in overlapping CR networks. In addition, these results serve as guidance for the deployment of multiple CR networks.

## I. INTRODUCTION

Cognitive Radio (CR) is a revolutionary technology aiming to increase spectrum utilization through dynamic secondary spectrum access. In many practical scenarios, multiple CR networks (CRNs) may coexist in the same geographical area. One example is disaster relief effort, where different organizations such as police, fire fighters, and emergency medical services are all deployed in the disaster area at the same time. All of these participating organizations use CRs to sweep a wide range of spectrum and find suitable spectrum for communications. Another example is in battlefield communications, where multiple wireless networks may coexist. These networks may belong to different military branches or organizations such as the army and the air force. With the advancement of CR technology, it is expected that many of the network elements will have cognitive capability enabled by a software defined radio platform, such as the Joint Tactical Radio System (JTRS) program being a prime example.

The network scenario for the case of two overlapping CR networks is demonstrated in Figure 1. The figure shows two coexisting CR networks,  $CRN_1$  and  $CRN_2$ , that are operating in the same spatial domain and on the same frequency with a Primary User (PU). The main problem is that the CR networks will interfere with each other in such situations, in addition to yielding to the PUs. In this paper we specifically study the impact of the interfering CR network on the performance of a given CR network.

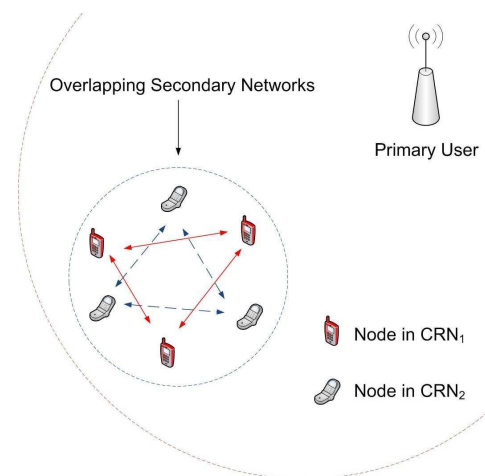


Figure 1: An example network scenario, where two overlapping CR networks are within the range of a primary user.

There are rich literatures on the coexistence of heterogeneous wireless networks on the ISM bands. Research work in this field has focused on the coexistence of WiFi (802.11) and Zigbee (802.15.4) radios, see, e.g., [1], [2], [3]. IEEE 802.11 b/g networks may interfere with IEEE 802.15.4 sensor networks and thereby introduce significant coexistence problems for low-power sensor nodes. Although intensive research has been carried out on CR technology and single CR networks, only a few studies address the coexistence of multiple CR networks [4], [5], [6], [7]. Furthermore, none of the existing works discuss the fundamental per-node throughput of a CR user when multiple CR networks coexist with the PUs.

The main objective of this paper is to find out how much throughput a node in a CR network can achieve in the presence of another CR network and a PU. The main contributions of this paper are as follows. We firstly derive an interference model for overlapping CR networks which takes into account spectrum sensing performance. This model is then used to deduce the probability that the received Signal to Interference and Noise Ratio (SINR) is larger than the required threshold for successful reception of a packet. These results are used to find out the per-node throughput and the bound for the probability of false alarm in case of coexisting CR

networks. This bound determines whether it is feasible to deploy multiple CR networks in the same region with the required quality-of-service, say, the minimum throughput. We analyze the performance of a node in detail by considering the effects of various CR network parameters such as transmission probability, performance of spectrum sensing (false alarm and detection probabilities), etc. This paper provides fundamental insights on the dominant factors of the per-node throughput in case of overlapping CR networks.

The paper is organized as follows. First, the used system model is defined in Section II. Then, we derive the interference model, the probabilistic per-node throughput, and the performance bound for a CR node in overlapping CR networks in Section III. It is also shown that the theoretical results agree with the simulation results. Fundamental results and detailed analysis on the performance of a CR node in coexisting CR networks are presented in Section IV. Section V gives the concluding remarks.

## II. SYSTEM MODEL

In this paper, we will focus on the case where two CR networks are uncoordinated and deployed in the same geographical area at the same time in addition to a PU. The presence of the PU is defined using the following hypotheses. Hypothesis  $H_0$  denotes the case in which the PU is not present and  $H_1$  stands for the case in which the PU is present. We further make the following assumptions:

- Each CR network performs its own spectrum sensing and the corresponding probabilities of detection and false alarm are taken into account in this paper. However, they do not coordinate their sensing nor share the sensing results. For example, although the organizations are collaborating on the disaster relief mission, each organization has its own CR network and their CR networks are not coordinated since the spectrum situation in the disaster area is not known a priori and each organization has its own administrative constraints such as security requirements.
- Since CSMA/CA is a well-established Media Access Control (MAC) protocol and has been adopted by many practical wireless networks, we presume that the CR networks use CSMA/CA as the basis of their MAC protocol. It is also assumed that CR nodes can detect others' transmissions by using CSMA/CA, where the RTS/CTS message exchange is carried out before data transmissions.
- The secondary CR networks are homogeneous in the sense that the nodes in the CR networks have similar capabilities and behaviors, such as the transmission power.
- Channels and signals from the PU to CRs are complex and the channels experience fast and shadow fading leading to Gaussian distributed signals [8].
- CR networks are located in an urban area. Since the CR nodes are typically less powerful than the primary nodes, have smaller transmission ranges and are located closer

to each other, we model the channel between CR nodes with Rayleigh fading.

- Noise is Additive White Gaussian Noise (AWGN).

In this paper, we focus on CR ad hoc networks instead of CR networks with infrastructure support such as the IEEE 802.22 [9] systems. There is a universal detector for PU signals in each CRN while each CR node uses CSMA/CA protocol by exploiting this detection result.

## III. THEORETICAL MODELING

In this section we first derive the interference model for overlapping CR networks which is then exploited to deduce the per-node probabilistic throughput for such scenario. Then, we enhance these results by taking into account the sensing parameters, the probability of false alarm and detection. The performance bound for the probability of false alarm is also derived. Finally, it is shown that the theoretical and simulated results match up well.

### A. Interference Modeling and Probabilistic Throughput per node

Both of the CR networks are uniformly random networks where nodes are independently distributed in an area according to a Poisson Point Process (PPP). Node densities of  $CRN_1$  and  $CRN_2$  are denoted by  $\lambda_1$  and  $\lambda_2$ , respectively. As a channel model we use deterministic distance-dependent path loss  $r^{-\alpha}$ , where  $r$  is the distance between the transmitter and the receiver and  $\alpha$  is the path loss exponent. We consider Rayleigh fading,  $x$  with  $E\{x^2\} = 1$ .

The Cartesian coordinates of a node are denoted by  $X$  and  $Y$ . These random variables are independent of the other nodes' locations and uniformly distributed in  $[-L, L]$ . By setting the node density  $\lambda = N/(4L^2)$ , where  $N$  is the number of nodes, the probability of finding  $k$  nodes in an area  $A$  in the plane is given by

$$\Pr\{k \in A\} = \frac{e^{-\lambda A} (\lambda A)^k}{k!}. \quad (1)$$

With these assumptions we can calculate the mean  $\mu$  and variance  $\sigma^2$  of interference  $\mathcal{I}$  for a random Poisson network with density  $\lambda$  as follows [10]

$$\mu = \frac{2\lambda p \pi d_0^{2-\alpha}}{\alpha - 2} \quad (2)$$

$$\sigma^2 = \frac{2\lambda p \pi d_0^{2(1-\alpha)}}{\alpha - 1}, \quad (3)$$

where  $p$  is the transmission probability and  $d_0$  the near field cut-off radius. The near field cut-off radius defines the distance in which other nodes in a network cannot transmit. For a large number of interferers, the interference can be modeled as Gaussian distributed due to the Central Limit Theorem, with parameters  $\mu$  and  $\sigma^2$  [11]. We call this as intra-network interference within one CR network.

In our case the problem is that nodes in the other CR network may decide to transmit as well (depending on the sensing results) and thus, create inter-network interference. We

can model inter-network interference similarly as before using Equations (2) and (3). The resulting interference  $\mathcal{I}$  is Gaussian distributed  $\mathcal{N}(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$  which gives

$$\mu = \frac{2\lambda_1 p_1 \pi d_{0,1}^{(2-\alpha)}}{\alpha - 2} + \frac{2\lambda_2 p_2 \pi d_{0,2}^{(2-\alpha)}}{\alpha - 2} \quad (4)$$

$$\sigma^2 = \frac{2\lambda_1 p_1 \pi d_{0,1}^{2(1-\alpha)}}{\alpha - 1} + \frac{2\lambda_2 p_2 \pi d_{0,2}^{2(1-\alpha)}}{\alpha - 1}. \quad (5)$$

Furthermore, the received SINR  $\gamma$  is calculated as follows

$$\gamma = \frac{Px^2R^{-\alpha}}{\mathcal{I} + \sigma_n^2}, \quad (6)$$

where  $P$  is the transmission power,  $R$  the distance between a transmitter and a receiver and  $\sigma_n^2$  is the noise power. Then, we can calculate the probabilistic throughput

$$\begin{aligned} \Pr\{\gamma > \theta\} &= \Pr\left\{\frac{x^2PR^{-\alpha}}{\mathcal{I} + \sigma_n^2} > \theta\right\} \\ &= \Pr\left\{x^2 > \frac{\theta(\mathcal{I} + \sigma_n^2)R^\alpha}{P}\right\}, \quad (7) \end{aligned}$$

where  $\theta$  is the required SINR for successful reception (threshold). By denoting  $z = x^2$  this can be deduced to the following form

$$\begin{aligned} \Pr\{\gamma > \theta\} &= E\left\{F_{c,z}\left(\frac{\theta(\mathcal{I} + \sigma_n^2)}{PR^{-\alpha}}\right)\right\} \\ &= E\left\{\exp\left(\frac{-\theta(\mathcal{I} + \sigma_n^2)}{PR^{-\alpha}}\right)\right\}, \quad (8) \end{aligned}$$

where  $F_c(\cdot)$  stands for the Complementary Cumulative Distribution Function (CCDF). Moreover, note that  $z$  is an exponential random variable and  $F_{c,z}(z) = e^{-z}$ . The expectation is taken over the Gaussian distribution which gives [11]

$$\begin{aligned} \Pr\{\gamma > \theta\} &= \exp\left(-\frac{\theta(\mu + \sigma_n^2)}{PR^{-\alpha}}\right) \exp\left(\frac{\theta^2\sigma^2}{2(PR^{-\alpha})^2}\right) \\ &\times Q\left(\frac{\theta\sigma^2}{PR^{-\alpha}} - \frac{\mu}{\sigma}\right). \quad (9) \end{aligned}$$

### B. Simultaneous Secondary Access

In CR networks the received interference depends on the sensing results. Furthermore, in case of overlapping CR networks the operations of the other CR network also affect the performance. Consequently, we have multiple scenarios listed in Table I depending on the PU's activities and the spectrum sensing results of the CRNs. For instance, if the PU is idle ( $H_0$ ), and only  $CRN_2$  has a false alarm, then  $CRN_1$  will be able to use that channel for transmission alone. By denoting the probability of false alarm and probability of detection as  $P_{f,i}$  and  $P_{d,i}$  for  $CRN_i$ , the probability of this scenario is  $(1 - P_{f,1})P_{f,2}P(H_0)$ . Other cases are determined using similar reasoning. The probability of miss for  $CRN_i$  is defined as  $P_{m,i} = 1 - P_{d,i}$ .

By using the scenarios defined in Table I we can derive the following equation for successful packet reception for a node

Table I: Possible transmission scenarios

Scenarios	$H_0$	$H_1$
Idle	$P_{f,1}P_{f,2}$	$P_{d,1}P_{d,2}$
$CRN_1$	$(1 - P_{f,1})P_{f,2}$	$P_{m,1}P_{d,2}$
$CRN_2$	$(1 - P_{f,2})P_{f,1}$	$P_{m,2}P_{d,1}$
$CRN_1$ & $CRN_2$	$(1 - P_{f,1})(1 - P_{f,2})$	$P_{m,1}P_{m,2}$

in  $CRN_1$

$$\begin{aligned} \Pr\{\gamma > \theta\} &= (1 - P_{f,1})P_{f,2}P(H_0) \Pr\left\{\frac{x^2P_1R^{-\alpha}}{\mathcal{I}_1 + \sigma_n^2} > \theta\right\} \\ &+ P_{m,1}P_{d,2}P(H_1) \Pr\left\{\frac{x^2P_1R^{-\alpha}}{\mathcal{I}_1 + \mathcal{I}_p + \sigma_n^2} > \theta\right\} \\ &+ (1 - P_{f,1})(1 - P_{f,2})P(H_0) \Pr\left\{\frac{x^2P_1R^{-\alpha}}{\mathcal{I}_1 + \mathcal{I}_2 + \sigma_n^2} > \theta\right\} \\ &+ P_{m,1}P_{m,2}P(H_1) \Pr\left\{\frac{x^2P_1R^{-\alpha}}{\mathcal{I}_1 + \mathcal{I}_2 + \mathcal{I}_p + \sigma_n^2} > \theta\right\}, \quad (10) \end{aligned}$$

where  $\mathcal{I}_1$ ,  $\mathcal{I}_2$ , and  $\mathcal{I}_p$  denote the received intra-network, inter-network, and PU's interference, respectively. Then, by formulating each term of Equation (10) in the same way as in Equation (8) and solving that we can find an exact value for  $\Pr\{\gamma > \theta\}$ , similar to Equation (9).

The main problem is that the performance of  $CRN_1$  will be determined by the operations of  $CRN_2$  and vice versa. By using these formulas we will analyze the throughput of overlapping CR networks to see what are the suitable bounds to guarantee reasonable performance.

We define the per-node throughput  $S$  such that the transmitter has a packet to transmit while a receiver is idle, i.e., the receiver does not have a packet to transmit. Moreover, the received SINR has to be larger than the threshold for successful packet reception. This can be mathematically formulated as follows

$$S = p(1 - p) \Pr\{\gamma > \theta\}. \quad (11)$$

In practice CR users should achieve reasonable throughput to enable feasibility from the economic perspective. We denote this throughput threshold by  $\hat{S}$ . Next, we derive the bound of the probability of false alarm that is required to achieve the desired throughput,  $S \geq \hat{S}$ . By analyzing Equation (10) we have concluded that in practice the second and the fourth term in Equation (10) have negligible influence on the performance of CR users, since both the miss rate and the probability of the PU being active are small. In addition, it is not practical to design CR networks by assuming that their transmissions would overlap with the transmissions of the PU's. Thus, we

use the following approximation

$$\begin{aligned}
S &\geq \hat{S} \Rightarrow \\
\hat{S} &\leq p(1-p)(1-P_{f,1})P_{f,2}P(H_0) \Pr \left\{ \frac{x^2 P_1 R^{-\alpha}}{\mathcal{I}_1 + \sigma_n^2} > \theta \right\} \\
&+ p(1-p)(1-P_{f,1})(1-P_{f,2})P(H_0) \\
&\times \Pr \left\{ \frac{x^2 P_1 R^{-\alpha}}{\mathcal{I}_1 + \mathcal{I}_2 + \sigma_n^2} > \theta \right\}. \quad (12)
\end{aligned}$$

The above inequality shows the maximum achievable throughput for a node in  $CRN_1$  given the PU's activity and the spectrum sensing performance of the two CRNs.

Moreover, let us define

$$\Omega_1 = P(H_0) \Pr \left\{ \frac{x^2 P_1 R^{-\alpha}}{\mathcal{I}_1 + \sigma_n^2} > \theta \right\} \quad (13)$$

$$\Omega_2 = P(H_0) \Pr \left\{ \frac{x^2 P_1 R^{-\alpha}}{\mathcal{I}_1 + \mathcal{I}_2 + \sigma_n^2} > \theta \right\}, \quad (14)$$

and assume that both CRNs have the same spectrum sensing performance, i.e.,  $P_{f,1} = P_{f,2} = P_f$ . Then,

$$\hat{S} \leq p(1-p)(1-P_f)[P_f\Omega_1 + (1-P_f)\Omega_2] \quad (15)$$

It is observed that when the false alarm probability  $P_f$  is very small, the achievable throughput approaches  $p(1-p)\Omega_2$ . It can be shown that as long as  $\frac{\Omega_1}{\Omega_2} \leq \frac{2-P_f}{1-P_f}$ , the achievable throughput will decrease when  $P_f$  increases.

If the spectrum sensing performance of  $CRN_2$  is given *a priori*, then we can find out the maximum probability of false alarm of  $CRN_1$  for achieving a certain throughput  $\hat{S}$ .

$$P_{f,1} \leq 1 - \frac{\hat{S}}{(P_{f,2}\Omega_1 + (1-P_{f,2})\Omega_2)p(1-p)}. \quad (16)$$

In other words, Equation (16) defines the upper bound for the probability of false alarm of  $CRN_1$ .

### C. Simulation Verification

Next, we show that the theoretical results agree with simulations by using Matlab simulations. We use the Monte Carlo simulation method and for each simulation run we randomly generate the received interference which is then used to produce the SINR according to Equation (6). SINR is further exploited to find out whether a packet was received properly or not by comparing it to the threshold. Finally, we took an average over all the runs so that the results are comparable with Equation (10). The number of simulation runs was set as 10 000.

For each simulation run the presence of the PU is first determined randomly using  $P(H_0)$ . We consider only the performance of  $CRN_1$  and hence, if a false alarm occurs in  $CRN_1$ , the secondary spectrum access opportunity is missed. However, if the PU is idle and  $CRN_1$  senses the situation correctly, the number of nodes in  $CRN_1$  is generated from a Poisson distribution with parameter  $\lambda_1 A$ . After this the nodes are positioned randomly according to the PPP and the received interference is calculated by taking into account  $d_0$  and  $p$ . In other words, only the nodes that are outside of  $d_0$  and

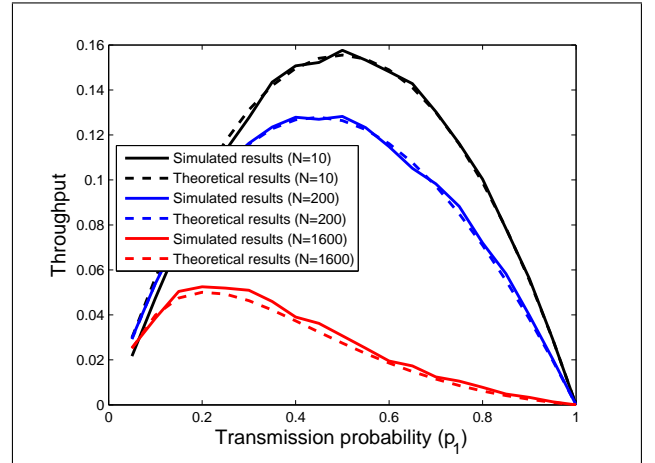


Figure 2: Throughput of CRN1 as a function of transmission probability.

have a packet to transmit will generate interference. The same procedure is repeated for  $CRN_2$  as well.

Furthermore, if the PU transmits and  $CRN_1$  does not detect the activity of the PU, the transmissions overlap. This happens with a small probability but still it has to be taken into account in the simulations. In this case the PPP is again used for positioning the nodes of  $CRN_1$  and the inter-network interference is produced as in the previous case but now the PU signal is also considered. If  $CRN_2$  also misses the transmission of the PU, interference from  $CRN_2$  is added. Finally, we calculate the mean of probability of successful transmission over all the simulation runs and use that to estimate the throughput.

The outcomes of the simulations are displayed in Figure 2 together with the theoretical results. In the figure we show the results for different numbers of nodes, i.e., for various network densities since the size of the area is fixed. As the figure demonstrates, theoretical and simulated results match up well even in case of small networks ( $N = 10$ ). The used network parameters are the same as in [10]:  $d_0 = 3m$ ,  $R = 1m$ ,  $P = 10nW$ ,  $\sigma_n = 5fW$ ,  $\theta = 10dB$ ,  $L = 40m$ ,  $\alpha = 4$ ,  $p_2 = 0.1$ , and  $N_2 = 10$ . Moreover, the used CR parameters are:  $P_{f,1} = P_{f,2} = 0.1$ ,  $P_m = 0.05$ , and  $P(H_0) = 0.7$ .

The influence of the PU's transmission power on the performance is negligible since the probabilities  $P_{m,1}P_{d,2}P(H_1)$  and  $P_{m,1}P_{m,2}P(H_1)$  are very small in general. On the other hand, the interfering CR network has a significant effect on the per-node throughput of  $CRN_1$ .

## IV. RESULTS AND ANALYSIS

The performance of overlapping CR networks is studied by investigating the effects of different parameters on the throughput of  $CRN_1$ . Unless otherwise stated, we use the following practical values for network parameters in this section:  $d_0 = 100$  m,  $R = 50$  m,  $P = 30$  dBm,  $\sigma_n = -70$  dBm,  $\theta = 10$  dB,  $L = 500$  m,  $\alpha = 4$ ,  $p_1 = p_2 = 0.5$ , and  $N_1 = N_2 = 100$ . Moreover, the used CR parameters are:  $P_{f,1} = P_{f,2} = 0.1$ ,  $P_m = 0.05$ , and  $P(H_0) = 0.9$ . For each

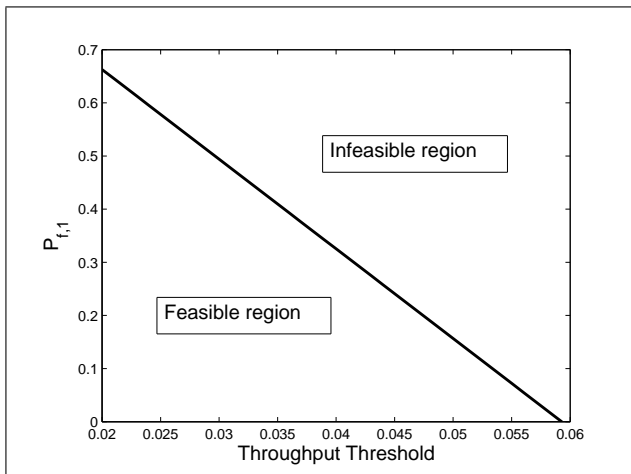


Figure 3: Maximum probability of false alarm as a function of throughput threshold.

result figure we vary different parameters to demonstrate their impact.

First of all, we study the upper bound of false alarm probability. By exploiting Equation (16) it is possible to determine the maximum value for the probability of false alarm that is required to achieve a certain throughput. This is demonstrated in Figure 3. Within the feasible region it is possible to implement a system which achieves the desired throughput. Outside of this region it is impossible to meet the requirements set for the throughput. The black line that divides these two regions is the upper bound for the probability of false alarm from Equation (16). This limit naturally depends on the network and sensing parameters of  $CRN_2$  as well.

Figure 4 shows the effect of sensing performance on the throughput in case of overlapping CR networks. The figure captures the fundamental nature of overlapping CR networks. As expected, the sensing performance of both networks has an effect and it seems that both networks have equal and linear influence on the throughput of  $CRN_1$ . These results imply that CR users would like to have as low probability of false alarm as possible to achieve the best performance. Whereas, the false alarm probability of the interfering CR network should be high such that the CR network in question would be able to access and use the spectrum alone as often as possible.

The activity of the interfering CR network has a significant impact on the performance in case of overlapping CR networks. Figure 5 demonstrates the effect of the transmission probability of  $CRN_2$  on the performance of  $CRN_1$ . From the figure it is possible to determine the optimal transmission probability for  $CRN_1$ . The results imply that the optimal transmission probability of the first network is independent of the transmission probability of the second network. However, as the network load of the second network is increased, the throughput of the first network decreases. The false alarm probability of  $CRN_2$  ( $P_{f,2}$ ) has a reversed effect on the performance of  $CRN_1$  since if  $P_{f,2}$  is increased,  $CRN_2$  will transmit more rarely which means that it is not as active from the perspective of  $CRN_1$ . Thus, for  $CRN_1$  it would

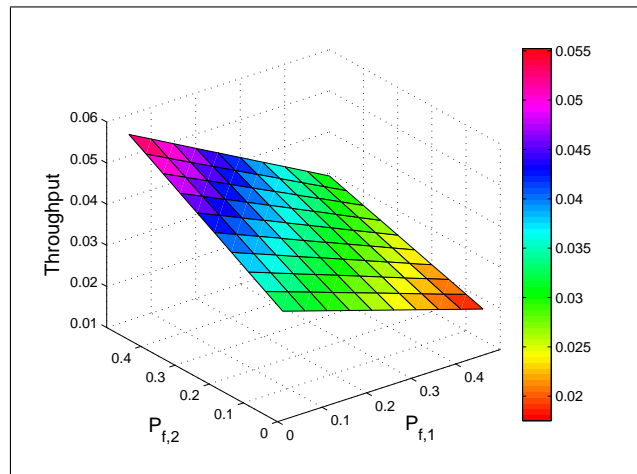


Figure 4: Effect of false alarm probabilities on the throughput of  $CRN_1$ .

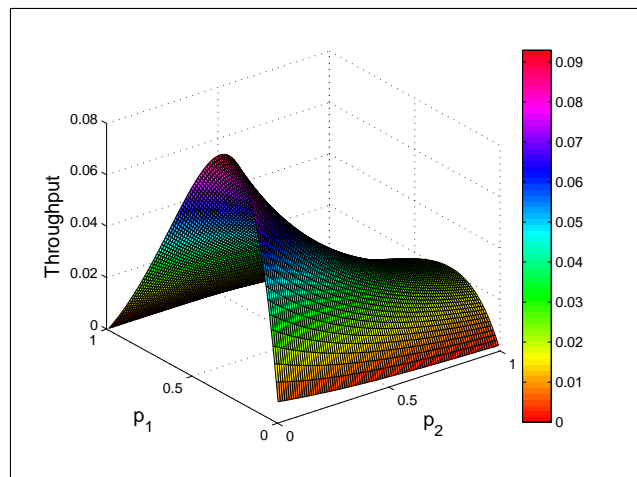


Figure 5: Impact of transmission probabilities of CR networks on the throughput of  $CRN_1$ .

be beneficial to have as low  $p_2$  and as high  $P_{f,2}$  as possible.

Nevertheless, it should be noted that the optimal value of  $p_1$  depends on the amount of intra-network interference. With these parameters the term  $p(1-p)$  in Equation (11) dominates the performance of  $CRN_1$  since the throughput is maximized when  $p_1 = 0.5$ . Whereas, if the amount of received intra-network is larger, i.e., the network density of  $CRN_1$  is higher,  $\Pr\{\gamma > \theta\}$  becomes dominant. Consequently, smaller values of  $p_1$  will give the best performance in that case.

In case of secondary spectrum usage, the activity of the PU determines the amount of transmission opportunities for CR users. Even though there would be large portions of available spectrum in time, high false alarm probabilities of CR users will restrict the achievable throughput. This is shown in Figure 6 where the throughput of  $CRN_1$  is plotted as a function of  $P(H_0)$  and  $P_{f,2}$ . If the PU is active for the most of the time, high probabilities of false alarm have only a minor effect on the throughput. Nevertheless, if the PU is inactive often, the probability of false alarm affects the performance significantly. In any case it is beneficial for  $CRN_1$  to have as high  $P(H_0)$

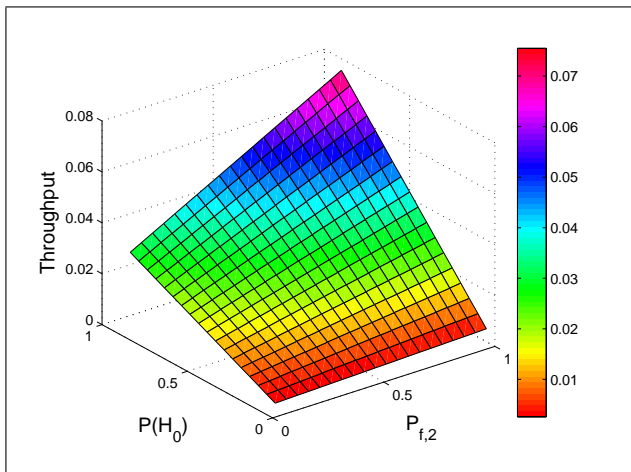


Figure 6: Combined effect of primary user's activity and false alarm probability of  $CRN_2$  on the performance of  $CRN_1$ .

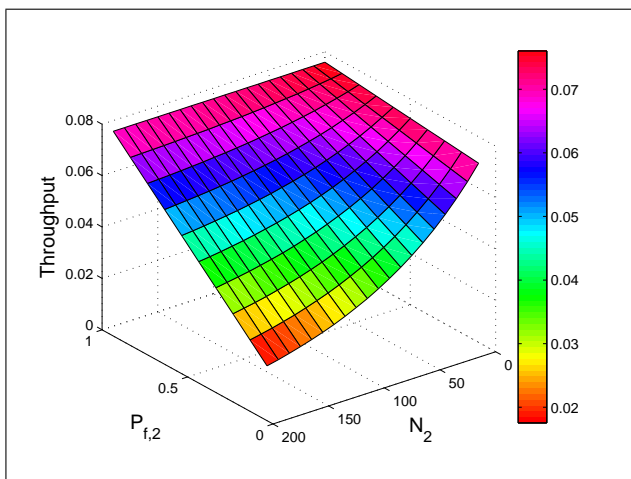


Figure 7: Effects of the network density and probability of false alarm of  $CRN_2$  on the performance of  $CRN_1$ .

and  $P_{f,2}$  as possible for throughput maximization.

Network density is another important parameter that should be taken into account when embarking on network design. In Figure 7 the combined effect of the network density and probability of false alarm of  $CRN_2$  on the throughput of  $CRN_1$  is displayed. The network density has an important impact on the achievable performance since the throughput diminishes significantly as the number of nodes within the area is increased. According to these results it seems that the effect of network density on the performance is smaller in case of high false alarm probabilities. This is due to the fact that in such cases transmissions take place rarely. To achieve high per-node throughput small networks are preferred. Nevertheless, this may not be true if the total throughput of the network would be considered but we leave this as future work since the focus of this paper is on per-node throughput.

## V. CONCLUSIONS

This paper studied the performance of overlapping CR networks which coexist together with a PU. The performance

of CR networks in such situations was evaluated by investigating the achievable per-node throughput. For the analysis, an interference model was derived which was then used to find out the per-node throughput. It was shown that the theoretical results agree with simulation results. The results show the impact of the interfering CR network on the performance of a CR network. Furthermore, the feasibility of a CR network was analyzed in the presence of another CR network. These results can be exploited to study the achievable throughput of overlapping CR networks in different scenarios and they serve as guidance for the deployment of multiple CR networks as well.

## VI. ACKNOWLEDGMENT

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