GLRT-Based Cooperative Sensing in Cognitive Radio Networks with Partial CSI

Xitao Gong, Adrian Ispas, and Gerd Ascheid
Institute for Communication Technologies and Embedded Systems
RWTH Aachen University, Templergraben 55, 52056 Aachen, Germany
Email:{gong, ispas, ascheid}@ice.rwth-aachen.de

***Abstract*—This paper studies cooperative spectrum sensing in cognitive radio networks. Both the sensing and the reporting channels are assumed to be either slow or fast fading channels. In a practical system, due to the lack of cooperation between the primary and secondary users, the power of the primary signal, the channel state information of the sensing channels as well as the noise power level are unknown. We first provide an analytically tractable signal model using a Gaussian approximation and then propose generalized likelihood ratio test (GLRT) methods for the design of the corresponding detectors. The basic idea lies in the fact that the unknown parameters can be estimated by exploiting hidden information in the sample covariance matrix of the received signals. The effectiveness of the proposed GLRT-based sensing methods are validated through numerical results.**

I. Introduction

Cognitive radio (CR) is a promising technique to solve the inefficient spectrum usage problem by exploiting dynamic spectrum access [1]. The key requirements are the effectiveness in learning from the environment and the guarantee that the performance degradation perceived by the primary user (PU) is controlled. Spectrum sensing is a basic learning function in different paradigms of CR systems [2], [3].

Various individual sensing algorithms for a single secondary user (SU) have been proposed, see [4] and [5] for an overview. In cooperative CR networks, the sensing performance is further improved by exploiting spatial diversity among multiple SUs [6]. Specifically, each SU transmits its local decision or observation to the secondary fusion center (FC) to make a more reliable global decision. The imperfect sensing and reporting links can deteriorate the cooperative sensing performance [7]. The works in [8]–[10] investigated optimal linear cooperative sensing algorithms under fading sensing channels and only noisy reporting channels. If fading effects of both sensing and reporting channels are considered, a near-optimal sensing rule with statistical channel state information (CSI) is designed in [11]. However, most of the previous works did not consider the practical issue where the information of the sensing channels and the primary signal is missing. In such a scenario, the authors in [12] studied a practical method to estimate the unknown parameters based on the idea to categorize the sensing observations into two classes. Each class represents the presence or absence of the PU according to the final global decision made at the FC. Therefore, the performance highly depends on the reliability of the global decision. Alternatively, the generalized likelihood ratio test (GLRT) principle [13] is widely used to tackle the binary hypothesis testing problem with unknown parameters, e.g., multi-antenna based sensing for a single SU with unknown transmit covariance matrix [14]. It can also be applied in cooperative sensing when part of the *a priori* information on the signal and the channel is not available.

This paper investigates cooperative sensing methods when both sensing and reporting channels are modeled as fading channels. Two kinds of fading channels are applied and the corresponding CSI type is considered. For slow fading channels, the CSI refers to the block fading, while for fast fading channels, the CSI refers to the statistical CSI. We focus on the study in a practical scenario where only the CSI and noise variances of the reporting channels are available. This assumption is justified since the SUs may transmit pilots to the FC for the estimation of the CSI of the reporting channels, while the information of the sensing channels are difficult to obtain due to the lack of cooperation between the PUs and the SUs. We use a Gaussian approximation to provide an analytically tractable signal model and then propose GLRT-based approaches for the design of sensing rules. The unknown parameters are embedded in the sample covariance matrix of the received signal and can be estimated. The effectiveness of the GLRT-based methods is validated by the comparison with the developed near-optimal approximated likelihood ratio test (ALRT) method and an energy detection (ED).

Notation: Let $(\mathbf{A})^T$, $(\mathbf{A})^H$, $(\mathbf{A})^*$, $\mathbb{E}\{\mathbf{A}\}$, and $\mathrm{tr}\{\mathbf{A}\}$ refer to the transpose, conjugate transpose, conjugate, expectation and trace of $\mathbf{A}$, respectively. $[\mathbf{A}]_{i,j}$ is the entry in the ith row and the jth column of $\mathbf{A}$. $\mathrm{diag}\{\mathbf{a}\}$ returns a square matrix with the elements of $\mathbf{a}$ on the diagonal and the other entries are all zero. $\mathbf{I}_N$ denotes the identity matrix of size N. $(a)^+$ represents $\max(a, 0)$. $\odot$ indicates the Hadamard product. $\mathbf{x} \sim \mathcal{CN}(\boldsymbol{\mu}, \mathbf{C})$ means $\mathbf{x} - \boldsymbol{\mu}$ is zero-mean circularly symmetric complex Gaussian distributed with covariance matrix $\mathbf{C}$.

II. System Model

Fig. 1 depicts a cooperative CR network composed of K SUs and a secondary FC. The binary hypotheses indicating the absence and presence of the PU are given by $\mathcal{H}_0$ and $\mathcal{H}_1$, respectively.

The cooperative sensing includes two parts: sensing and reporting. In the sensing procedure, each SU performs spectrum sensing based on the observation over M sampling times. Under $\mathcal{H}_1$, the primary signal is assumed to be a zero-mean temporally and spatially white signal with unit power for each transmit antenna. The primary transmit signal at the mth time instant is $\tilde{\mathbf{s}}(m) \in \mathbb{C}^{N_t \times 1}$, where N_t is the number of transmit

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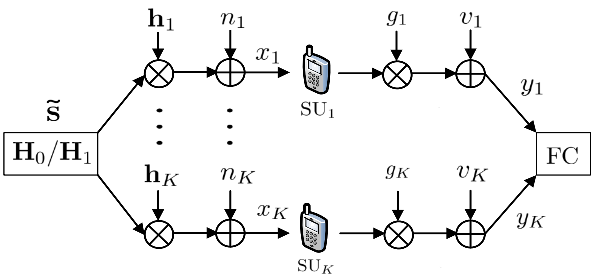

Fig. 1. Cooperative sensing in the secondary network with K SUs and FC at time instant m.

antennas at the PU. Assuming each SU has a single receive antenna and denoting $\mathbf{h}_k(m) \in \mathbb{C}^{1\times N_t}$ as the stationary Rayleigh fading channel from the PU to the kth SU at the mth instant, we obtain the matrix $\mathbf{H}(m) = [\mathbf{h}_1(m)^T, \cdots, \mathbf{h}_K(m)^T]^T$. The signal part received by the SUs at the mth instant is

$$\mathbf{s}(m) = \mathbf{H}(m)\tilde{\mathbf{s}}(m) \tag{1}$$

and $\mathbf{\Sigma}_S$ is the covariance matrix of $\mathbf{s}(m)$.

Assume that $x_k(m)$ is the mth received signal sample at the kth SU, the received signals at K SUs are stacked into a vector $\mathbf{x}(m) = [x_1(m), \cdots, x_K(m)]^T$ as

$$\mathbf{x}(m) = \begin{cases} \mathbf{n}(m), & \mathcal{H}_0 \\ \mathbf{s}(m) + \mathbf{n}(m), & \mathcal{H}_1 \end{cases}, \quad m = 1, \cdots, M \tag{2}$$

where the temporally and spatially white noise is $\mathbf{n}(m) = [n_1(m), \cdots, n_K(m)]^T$ with $n_k(m), k = 1, \cdots, K$ being the noise sample for the kth SU at the mth time instant. We assume $\mathbf{n}(m) \sim \mathcal{CN}(0, \sigma_n^2 \mathbf{I}_K)$. $\{\mathbf{s}(m)\}$ and $\{\mathbf{n}(m)\}$ are mutually independent.

In the reporting procedure, the FC receives the signals relayed by each SU through the reporting channels in an orthogonal manner [8],

$$\mathbf{y}(m) = \mathbf{G}(m)\mathbf{x}(m) + \mathbf{v}(m) \tag{3}$$

where $\mathbf{G} = \mathrm{diag}\{\mathbf{g}(m)\}$ with $\mathbf{g}(m) = [g_1(m), \cdots, g_K(m)]^T$ and $g_k(m)$ representing the stationary fading coefficient from the kth SU to the FC. We remark that $g_k(m)$ is not restricted to Rayleigh fading channels, e.g., a more general Rician fading channel model can be applied if there is a line of sight between the SU and the FC. $\mathbf{v}(m) = [v_1(m), \cdots, v_K(m)]^T$ is the vector containing the temporally and spatially white noise samples of the reporting channel and $\mathbf{v}(m) \sim \mathcal{CN}(0, \sigma_v^2 \mathbf{I}_K)$.

Two kinds of fading channel models are applied to both sensing and reporting channels. We also assume different types of available CSI for two channel models, respectively.

- Fast fading channels: the instantaneous CSI varies for different time instants and is hard to obtain. Thus, only statistical CSI is assumed to be available. We use $\mathbf{\Sigma}_H = \mathbb{E}\{\mathbf{H}(m)\mathbf{H}(m)^H\}$, $\mathbf{g}(m) \sim \mathcal{CN}(\bar{\mathbf{g}}, \mathbf{\Sigma}_g)$. Consequently, we obtain
$$\mathbf{\Sigma}_S = \mathbf{\Sigma}_H, \quad \mathbf{R}_g = \mathbf{\Sigma}_g + \bar{\mathbf{g}}\bar{\mathbf{g}}^H \tag{4}$$
where $\mathbf{R}_g$ is the correlation matrix of $\mathbf{g}(m)$.
- Slow fading channels, i.e., $\{\mathbf{H}(m)\}$ and $\{\mathbf{g}(m)\}$ are the block fading values that are fixed for $m = 1, \cdots, M$. Therefore, the time argument m in $\mathbf{H}(m)$ and $\mathbf{g}(m)$ is dropped for simplicity. We obtain
$$\mathbf{\Sigma}_S = \mathbf{H}\mathbf{H}^H, \quad \mathbf{R}_g = \mathbf{g}\mathbf{g}^H. \tag{5}$$

III. Sensing Algorithms

In this section, we design the sensing rule at the FC to combine $\mathbf{y}(m)$, $m = 1, \cdots, M$. Two sensing performance measures are used: the false alarm rate P_{FA} and the detection probability P_{D}. The optimization is aiming at increasing the detection probability given a certain P_{FA}. In the first IEEE CR standard 802.22 Wireless Regional Area Network (WRAN), the required P_{FA} should be smaller than 0.1 [15].

According to the Neyman-Pearson optimality criterion, for the binary hypothesis testing problem, the optimal test statistic to maximize the P_{D} given a fixed P_{FA} is the following log LRT (LLRT) scheme [13]:

$$\Lambda_{LLRT}(\tilde{\mathbf{y}}) = \ln \frac{p(\tilde{\mathbf{y}}|\mathcal{H}_1)}{p(\tilde{\mathbf{y}}|\mathcal{H}_0)} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \gamma \tag{6}$$

where γ is the threshold and $\tilde{\mathbf{y}} = [\mathbf{y}^T(1), \cdots, \mathbf{y}^T(M)]^T$. The conditional probability density function (PDF) $p(\tilde{\mathbf{y}}|\mathcal{H}_i)$, $i = 0, 1$ is difficult to calculate since each $\mathbf{y}(m)$ contains, e.g., a multiplication of two Gaussian distributions [11]. Instead, we approximate $p(\tilde{\mathbf{y}}|\mathcal{H}_i)$ by using a Gaussian distribution $p_G(\tilde{\mathbf{y}}|\mathcal{H}_i)$. Since the observations $\{\mathbf{y}(m)\}$ are independent with the Gaussian assumption, we have

$$p_G(\tilde{\mathbf{y}}|\mathcal{H}_i) = \prod\nolimits_{m=1}^{M} p_G(\mathbf{y}(m)|\mathcal{H}_i).$$

Dropping the time argument m for $\mathbf{y}(m)$, we obtain $p_G(\mathbf{y}|\mathcal{H}_i) \sim \mathcal{CN}(\boldsymbol{\mu}_i, \mathbf{\Sigma}_i)$, where $\boldsymbol{\mu}_i$ and $\mathbf{\Sigma}_i$ are

$$\boldsymbol{\mu}_i = \mathbb{E}\{\mathbf{y}|\mathcal{H}_i\} \tag{7}$$
$$\mathbf{\Sigma}_i = \mathbb{E}\{\mathbf{y}\mathbf{y}^H|\mathcal{H}_i\} - \boldsymbol{\mu}_i\boldsymbol{\mu}_i^H. \tag{8}$$

In Appendix A, (7) and (8) are calculated as

$$[\boldsymbol{\mu}_i]_k = \bar{g}_k \bar{x}_{k,i} \tag{9}$$
$$[\mathbf{\Sigma}_i]_{k,k'} = \mathbb{E}\{x_k x_{k'}^*\}\mathbb{E}\{g_k g_{k'}^*\} + \sigma_v^2 \delta_{k,k'} - [\boldsymbol{\mu}_i]_k [\boldsymbol{\mu}_i]_{k'} \tag{10}$$

where $\bar{g}_k = \mathbb{E}(g_k)$, and $\bar{x}_{k,i} = \mathbb{E}(x_k|\mathcal{H}_i)$.

According to (2), the received signal $\mathbf{x}$ can be approximated by the following Gaussian distribution

$$\mathbf{x} \sim \begin{cases} \mathcal{CN}(\mathbf{0}, \sigma_n^2 \mathbf{I}_K), & \mathcal{H}_0 \\ \mathcal{CN}(\mathbf{0}, \mathbf{\Sigma}_S + \sigma_n^2 \mathbf{I}_K), & \mathcal{H}_1 \end{cases} \tag{11}$$

Integrating (11) into (9) and (10), the mean and covariance matrices of $\mathbf{y}$ are obtained as

$$\boldsymbol{\mu}_0 = \mathbf{0},\ \mathbf{\Sigma}_0 = \sigma_n^2 \mathbf{I}_K \odot \mathbf{R}_g + \sigma_v^2 \mathbf{I}_K \tag{12}$$
$$\boldsymbol{\mu}_1 = \mathbf{0},\ \mathbf{\Sigma}_1 = (\mathbf{\Sigma}_S + \sigma_n^2 \mathbf{I}_K) \odot \mathbf{R}_g + \sigma_v^2 \mathbf{I}_K \tag{13}$$

where $\mathbf{\Sigma}_\mathbf{S}$ and $\mathbf{R}_g$ are given in (4) and (5) for two kinds of fading channels.

A. Approximated Likelihood Ratio Test

If the exact PDF $p(\tilde{\mathbf{y}}|\mathcal{H}_i)$ is replaced with the Gaussian approximation $p_G(\tilde{\mathbf{y}}|\mathcal{H}_i)$, it yields the near-optimal ALRT method for known CSI and noise variances of both sensing and reporting channels. Specifically, substituting $p_G(\tilde{\mathbf{y}}|\mathcal{H}_i)$ into (6) and integrating the constant items into the threshold design, the test statistic of the ALRT detector is given as

$$\Lambda_{ALRT}(\tilde{\mathbf{y}}) = \sum_{m=1}^{M} \mathbf{y}^H(m)(\mathbf{\Sigma}_0^{-1} - \mathbf{\Sigma}_1^{-1})\mathbf{y}(m).$$

B. Energy Detection

If the FC sums up the energy of the signals relayed by each SU, the detection scheme becomes the widely-used ED

$$\Lambda_{ED}(\tilde{\mathbf{y}}) = \sum_{m=1}^{M} \mathbf{y}^H(m)\mathbf{y}(m). \tag{14}$$

Λ_{ED} can be considered as the special case of ALRT when $\mathbf{\Sigma}_0^{-1} - \mathbf{\Sigma}_1^{-1}$ is an identity matrix. ED requires minimum knowledge of the primary signal such as the center frequency and bandwidth, but no *a priori* information on the signal nor on the CSI.

C. GLRT-Based Detection Algorithms

In a practical CR system, the power of the primary signal and the CSI of the sensing channels are always hard to obtain due to the lack of cooperation between the PU and the SUs. Therefore, the sensing problem becomes a binary hypothesis testing problem with unknown parameters, which can be solved with the GLRT principle [13]. Asymptotically, GLRT is equivalent to the uniformly most powerful among all tests that are invariant [16]. The GLRT-based method substitutes the unknown parameters with their maximum likelihood estimates (MLE)s and applies the LLRT principle in (6). Specifically, if θ_0 and θ_1 are the unknown parameters under $\mathcal{H}_0$ and $\mathcal{H}_1$, respectively, the GLRT test statistic is written as

$$\Lambda_{GLRT}(\tilde{\mathbf{y}}) = \ln \frac{p_G(\tilde{\mathbf{y}}|\mathcal{H}_1, \hat{\theta}_1)}{p_G(\tilde{\mathbf{y}}|\mathcal{H}_0, \hat{\theta}_0)} \tag{15}$$

with
$$\hat{\theta}_0 = \arg\max_{\theta_0} p_G(\tilde{\mathbf{y}}|\mathcal{H}_0, \theta_0) \tag{16}$$

$$\hat{\theta}_1 = \arg\max_{\theta_1} p_G(\tilde{\mathbf{y}}|\mathcal{H}_1, \theta_1). \tag{17}$$

Note that the exact distribution $p(\tilde{\mathbf{y}}|\mathcal{H}_i)$ with $i = 0, 1$ is replaced with the approximated Gaussian distribution $p_G(\tilde{\mathbf{y}}|\mathcal{H}_i)$ for the sake of computational tractability.

1) GLRT1, Unknown $\mathbf{\Sigma}_S$: If the covariance matrix $\mathbf{\Sigma}_S$ is unknown, i.e., the power of the primary signal and the CSI of the sensing channel are unknown, they can be estimated by exploiting the structure of the sample covariance matrix of the received signals. Specifically, the log-likelihood function (LLF) under $\mathcal{H}_0$ is

$$\begin{aligned}\ln p_G(\tilde{\mathbf{y}}|\mathcal{H}_0) = &- KM\ln\pi - M\ln\det\mathbf{\Sigma}_0 \\ &- \sum_{m=1}^{M} \mathbf{y}^H(m)\mathbf{\Sigma}_0^{-1}\mathbf{y}(m)\end{aligned} \tag{18}$$

where all parameters in (18) are known, i.e., there is no parameter θ_0 in (15). By using a decomposition of the form

$$\mathbf{\Sigma}_0 = \mathbf{L}_1^H\mathbf{L}_1$$

(18) is reformulated as

$$\ln p_G(\tilde{\mathbf{y}}|\mathcal{H}_0) = -KM\ln\pi - M\ln\det\mathbf{L}_1^H\mathbf{L}_1 - M\text{tr}\{\mathbf{R}_y\} \tag{19}$$

where

$$\mathbf{R}_y = \mathbf{L}_1^{-H}\left(\frac{1}{M}\sum_{m=1}^{M}\mathbf{y}(m)\mathbf{y}^H(m)\right)\mathbf{L}_1^{-1}.$$

Similarly, the LLF under $\mathcal{H}_1$ is reformulated as follows due to the temporal whiteness of $\tilde{\mathbf{s}}(m)$, $m = 1, \cdots, M$

$$\begin{aligned}&\ln p_G(\tilde{\mathbf{y}}|\mathcal{H}_1) \\ &= -KM\ln\pi + M\ln\det\left(\mathbf{L}_1^{-1}\mathbf{B}\mathbf{L}_1^{-H}\right) - M\text{tr}\{\mathbf{R}_y\mathbf{B}\}.\end{aligned} \tag{20}$$

where

$$\mathbf{B} = \left(\mathbf{L}_1^{-H}\left(\mathbf{\Sigma}_S \odot \mathbf{R}_g\right)\mathbf{L}_1^{-1} + \mathbf{I}_K\right)^{-1} \tag{21}$$

According to (21), the unknown parameter $\mathbf{\Sigma}_S$ is only included in $\mathbf{B}$, i.e., the unknown parameter θ_1 in (15) is $\mathbf{B}$. The MLE of $\mathbf{B}$ is obtained by solving the following constrained optimization problem

$$\begin{aligned}\max_{\mathbf{B}} \quad & \ln\det\mathbf{B} - \text{tr}\{\mathbf{R}_y\mathbf{B}\} \\ \text{s.t.} \quad & \mathbf{B} \succeq \mathbf{0} \\ & \mathbf{B} \preceq \mathbf{I}_K.\end{aligned}$$

Similar to [14, Appendix], we obtain the MLE of $\mathbf{B}$

$$\hat{\mathbf{B}} = \mathbf{U}_y\text{diag}\left\{\min\left(\frac{1}{\lambda_{1,y}}, 1\right), \cdots, \min\left(\frac{1}{\lambda_{K,y}}, 1\right)\right\}\mathbf{U}_y^H \tag{22}$$

where $\mathbf{U}_y$ contains the eigenvectors of $\mathbf{R}_y$ and $\lambda_{k,y}$, $k = 1, \cdots, K$ are the corresponding K eigenvalues listed in descending order, i.e., $\lambda_{1,y} \geq \lambda_{2,y} \geq \cdots \geq \lambda_{K,y}$.

Taking the MLE result (22) into (20) and subtracting (19), results in the test statistic of GLRT1

$$\mathbf{\Lambda}_{GLRT1}(\tilde{\mathbf{y}}) = M\ln\det\hat{\mathbf{B}} - M\text{tr}\{\mathbf{R}_y\hat{\mathbf{B}}\} + M\text{tr}\{\mathbf{R}_y\}. \tag{23}$$

Using (22), we reformulate the two terms containing $\hat{\mathbf{B}}$ as

$$\ln\det\hat{\mathbf{B}} = -\ln\prod_{k=1}^{m_1}\lambda_{k,y} \tag{24}$$

$$\text{tr}\{\mathbf{R}_y\hat{\mathbf{B}}\} = m_1 + \sum_{k=m_1+1}^{K}\lambda_{k,y} \tag{25}$$

where m_1 refers to the largest m_1 such that $\lambda_{m_1,y} \geq 1$. Integrating (24) and (25) into (23) results in the final test statistic of GLRT1

$$\mathbf{\Lambda}_{GLRT1}(\tilde{\mathbf{y}}) = Mm_1\left(\frac{1}{m_1}\sum_{k=1}^{m_1}\lambda_{k,y} - \ln\left(\prod_{k=1}^{m_1}\lambda_{k,y}\right)^{\frac{1}{m_1}} - 1\right).$$

2) GLRT2, Unknown $\mathbf{\Sigma}_S$ and σ_n^2: In this case, both noise variance σ_n^2 of the sensing channels and $\mathbf{\Sigma}_S$ need to be estimated under both hypotheses $\mathcal{H}_0$ and $\mathcal{H}_1$.

First, the LLF under $\mathcal{H}_0$ with the unknown σ_n^2 is written as

$$\ln p_G(\tilde{\mathbf{y}}|\mathcal{H}_0) = -KM\ln\pi - M\ln\det\mathbf{\Sigma}_0 - M\text{tr}\{\mathbf{\Sigma}_0^{-1}\mathbf{R}_{\bar{y}}\}. \tag{26}$$

where

$$\mathbf{R}_{\bar{y}} = \frac{1}{M}\sum_{m=1}^{M}\mathbf{y}(m)\mathbf{y}^H(m) \overset{(a)}{=} \mathbf{U}_{\bar{y}}\mathbf{\Lambda}_{\bar{y}}\mathbf{U}_{\bar{y}}^H. \tag{27}$$

The step (a) in (27) indicates the eigenvalue decomposition of $\mathbf{R}_{\bar{y}}$, in which $\mathbf{\Lambda}_{\bar{y}} = \text{diag}\{\lambda_{1,\bar{y}}, \cdots, \lambda_{K,\bar{y}}\}$ contains all the eigenvalues of $\mathbf{R}_{\bar{y}}$ as diagonal entries in descending order. By applying a similar mathematical manipulation as in Section III-C1, the MLE of σ_n^2 is derived as

$$\widehat{\sigma_n^2}|\mathcal{H}_0 = \frac{1}{K}\mathrm{tr}\{\mathbf{C}\} \tag{28}$$

with $[\mathbf{C}]_{i,j} = \dfrac{[\mathbf{U}_{\tilde{y}}\mathrm{diag}\{(\lambda_{1,\tilde{y}}-\sigma_v^2)^+,\cdots,(\lambda_{K,\tilde{y}}-\sigma_v^2)^+\}\mathbf{U}_{\tilde{y}}^H]_{i,j}}{[\mathbf{R}_g]_{i,j}}$

Similarly, the LLF under $\mathcal{H}_1$ with the unknown $\mathbf{\Sigma}_S$ and σ_n^2 is

$$\ln p_G(\tilde{\mathbf{y}}|\mathcal{H}_1) = -KM\ln\pi - M\ln\det\mathbf{\Sigma}_1 - M\mathrm{tr}\{\mathbf{\Sigma}_1^{-1}\mathbf{R}_{\tilde{y}}\}. \tag{29}$$

The MLE of the unknown parameters under $\mathcal{H}_1$ is

$$\widehat{\mathbf{\Sigma}_S+\sigma_n^2\mathbf{I}_K}|\mathcal{H}_1 = \mathbf{C}. \tag{30}$$

Integrating (28) and (30) into (26) and (29), the GLRT test statistic is given as

$$\Lambda_{GLRT2}(\tilde{\mathbf{y}}) = -M\ln\prod_{k=1}^{m_2}\frac{\lambda_{k,\tilde{y}}}{\sigma_v^2} - Mm_2 - M\sum_{k=m_2+1}^{K}\frac{\lambda_{k,\tilde{y}}}{\sigma_v^2} + M\ln\prod_{k=1}^{K}\lambda_{k,e} + M\mathrm{tr}\{\mathbf{E}^{-1}\mathbf{R}_{\tilde{y}}\}$$

where $\mathbf{E} = (\widehat{\sigma_n^2}|\mathcal{H}_0)\mathbf{I}_K \odot \mathbf{R}_g + \sigma_v^2\mathbf{I}_K$

and $\lambda_{k,e}$, $k=1,\cdots,K$ are the eigenvalues of $\mathbf{E}$. m_2 refers to the largest m_2 such that $\lambda_{m_2,\tilde{y}} \geq \sigma_v^2$.

3) Remarks on the GLRT-Based Methods: The performance of the GLRT-based methods depends on whether the statistical properties of the estimated parameters are distinguishable under both hypotheses. Specifically, the following two properties are exploited: under $\mathcal{H}_1$, the received signals from multiple SUs at the FC are correlated and have unbalanced power levels due to different properties of the sensing and the reporting channels; under $\mathcal{H}_0$, the received noises are uncorrelated and their variances are same. However, in fast fading scenario, if the reporting channel has low spatial correlation, i.e., $\mathbf{R}_g$ is a scaled identity matrix, the correlation effect diminishes at the FC according to (12) and (13). This results in a performance degradation of the GLRT-based methods, since only the property of unbalanced power levels is used to identify the presence/absence of the PU signal. Nevertheless, the performance can be improved by other approaches, e.g., prolonging the sensing length.

IV. Numerical Results

We consider a CR network with $K = 4$ SUs detecting one PU with a single transmit antenna. The primary signals are quadrature phase-shift keying modulated signals with unit variance. Each channel $\mathbf{h}_k(m)$ or $\mathbf{g}_k(m)$ is assumed to be a Rayleigh fading channel. The SNRs of the sensing and the reporting channels are specified in each figure. The noise variance of the reporting links is $\sigma_v^2 = 0.8$. The noise uncertainty effect of the sensing links is considered, i.e., the estimated noise variance is $\sigma_n^2 = 1.5$ and the true noise variance is $\tilde{\sigma}_n^2 = \frac{1}{\alpha}\sigma_n^2$, where $10\log_{10}\alpha$ is uniformly distributed in $[-B, B]$ with noise uncertainty factor B in dB [17]. $B = 0\,$dB indicates that the noise variance is precisely known.

We compare four sensing algorithms for the performance evaluation. More specifically, we have

- ALRT: known matrix $\mathbf{\Sigma}_S$, CSI of the reporting channels $\mathbf{\Sigma}_g$, and noise variances σ_n^2 (estimated) and σ_v^2.

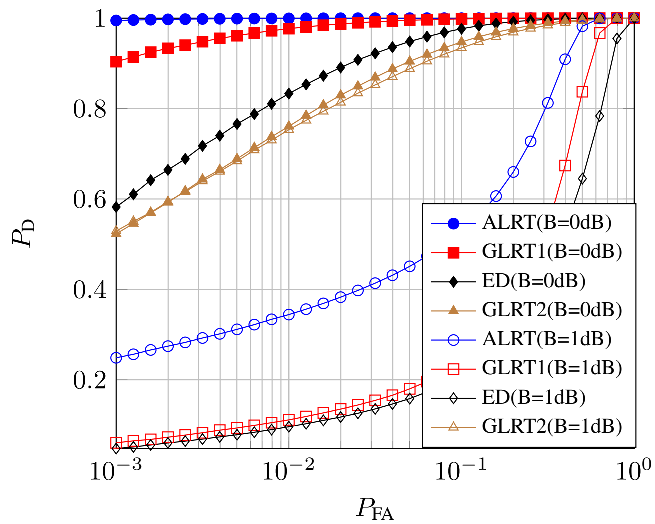

Fig. 2. ROC curve with noise uncertainty for slow fading channels. The SNRs of the sensing links are set to [-18, -11, -16, -10] dB and the SNRs of the reporting links are [5, 8, 6, 7] dB, M=1000.

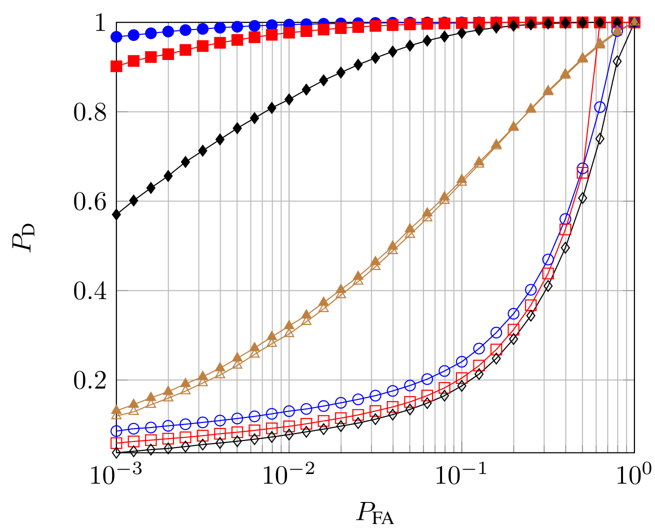

Fig. 3. ROC curve with noise uncertainty for fast fading channels. The SNRs of the sensing links are set to [-18, -11, -16, -10] dB and the SNRs of the reporting links are [8, 10, 12, 6] dB, M=5000. The legend is the same as in Fig. 3.

- ED: no *a priori* information on the primary signal and the CSI.
- GLRT1: known CSI of the reporting channels $\mathbf{\Sigma}_g$ and noise variances σ_n^2 (estimated) and σ_v^2, but unknown $\mathbf{\Sigma}_S$.
- GLRT2: known CSI and noise variance of the reporting channels $\mathbf{\Sigma}_g$ and σ_v^2, but unknown $\mathbf{\Sigma}_S$ and σ_n^2.

Fig. 2 plots the receive operating characteristic (ROC) curves for slow fading channels. The observation length is $M = 1000$. Two cases of noise uncertainty are considered: $B = 0$ dB and $B = 1$ dB. In the legend, the value of B is denoted in parentheses. We observe that if the noise variance σ_n^2 is perfectly known, the ALRT method performs the best due to complete *a priori* knowledge. Without the knowledge of $\mathbf{\Sigma}_S$, GLRT1 uses the hidden knowledge in the received signals and outperforms ED. If a noise uncertainty exists, GLRT2 always performs the best in the region of interest since the noise variance is estimated. As expected, GLRT2 is shown to be insensitive to the uncertainty, while the performance of the other three methods degrades severely. Similar observation are obtained from Fig. 3, where the ROC curves for fast fading channels are depicted. The corresponding correlation matrix $\mathbf{\Sigma}_H$ or $\mathbf{\Sigma}_g$ is a Toeplitz matrix $[\mathbf{\Sigma}_H]_{i,j} = 0.1^{|i-j|}$ and $[\mathbf{\Sigma}_g]_{i,j} = 0.1^{|i-j|}$, respectively. The observation length is $M = 5000$. The requirement of a long sensing length is due to the effect explained in Section III-C3.

V. Conclusion

In this paper, we have investigated cooperative spectrum sensing in a CR network with both sensing and reporting channels modeled as slow or fast fading channels. We considered two practical scenarios: first, the power of the primary signal and the CSI of the sensing channels are unknown, secondly, additionally to the first case, the noise variances of the sensing channels are unknown. A GLRT algorithm was proposed for each scenario by estimating the unknown parameters with the sample covariance matrix of the received signals and then applying the LLRT principle. Numerical results showed the effectiveness of both proposed methods, as well as the robustness to noise uncertainty for the second GLRT method.

Appendix A
Derivation of (9) and (10)

The mean and the covariance matrix of the received signals at the FC are derived under both hypotheses $\mathcal{H}_i$ with $i = 0, 1$. Specifically, the k-th element of $\boldsymbol{\mu}_i$ is

$$\begin{aligned}
&[\boldsymbol{\mu}_i]_k = \mathbb{E}(y_k|\mathcal{H}_i)\\
&= \int_{x_k,g_k,y_k} y_k p(y_k|x_k, g_k, \mathcal{H}_i) p(x_k, g_k|\mathcal{H}_i) dy_k dg_k dx_k\\
&= \int_{x_k} p(x_k|\mathcal{H}_i) \int_{g_k} p(g_k) \int_{y_k} y_k p(y_k|x_k, g_k, \mathcal{H}_i) dy_k dg_k dx_k\\
&= \int_{x_k} p(x_k|\mathcal{H}_i) \int_{g_k} g_k x_k p(g_k) dg_k dx_k\\
&= \bar{g}_k \bar{x}_{k,i}.
\end{aligned}$$

where $\bar{g}_k = \mathbb{E}(g_k)$ and $\bar{x}_{k,i} = \mathbb{E}(x_k|\mathcal{H}_i)$.

The k-th row and $k^{'}$-th column of the covariance matrix is

$$\begin{aligned}
&[\mathbf{\Sigma}_i]_{k,k'} = [\mathbb{E}\{\mathbf{y}\mathbf{y}^H|\mathcal{H}_i\} - \boldsymbol{\mu}_i\boldsymbol{\mu}_i^H]_{k,k'}\\
&= \int_{y_k,y_{k'}} (y_k - [\boldsymbol{\mu}_i]_k)(y_{k'} - [\boldsymbol{\mu}_i]_{k'})^* p(y_k, y_{k'}|\mathcal{H}_i) dy_k dy_{k'}\\
&\overset{(a)}{=} \int_{y_k,y_{k'}} (y_k - [\boldsymbol{\mu}_i]_k)(y_{k'} - [\boldsymbol{\mu}_i]_{k'})^*\\
&\quad \int_{x_k,x_{k'},g_k,g_{k'}} p(y_k, y_{k'}|x_k, x_{k'}, g_k, g_{k'}, \mathcal{H}_i)\\
&\quad p(x_k, x_{k'}, g_k, g_{k'}|\mathcal{H}_i) dx_k dx_{k'} dg_k dg_{k'} dy_k dy_{k'}\\
&\overset{(b)}{=} \int_{x_k,x_{k'}} p(x_k, x_{k'}|\mathcal{H}_i) \int_{g_k,g_{k'}} p(g_k, g_{k'})\\
&\quad \int_{y_k,y_{k'}} (y_k - g_k x_k + g_k x_k - [\boldsymbol{\mu}_i]_k)\\
&\quad (y_{k'} - g_{k'} x_{k'} + g_{k'} x_{k'} - [\boldsymbol{\mu}_i]_{k'})^*\\
&\quad p(y_k, y_{k'}|x_k, x_{k'}, g_k, g_{k'}, \mathcal{H}_i) dy_k dy_{k'} dg_k dg_{k'} dx_k dx_{k'}\\
&\overset{(c)}{=} \int_{x_k,x_{k'}} p(x_k, x_{k'}|\mathcal{H}_i) \int_{g_k,g_{k'}} p(g_k, g_{k'})\\
&\quad \int_{y_k,y_{k'}} (v_k v_{k'*} + (g_k x_k - [\boldsymbol{\mu}_i]_k)(g_{k'} x_{k'} - [\boldsymbol{\mu}_i]_{k'})^*)\\
&\quad p(y_k, y_{k'}|x_k, x_{k'}, g_k, g_{k'}, \mathcal{H}_i) dy_k dy_{k'} dg_k dg_{k'} dx_k dx_{k'}\\
&= \sigma_v^2 \delta_{k,k'} + \mathbb{E}(x_k x_{k'}^*)\mathbb{E}(g_k g_{k'}^*) - [\boldsymbol{\mu}_i]_k [\boldsymbol{\mu}_i]_{k'}
\end{aligned}$$

where step (a) uses the Bayesian rule to expand the conditional probability $p(y_k, y_{k'}|\mathcal{H}_i)$ as a function of $x_k, x_{k'}, g_k$, and $g_{k'}$. Step (b) expresses $p(x_k, x_{k'}, g_k, g_{k'}|\mathcal{H}_i)$ as the product of $p(x_k, x_{k'}|\mathcal{H}_i)$ and $p(g_k, g_{k'})$ by exploiting the mutual independency between the received signals and the reporting channels. The next step (c) simplifies the third integral in the last step by considering that $y_k - g_k x_k = v_k$ for all k and removes the items including the first order of v_k and $v_k^{'}$ because theirs means are both equal to zero.

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