Dynamic Spectrum Allocation for Heterogeneous Cognitive Radio Networks from Auction Perspective

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***Abstract*—For attracting primary users to participate in secondary spectrum market, auction was proposed as an alternative for spectrum trade. Existing auction schemes are either to be single-sided trade which only supports heterogeneous cognitive radio networks without guarantee of bid truthfulness, or to be truthful single-unit auction which only supports homogeneous channels. Few of them comprehensively take all these factors in practical spectrum trade into consideration such as spectrum allocation and reusability, channel diversity and economic property. A Truthful Bilateral Multi-unit Auction with features of supporting Heterogeneous networks(TBMAH) is proposed in this paper. We do experiments with both simulation and real networks, and the results show that TBMAH trades more spectrum resources than TRUST by 13.01% in average.**

I. Introduction

Many researchers are interested and engaged in solving how to dynamically utilize spectrum resources efficiently in recent years. So far many dynamic spectrum access technologies have been proposed and auction is one of the best-known market-driven mechanism among them. In contrast to the primary spectrum market conducted by spectrum administration department like FCC and its counterparts in other countries, we are mainly focused on secondary spectrum market, which is different from the primary one mainly in validation of time periods and regions.

Zhou[1] proposes the first truthful spectrum auction-VERITAS which supports diverse bidding formats and multiple objects auction, but VERITAS only addresses single-sided auction. Subramanian[2] presents a coordinated dynamic spectrum access architecture which is composed of multiple buyers characterized by heterogeneous channel width and one spectrum broker. This architecture adopts physical interference model and improves spectrum utility by multiplexing channel among non-conflict base stations. However, it is a single-sided auction and has no guarantee of truthfulness of bidding function. Zhou puts forward a general framework-TRUST for truthful bilateral spectrum auctions in[3][4]. For the purpose of avoiding bid manipulation, TRUST adopts a bid-independent buyer grouping method and employs a truthful auction mechanism like McAfee double auction. However, TRUST is a single-unit auction and only supports homogeneous channel. Other efforts either lead to market manipulation due to the loss of property of truthfulness like[5][6][7][8] or have no consideration of reusability like[9][10].

In order to build up a more practical secondary spectrum market, we analyze the problem comprehensively and propose a truthful bilateral multi-unit auction which supports participation of heterogeneous networks. The rest of our paper is organized as follows: section II describes spectrum auction problem formally and shows assumptions in the design of spectrum auction. We make detailed description of our auction scheme in section III and illustrate results in section IV with both simulation and real networks, and finally conclude in section V.

II. Spectrum Auction Problem Description

The scenario we considered consists of multiple spectrum providers and demanders. Spectrum providers want to earn additional revenues by leasing spectrum at their free time. Spectrum demanders require more spectrum resources to alleviate their heavy business load, and accordingly they should pay sellers some money as compensation. For the purpose of trading efficiently, we assume that there is a broker taking charge of building relationship between two sides, executing spectrum allocation, charging buyers and paying sellers in secondary spectrum market.

A. Spectrum Auction Problem Definition

Definition 1. *Spectrum auction is the process of determining winners and calculating clearing price while maximizing the total number of resources transacted. Each winning buyer is charged for the resources he obtained and each winning seller is payed for what he leased.*

We assume that the auction is bid-sealed and carried out periodically, neither sellers nor buyers collude, spectrum resources collected from different sellers are not necessarily continuous and demands of all buyers are not channel-specific. The auctioneer will stop auction process if ends up with an extra profit. For the purpose of simplicity, we adopt coverage-based interference model like [3], so we assume that the auctioneer is capable of obtaining coordinates and transmitted power of each network.

III. Design Details of TBMAH

A. Buyer Grouping problem

In order to utilize spectrum efficiently, we divide those nodes which can use the same spectrum into groups. Ac-

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cording to the coverage-based interference model we adopt, any two nodes within the same group are not in conflict with each other. Buyer grouping problem can be transformed into the problem of finding chromatic number or maximum independent set of a graph, which is NP-hard and many approximate algorithms have been proposed.

B. Bid Series Generation

These bidding groups are real participants of the auction. Bid series generation algorithm is described as follows.

Algorithm 1 Bid Series Generation Algorithm

Input: Buyer group set G with each element G_i consists of a certain number of non-conflict buyers.
Output: Bids of each group G_i in buyer group set G.
for G_i in G **do**
  $l_i = \max_{j=1}^{|G_i|} \left\{ B_j^n \times B_j^w \right\}$
  for $k = 1$ to l_i **do**
    for each buyer B_j $(1 \le j \le |G_i|)$ in group G_i **do**
      if $\lfloor (k-1)/B_j^w \rfloor + 1 = \lfloor k/B_j^w \rfloor$ **then**
        $total \mathrel{+}= B_j^w$ {total:net increase channel resource amount}
        $l_i = \min_{j=1}^{|G_i|} B_j^p$
      end if
    end for
    $Bid_i[k] \leftarrow Bid_i[k-1] + total \times p_i^k$
  end for
  for $k > l_i$ **do**
    $Bid_i[k] \leftarrow Bid_i[l_i]$
  end for
end for

As described in algorithm 1, we first check the largest amount of resources requested by each group. For any $k < l_i$ resource, divide the k resource into channels of buyer B_j. If k is wide enough to accommodate one more channel of buyer B_j, the money he is willing to pay should be accumulated with $B_j^w \times p_i^k$, in which p_i^k denotes the least unit spectrum price willing to pay by members of this group when k resources assigned. Assigning redundant resource is meaningless for this group, so they are not likely to pay more if allocated more resources than what they demanded.

C. Winners Determination

1) Winning buyer determination: With bidding series of competitive groups and the number of resources on sale, the auctioneer assigns spectrum with the goal of maximizing total revenues from these bidders. Variable SW_m^n is defined as the maximal revenue which can be obtained when m bidders waiting for allocation result and n resources left.

$$SW_m^n = \max_{i=0}^{n} \left(Bid_m^i + SW_{m-1}^{n-i} \right) \tag{1}$$

From descriptions in algorithm 2, it is obvious that we solve this problem with dynamic programming algorithm and record

Algorithm 2 Winning Buyer Determination Algorithm

Input: Bidding series generated in III-B for bidding group set G and the number of resources R for sale.
Output: Assignment for each group G_i in G.
for $i = 1$ to $|G|$ **do**
  for $j = 1$ to R **do**
    for $k = 1$ to j **do**
      if $Bid[i][k] + SW[i-1][j-k] > SW[i][j]$ **then**
        $SW[i][j] \leftarrow Bid[i][k] + SW[i-1][j-k]$
        $Assign[i][j] \leftarrow k$
      end if
    end for
  end for
end for
Initialize temporary variable $j = 0$.
for $i = |G|$ to 1 **do**
  $N[i] = Assign[i][R-j]$
  $j \mathrel{+}= N[i]$
end for

each intermediate state with an auxiliary two dimensional array $SW[|G|][R]$ for avoiding duplicated computations. Variable $Assign[i][j]$ denotes the optimal amount should be allocated to bidder i when j resource available and $SW[i][j]$ means the corresponding maximal revenue.

2) Winning seller determination: Our winning seller determination follows what McAfee proposed in [10], which is a VCG scheme in nature and is commonly used in truthful auction. Sort sellers by their bidding prices in non-decreasing order first, then find the proper argument k value.

D. Pricing

1) Costs of buyers: Before charing a single buyer, we need to calculate firstly how much each bidding group should pay when k resources assigned, then accumulate each buyer's charge with different k. The pricing algorithm for each group with k resources assigned is stated as algorithm3 shows.

According to descriptions of algorithm 3, we first derive net bid of each group G_i when k resources assigned from initial bid series generated in algorithm III-B. We can see that charge of bidding group G_i when k resources assigned equals to the difference of total revenue obtained under two situations. The first one is accumulated revenues when he is absent from this auction, and the other is sum of revenues from all other bidders when he participates. This is also the main idea of VCG truthful auction. Charge of each buyer B_j in group G_i can be computed with the following equation.

$$p_{ij} = \sum_{k=1, k\% B_j^w = 0}^{\min\left(N[i], B_j^n \times B_j^w\right)} \frac{C_i^k}{Bid_{net}[i][k]} \times p_i^k \times B_j^w \tag{2}$$

In equation 2, we accumulate charge of buyer B_j with different k which meets $1 \le k \le \min\left(N[i], B_j^n \times B_j^w\right)$ and

Algorithm 3 Pricing Algorithm for Bidding Groups

Input: Bidding series Bid, assignment solution N and the amount of available spectrum resource R.
Output: Pricing scheme C_i^k for group G_i in buyer groups set G when k resources assigned.
Derive net bid of each group G_i when k resources assigned Bid_{net} from bid series Bid.
for G_i in G **do**
  for $k = 1$ to $N[i]$ **do**
    Replace bid of group G_i when k resources assigned $Bid_{net}[i][k]$ with zero to construct a new bidding series Bid' and new net bidding series Bid'_{net}.
    Substitute Bid' and available resource R into algorithm 2, calculate out a new assignment N'.
    $rev1 = \sum_{j=1}^{|G|} \sum_{t=1}^{N'[j]} Bid'_{net}[j][t]$
    $rev2 = \sum_{j=1, j\neq i}^{|G|} \sum_{t=1}^{N[j]} Bid_{net}[j][t] + \sum_{t=1, t\neq k}^{N[j]} Bid_{net}[i][t]$
    $C_i^k = rev1 - rev2$
  end for
end for

is exactly divisible by B_j^w. $C_i^k / Bid_{net}[i][k]$ means the scaling factor between real charge and promising bid of group G_i with k resources allocated, and p_i^k is the least price for unit resource used for calculating net bid $Bid_{net}[i][k]$.

2) Revenues of sellers: For the sellers side auction, we simplify McAfee's double auction into single side auction. Revenues of each seller S_i is calculated with the following equation.

$$P_i = S_i^n \times {S'}_k^p \tag{3}$$

In equation 3, S_i^n denotes the number of resources S_i sold and ${S'}_k^p$ is the kth sorted price for unit spectrum resource.

E. Proof of Economic Properties

1) Individual Rationality: We prove individual rationality of winning buyers and winning sellers separately. If payments of all winning buyers are not bigger than their bid and revenues of all winning sellers are not less than their expected earnings, we say that they are rational.

Proof: According to equation 2 and algorithm 1, the difference between real charged value of buyer B_i^j and his promise equals:

$$\Delta_{ij} = \sum_{k=1, k\%B_j^w=0}^{\min\left(N[i], B_j^n \times B_j^w\right)} \left(\frac{C_i^k}{Bid_{net}[i][k]} - 1\right) \times p_i^k \times B_j^w < 0 \tag{4}$$

From calculation of C_i^k in algorithm 3, we know that C_i^k is critical value of net bid $Bid_{net}[i][k]$. According to definition of critical value and theorem 3.3 in [9] , we know that $C_i^k < Bid_{net}[i][k]$, so $\Delta_{ij} < 0$, that is to say, each buyer B_i^j is rational and will not be charged more than what he bids.

For each winning seller, the clearing price is the kth bidding price and sellers are sorted by their bidding price in non-decreasing order, so winning sellers are payed more than what they desire. ■

2) Truthfulness: In this section, we try to prove that each bidding buyer B_j can not improve its utility by bidding untruthfully. We examine four possible cases one by one when bidder B_j bids truthfully and untruthfully as the following parts describe.

Proof:

- Case 1: Either bidding truthfully or untruthfully in this case will make buyer B_j lose the auction, so buyer B_j will be charged with zero under both situations and his utility remains to be zero.
- Case 2: Buyer B_j loses when he bids untruthfully and wins when bids truthfully, as TBMAH always tries to maximize its revenue from potential buyers, so it is easy to infer that our auction scheme is monotonic. So his untruthful bid ${B'}_j^p$ must meet ${B'}_j^p < B_j^p$ in this case, because our auction is individual rational for each winning buyer, so every winner's utility is non-negative, which is not less than that when he bids untruthfully with utility equals to zero.
- Case 3: For the same reason of monotonicity of our auction scheme, this case only happens when ${B'}_j^p > B_j^p = V_j$. Suppose that B_j^p is not the lowest bid price of unit spectrum, according to algorithm 1, there must be a price $p < B_j^p$ which is used for calculating bidding series. When buyer B_j bids untruthfully with ${B'}_j^p$, bidding series and final assignment result should not change, so buyer B_j loses again. However, this is in contradiction with the fact that buyer B_j wins by bidder higher. So our hypothesis does not hold and B_j^p is the lowest bid when he bids truthfully. Let symbol δ denotes the net increasing amount of group G_i with k resources assigned. The following non-equation must exist.

$$\begin{aligned} & Bid_{net}[i][k] < C_i^k < Bid'_{net}[i][k] \\ & \Rightarrow \delta \times B_j^p < C_i^k < \delta \times {p'}_i^k \\ & \Rightarrow B_j^p < C_i^k / \delta < {p'}_i^k \end{aligned} \tag{5}$$

  In the above non-equation, B_j^p equals to estimated true value V_j of buyer B_j, ${p'}_i^k$ is the least unit spectrum price used for calculating bidding series,C_i^k/δ is the clearing price of unit spectrum. So when buyer B_j wins by bidding higher than its true value, his utility becomes $V_j - C_i^k/\delta$ is negative and less than the utility when he bids truthfully which equals to zero.
- Case 4: No matter buyer B_j bids truthfully or not, he will always win the auction, and will be charged with the same value which equals to his critical value under both situations. So his utility which is defined as the difference between his value and payment is invariant.

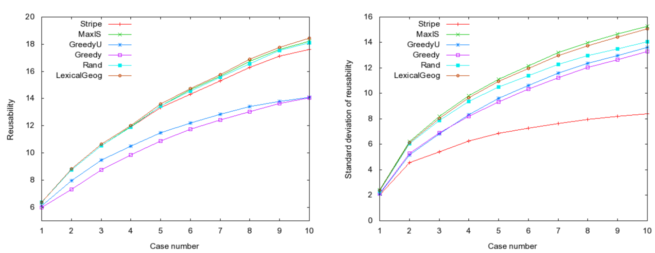
In summary, each potential buyer has no motivation to bid untruthfully and bidding truthfully becomes their dominating strategy. Proof of truthfulness for sellers side in TBMAH is similar to McAfee double auction, for the sake of saving space we omit this part, anyone who wants to see details, please reference to [10]. ■

3) Ex-post Budget Balance: As the auctioneer stops spectrum auction when its income is not less than outcome, so TBMAH keeps ex-post budget balance and the final transacted resources R is maximal under current bidding functions of bidding buyers.

IV. Experimental Results

A. Group Algorithms

Grouping algorithms we implemented include Stripe[11], Max-IS[12], Greedy-U[13], Greedy[13], Rand and Lexicographic[14]. Potential buyers with coverage radius equals to 50 are uniformly distributed within location 1000×1000, and each potential buyer with a circle coverage stands for a network. Network scale varies from 10 buyers 1 seller to 100 buyers 10 sellers. Experimental results are averaged over 500 times with each network size.



(a) Reusability (b) Standard deviation of reusability

Fig. 1. Group with different algorithms

Performances of different grouping algorithms are measured with reusability which is defined as average number of nodes in each group. Standard deviation of reusability characterizes uniformity among different groups. The X-coordinates of both figures are case index of different network scale. From Fig. 1(a) we can see that reusability of Greedy-U and Greedy algorithm are relatively lower than the other four algorithms. Result shown in Fig. 1(b) tells us that standard deviation of Stripe is minimum among all these algorithms.

B. Performance comparisons in different scenarios

In a practical spectrum auction, auction patterns can be single-unit homogeneous, single-unit heterogeneous, multi-unit homogeneous and multi-unit heterogeneous. However, TRUST can only support single-unit homogeneous, for the sake of comprehensive comparisons with our scheme we make small extensions of TRUST.

For single-unit homogeneous scenario, our spectrum auction scheme TBMAH is degraded to TRUST. Experimental result also validates that the number of resources transacted of TBMAH is the same as that of TRUST. However, for the other three scenarios which are commonly seen in practical auction, TBMAH outperforms extended version of TRUST with Stripe grouping algorithm, as can be seen in Fig.2. The X-coordinate values are still case index of different network scales. In Fig.2(b), the reason of extended version of TRUST exceeds TBMAH at the beginning is that increasing nodes by duplicating nodes with request large than 1 is favor of trade with sellers. Moreover, trades with different grouping algorithms obtain similar results, and we omit results of other grouping algorithms for the sake of saving space.

C. Impact of Distribution

From Fig.3(a) we can see that with seller's asking price increasing from 0.5 to 5, the number of resources transacted of both TBMAH and TRUST decreases due to elevation of spectrum price goes against transaction. With the amount of spectrum buyer requested increases from 1 to 10 in Fig.3(b), the transacted amount of both TBMAH and TRUST increases at the beginning due to abundant free spectrum resources and relative few requests, then both curves go flat because of reaching system's maximal capacity and not being able to accommodate any more requests. With buyer's request channel width increases, the number of resources transacted of TBMAH rises then becomes saturated. While the number of resources transacted of TRUST rises at the beginning due to oversupply. Due to increase of request channel width, the number of super sellers decreases and asking prices of them increase, the number of successful trade of extended version of TRUST declines afterwards.

D. Comparisons with Real Network

Besides making detailed comparisons between TBMAH and TRUST with simulation networks, we also study performance of them with real networks. We use locations of real cellular base stations available in FCC public GIS database [15], both TBMAH and TRUST are set to support multi-unit heterogeneous channel width.

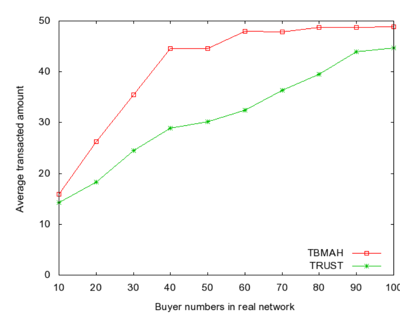
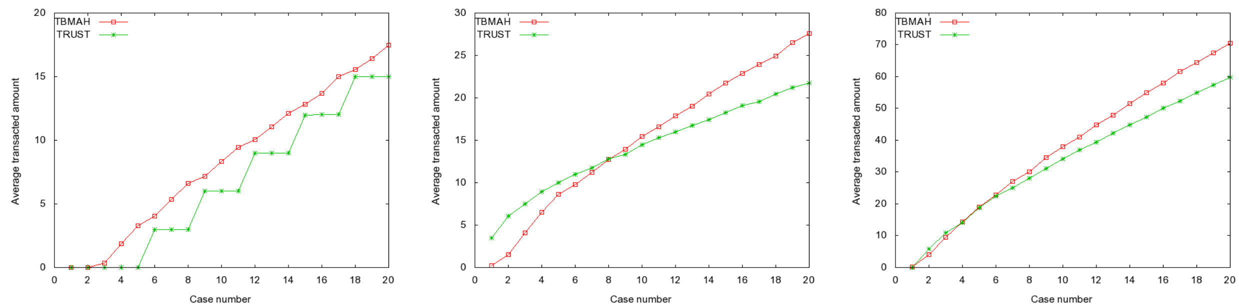


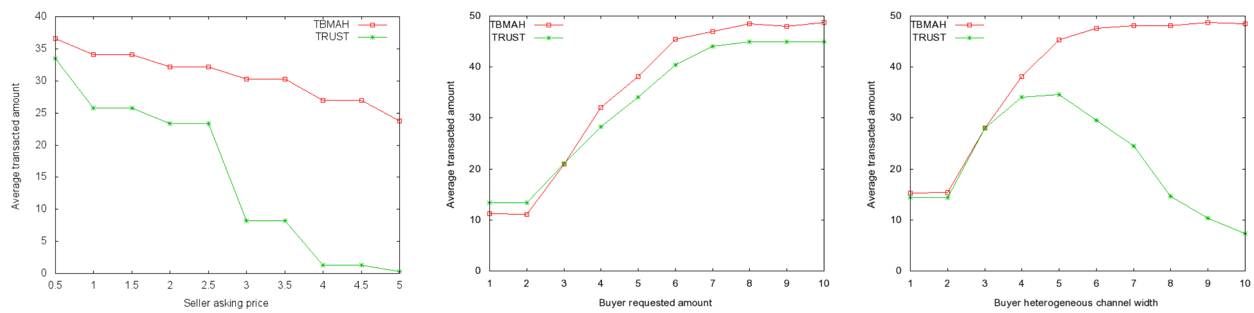
Fig. 4. TBMAH vs. TRUST with real networks

Fig.4 shows that TBMAH outperforms TRUST. With the increasing number of potential buyers, the number of transacted resources of both TBMAH and TRUST increases and gets saturated finally. It is obvious that due to limited available resources, the number of buyers required to get TBMAH saturated is less than that of TRUST. This is because that it is



(a) Single unit heterogeneous (b) Multi unit homogeneous (c) Multi unit heterogeneous

Fig. 2. TBMAH vs. TRUST in different scenarios



(a) Seller asking price distribution (b) buyer request amount distribution (c) buyer channel width distribution

Fig. 3. TBMAH vs. TRUST with different distributions

unnecessary for TBMAH to format diverse channel widths with the largest one and reconstruct the graph by making mirrors of nodes with multiple resources request.

V. Conclusion

We propose a truthful bilateral multi-unit auction with characteristic of supporting heterogeneous networks(TBMAH) in this paper. Definition of dynamic spectrum allocation problem from auction perspective and design details of TBMAH are described. Results of both simulation and real networks experiments show that TBMAH outperforms TRUST on the whole, especially in different scenarios, TBMAH trades more spectrum resources than TRUST by 13.01% in average.

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References

[1] X. Zhou, S. Gandhi, S. Suri, and H. Zheng. eBay in the sky: strategy-proof wireless spectrum auctions. In *Proceedings of the 14th ACM international conference on Mobile computing and networking*, pages 2–13. ACM, 2008.

[2] A.P. Subramanian, M. Al-Ayyoub, H. Gupta, S.R. Das, and M.M. Buddhikot. Near-optimal dynamic spectrum allocation in cellular networks. In *New Frontiers in Dynamic Spectrum Access Networks, 2008. DySPAN 2008. 3rd IEEE Symposium on*, pages 1–11. IEEE, 2008.

[3] X. Zhou and H. Zheng. TRUST: A general framework for truthful double spectrum auctions. In *INFOCOM 2009, IEEE*, pages 999–1007. IEEE, 2009.

[4] X. Zhou and H. Zheng. TRUST: A general framework for truthful double spectrum auctions (extended). Technical report, Citeseer, 2009.

[5] S. Sengupta, M. Chatterjee, and S. Ganguly. An economic framework for spectrum allocation and service pricing with competitive wireless service providers. In *New Frontiers in Dynamic Spectrum Access Networks, 2007. DySPAN 2007. 2nd IEEE International Symposium on*, pages 89–98. IEEE, 2007.

[6] A. Al Daoud, M. Alanyali, and D. Starobinski. Secondary pricing of spectrum in cellular CDMA networks. In *New Frontiers in Dynamic Spectrum Access Networks, 2007. DySPAN 2007. 2nd IEEE International Symposium on*, pages 535–542. IEEE, 2007.

[7] H. Mutlu, M. Alanyali, and D. Starobinski. Spot pricing of secondary spectrum usage in wireless cellular networks. In *INFOCOM 2008. The 27th Conference on Computer Communications. IEEE*, pages 682–690. IEEE, 2008.

[8] Y. Xing, R. Chandramouli, and C. Cordeiro. Price dynamics in competitive agile spectrum access markets. *Selected Areas in Communications, IEEE Journal on*, 25(3):613–621, 2007.

[9] M. Babaioff and N. Nisan. Concurrent auctions across the supply chain. In *Proceedings of the 3rd ACM conference on Electronic Commerce*, pages 1–10. ACM, 2001.

[10] R.P. McAfee. A dominant strategy double auction. *Journal of Economic Theory*, 56(2):434–450, 1992.

[11] A. Graf, M. Stumpf, and G. Weißenfels. On coloring unit disk graphs. *Algorithmica*, 20(3):277–293, 1998.

[12] A.P. Subramanian and H. Gupta. Fast spectrum allocation in coordinated dynamic spectrum access based cellular networks. In *New Frontiers in Dynamic Spectrum Access Networks, 2007. DySPAN 2007. 2nd IEEE International Symposium on*, pages 320–330. IEEE, 2007.

[13] S. Ramanathan. A unified framework and algorithm for channel assignment in wireless networks. *Wireless Networks*, 5(2):81–94, 1999.

[14] R. Peeters. *On coloring j-unit sphere graphs*. Tilburg University, 1991.

[15] Fcc geographic information systems, Jan 2011. `http://wireless.fcc.gov/geographic/`.