Path Diversity with Imaging Components on Separate Frequency Channel in Fractional Sampling OFDM

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***Abstract*—In this paper, a diversity scheme with imaging components on a separated channel for a fractional sampling (FS)-OFDM receiver is proposed. Cognitive radio is an emerging technology to utilize frequency spectrum flexibly through dynamic spectrum access. As one of the dynamic spectrum access scheme, here, the transmission of the imaging components of the desired signal over a separated channel is investigated. FS path diversity makes use of the imaging components of the desired signal transmitted on the separate channel. With the assumption of the dynamic spectrum access, this paper evaluates the performance of FS path diversity with the imaging components on a frequency channel apart from the main channel. Numerical results show that the proposed scheme improves bit error rate performance of the OFDM system through path diversity.**

***Index Terms*—OFDM, Fractional Sampling, Cognitive Radio, Imaging Component**

I. Introduction

Frequency spectrum is getting scarce since more numbers of wireless systems have been launched in order to realize ubiquitous network society. The ISM band in 2.4 GHz, for example, has been shared by various wireless systems such as WLANs, Bluetooth, and so on. Since the demands for these wireless systems are increasing, it is expected that radio channels in the ISM band will be crowded and the throughput performance of these systems is deteriorated. In this band, various radio systems and RF (radio frequency) equipments work independently. Since the frequency spectrum on the ISM band is occupied randomly by those wireless systems, the available vacant spectrum is scattered over the whole band [1], [2].

Cognitive radio is a technology that enables flexible usage of frequency spectrum and improves the capacity of a system. One type of the cognitive radio technologies is dynamic spectrum access that utilizes frequency spectrum which is not in use by the system primarily assigned to the bands [3]. There are a lot of literatures regarding the cognition of radio frequency environments and the cognitive radio systems that employ unused spectrum [4], [5].

On the other hand, fractional sampling (FS) has been then proposed for OFDM that can achieve path diversity with a single antenna by oversampling the received signal [6]. FS can resolve the multipath by sampling the received signal at a rate higher than the Nyquist rate and taking DFTs of the sampled signals in parallel. It is known that the diversity gain with FS greatly depends on the imaging components of the desired signal over the excess bandwidth of the filter. When the signal is upsampled at the transmitter, the images of the desired signal is generated outside of the main channel. The path diversity with FS is achieved through these imaging components on the adjacent channel that passes the baseband filter and is downconverted and folded to the main channel in the baseband. However, if the adjacent channel is not vacant, a sharp baseband filter has to be applied in order to avoid the adjacent channel interference. The sharp filter then limits the diversity gain through the FS scheme.

In this paper a novel usage of the frequency spectrum with the FS scheme has been investigated. With the assumption of cognitive radio, the imaging components of the desired signal are transmitted through a channel apart from the main channel. The path diversity effect of the proposed spectrum usage with the FS scheme is investigated. The proposed scheme may also be employed in order to enhance the performance of the primary wireless system.

This paper is organized as follows. Section 2 explains the system model. Section 3 shows numerical results. In Section 4, our conclusions are presented.

II. System Model

A. Fractional Sampling

Suppose an information symbol on the kth subcarrier is $s[k](k = 0, ..., N-1)$, an OFDM symbol is then given as

$$u[n] = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} s[k] e^{j\frac{2\pi nk}{N}}, \qquad n = 0, ..., N-1 \quad (1)$$

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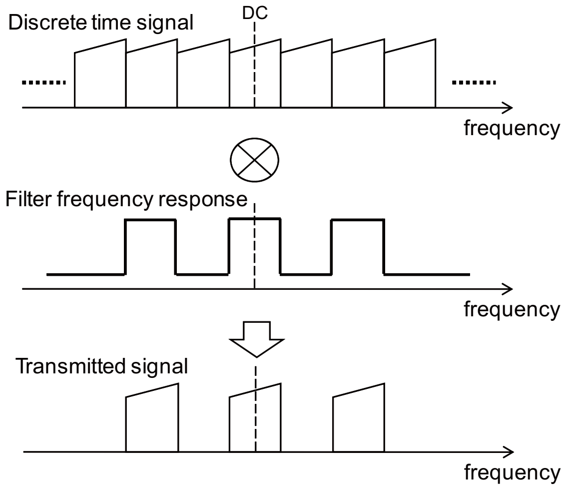

Fig. 1. Transmission of the imaging components with baseband filtering.

where n $(n = 0, 1, ..., N - 1)$ is the time index. A guard interval (GI) with the length of N_{GI} is appended before transmission. The length of one OFDM symbol is $P := N + N_{GI}$. $u[n]$ is converted from digital to analog and filtered by the baseband filter of the transmitter. As shown in Fig. 1, because the digital-analog (D/A) outputs are discrete analog samples, the imaging components are generated over the frequency band. After filtering by the baseband filter with the impulse response of $p(t)$, only the desired imaging components are transmitted. The discrete analog samples, $u[n]$, have images over the frequency domain and the baseband filter passes some of those components.

This signal is then upconverted and transmitted through a multipath channel with an impulse response, $c(t)$. The received signal after downconversion is given as

$$y(t) = \sum_{n=0}^{P-1} u[n]h(t - nT_s) + v(t) \tag{2}$$

where $h(t)$ is the impulse response of the composite channel and is given by $h(t) := p(t) \star c(t) \star p(-t)$, $\star$ denotes convolution, and $v(t)$ is the additive Gaussian noise filtered at the receiver. For the multipath channel, $h(t)$ can be expressed in a baseband form as

$$h(t) = \sum_{i=0}^{L-1} \alpha_i p_2(t - \tau_i) \tag{3}$$

where $p_2(t) := p(t) \star p(-t)$ is the deterministic correlation of $p(t)$. It is assumed that the channel has L path components, α_i is the amplitude of the i^{th} path that is time-invariant during one OFDM symbol (a so-called quasi-static channel model), and τ_i is the path delay. If $y(t)$ is sampled at the rate of T_s/G, its polyphase components can be expressed as

$$y_g[n] = \sum_{l=0}^{P-1} u[l]h_g[n-l] + v_g[n], \quad g = 0, ..., G-1 \tag{4}$$

where $y_g[n]$, $h_g[n]$, and $v_g[n]$ are polynomials of sampled $y(t)$, $h(t)$, and $v(t)$, respectively, and are given as

$$y_g[n] := y(nT_s + gT_s/G), \tag{5}$$
$$h_g[n] := h(nT_s + gT_s/G), \tag{6}$$
$$v_g[n] := v(nT_s + gT_s/G). \tag{7}$$

These samples are put into G DFT demodulators in parallel and the sampling rate on each branch reduces to $1/T_s$. Therefore, the images fold dawn to the main channel as alias components. After removing the GI and taking DFT at each subcarrier, the received symbol is given by

$$\mathbf{z}[k] = \mathbf{H}[k]\mathbf{s}[k] + \mathbf{w}[k] \tag{8}$$

where $\mathbf{z}[k] = [z_0[k]...z_{G-1}[k]]^T$, $\mathbf{w}[k] = [w_0[k]...w_{G-1}[k]]^T$, and $\mathbf{H}[k] = [H_0[k]...H_{G-1}[k]]^T$ are $G \times 1$ column vectors, each gth component representing

$$[\mathbf{z}[k]]_g := z_g[k] = \sum_{n=N_{GI}}^{P-1} y_g[n]e^{-j\frac{2\pi kn}{N}}, \tag{9}$$

$$[\mathbf{w}[k]]_g := w_g[k] = \sum_{n=N_{GI}}^{P-1} v_g[n]e^{-j\frac{2\pi kn}{N}}, \tag{10}$$

$$[\mathbf{H}[k]]_g := H_g[k] = \sum_{n=N_{GI}}^{P-1} h_g[n]e^{-j\frac{2\pi kn}{N}}, \tag{11}$$

respectively.

As already stated, $v(t)$ in Eq. (2) is the filtered noise. When $v(t)$ is sampled at the Nyquist rate of $1/T_s$, the samples of $v(t)$ are independent one another. However, when the sampling rate is a multiple of the baud rate, the noise samples are correlated. Consequently, it is necessary to whiten the colored noise samples. In order to perform noise-whitening, it is required to calculate the noise covariance matrix on each subcarrier whose (g_1, g_2)th element is given as

$$\begin{aligned} R_{w_{g1,g2}}[k] &= E[w_{g1}[k]w^*_{g2}[k]] \\ &= \sigma_v^2 \frac{1}{N} \sum_{n_1=0}^{N-1} \sum_{n_2=0}^{N-1} p_2((n_2 - n_1 + (g_2 - g_1)/G)T_s) \\ &\quad \times e^{j\frac{2\pi k(n_2-n_1)}{N}} \end{aligned} \tag{12}$$

where σ_v^2 is the noise variance. Multiplying both sides of Eq. (8) by $\mathbf{R}_w^{-\frac{1}{2}}[k]$,

$$\begin{aligned} \mathbf{R}_w^{-\frac{1}{2}}[k]\mathbf{z}[k] &= \mathbf{R}_w^{-\frac{1}{2}}[k](\mathbf{H}[k]\mathbf{s}[k] + \mathbf{w}[k]), \\ \mathbf{z}'[k] &= \mathbf{H}'[k]\mathbf{s}[k] + \mathbf{w}'[k] \end{aligned} \tag{13}$$

where $\mathbf{Z}'[k] = \mathbf{R}_w^{-\frac{1}{2}}[k]\mathbf{Z}_[k]$, $\mathbf{H}'[k] = \mathbf{R}_w^{-\frac{1}{2}}[k]\mathbf{H}_[k]$ and $\mathbf{w}'[k] = \mathbf{R}_w^{-\frac{1}{2}}[k]\mathbf{w}_[k]$. The output of the subcarrier base maximum ratio combining (MRC) among the whitened samples is given as

$$\begin{aligned} \hat{s}[k] &= \frac{\mathbf{H}'^H[k]\mathbf{z}'[k]}{\mathbf{H}'^H[k]\mathbf{H}'[k]} \\ &= \frac{(\mathbf{R}_w^{-\frac{1}{2}}[k]\mathbf{H}[k])^H(\mathbf{R}_w^{-\frac{1}{2}}[k]\mathbf{z}[k])}{(\mathbf{R}_w^{-\frac{1}{2}}[k]\mathbf{H}[k])^H(\mathbf{R}_w^{-\frac{1}{2}}[k]\mathbf{H}[k])}. \end{aligned} \tag{14}$$

B. Bit Error Rate Performance

BER performance through FS and MRC at each subcarrier is evaluated through a channel covariance matrix [6]. The channel covariance matrix after noise whitening, $\mathbf{R}'_H[k]$, is

$$\mathbf{R}'_H[k] = \mathbf{R}_w^{-1/2}[k]\mathbf{R}_H[k]\mathbf{R}_w^{-H/2}[k] \quad (15)$$

where $\mathbf{R}_H[k]$ is the channel covariance matrix on the kth subcarrier that is given as $\mathbf{R}_H[k] = E\left[\mathbf{H}[k]\mathbf{H}^H[k]\right]$. In the case of QPSK modulation, the BER on the kth subcarrier is calculated from the set of eigenvalues $\{\lambda_0, \lambda_1 \ldots, \lambda_{G-1}\}$ of $\mathbf{R}_H[k]$ as

$$P_b[SNR_k/2, k] = \frac{1}{2}\sum_{g=0}^{G-1}\rho_g\left[1 - \sqrt{\frac{\lambda_g}{1+\lambda_g}}\right], \quad (16)$$

$$\rho_g = \Pi_{m\neq g}(1 - \frac{\lambda_m}{\lambda_g})^{-1}, \quad (17)$$

and the SNR per symbol on the kth subcarrier is given by

$$SNR_k = \text{trace}[\mathbf{R}_H[k]] = \sum_{g=0}^{G-1}\lambda_g. \quad (18)$$

C. Correlation among Sampling Points

The correlation between the FS branches depends on the frequency channel response, the path delay profile, and the oversampling ratio. Suppose that the noise is neglected for simplicity, from the sampling theorem, the sampled signal on the g^{th} branch is given as

$$y_g(t) = \sum_{n=-\infty}^{\infty}\sum_{i=0}^{L-1}\alpha_i x(nT_s + (gT_s/G - \tau_i)) \cdot\delta(t - (nT_s + (gT_s/G - \tau_i))) \quad (19)$$

where $x(t)$ is the baseband signal that passes through the transmit and receive filters as $x(t) = \sum_{n=0}^{P-1} u[n]p_2(t - nT_s)$, δ is the Dirac delta function, L is the number of multipath, α_i and τ_i is the gain and delay time of the i^{th} delay path and $y_g[n] = y_g(nT_s)$. It is then given in the frequency domain as

$$Y_g(f) = \sum_{q=-\infty}^{\infty}\sum_{i=0}^{L-1}\alpha_i f_s \exp(j2\pi(f + qf_s)(gT_s/G - \tau_i)) \cdot X(f - qf_s) \quad (20)$$

where $f_s = 1/T_s$ is the sampling frequency and $X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi tf}dt$. If the received signal has been transmitted over two channels, the center channel ($q = 0$) and the separate channel ($q = Q$) where Q is an integer, i.e. $X(f) = X(f + Qf_s)$, and the received signal on the g^{th} FS branch is derived as

$$Y_g(f) = \sum_{i=0}^{L-1}\alpha_i f_s \exp(-j2\pi f\tau_i)\exp(j2\pi fgT_s/G)X(f) \cdot\{1 + \exp(j2\pi Qg/G)\exp(-j2\pi Qf_s\tau_i)\} \quad (21)$$

If $Q = CG$ where C is an integer,

$$Y_g(f) = \sum_{i=0}^{L-1}\alpha_i f_s \exp(-j2\pi f\tau_i)\exp(j2\pi fgT_s/G)X(f) \cdot\{1 + \exp(-j2\pi CGf_s\tau_i)\}. \quad (22)$$

As observed in the above equation, the use of the $(CG)^{th}$ separated channel makes the FS branches be coherent because every branch receives the same amounts of the path components.

Suppose that $G = 2$, the received time signal sampled on the 0^{th} branch ($g = 0$) is given as

$$y_0(t) = \sum_{n=-\infty}^{\infty}\sum_{i=0}^{L-1}\alpha_i x(nT_s - \tau_i)\delta(t - (nT_s - \tau_i)). \quad (23)$$

It is given in the frequency domain as

$$Y_0(f) = \sum_{q=-\infty}^{\infty}\sum_{i=0}^{L-1}\alpha_i f_s \exp(j2\pi(f + qf_s)(-\tau_i)) \cdot X(f - qf_s). \quad (24)$$

Similarly, the received signal on the 1^{st} branch ($g = 1$) is given as

$$y_1(t) = \sum_{n=-\infty}^{\infty}\sum_{i=0}^{L-1}\alpha_i x(nT_s + (T_s/2 - \tau_i)) \cdot\delta(t - (nT_s + (T_s/2 - \tau_i))). \quad (25)$$

It is also transformed on the frequency domain as

$$Y_1(f) = \sum_{q=-\infty}^{\infty}\sum_{i=0}^{L-1}\alpha_i f_s \exp(j2\pi(f + qf_s)(T_s/2 - \tau_i)) \cdot X(f - qf_s). \quad (26)$$

Here, for simplicity, it is assumed that the received signal has been transmitted over two channels; the center channel ($q = 0$) and the separate channel ($q = Q$). Because the signal generated on the outside of the center channel is the imaging component of the desired signal ($X(f) = X(f + qf_s)$), the frequency component of the received signal after sampling on the 0^{th} branch is

$$Y_0(f) = \sum_{i=0}^{L-1}\alpha_i f_s \exp(-j2\pi f\tau_i) \cdot\{1 + \exp(-j2\pi Qf_s\tau_i)\}X(f). \quad (27)$$

Also, the frequency component of the received signal after sampling on the 1^{st} branch is

$$Y_1(f) = \sum_{i=0}^{L-1}\alpha_i f_s \exp(-j2\pi f\tau_i)\{1 - \exp(-j2\pi Qf_s\tau_i)\} \cdot X(f)\exp(j2\pi fT_s/2), \qquad Q = odd, \quad (28)$$

or,

$$Y_1(f) = \sum_{i=0}^{L-1}\alpha_i f_s \exp(-j2\pi f\tau_i)\{1 + \exp(-j2\pi Qf_s\tau_i)\} \cdot X(f)\exp(j2\pi fT_s/2), \qquad Q = even. \quad (29)$$

Therefore, if Q is an even number, the sampled signals on the 0^{th} and 1^{st} branches are highly correlated while it is independent when Q is an odd number.

As an example, it is assumed that f_d is the frequency separation of the subcarriers, $L = 2, \tau_0 = 0, \tau_1 = T_s/2, X(kf_d) = X(kf_d + qf_s)$, and the imaging components are transmitted on the center channel and the adjacent channel ($q = 0, -1$) as shown in Fig. 2. The received signal on each branch is then given in the frequency domain as

$$Y_0(kf_d) = 2\alpha_0 f_s X(kf_d), \tag{30}$$

$$Y_1(kf_d) = 2\alpha_1 f_s X(kf_d), \tag{31}$$

respectively. It is found that the received signals are uncorrelated. When the imaging components are ideally limited to

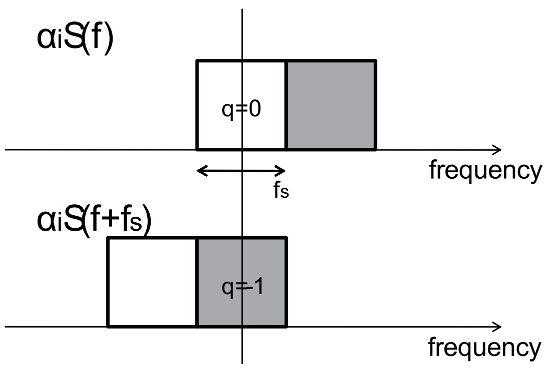

Fig. 2. Frequency component of the received signal ($q = 0, -1$).

the center channel and the next adjacent channel ($q = 0, -2$) as shown in Fig. 3, the received signal is derived as

$$Y_0(kf_d) = 2\left\{\alpha_0 + \alpha_1 \exp(-j2\pi(kf_d)T_s/2)\right\} \cdot f_s X(kf_d), \tag{32}$$

$$Y_1(kf_d) = 2\left\{\alpha_0 \exp(j2\pi(kf_d)T_s/2) + \alpha_1\right\} \cdot f_s X(kf_d), \tag{33}$$

respectively. As seen above, the sampled signals on the 0^{th} and 1^{st} branches are coherent.

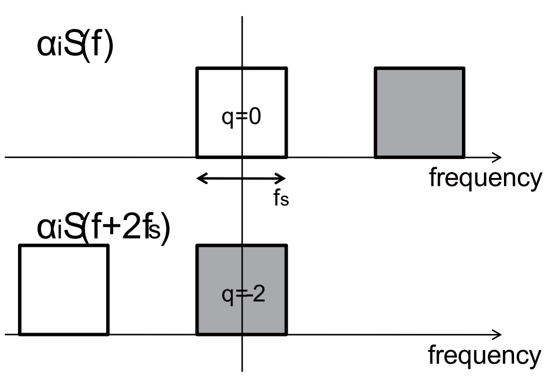

Fig. 3. Frequency component of the received signal ($q = 0, -2$).

III. Numerical Results

A. Simulation Conditions

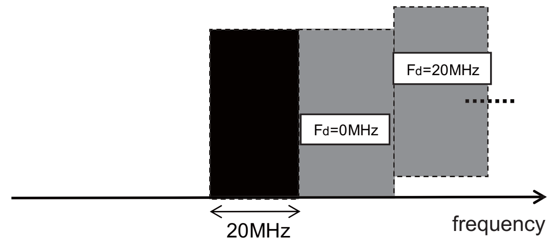

Fig. 4. Composite frequency response of the transmit and receive filter.

Table I shows the simulation conditions. The FS order is set to 2. Information symbols are modulated with QPSK-OFDM. The DFT size is 64. As a channel model a 16 path Rayleigh fading model is assumed. The interval of the path delays is set to $T_s/16$. The composite impulse response of the pulse shaping filters, $p_2(t)$, has the duration of $64T_s$. The deterministic correlation of the transmit and receive filters is assumed to have asymmetric frequency response as shown in Fig. 4. The imaging components have the bandwidth of 20 MHz with the frequency separation of F_d from the main channel. Unless it is specified the frequency responses on the pass band and the stop band of each filter are set to 0 dB and -50 dB. The channel covariance matrix on the kth subcarrier, $\mathbf{R}_H[k]$, is obtained through the average of 1000 times of channel response generation in Monte Carlo simulation.

TABLE I
SIMULATION CONDITIONS

FS order	2
Modulation scheme	QPSK/OFDM
Bandwidth of channel	20 MHz
DFT size	64
No. subcarriers	64
Channel model	16 path Rayleigh fading model
Path delay interval	$T_s/16$
Fading model	Static over one OFDM symbol
OFDM symbol length	$64Ts$
Impulse response of $p_2(t)$	$64Ts$
No. trials	1000

B. Simulation Result

Figure 5 shows the BER versus the relative signal power of the imaging components. The frequency separation, F_d, is set to 0 MHz and the bandwidth of the imaging components is 20 MHz on the separated channel. The average E_b/N_0 over all the subcarriers after combining is 25 dB. Since $G = 1$ implies no diversity, the BER is constant over any relative signal power. The FS scheme achieves almost the same BER performance as that with the 2^{nd} order diversity. Therefore, the proposed FS receiver can realize diversity without increasing the number of the RF branches. The diversity gain is almost lost when the relative power level is less than -40 dB. The slight difference between the FS and the frequency diversity comes

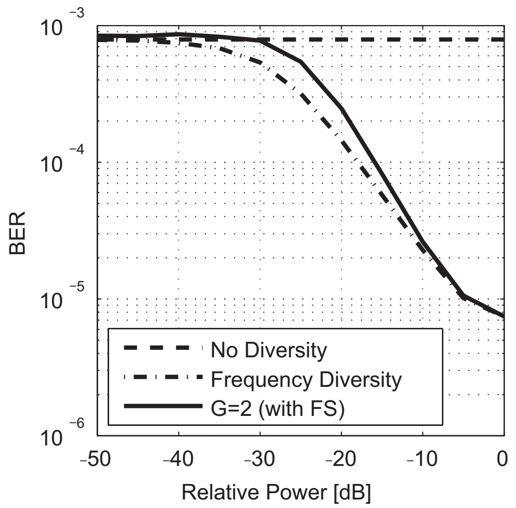

Fig. 5. BER vs. relative signal power of the imaging components ($F_d = 0$ MHz, $E_b/N_0 = 25$ dB).

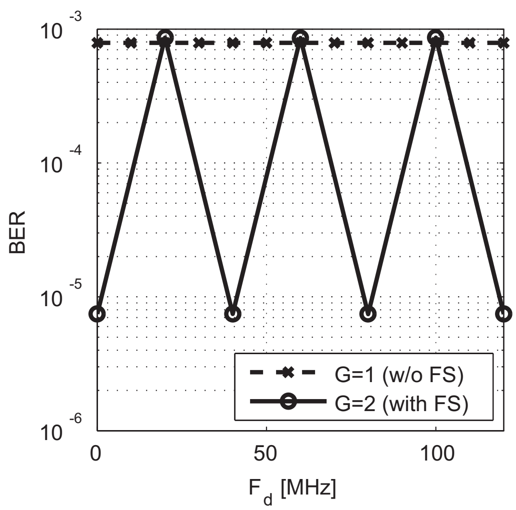

Fig. 6. BER vs. frequency separation (Relative signal power : 0 dB, $E_b/N_0 = 25$ dB).

from the nonideality of the filter with the limited duration. The nonideality of the whitening filter reduces the SNR of the subcarriers on the channel edges.

Figure 6 is the BER performance when the frequency response of the filters is shown in Fig. 4. The 2^{nd} order diversity can be achieved through FS when the frequency separation is $40q + 20$ MHz (q is 0 or positive integer). This is because of the correlation between the branches becomes 1 as explained in Sec. II-C. Figure 7 shows the BER versus average E_b/N_0 over all the subcarriers when the imaging components have 20 MHz bandwidth on the separated band. In the case of $F_d = 0$ MHz, the BER performance is almost the same as that of the theoretical 2^{nd} order diversity on a Rayleigh fading channel. This is also due to FS path diversity. On the

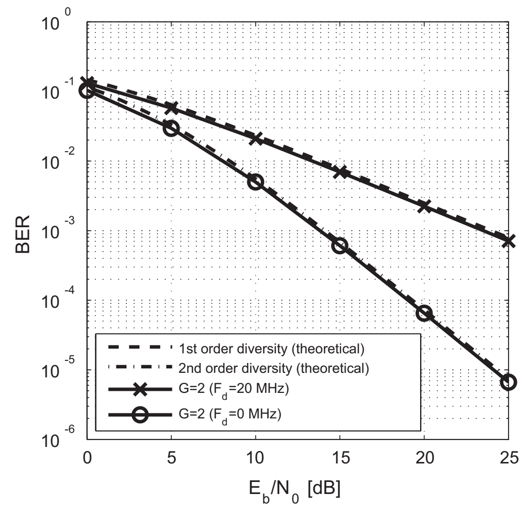

Fig. 7. BER vs. E_b/N_0 (Relative signal power : 0 dB)

other hand, in the case of $F_d = 20$ MHz, the BER is the same performance as that of the theoretical 1^{st} order diversity on a Rayleigh fading channel because the FS branches are coherent.

IV. CONCLUSIONS

In this paper, the diversity scheme with the imaging components on the channels apart from the main channel through FS has been investigated. It has been evaluated that FS achieves path diversity with the imaging components. It has also been confirmed that the diversity gain depends on the frequency separation of the imaging components from the main channel as well as the path delays. If the frequency separation is CGf_s where C is an integer and f_s is the bandwidth of the desired signal, all the demodulation branches of the FS receiver is coherent and no diversity gain is observed. The proposed scheme may be useful in conjunction with the dynamic spectrum access scheme in the cognitive radio technology.

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