# Statistical Test for Multiple Primary User Spectrum Sensing

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***Abstract*—Spectrum sensing is a key component in cognitive radio networks. The existing results so far primarily focus on single primary user detection. Little is known in the most practical and critical setting when multiple primary users exist. In this paper, we aim to address this problem by studying an optimal detector in the presence of multiple primary users. Specifically, a simple and accurate analytical formula for its test statistics distribution is derived, which yields a useful tool in determining the decision threshold. Simulations are provided to show both the accuracy of the derived result and the superior detection performance in realistic sensing scenarios.**

***Index Terms*—Cognitive radio; spectrum sensing; multiple primary users; the spherical test.**

## I. Introduction

Cognitive radio (CR) is a promising technique for future wireless communication systems. In CR networks, dynamic spectrum access is implemented to mitigate spectrum scarcity. Namely, a secondary (unlicensed) user is allowed to utilize the spectrum resources when it does not cause intolerable interference to the primary (licensed) user. A key requirement for this is the secondary user's ability to detect the presence of the primary user. Thus spectrum sensing is considered as a fundamental task in CR networks.

Prior work on cooperative spectrum sensing predominately employ the assumption of a single active primary user. Based on this assumption, several eigenvalue based sensing algorithms have been proposed recently [1], [2], [3], [4], [5], [6]. These algorithms achieve optimality under different criteria. Besides analytical tractability, this assumption is made since early research mainly focuses on CR networks, where the primary systems are TV or DVB. In these systems the single active primary user assumption is, to some extent, justifiable. However, this assumption may fail to reflect the situation in forthcoming CR networks, where the primary system could be a cellular network, and the existence of more than one primary user would be prevailing. Using the existing single primary user detection algorithms in such a scenario will clearly induce performance loss. Despite the need to understand multiple primary user detection, the results in this direction are rather limited. An heuristic detection algorithm is investigated in [7], [8], [9], yet its detection performance turns out to be sub-optimal [6]. Recently an optimal detection algorithm in the presence of multiple primary users has been proposed in [10], however no analytical results pertaining to its statistical performance are presented. In this paper, we study the performance of this optimal multiple primary user detection by deriving a closed-form expression for its test statistics distribution. The derived result can be utilized to accurately set a decision threshold according to a required false alarm probability in an computationally affordable manner.

The rest of this paper is organized as follows. In Section II we propose the test statistics for the multiple primary user detection after outlining the signal model. Analytical performance of the proposed detection scheme is addressed in Section III. Section IV presents numerical examples to examine the detection performance in the multiple primary users scenarios. Finally in Section V we conclude the main results of this paper.

## II. Problem Formulation

### A. Signal Model

Consider the standard model for $K$-sensor cooperative detection in the presence of $P$ primary users,

$$\mathbf{x} = \mathbf{H}\mathbf{s} + \sigma\mathbf{n} \tag{1}$$

where $\mathbf{x} \in \mathbb{C}^K$ is the received data vector. These sensors may be e.g. $K$ receive antennas in one secondary terminal or $K$ secondary devices each with a single antenna, or any combination of these. The $K \times P$ matrix $\mathbf{H} = [\mathbf{h}_1, \ldots, \mathbf{h}_P]$ represents the channels between the $P$ primary users and the $K$ sensors. The $P \times 1$ vector $\mathbf{s}$ denotes zero mean transmit signals from the primary users. The $K \times 1$ vector $\mathbf{n}$ is the complex Gaussian noise with zero mean and identity covariance matrix[1] $E[\mathbf{n}\mathbf{n}^\dagger] = \sigma^2\mathbf{I}_K$, where $\sigma^2$ is the noise power.

We collect $N$ i.i.d observations from model (1) to a $K \times N$ matrix $\mathbf{X} = [\mathbf{x}_1, \ldots, \mathbf{x}_N]$. The problem of interest is to use the data matrix $\mathbf{X}$ to decide whether there are primary users. This collaborative sensing scenario is more relevant when the $K$ sensors are in one device[2], since for multiple collaborating devices, communications to the fusion center becomes a problem even with a small sample size $N$. For the ease of analysis we make the following typical assumptions

[1]The operator $(\cdot)^\dagger$ denotes conjugate-transpose.
[2]In practise $K$ can be as large as eight, since eight antenna terminals are an option considered in 3GPP standards.

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1) The channel $\mathbf{H}$ is constant during sensing time.
2) Signals from the $P$ primary users are mutually uncorrelated and are uncorrelated with the noise.

Due to the first assumption the channel model for $\mathbf{H}$ may not need to be specified. In the absence of primary users, the sample covariance matrix $\mathbf{R} = \mathbf{X}\mathbf{X}^\dagger$ follows an uncorrelated (white) complex Wishart distribution $\mathcal{W}_K(N, \mathbf{\Sigma})$ with population covariance matrix

$$\mathbf{\Sigma} := E[\mathbf{X}\mathbf{X}^\dagger]/N = \sigma^2 \mathbf{I}_K. \tag{2}$$

In the presence of primary users, the sample covariance matrix $\mathbf{R}$ follows a correlated complex Wishart distribution. The covariance matrix, by the above two assumptions, equals

$$\mathbf{\Sigma} = \sigma^2 \mathbf{I}_K + \sum_{i=1}^{P} \gamma_i \mathbf{h}_i \mathbf{h}_i^\dagger, \tag{3}$$

where $\gamma_i$ is the transmission power of the $i$-th primary user. The received Signal to Noise Ratio (SNR) of primary user $i$ across the $K$ sensors is

$$\mathrm{SNR}_i := \frac{\gamma_i ||\mathbf{h}_i||^2}{\sigma^2}. \tag{4}$$

Finally, we denote the ordered eigenvalues of the sample covariance matrix $\mathbf{R}$ as $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_K$.

### *B. Test Statistics*

The differences between the population covariance matrices (2) and (3) can be explored to detect the primary user. This detection problem can be formulated as a binary hypothesis test, where hypothesis $\mathcal{H}_0$ denotes the absence of primary user and hypothesis $\mathcal{H}_1$ denotes the presence of primary user. The probability that $\mathcal{H}_0$ and $\mathcal{H}_1$ is wrongly declared defines the false alarm probability $P_{\text{fa}}$ and the missed detection probability $P_{\text{m}}$ respectively. It is a well-known fact that the sample covariance matrix $\mathbf{R}$ is sufficient statistics for the population covariance matrix $\mathbf{\Sigma}$ [11]. This fact leads to various test statistics as functions of $\mathbf{R}$ with different assumptions on the number of primary users $P$ and knowledge of the noise power $\sigma^2$.

In the case of *a single* primary user ($P = 1$) the hypothesis test can be expressed as

$$\mathcal{H}_0 \quad : \quad \mathbf{\Sigma} = \sigma^2 \mathbf{I}_K \tag{5}$$
$$\mathcal{H}_1 \quad : \quad \mathbf{\Sigma} = \sigma^2 \mathbf{I}_K + \gamma_1 \mathbf{h}_1 \mathbf{h}_1^\dagger. \tag{6}$$

Assuming known noise power $\sigma^2$, the Largest Eigenvalue based (LE) detection ($T_{\text{LE}} = \lambda_1$) is shown to be optimal under the Likelihood Ratio Test (LRT) criterion [12]. Performance analysis of the LE detection can be found, e.g., in [1], [2]. Assuming unknown noise power, the Scaled Largest Eigenvalue based (SLE) detection ($T_{\text{SLE}} = \lambda_1/\mathrm{tr}(\mathbf{R})$) is obtained in [3], [4], [5], [6], which is optimal in the General Likelihood Ratio Test (GLRT) sense.

In the presence of *multiple* primary users ($P \geq 2$), neither LE nor SLE detection are optimal. To formulate a hypothesis test in this setting, a natural idea is to consider the following hypothesis test, which is a direct extension from the single primary user detection

$$\mathcal{H}_0 \quad : \quad \mathbf{\Sigma} = \sigma^2 \mathbf{I}_K \tag{7}$$
$$\mathcal{H}_1 \quad : \quad \mathbf{\Sigma} = \sigma^2 \mathbf{I}_K + \sum_{i=1}^{P} \gamma_i \mathbf{h}_i \mathbf{h}_i^\dagger. \tag{8}$$

Since the number of primary users $P$ is not known a priori, a separate procedure is needed to estimate it. With the estimated $P$, the model in (8) is specified, and a test statistics is chosen accordingly. The main issue with this detection scheme is the additional delay induced by estimating the number of primary users, which may not be affordable for real-time detection. Moreover, estimation of $P$ is possible only when $P$ is less than the sensor size $K$, which is also the maximal rank of $\mathbf{\Sigma}$. Further, the estimated $P$ may deviate severely from its true value in e.g. low SNR or small sample size situations [12], [13].

For a secondary user, the most critical information is whether or not there are active primary users. Knowledge of the number of active primary users may not be relevant from the secondary user's perspective. Therefore, in this paper, we propose the following hypothesis test in the multiple primary users

$$\mathcal{H}_0 \quad : \quad \mathbf{\Sigma} = \sigma^2 \mathbf{I}_K \tag{9}$$
$$\mathcal{H}_1 \quad : \quad \mathbf{\Sigma} \neq \sigma^2 \mathbf{I}_K, \tag{10}$$

where the noise power $\sigma^2$ is assumed to be unknown. Essentially, we are now testing a null hypothesis $\mathbf{\Sigma} = \sigma^2 \mathbf{I}_K$ against all the other possible alternatives of $\mathbf{\Sigma}$. Thus no a priory assumption on the structure of $\mathbf{\Sigma}$ is required, except for its positive definiteness ($\mathbf{\Sigma} \succ 0$). In particular, deciding the number of primary users $P$ is no longer needed. Thus it is not necessary that $P$ is smaller than the sensor size $K$. Intuitively, this hypothesis test is to reject $\mathcal{H}_0$ if we have reason to believe that the population covariance matrix $\mathbf{\Sigma}$ departs from the matrix $\sigma^2 \mathbf{I}_K$.

In statistics literatures this hypothesis test is known as the sphericity test, which was first studied in [14]. There, test statistics of this Spherical Test based (ST) detector was derived under the GLRT criterion as

$$T_{\text{ST}} = \frac{\det(\mathbf{R})}{\left(\frac{1}{K}\mathrm{tr}(\mathbf{R})\right)^K} = \frac{\prod_{i=1}^{K} \lambda_i}{\left(\frac{1}{K}\sum_{i=1}^{K} \lambda_i\right)^K}. \tag{11}$$

For completeness, the essential steps of the derivation is outlined here. Apart from a constant, the likelihood function of the data matrix $\mathbf{X}$ is

$$L(\mathbf{X}|\mathbf{\Sigma}) = (\det(\mathbf{\Sigma}))^{-N} \, \mathrm{e}^{\mathrm{tr}(-\mathbf{\Sigma}^{-1}\mathbf{R})}. \tag{12}$$

The likelihood ratio statistics is

$$\rho := \frac{\sup_{\sigma^2 > 0} L(\mathbf{X}|\sigma^2 \mathbf{I}_K)}{\sup_{\mathbf{\Sigma} \succ 0} L(\mathbf{X}|\mathbf{\Sigma})}. \tag{13}$$

Inserting the maximum likelihood estimation of $\sigma^2$ and $\mathbf{\Sigma}$ [11]

$$\hat{\sigma^2} = \frac{\mathrm{tr}(\mathbf{R})}{KN}, \qquad \hat{\mathbf{\Sigma}} = \frac{\mathbf{R}}{N} \tag{14}$$

into (13) we obtain

$$\rho^{1/N} = \frac{\det(\mathbf{R})}{\left(\frac{1}{K}\mathrm{tr}(\mathbf{R})\right)^K} := T_{\mathrm{ST}}. \tag{15}$$

Hypothesis $\mathcal{H}_0$ is rejected if $\rho$ is small, i.e. when $\rho^{1/N}$ is small. Thus if $T_{\mathrm{ST}}$ is greater that some threshold $\zeta$ the detector declares $\mathcal{H}_0$, otherwise $\mathcal{H}_1$

$$T_{\mathrm{ST}} \underset{\mathcal{H}_1}{\overset{\mathcal{H}_0}{\gtrless}} \zeta. \tag{16}$$

Recently the spherical test is formulated in [10] as a spectrum sensing algorithm. However the detection performance analysis in [10] relies on simulations only. In this paper we analytically investigate the detection performance by deriving a closed-form distribution for the test statistics $T_{\mathrm{ST}}$. As a result we obtain analytical formulae for the false alarm probability and the decision threshold. The derived results are easily computable and uniformly accurate for all sensor sizes and number of samples.

Besides the ST detection, another detection algorithm in the presence of multiple primary users is the Eigenvalue Ratio based (ER) detection ($T_{\mathrm{ER}} = \lambda_1/\lambda_K$). Performance analysis of the ER detection can be found in [7], [8], [9]. It will be shown in Section IV that the proposed ST detector attains better detection performance than that of the ER detector. One intuitive reason is that the ER detection is unable to make full use of the information from the sample covariance matrix $\mathbf{R}$. Moreover the computational complexity of the ER detection is higher than that of the ST detection since the latter does not need to perform eigenvalue decomposition.

## III. Performance Analysis

In this section we derive test statistics distribution for the spherical test based detection, which leads to closed-form expressions for the false alarm probability and the decision threshold.

We first derive the exact moments of random variable $T_{\mathrm{ST}}$ denoted by

$$X := \frac{\det(\mathbf{R})}{\left(\frac{1}{K}\mathrm{tr}(\mathbf{R})\right)^K}, \quad x \in [0,1]. \tag{17}$$

Under $\mathbf{H_0}$, the sample covariance matrix $\mathbf{R}$ follows uncorrelated complex Wishart distribution $\mathcal{W}_K(N, \mathbf{I}_K)$ with density function[3]

$$\mathbf{R} \sim \frac{1}{\Gamma_K(N)} \left(\det(\mathbf{R})\right)^{N-K} \mathrm{e}^{\mathrm{tr}(-\mathbf{R})}, \tag{18}$$

where

$$\Gamma_K(N) = \pi^{\frac{1}{2}K(K-1)}\Gamma(N)\Gamma(N-1)\cdots\Gamma(N-K+1) \tag{19}$$

and $\Gamma(\cdot)$ denotes the Gamma function. Since $X$ is a scalar function of matrix argument $\mathbf{R}$, its $n$-th moment can be calculated as

$$\begin{aligned}\mathbb{E}[x^n] &= C\int_{\mathbf{R}\succ 0} \left(\det(\mathbf{R})\right)^{N-K+n} \mathrm{e}^{\mathrm{tr}(-\mathbf{R})} \left(\mathrm{tr}(\mathbf{R})\right)^{-Kn} \mathrm{d}\mathbf{R} \\ &= C'\int_{\mathbf{R}\succ 0} \frac{\left(\det(\mathbf{R})\right)^{N-K+n} \mathrm{e}^{\mathrm{tr}(-\mathbf{R})}}{\Gamma_K(N+n)} \left(\mathrm{tr}(\mathbf{R})\right)^{-Kn} \mathrm{d}\mathbf{R} \\ &= C'\mathbb{E}[(\mathrm{tr}(\mathbf{R}'))^{-Kn}],\end{aligned}$$

where $C = K^{Kn}/\Gamma_K(N)$ and $C' = K^{Kn}\Gamma_K(N+n)/\Gamma_K(N)$ are constants. The last expectation is with respect to the Wishart matrix $\mathbf{R}'$ distributed as $\mathcal{W}_K(N+n, \mathbf{I}_K)$. Since the trace of $\mathbf{R}'$ follows Chi-square distribution with $2K(N+n)$ degrees of freedom[4], its $(-Kn)$-th moment equals

$$\mathbb{E}[(\mathrm{tr}(\mathbf{R}'))^{-Kn}] = \frac{\Gamma(KN)}{\Gamma\left(K(N+n)\right)}. \tag{20}$$

The $n$-th moment of $X$ is now

$$\mathbb{E}[x^n] = \frac{\Gamma(KN)}{\Gamma_K(N)} \frac{K^{Kn}\Gamma_K(N+n)}{\Gamma(K(N+n))} := \mathcal{M}_n. \tag{21}$$

Note that the expression for the exact moments can be also obtained by exploiting the independence of random variables $X$ and $\mathrm{tr}(\mathbf{R})$ [14]. From the expression of the moments (21) the exact $T_{\mathrm{ST}}$ distribution can, in principle, be obtained by using the Mellin inversion integral [15]. The resulting density function involves the Meijer G-function or the Fox H-function. This result, although theoretically interesting, appears to be of limited usefulness due to its complicated form. Since an explicit expression for the distribution of $X$ may not be easily obtained, it is more desirable to approximate its distribution by some known distribution based on fitting the first few moments. Motivated by the results for $K=3$ with arbitrary $N$ [14], in this paper we consider to employ the Beta distribution to approximate the distribution of $X$ for any $K$ and $N$. Specifically, for a Beta distribution with density function

$$\frac{1}{B(\alpha,\beta)} x^{\alpha-1}(1-x)^{\beta-1} \quad x \in [0,1], \tag{22}$$

where $B(\alpha,\beta) = \Gamma(\alpha)\Gamma(\beta)/\Gamma(\alpha+\beta)$ is the Beta function, equaling the first two moments to the moments of $X$ we have

$$\mathcal{M}_1 = \frac{\alpha}{\alpha+\beta}, \tag{23}$$

$$\mathcal{M}_2 = \frac{\alpha(\alpha+1)}{(\alpha+\beta)(\alpha+\beta+1)}. \tag{24}$$

The parameters $\alpha$ and $\beta$ are solved from the above equations as

$$\alpha = \frac{\mathcal{M}_1(\mathcal{M}_1-\mathcal{M}_2)}{\mathcal{M}_2-(\mathcal{M}_1)^2}, \tag{25}$$

$$\beta = \frac{(1-\mathcal{M}_1)(\mathcal{M}_1-\mathcal{M}_2)}{\mathcal{M}_2-(\mathcal{M}_1)^2}. \tag{26}$$

[3] Since $T_{\mathrm{ST}}$ is independent of $\sigma^2$, without loss of generality we set $\sigma^2 = 1$.

[4] Here the degrees of freedom refers to the number of Gaussian random variables with mean zero and variance one half.

Note that both $\alpha$ and $\beta$ are simple functions of the sensor size $K$ and sample size $N$ only. Thus, for any $K$ and $N$ the CDF of $T_{\text{ST}}$ under $\mathcal{H}_0$, $F_{\text{ST}}(y)$, is approximated by

$$F_{\text{ST}}(y) \approx \frac{B_y(\alpha,\beta)}{B(\alpha,\beta)}, \quad y \in [0,\infty] \tag{27}$$

where $B_y(\alpha,\beta) = \int_0^y x^{\alpha-1}(1-x)^{\beta-1}\mathrm{d}x$ is the incomplete Beta function.

By (16) the false alarm probability can now be obtained, for any threshold $\zeta$, as

$$P_{\text{fa}} = F_{\text{ST}}(\zeta). \tag{28}$$

In order to implement spectrum sensing algorithms, we need to accurately determine a decision threshold for a given detection requirement in an affordable manner. By using the derived $P_{\text{fa}}$ expression (28) we can analytically obtain the decision threshold. Namely, for a target false alarm probability, a corresponding threshold is obtained by solving (28) as

$$\zeta = F_{\text{ST}}^{-1}(P_{\text{fa}}). \tag{29}$$

Note that if we further approximate $\alpha$ and $\beta$ to their respect nearest integer, (28) reduces to a simple polynomial equation in $\zeta$. Thus the computational complexity of threshold calculation becomes quite affordable for on-line implementations.

## IV. Numerical Results

In this section we first validate the derived analytical $P_{\text{fa}}$ expression by Monte-Carlo simulations. Then we investigate the performance of the ST detection by comparing with several detection algorithms in realistic spectrum sensing scenarios. The considered parameters $K$ and $N$ in the following plots reflect practical spectrum sensing scenarios. The sample size $N$ can be as large as a couple of hundred whereas the number of sensors $K$ is typically less than eight due to physical constraints of the device size. Note that the accuracy of derived results are not limited to the $K$, $N$ considered here.

In Figure 1 we compare the analytical and simulated false alarm probabilities as a function of threshold for various $K$ and $N$. The analytical $P_{\text{fa}}$ curves are calculated by using (28). We can see that the proposed analytical $P_{\text{fa}}$ matches the simulations well.

We now compare the detection performance of the spherical test based detector with other known detectors by means of Receiver Operating Characteristic (ROC) curves. Since ROC curve shows the achieved missed detection probability as a function of the false alarm probability, it reflects the overall detection performance for a given detector. We consider for comparison the cooperative Energy Detector $T_{\text{ED}} = ||\mathbf{X}||_F^2$ [5] [16], [17]. In addition, the previously discussed Eigenvalue Ratio based detector $T_{\text{ER}} = \lambda_1/\lambda_K$, which is also a candidate detector in the presence of multiple primary users, needs to be compared to. Optimal detectors for a single primary user, such as the LE detector $T_{\text{LE}} = \lambda_1$ and the SLE detector $T_{\text{SLE}} = \lambda_1/\mathrm{tr}(\mathbf{R})$ are considered for comparison as well.

[5] $||\cdot||_F$ denotes the Frobenius norm.

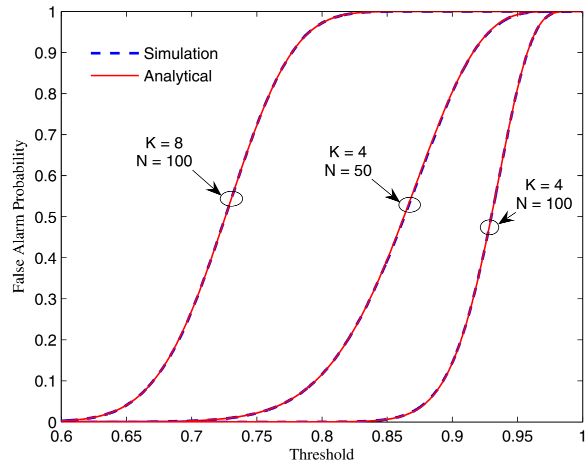


Fig. 1. False alarm probability: analytical versus simulation.

In practical systems we may not have perfect knowledge of the noise power [17]. The noise uncertainty may arise due to interference, noise estimation errors or noise variation during sensing. Hence for practical systems, modeling the noise uncertainty is unavoidable. The energy detector and the LE detector are subject to noise uncertainty due to the dependence of the test statistics on the noise power [17], [6]. The SLE, ER and the proposed ST detectors are immune to noise uncertainty since the noise powers are replaced by their respective ML estimates in constructing the test statistics. Robustness to noise uncertainty is of fundamental importance because the uncertainty may severely degrade detection performance, especially at low SNR. If $\mu$ denotes the degree of noise uncertainty in dB, the actual noise power falls in the interval $\Omega = [\sigma^2/\rho, \rho\sigma^2]$, where $\rho = 10^{\mu/10}$. In the following plots we consider the worst performance degradation due to noise uncertainty, where the noise power is $\rho\sigma^2$ under $\mathcal{H}_0$ and $\sigma^2/\rho$ under $\mathcal{H}_1$ [17]. Also, we assume that the channels stay constant during sensing time.

In Figure 2 we assume a scenario of three simultaneously transmitting primary users with $\text{SNR}_1 = -1$ dB, $\text{SNR}_2 = -3$ dB and $\text{SNR}_3 = -10$ dB. The number of sensors, samples per sensor and noise uncertainty are chosen to be $K = 4$, $N = 200$ and $\mu = 0.5$ dB respectively. It can be seen that the ST detector outperforms all other detectors in the presence of multiple primary users. When the false alarm probability equals $P_{\text{fa}} = 0.01$, the missed detection probability of the ST detector is about ten times lower than that of the single primary user detectors, such as the LE and SLE detectors.

As we have argued, the ST detector works also when the the number of active primary users $P$ is larger than the number of sensors $K$. This fact is illustrated in Figure 3, where we assume the existence of six primary users with $\text{SNR}_1 = 0$ dB, $\text{SNR}_2 = -1$ dB, $\text{SNR}_3 = -3$ dB, $\text{SNR}_4 = -8$ dB, $\text{SNR}_5 = -10$ dB and $\text{SNR}_6 = -22$ dB. We consider $K = 4$ cooperating sensors with sample size $N = 100$ and noise

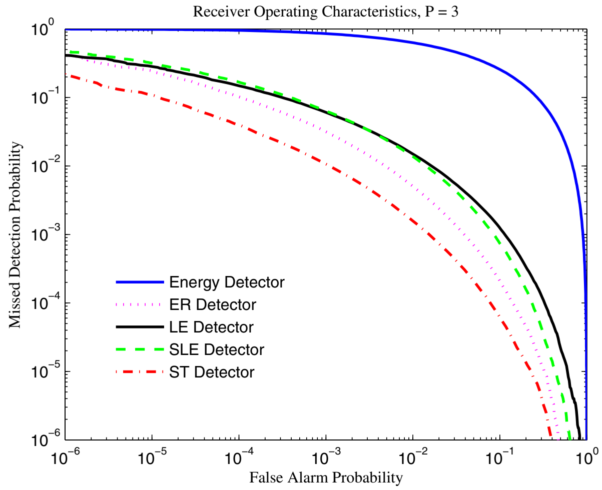


Fig. 2. ROC: assuming three primary users with $\text{SNR}_1 = -1$ dB, $\text{SNR}_2 = -3$ dB and $\text{SNR}_3 = -10$ dB.

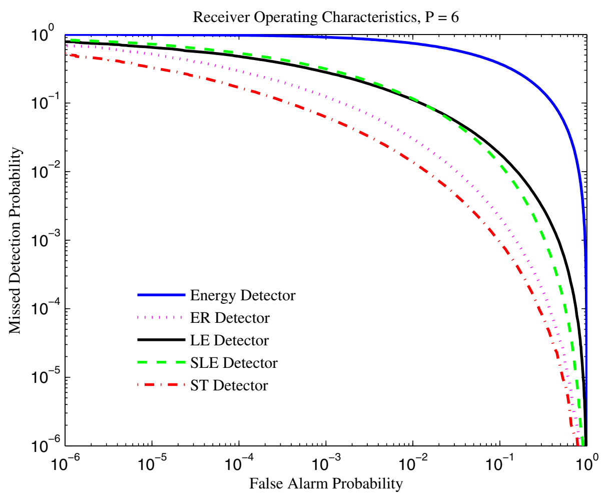


Fig. 3. ROC: assuming six primary users with $\text{SNR}_1 = 0$ dB, $\text{SNR}_2 = -1$ dB, $\text{SNR}_3 = -3$ dB, $\text{SNR}_4 = -8$ dB, $\text{SNR}_5 = -10$ dB and $\text{SNR}_6 = -22$ dB.

uncertainty level $\mu = 0.9$ dB. We can see from this figure that the ST detector still uniformly outperforms single primary user detectors when $P > K$. Many other testing examples were carried out in the multiple primary users settings and the superior performance of the ST detector persists.

In both the two ROC figures we can see that the ST detector always achieves higher performance than the ER detector. This is intuitively clear by examining their test statistics. For the ER detector, the test statistics depends only on the extreme eigenvalues of the sample covariance matrix $\mathbf{R}$, whereas the test statistics of the ST detector is a function of all the eigenvalues of $\mathbf{R}$. Finally we emphasize that the superior performance of the ST detector in the presence of multiple primary users does not depend on a specific channel model, as long as the channel matrix is constant during detection. Also, the test statistics of ST detector is independent of the noise power, thus its performance will not be affected by noise uncertainty.

## V. Conclusion

In this paper, we investigated the sensing performance of a multiple primary user detector. Analytical formulae have been found for its false alarm probability and decision threshold. The derived results are simple to calculate and yield almost-exact fit to simulations. Numerical examples show significant performance gain over several detection algorithms in scenarios with realistic parameters.

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