

Efficient Shortest Paths on Massive Social Graphs (Invited Paper)

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Abstract—Analysis of large networks is a critical component of many of today’s application environments, including online social networks, protein interactions in biological networks, and Internet traffic analysis. The arrival of massive network graphs with hundreds of millions of nodes, *e.g.* social graphs, presents a unique challenge to graph analysis applications. Most of these applications rely on computing distances between node pairs, which for large graphs can take minutes to compute using traditional algorithms such as breadth-first-search (BFS).

In this paper, we study ways to enable scalable graph processing for today’s massive networks. We explore the design space of *graph coordinate systems*, a new approach that accurately approximates node distances in constant time by embedding graphs into coordinate spaces. We show that a hyperbolic embedding produces relatively low distortion error, and propose *Rigel*, a hyperbolic graph coordinate system that lends itself to efficient parallelization across a compute cluster. *Rigel* produces significantly more accurate results than prior systems, and is naturally parallelizable across compute clusters, allowing it to provide accurate results for graphs up to 43 million nodes. Finally, we show that *Rigel*’s functionality can be easily extended to locate (near-) shortest paths between node pairs. After a one-time preprocessing cost, *Rigel* answers node-distance queries in 10’s of microseconds, and also produces shortest path results up to 18 times faster than prior shortest-path systems with similar levels of accuracy.

Index Terms—Online Social Network, Graph Coordinate System.

I. INTRODUCTION

Fast and scalable analysis of large networks is a critical component of many of today’s application environments, including online social networks, biological protein interaction networks, and analysis of the Internet router backbone. For example, a social game network might search for “central” users to help deploy new games, while a social auction site [1] wants to tell a buyer if a specific item is being auctioned by someone in her social circles. Ideally, such queries should be answered quickly, regardless of the size of the graph, or even if graphs themselves are changing over time.

Unfortunately, these goals are simply unattainable for today’s online social networks. This is because numerous graph analysis problems such as centrality computation, node separation, and community detection all rely on the simple *node distance* (length of shortest path) primitive, which scales badly with graph (or network) size. For graphs generated from social networks such as Facebook (700 million nodes), LinkedIn (100 million) and Twitter (200 million), computing the shortest path distance between a single pair of nodes can take a minute or

more using traditional algorithms such as breadth-first-search (BFS) [2]. Similarly, variants such as Dijkstra and Floyd-Warshall also fail to scale to these network sizes.

Without an efficient alternative for node distance computation, recent work has focused on exploring efficient approximation algorithms [2], [3], [4]. Our prior work [4], described the idea of *graph coordinate systems*, which embeds graph nodes into points on a coordinate system. The resulting coordinates can be used to quickly approximate node distance queries on the original graph. Our initial system, Orion, was a centralized system that approximated node distances by mapping nodes to the Euclidean coordinate system [4]. It has several limitations in practice. First, Orion’s initial graph embedding process is centralized and computationally expensive, which presents a significant performance bottleneck for larger graphs. Second, Orion’s results produce error rates between 15% and 20%, which limits the types of applications it can serve. Finally, it is unable to produce actual paths connecting node pairs, which is often necessary for a number of graph applications.

In this work, we seek to extend work on *graph coordinate systems* by developing a practical system that provides significant improvement in accuracy, scalability, and extended functionality. We systematically explore decisions in the design of a graph coordinate system, and make two key observations. *First*, we propose to extend our work on graph coordinate systems, by embedding large graphs in a hyperbolic space for lower distance distortion errors. Our embedding algorithm naturally parallelizes the costly embedding process across multiple servers, allowing our system to quickly embed multi-million node graphs. *Second*, we propose a novel way to use graph coordinates to efficiently locate shortest paths between node pairs. The result of our work is *Rigel*, a hyperbolic graph coordinate system that supports queries for both node distance and shortest paths on today’s large social graphs. After a one-time, easily parallelizable, preprocessing phase, *Rigel* can resolve queries in tens of microseconds, even for massive social graphs up to 43 million nodes.

Our paper describes four key contributions.

- In Sections III and IV, we describe the detailed design of *Rigel*, and show how we can minimize embedding time by effectively parallelizing the most computationally expensive parts of the graph embedding process.
- We evaluate a distributed prototype of *Rigel* using social

TABLE I
A VARIETY OF SOCIAL GRAPHS USED IN OUR WORK.

Graphs	Nodes	Edges	Avg. Path Len.
Egypt	246K	1,618K	5.0
Norway	293K	5,589K	4.2
L.A.	275K	2,115K	5.2
Flickr	1,715K	15,555K	5.1
Orkut	3,072K	117,185K	4.1
Livejournal	5,189K	48,942K	5.4
Renren	43,197K	1,040,429K	5.0

graphs of different sizes from several OSNs, including Facebook, Flickr, Orkut, LiveJournal, and Renren. Our results show that Rigel achieves consistently improved accuracy compared to Orion, and scales to large graphs of up to 43 million nodes.

- In Section V, we implement three different social graph analysis applications on top of the Rigel system. Our results illustrate both the accuracy and scalability of the Rigel system for use in real graph analysis applications.
- Finally, we propose an approach to approximate shortest paths for any node pair using graph coordinates. We compare Rigel’s shortest path results to those from recently proposed techniques. Rigel paths provide accuracy similar to the most accurate of prior schemes, while resolving queries up to 18 times faster.

A. Social Network Graph Datasets

Throughout our paper, we use a number of anonymized social graph datasets gathered from measurements of online social networks to guide and evaluate our system design. We utilize a total of 7 social graphs, ranging in size from 246,000 nodes and 1.6 million edges, to 43.2 million nodes and 1 billion edges. We list their key characteristics in Table I.

Three of these graphs, Egypt, Los Angeles (LA) and Norway, are Facebook regional networks shared by the authors of [5]. The remaining four graphs are significantly larger graphs crawled from the Flickr, Orkut, LiveJournal, and Renren social networks, each with millions of nodes and edges. We use them to test the efficiency and scalability of our system. The Livejournal, Flickr and Orkut are datasets shared by the authors of [6]. With 43 million nodes and more than 1 billion edges, our largest dataset is a snapshot of Renren, the largest online social network in China. We obtained this graph after seeking permission from Renren and the authors of [7]. While these graphs are still significantly smaller than the current user populations of Facebook (600 million) and LinkedIn (80 million), we believe our graphs are large enough to demonstrate the scalability of our mechanisms.

II. BACKGROUND AND RELATED WORK

Our goal is to develop a practical system that quickly answers node distance queries for today’s massive social graphs. To do so, we will use our proposed concept of *graph coordinate systems* (GCS), an approach that tolerates an initial computational overhead in order to provide node-distances approximations that take constant time regardless of

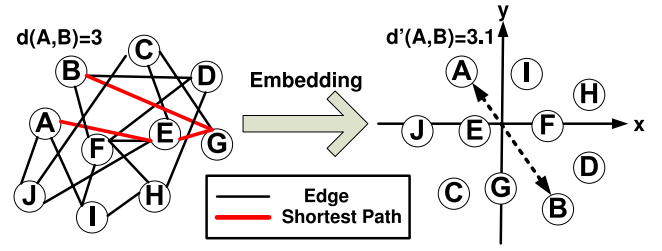


Fig. 1. An example of graph embedding to an Euclidean space. For example, the shortest path distance between nodes A and B is 3 in the graph (left), and the Euclidean distance between their coordinates is 3.1 (right).

graph size. In this section, we introduce the concept of graph coordinate systems, and related work on graph embedding and social networks.

A. Background

Graph coordinate systems, a concept we first proposed in Orion [4], seek to provide accurate estimates of distances between any pair of graph nodes. At a high level, this approach captures the complex structure of a high dimensional graph, and computes a lossy representation for it in the form of a fixed position for each graph node in a coordinate space. Each node’s coordinate position is chosen such that its distance to another node in the coordinate space matches its real shortest path distance to that node in the actual graph. In Figure 1 for example, the shortest path distance between nodes A and B is 3 in the graph, and the Euclidean distance between their coordinate positions is 3.1.

Pros and Cons. The advantage of using a GCS is that, once a graph is embedded, the system can answer each node distance query using a small amount of time independent of the graph size, *i.e.* $O(1)$ time. In practice, each query takes only a few microseconds (μs) to compute. This is very attractive for applications that require large numbers of node distance computations, such as computing graph-wide metrics like graph diameter and average path length. To process queries on a given graph G , however, a GCS must first compute a one-time embedding of G into the coordinate space, *i.e.* compute the coordinate positions of each graph node. This initial step can be computationally expensive, and scales roughly linearly with graph size, *i.e.* $O(n)$ for a graph with n nodes. Finally, a graph coordinate system provides good approximations to graph queries, but does not provide perfect results.

Goals. We focus on two goals in our exploration of the GCS design space. First, we seek to optimize the graph embedding to maximize accuracy. Second, since graph embedding is by far the biggest source of computational cost in a GCS system, our goal is to ensure that we can take advantage of distributed computing resources, *e.g.* server clusters, to ensure scalability as network graphs continue to grow in size.

B. Work on Embedding in Geometric Spaces

Embedding techniques have been used in a variety of application contexts. The most recent and well-known use

of embedding techniques was in the context of network coordinate systems used to estimate Internet latencies without performing exhaustive end-to-end measurements [8], [9], [10]. We summarize prior experiences in embedding in geometric spaces from both measurement and theoretical studies.

Euclidean embedding was first used on simple graphs [11], and was widely used to predict routing latency between Internet hosts [8], [10], [12], [13]. These systems calibrate nodes’ geometric positions based on Internet round-trip time (RTT). Recent result in [14] proves the tightest upper bound, $O(\sqrt{\log n \log \log n})$ for an n -point Euclidean embedding.

Vivaldi [15] was the first to investigate the accuracy of embedding a network into a spherical space. While morphing on spherical spaces is widely used in computer vision [16], there is little theoretical work investigating spherical coordinate systems.

A hyperbolic space can be thought of a space with a tightly connected core, where all paths between nodes pass through. Experimental systems for embedding Internet distances [17], [18], [15] generally showed improved accuracy over analogous systems that used Euclidean spaces.

Kleinberg proposed a routing algorithm in ad hoc networks that works by *greedy embedding* the network into a hyperbolic space, and [19] proposed a similar approach for dynamic graphs. However, their focus is on smaller graphs of wireless or synthetic networks (~ 50 nodes as in [19]). [20] proposes a model using Hyperbolic spaces to *produce* synthetic graphs.

C. Social Network Applications and Studies

Here we briefly summarize other related projects on social applications and social network measurements.

Shortest-path based Applications. Recently, social networks have inspired a numerous security protocols and social applications in a number of fields. In Section V, we will evaluate our proposed system using three of the most common social analysis applications: graph separation metrics, graph centrality, and distance-ranked social search [2], [21].

Many other social applications rely on shortest path computations. For instance, information dissemination [22] can use node distances to find the most influential nodes. Community detection algorithms [23] can use distance between nodes to cluster them. Algorithms for detecting Sybil attacks rely on strategies similar to community detection [24], and hence can also leverage node distance information. Neighborhood function [25] uses node distance distributions to predict whether two graphs are similar. Finally, users in the Overstock auction site query the social graph to see how they are connected to sellers of a given product [1]. All these applications rely heavily on shortest path computations, and therefore can benefit significantly from our system.

Studies of Online Social Networks. Recently, a number of large measurement studies have studied the structure of online social networks through graph measurement and analysis. For example, Mislove et al. published a comprehensive paper to analyze data crawled from Flickr, Livejournal, Orkut and

Youtube [6]. Wilson et al. generated large social graphs and interaction graphs by crawling the Facebook network [5]. Jiang et al. [7] used the same methodology to generate a large social graph of 43 million users on Renren, the Chinese Facebook clone. Finally, Twitter was analyzed in [26], and other studies modeled behavior of social network users using network level data measurements [27], [28].

Our focus. We focus on the problem of designing and building a real system for analyzing today’s massive networks. As with prior work [14], [29], [19], it is extremely challenging to prove bounds on these probabilistic approaches. Instead, we use a wide range of empirical data to verify that our system works accurately for network graphs up to tens of millions of nodes.

III. A HYPERBOLIC GRAPH COORDINATE SYSTEM

A number of recent projects have shown that hyperbolic spaces can more accurately capture distances on a network graph [18], [19], [20]. We also empirically compute distortion metrics [30] on our social graphs for different coordinate systems, and find that the hyperbolic space is in fact significantly more accurate than Euclidean and spherical alternative. The results are omitted here for brevity, but available online [31].

In this section, we describe *Rigel*, a hyperbolic graph coordinate system (GCS) for estimating node distance queries. Before answering queries on a particular graph, the graph must first be embedded into a hyperbolic coordinate space, a process that involves computing ideal coordinate values for each node in the graph. We describe hyperbolic coordinate computation in Rigel, present details of Rigel’s graph embedding process, and explore the impact of system parameters on embedding accuracy. Wherever possible, we compare Rigel’s results directly to comparable results obtained from running Orion [4], our Euclidean GCS.

A. Distance Computation in the Hyperboloid

There are five known “Hyperbolic models” that have been proposed for different graph structures, including the Half-plane, the Poincaré disk model, the Hemisphere model, the Klein model and the Hyperboloid model [18]. In designing Rigel, we chose the *Hyperboloid* model for two practical reasons. First, computing distances between two points in this model is computationally much simpler than alternative models. Second, the computational complexity of calculating distances is independent of the space curvature. This gives us additional flexibility in tuning the structure of the hyperbolic space for improved embedding accuracy.

The curvature parameter c ($c \leq 0$) is an important parameter in the definition of the Hyperbolic space [18]. When $c = 0$, the Hyperbolic space reduces to the Euclidean space. The choice of c has significant impact on the level of distortion between the real node distances and their images on the Hyperbolic space. For a Hyperboloid model with curvature c , the distance between two n -dimension points x and y is defined as follows:

$$\delta(x, y) = \operatorname{arccosh} \left(\sqrt{\left(1 + \sum_{i=1}^n x_i^2\right) \left(1 + \sum_{i=1}^n y_i^2\right) - \sum_{i=1}^n x_i y_i} \right) \cdot |c| \quad (1)$$

B. Computing a Hyperbolic Embedding

We now describe a basic (centralized) algorithm for embedding a graph into our Hyperbolic space. At a high level, we follow the “landmarks” approach proposed in [4], where we first choose a small number of l nodes as landmarks, where $l \ll N$ and N is the number of nodes in G . We first use a global optimization algorithm to fix the coordinates of these landmarks, such that their distances to each other in the coordinate space are as close as possible to their matching path distances in the graph. We refer to this step as “bootstrapping.” Once the landmarks are set, we compute the positions of all remaining nodes, such that each node’s distances to all landmarks in the coordinate space closely match its actual node distances to those landmarks in the graph.

The rationale behind this approach is that computing “ground truth,” *i.e.* the shortest path length between any two nodes, is an expensive task. Thus “calibrating” node positions in a pairwise fashion would generate a large number ($O(N^2)$) of breadth-first-search (BFS) computations. By choosing a small, constant number of landmarks, we only need to compute a BFS tree for each landmark. The resulting values represent shortest path lengths from all remaining nodes to these landmarks, and are sufficient to calibrate their coordinates. As in [4], we choose the landmarks as nodes with highest degree, as a way to efficiently approximate nodes with high centrality.

Next, to compute the coordinate position for a graph node, we randomly select 16 out of the l ($l = 100$) landmarks. Since we know the actual distances in the graph between the new node and its 16 selected landmarks, we apply the Simplex method [32] to compute an optimal coordinate to minimize the deviation in distances between the node and its landmarks in the coordinate space and their actual distances.

Optimizing Local Paths. It has been shown in Internet embedding systems [17] that the largest errors are introduced when estimating paths or node distances for nearby nodes, *i.e.* nodes separated only by 1 or 2 hops. In addition, accuracy in resolving “local” graph queries is critical to many graph operations. In the context of graphs, this is an easy limitation to overcome, since 1-hop neighbors are easily accessible via graph representations, *e.g.* edge lists or adjacency matrices. Rigel uses local neighbor information to augment the node knowledge about its close-by topology. Before answering a query for a pair of nodes, Rigel first checks their adjacency lists to detect if they are direct neighbors or 2 hop neighbors (share a node in their adjacency list).

C. Embedding Accuracy on Real Graphs

We now investigate the impact on embedding accuracy by two important parameters, curvature of the space c and number

TABLE II
RESPONSE TIME FOR ORION, RIGEL-S, RIGEL AND BFS.

Graphs	Orion	Rigel-S	Rigel	BFS
Egypt	0.2 μ s	0.33 μ s	6.8 μ s	0.75s
L.A.	0.18 μ s	0.33 μ s	8.5 μ s	1.027s
Norway	0.19 μ s	0.33 μ s	17.8 μ s	1.44s

of dimensions of the space n . We then study the magnitude of approximation errors as a function of the actual path lengths, and examine the efficiency of our system by using the average query latency. Here, we show the results on the three Facebook graphs in Table I and omit other similar results for brevity.

1) *Impact of Curvature and Dimension:* In order to derive the parameters that maximize the accuracy of our system, we evaluate the impact of two important parameters of Hyperbolic space: curvature and number of dimensions.

Impact of Curvature. The curvature c of a Hyperbolic space is an important parameter that determines the structure of the space. We vary curvature from -50 to 0 and investigate the effect on the accuracy of the distance estimation using our three Facebook social graphs.

Figure 2 plots the average relative error when the curvature varies between -50 and 0 . When the curvature is 0 , the Hyperbolic space is equivalent to a Euclidean space. We include this value as the rightmost point in our plot. From our results, we see that the average error decreases significantly as the curvature approaches -1 . We performed further fine grain tests with curvature values around -1 , and find that the accuracy of our system reaches a plateau near -1 . Results at curvature of -1 are 30% more accurate than results from a Euclidean system, shown in the plot as curvature of 0 . Thus we use the curvature value at -1 in the rest of this paper.

Impact of Dimensions. The number of dimensions of a geometric space plays an important role in determining the accuracy level in the estimate of distances between nodes. We vary the number of dimensions from 2 and 14 to evaluate accuracy. Since the results are similar to [4], *i.e.* accuracy improves as the number of dimensions increases, we omit the result for brevity. As in Orion, 10-dimensions provides a good balance between complexity and accuracy in our Hyperbolic graph coordinate system.

2) *Accuracy and Per-query Latency:* In this section, we examine accuracy as a function of path length, and also compare per-query latency across a number of systems. We use a 10-dimensional Hyperbolic space with curvature of -1 .

Accuracy vs Path Length. We explore the accuracy of predictions for paths of different lengths. Figure 3 shows the average absolute errors per path length on three Facebook graphs by leveraging two embedding systems: Orion (using an Euclidean space) and Rigel. The bottom three lines are the results of Rigel where the average absolute error per path length ranges between 0 and 0.9. Orion results are the top three lines in Figure 3, and the average absolute error per path length range from 0.6 and 3.4, significantly higher than Rigel. This clearly shows that using hyperbolic spaces improves accuracy.

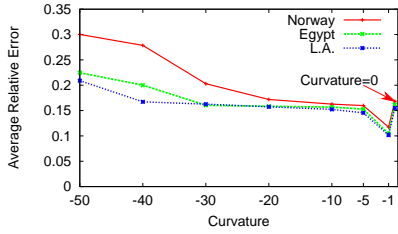


Fig. 2. Impact of hyperbolic curvature on accuracy.

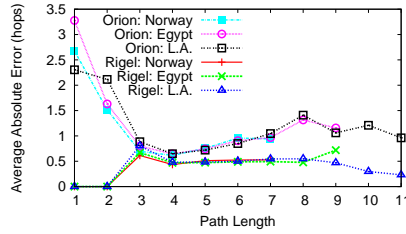


Fig. 3. Average absolute errors for paths of different lengths comparing Rigel and Orion.

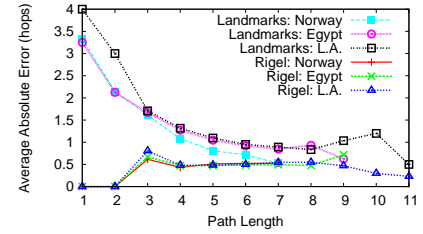


Fig. 4. Average absolute errors comparing Rigel and the “Landmark” scheme from [2].

Also note that Orion produces extremely large errors for close node pairs. These errors are completely eliminated by Rigel’s local path optimization.

We also compare Rigel’s accuracy against the “Landmark” scheme in [2]. Figure 4 shows that Rigel significantly outperforms [2] regardless of the real distance between nodes.

Query Latency. Table II shows the average per-query response time to compute the distance of two random nodes using Orion, Rigel, and BFS. We also list the query time of Rigel without the local path optimization labeled as “Rigel-S.” Rigel-S requires slightly longer time than Orion, because of the increased complexity of the hyperboloid coordinate computation. Memory access in Rigel’s local path optimization adds several microseconds to each query. But overall, Rigel’s per-query time is still 5 orders of magnitude faster than BFS.

IV. EMBEDDING MASSIVE GRAPHS

Since the complexity of Rigel embedding scales linearly with graph size, this processing overhead presents a significant performance bottleneck for large graphs with millions of nodes, and prevents practical applications of Rigel on large social graphs. Here, we describe a mechanism to address this challenge by parallelizing Rigel’s embedding process across multiple servers, named as “Parallel Rigel”. We then evaluate its impact using four large social graphs.

A. Parallelizing Graph Embedding

Parallelizing Rigel is feasible because of two reasons. First, landmark bootstrapping requires computing BFS trees rooted from each landmark, which can be run independently and in parallel on different servers. Second, after bootstrapping, each graph node u can also be embedded independently and in parallel based on the coordinates of the global landmarks. Because the number of nodes is large, we just need to distribute nodes across servers to ensure load balancing.

Parallel Rigel. We integrate the above mechanisms with the original Rigel design, called *Parallel Rigel*. Figure 5 demonstrates the Parallel Rigel system on top of and contrasts it to the original Rigel design. It consists of three components: *parallel bootstrapping*, *graph partitioning* and *parallel embedding*. The parallel bootstrapping module distributes BFS tree computation related to each landmark across servers, one or more landmarks per server. The graph partitioning module provides a balanced distribution of nodes across servers. The

cost of this operation is negligible since simple partitioning schemes are sufficient. Finally, the parallel embedding module embeds all graph nodes in parallel across the servers, allowing Parallel Rigel to achieve significant speedup.

We have implemented a fully functional prototype of parallel Rigel, and used it to embed the largest graph we have, the 43 million-node graph from the Renren online social network. As seen in Figure 5, running the centralized version of Rigel on a single large memory server (Dell PowerEdge server with 32GB of RAM) required 136 hours to perform initial bootstrapping (computing BFS trees), and more than 10 days to do the actual node embedding of all graph nodes. Applying parallel Rigel to the same graph over a cluster of 50 servers (Dell Xeon, 2GB) reduces the parallel bootstrap process to 2.7 hours, and embedding to only 6.4 hours.

B. Experimental Results

Here, we use four of the largest social graphs available today, Flickr, Orkut, Livejournal and Renren in Table I, to examine the accuracy and efficiency of Parallel Rigel.

Accuracy. We first examine the accuracy of Parallel Rigel by comparing it to Orion. In Figure 6 we plot the average absolute error for different path lengths using Parallel Rigel and Orion. Like our previous results on smaller Facebook graphs, Parallel Rigel not only significantly improves the accuracy of long distance prediction, but also reduces the error in short distance estimation. We also verify that Parallel Rigel performs similar to the original Rigel on these graphs.

Computation Efficiency. We evaluate the efficiency of Parallel Rigel by comparing its computation time to that of original Rigel. By utilizing a cluster of servers, Parallel Rigel distributes the tasks of landmark bootstrapping and graph embedding over multiple parallel servers. While Parallel Rigel does require an extra step of graph partitioning by distributing nodes among machines, it only leads to a minor increase in time complexity, less than 0.1% of the original bootstrapping time. Table III shows the comparison when Parallel Rigel runs on a cluster of 50 servers. We see that Parallel Rigel achieves close to linear speedup, even slightly better due to reduced virtual memory paging on each server.

To examine the impact of the cluster size, we compare the speedup of Parallel Rigel by using 5, 10, 20 and 50 servers, where speedup is the decrease in embedding time. Figure 7 shows that run time decreases almost linearly with cluster size.

TABLE III
COMPARING THE TIME COMPLEXITY OF RIGEL AND PARALLEL RIGEL (P-RIGEL) USING A CLUSTER OF 50 SERVERS.

Graphs	Bootstrap (hours)		Graph Partitioning (hours)	Embedding (hours)		Response	
	Rigel	P-Rigel	P-Rigel	Rigel	P-Rigel	BFS	Rigel
Flickr	1.4	0.028	0.003	9.7	0.24	24.5s	12.9 μ s
Orkut	7.5	0.15	0.005	19.4	0.42	56.2s	36.6 μ s
Livejournal	4.8	0.096	0.008	32.2	0.66	65.2s	8.4 μ s
Renren	136.2	2.7	0.07	250	6.4	1598.5s	28.9 μ s

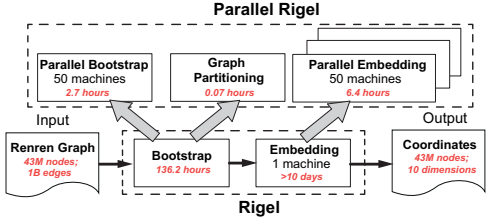


Fig. 5. A high-level view of how embedding is parallelized and its net impact on embedding latency for Renren, our largest graph.

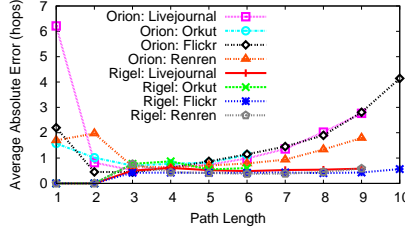


Fig. 6. Average Absolute Error for different path lengths computed by Parallel Rigel and Orion.

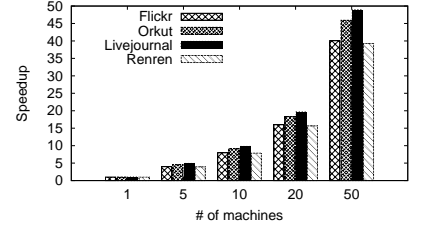


Fig. 7. Average speedup achieved by Parallel Rigel on different cluster configurations.

V. APPLICATIONS

We demonstrate the effectiveness and efficiency of Rigel in social network analysis and applications by implementing several common graph applications. In each case, we compare the accuracy of Rigel against that of Orion [4].

A. Computing Separation Metrics

Social network graphs are known for displaying the “Small World” behavior. Graph separation metrics such as diameter, radius and average path length, have been widely used to examine and quantify the Small World behavior. But since each of these metrics relies on large numbers of node distance computations, computing them for large graphs can become extremely costly or even intractable.

Using Rigel, we build an application to compute the graph separation metrics listed above, and examine their accuracy by comparing their results to ground truth. Since computing shortest path length between all node pairs takes several days even for our smallest graph (Facebook Egypt), we take a random sampling approach to compute the ground truth. We randomly sample 5000 nodes from the three Facebook graphs, 500 nodes from Flickr, Livejournal and Orkut, and 100 nodes from Renren, and use shortest path lengths between these pairs to derive the separation metrics.

We report the results in Table IV for Radius, Diameter and Average Path Length on seven different graphs, for Rigel, Orion and Ground Truth. In general, Rigel consistently provides more accurate results compared to Orion. More importantly, Rigel provides results across all three metrics that are extremely close to ground truth values.

B. Computing Graph Centrality

Graph centrality is an extremely useful metric for social applications such as influence maximization [22] and social search. For example, application developers can use node centrality values to identify the most influential nodes for

propagating information in an online social network. Formally, the most “central” node is defined as the node that has the lowest average node distance to all other nodes in the network.

Using Rigel, we implement a simple application to compute node centrality directly from node distance values, where a node with a small average path length has a high centrality score. As before, we examine the accuracy of our Rigel-enabled application by computing the centrality of $x = 5000$ randomly chosen nodes on the three Facebook graphs, $x = 500$ randomly chosen nodes each for Flickr, Livejournal Orkut, and $x = 100$ nodes for Renren. For each graph, we sort these x nodes by centrality, and select the top k nodes. We compute the “accuracy” of Rigel’s centrality ordering by counting the number of overlapping nodes (m) in Rigel’s top k nodes and actual top k centrality nodes as computed by BFS on the original graph. We study the accuracy of our Rigel-based system as the ratio of m to k .

We perform our experiments on all seven of our social graphs, and find the results to be highly consistent. For the rest of this section, we will only report results for three of them: Facebook Los Angeles, Orkut and Livejournal. Figure 8 shows the centrality accuracy results for different values of k . As expected, the accuracy of both Rigel and Orion increases with larger k values. In general, Rigel consistently outperforms Orion for different graphs and different values of k .

C. Distance-Ranked Social Search

Social networks such as Facebook and LinkedIn can best serve their users by ranking search results by the proximity of each result to the user in the social graph [21]. This is because users are likely to care about its social proximity to the origin of the search result as much as the quality of the result itself, *i.e.* a user would pay more interest to results from her close friend rather than those from an unrelated stranger.

Despite its usefulness, using social distance in search results is highly costly due to the number of node distance computa-

TABLE IV
COMPARING SEPARATION METRIC RESULTS, AS COMPUTED BY RIGEL, ORION, AND BFS (GROUND TRUTH).

Metric	Method	Egypt	L. A.	Norway	Flickr	Orkut	Livejournal	Renren
Radius	Ground Truth	9	11	8	13	6	13	12
	Rigel	8.7	11.0	7.5	12.7	6.4	12.2	12.0
	Orion	9.2	10.7	7.8	12.6	6.3	12.0	12.1
Diameter	Ground Truth	14	18	12	19	8	17	15
	Rigel	14.8	17.9	11.7	18.6	10.2	17.7	14.9
	Orion	14.4	17.8	12.2	17.3	10.0	16.8	14.9
Average Path Length	Ground Truth	5.0	5.2	4.2	5.1	4.1	5.4	5.0
	Rigel	4.9	5.1	4.2	5.0	4.3	5.5	4.9
	Orion	4.7	5.0	4.1	4.3	3.9	4.8	4.6

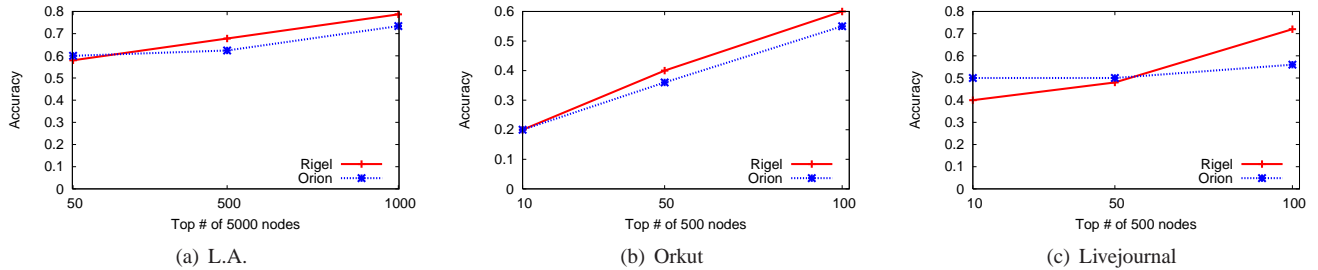


Fig. 8. Average accuracy of queries for the top k high centrality nodes. Rigel consistently outperforms Orion.

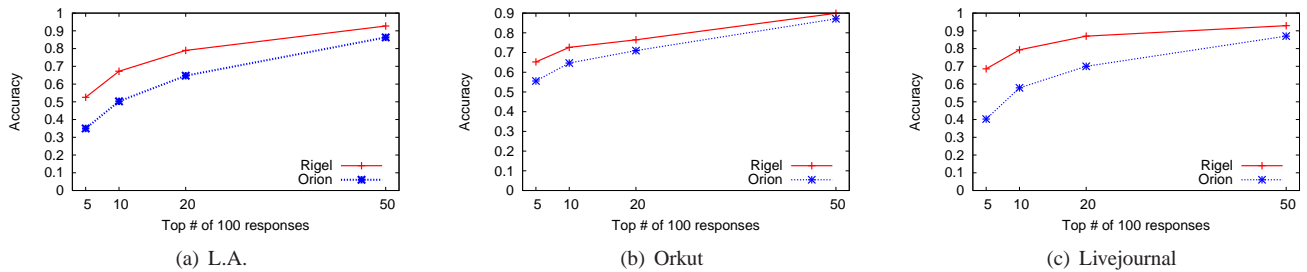


Fig. 9. Average accuracy of social search queries that return top k ranked nodes

tions necessary for each social search query. Instead, we can leverage Rigel’s constant time node-distance functionality to build powerful distance-based social search applications.

To verify the impact of Rigel on distance-ranked social search, we perform the following experiment. For each node initiating a query, we select 100 random nodes in the graph to respond to the query. We sort the responses by their distances to the query node, computed via Rigel and Orion, and return the top k nodes to the user. We then compute the same top k nodes using BFS for distance computation, and examine the percent of overlapping nodes between the result sets as a measure of accuracy. We repeat this experiment 5000 times on smaller graphs, *e.g.* Facebook graphs, and 100 times on our largest graph, *i.e.* Renren. We vary k from 5 to 50, and show the results of L.A., Orkut and Livejournal in Figure 9. It shows that Rigel’s hyperbolic coordinates consistently and significantly outperform Orion’s Euclidean coordinates. On Livejournal, for example, when we rank the top 5% search results, average accuracy of Rigel is 70% while Orion only achieves 40%.

VI. SHORTEST PATHS IN RIGEL

A number of critical graph-based applications require not only the length of the shortest paths, but also the actual shortest path between two nodes. For example, on the Overstock social auction system, users can search how they connect to the seller of a given object, and choose to buy from friends of friends instead of complete strangers [1]. On LinkedIn, a user browsing another’s profile automatically shows any social paths (3 hops or less) connecting them.

In this section, we describe a novel extension to Graph Coordinate Systems that produces accurate approximations of shortest paths by using node distance queries as a tool. We first describe how this extension to Rigel computes short paths between any two nodes. Next, we describe the Sketch algorithm [33], an efficient algorithm for shortest path estimation, and its followup algorithms including SketchCE, SketchCESC, and TreeSketch [34]. Finally, we compare Rigel’s shortest path algorithm against these algorithms on a variety of social graphs in both accuracy and per-query runtime. We show that while Rigel requires similar preprocessing times to these algorithms, Rigel’s shortest paths return query results 3-18 times faster,

while matching the best of these algorithms in accuracy.

A. Finding Shortest Paths using Rigel

We now describe a heuristic that uses our coordinate system to find a good approximation of the shortest path connecting any two nodes. Our algorithm, which we call *Rigel Paths*, uses techniques reminiscent of the routing algorithm in [20].

Given two nodes A and B , we start by computing the distance between them $d(A, B)$. If the distance is 1 or 2 hops, we can use simple lookup on their adjacency lists to determine the shortest path between them. If the predicted distance between the nodes is greater than 2 hops, then we begin an iterative process where we attempt to explore potential paths between the nodes using the coordinate space as a directional guide.

Starting from A , we use Rigel to estimate the distance of each of its neighbors N_i^A to B . The expected distance for a neighbor on the shortest path should be $d(A, B) - 1$. If any neighbor's estimated distance is within a δ factor of that prediction, it is considered a candidate to explore. For each of A 's neighbors that qualify as a candidate node, we repeat the process to obtain candidates for hop 2. This process iterates until one of the candidate nodes is a direct neighbor of B .

At each iteration of the algorithm, *i.e.* for the n^{th} hop, we keep a maximum number of candidates C_{max} to explore. Choosing this number manages the tradeoff between exploring too many paths (and extending processing latency) and exploring too few paths (and finding a dead end or inefficient paths). In practice we choose C_{max} to be 30, and δ to be 0.3.

B. Sketch-based Algorithms for Shortest Path

We first describe the state-of-the-art algorithms for locating shortest paths. There are four algorithms all based on variants of the Sketch algorithm [33], [34].

Sketch [33]. Sketch is a landmark-based solution where each node computes its shortest paths to the landmarks and then uses common landmarks between itself and another node in the graph to estimate their shortest paths. This method selects $r = \lceil \log N \rceil$ sets of landmark nodes, where N is the number of the graph nodes. For each node, Sketch computes its shortest paths to k ($k=2$) different landmarks in each set. Those shortest paths are precomputed by using the results of BFS trees rooted in each landmark. Therefore, for an undirected graph, each node is associated with $k \cdot r$ shortest paths.

Cycle Elimination, Short Cutting and TreeSketch [34]. These three algorithms are variants of the basic Sketch approach for finding shortest paths [34]. *First*, Cycle Elimination, called SketchCE, improves Sketch by simply removing cycles in the estimated paths computed by Sketch. *Second*, Short Cutting improves Sketch by searching for bridging edges between two nodes x and y , where x is on the path between the source and the landmark and y is on the path from the landmark to the destination. If such an edge is found, this edge replaces the sub-path through the landmark. This approach is called SketchCESC. It locates shorter paths, but dramatically increases computational time.

Finally, TreeSketch is a tree-based approach. At query time, TreeSketch builds two trees separately rooted in the source and the destination using precomputed paths to landmarks. Given the two trees, the path search starts from both root nodes, and iteratively explores more nodes from both trees. BFS computation starts from roots of both trees. For each visited node u in a tree, its neighbors are compared with any visited node v in the other tree. Once a common node is found, the shortest path between source and destination is constructed using the sub-path from source to node u , the edge (u, v) , and the sub-path from v to the destination. While producing very accurate paths, TreeSketch is computationally slow due to the tree construction and extensive search process.

C. Comparing Shortest Path Algorithms

We compare our *Rigel Paths* to Sketch, SketchCE, SketchCESC and TreeSketch in accuracy and query latency.

Experimental Settings. To compare Rigel Paths against prior work, we obtained the source code for the sketch-based algorithms from the authors [34]. The code runs on RDF-3X [35], a specialized database system optimized for efficient storage and computation of large graphs. All experiments were performed on Dell quad-core Xeon servers with 24GB of RAM, except for Renren experiments, which were performed on similarly configured Dell servers with 32GB of RAM.

Accuracy. For each graph in Table I, we randomly sample 5000 node pairs, and compare the shortest path results of Rigel Paths, Sketch, SketchCE, SKetchCESC, and TreeSketch algorithms against the actual shortest paths computed via BFS.

Figure 10 shows the average absolute error of the five different algorithms broken down by length of the actual shortest path. Here we define the absolute error as the additional number of hops in the estimated path compared to the real path. As before, we only show the Facebook Los Angeles, Orkut and Livejournal graphs for brevity, because their results are representative of results on other graphs. The results show consistent trends across the graphs. The Sketch and SketchCE algorithms are highly inaccurate, and generally produce shortest paths that are roughly 2 hops longer than the real path. TreeSketch and Rigel Paths are the most accurate algorithms and often indistinguishable from each other.

We show the CDF of absolute errors of the different algorithms in Figure 11. This shows a clear picture of the distribution of errors. Rigel paths and TreeSketch are by far the most accurate algorithms. Both produce exact shortest paths for a large majority of node pairs. Both are significantly better than SketchCESC. SketchCE and Sketch are fairly inaccurate, and provide paths with multiple hop errors for the overwhelming majority of node pairs. While Rigel Paths provides accuracy that matches or beats all of the Sketch based algorithms, we will show later that it is significantly faster than both SketchCESC and TreeSketch (ranging from a factor of 3 to a factor of 18 depending on the specific graph).

Finally, we also compared the length of the shortest paths found by our Rigel Paths algorithm to node distance values

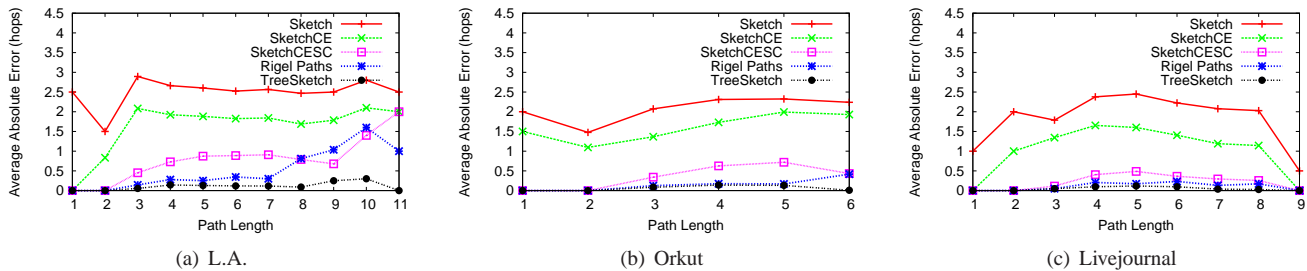


Fig. 10. Absolute error (in hops) of shortest paths returned by Rigel Paths, Sketch, SketchCE, SketchCESC and TreeSketch.

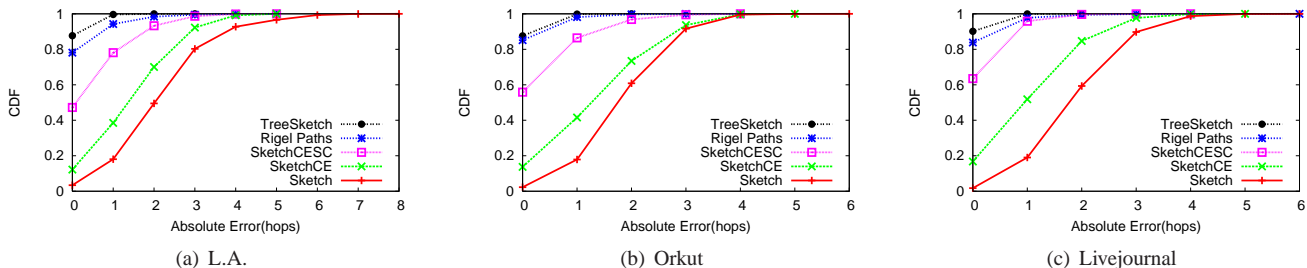


Fig. 11. CDF of the absolute error in path finding among Rigel Paths, Sketch, SketchCE, SketchCESC and TreeSketch.

TABLE V
COMPARING THE PREPROCESSING TIMES AND PER-QUERY RESPONSE TIMES OF RIGEL PATHS, SKETCH AND VARIANTS SKETCHCE, SKETCHCESC AND TREESKETCH. PREPROCESSING/EMBEDDING TIME FOR RIGEL (AND RIGEL PATHS) IS FOR SINGLE SERVER (NON-PARALLEL VERSION).

Graphs	Preprocessing (Hours)		Per-Query Response Time (μ s)						
	Rigel	Sketch	Rigel	Sketch	SketchCE	Rigel Paths	SketchCESC	TreeSketch	
Egypt	1.3	0.43	6.8	1781	1792	3667	38044	62407	
L.A.	1.5	0.54	8.4	936	946	4008	20597	56828	
Norway	1.4	0.67	17.8	1492	1501	4621	21472	59635	
Flickr	9.7	3.3	12.9	17157	17178	41279	732332	630890	
Orkut	19.4	13.1	36.6	21043	21054	49470	273586	730284	
Livejournal	32.2	14.2	8.4	75101	75114	28355	253976	348464	
Renren	250	348	28.9	124327	124334	181814	546925	2594756	

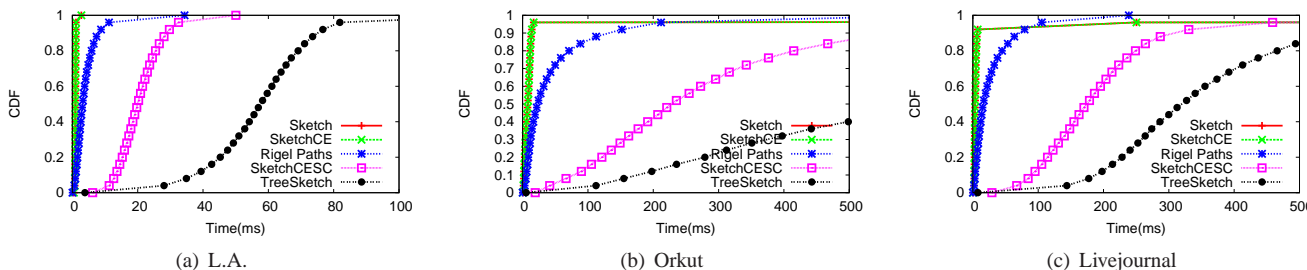


Fig. 12. CDF of computing time in path finding among Rigel Paths, Sketch, SketchCE, SketchCESC and TreeSketch.

estimated by Rigel. Interestingly, Rigel Paths is more accurate, with absolute errors below 0.3, compared to errors between 0.5 and 1 hop by comparing Figure 10 and Figure 6. Rigel Paths achieves this higher level of accuracy because it leverages actual graph structure to compute its shortest paths.

Computational Costs. We now compare Rigel Paths and the four Sketch algorithms on computational time complexity. We break down our analysis into two components. First, we measure each algorithm’s *preprocessing time*. For Rigel Paths (and Rigel), this represents the time required to embed the

graph into the coordinate space. For all Sketch algorithms, this is the time to compute shortest paths (using BFS) to all of their landmark nodes [34]. Our second component measures the computational latency required to resolve each query. All experiments are run on a single server.

In Table V, we see that Rigel takes roughly 2–3 times longer preprocessing time. Note, however, that these measurements were run on only a single server. As shown in Figure 7, we can distribute Rigel’s preprocessing phase across multiple machines with close to linear speedup. Thus, we can reduce Rigel preprocessing by spreading the load over 2 or 3 machines.

Again, we choose 5000 random node pairs in each graph, and compare the average query response time for each algorithm in Table V. Recall that Sketch and SketchCE produce paths that are highly inaccurate, *i.e.* introduce an average of 2-3 additional hops in each path. Of the two best algorithms, Rigel Paths and TreeSketch, Rigel paths returns results in a fraction of the time required by TreeSketch and SketchCESC. The latency reduction ranges from ~ 3 (against SketchCESC on Renren) to a factor of 18 (against SketchCESC on Flickr). We show a CDF of these results in Figure 12. Rigel Paths is clearly much faster than both TreeSketch and SketchCESC.

Finally, we also include the node-distance computation time from Rigel as a point of reference. Clearly, finding actual shortest paths is orders of magnitude more expensive than simply computing node distance. Luckily, the large majority of graph analysis tasks only require node-distance computation, and only user-interactive queries require the full shortest path between node pairs.

VII. CONCLUSION

Traditional algorithms for performing graph analytics no longer scale to today’s massive graphs with millions of nodes and billions of edges. Computing distances and shortest paths between nodes lies at the heart of most graph analysis metrics and applications, and is often responsible for making them intractable on large graphs.

We propose Rigel, a hyperbolic graph coordinate system that approximates node distances by first embedding graphs into a hyperbolic space. Even for graphs with 43 million nodes and 1+ billion edges, Rigel not only produces significantly more accurate results than prior system, but also answers node distance queries in 10’s of microseconds using commodity computing servers. For the more challenging task of computing shortest paths, we propose Rigel Paths, a highly efficient algorithm that leverages Rigel’s node distance estimates to locate shortest paths. The results are impressive. Rigel Paths produces exact shortest paths for the large majority of node pairs, matching the most accurate of prior systems. And it does this quickly, returning results up to 18 times faster than state-of-the-art shortest-path systems with similar levels of accuracy.

Finally, we are releasing the code to Rigel for download at <http://current.cs.ucsb.edu/rigel>. To the best of our knowledge, our system has already been deployed at several social networking and gaming companies.

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