Sum-Rate Distortion Bound for Suboptimal Multiterminal Source Coding Applied in Medical Wireless Sensor Networks

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ABSTRACT

In this paper, we derive the suboptimal sum-rate distortion bound for encoding correlated Gaussian sources for which the uniform scalar quantizers are used to convert the signals from analog to the discrete form. The correlated discrete signals are further compressed using the lossless Slepian and Wolf encoder. The gap between the suboptimal bound and the inner/outer bounds is also investigated. The result shows that the gap is at most 0.255 bits/sample/source which is far less than the universal bound of 2.45 bits/sample given in [1] for the three tree-structured Gaussian sources.

Keywords

Multiterminal source coding, high fidelity distortion, scalar quantization, asymptotic performance.

1. INTRODUCTION

Medical wireless body area networks using Ultra WideBand (UWB) technology have recently been proposed for surgical and intensive care units at hospitals [2]. The network architecture includes an in-body and on-body sensor, plus a network coordinator. The in-body sensor is implanted inside a patient's body in order to capture important medical information which would help doctors and nurses provide better cares or services for patients. The on-body sensor, on the other hand, is attached on the patient's body not only to sense medical parameters but also to function as a communication relay for the in-body sensor since the in-body sensor's transmission distance is often short due to power limitation [2].

The signals from the on-body sensors may have a certain degree of correlation because: (1) One copy of the transmitted signal from the in-body will be received by a number of the on-body sensors. (2) The on-body sensors are placed physically close to sense the same medical parameters. It has been theoretical established under the topic of the distributed lossy source coding problem that if the correlation is taken into account to compress the source signal (see [**?**], [3] and references therein), the compression efficiency is significantly improved.

However, the cited works focus on deriving inner and outer bounds for rate distortion regardless of delay and complexity issues. In our engineering source coding problem for medical wireless sensor networks, the complexity and delay are both critically limited. Hence, in this paper, we consider a suboptimal multi-terminal source coding system, where M encoders compress their own signals without cooperating with one another. The compressed messages are then delivered to a common decoder over the constrained-rate and noiseless channels. The considered system is suboptimal in the sense that:

i) A scalar quantizer is used to convert the analog signal to discrete form before applying lossless Slepian and Wolf encoding.

ii) The design of the scalar quantizer in one on-body sensor is independent of the others.

iii) The decoder restores each individual source with different distortion level.

The multi-terminal source coding has been considered since the early 1970s. However, characterization of the rate distortion region is known only for some particular cases. For the lossless discrete source coding problem, the rate distortion region was completely solved by Slepian and Wolf [4] in 1973. Some practical code designs based on their idea that perform closely to the established theoretical limit are now available and ready for implementation. However, the characterization of the rate distortion region for the lossy source coding problem is still open. In Beger and Tung's early work,[5],[6], the inner bound was derived, and later Y. Oohama, [7], proved that the inner bound is partially tight for a special case of Gaussian sources.

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BODYNETS 2012, September 24-26, Oslo, Norway

DOI 10.4108/icst.bodynets.2012.249975

Recently, Wagner and et al.,[3], have investigated the rate distortion region for the special case of two Gaussian sources. They proved that the source coding architecture based on the vector quantization and the Slepian and Wolf encoding techniques are optimal for this specific case. In another direction, by using the approximation method, M. Ali and et al [1] have derived the rate distortion region for three Gaussian tree structured sources. That is, for this special case, the rate distortion region of the approximation method is at most 2.45 bits/sample from the cooperative bound.

In this paper, we use the technique in [8] to derive the sum-rate distortion bound for the suboptimal multiterminal source coding problem of general M Gaussian sources at high fidelity region. The asymptotic gaps between the suboptimal bound and the optimal one for two Gaussian sources and three Gaussian tree structured sources are also included. The remaining of this paper is organized as follows. In Section 2, we briefly describe the suboptimal system. In Section 3, the sum-rate distortion is derived. The asymptotic gaps for two special cases are presented in Section 5. Section 6 concludes the paper.

2. SYSTEM DESCRIPTION

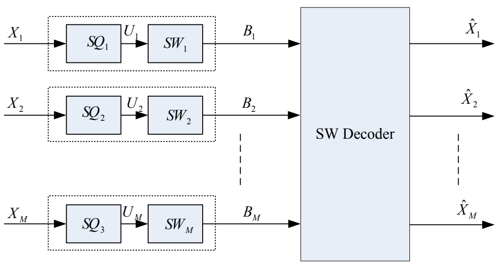

Figure 1: Encoder of the considered system

In this paper, we consider a suboptimal source coding system where M jointly Gaussian sources, denoted $X_1, X_2, \cdots, X_M$ with zero mean the correlation matrix K_X, are the sources to be transmitted to the same destination. The sources are encoded separately at the encoders and jointly decoded at the destination. Furthermore, each source is restored with a different fidelity requirement.

The considered source coding system is depicted in Fig. 1 where each source sequence is first uniformly quantized by a scalar quantizer to obtain a finite-level sequences $B_1, B_2, \cdots, B_M$ and then the quantized outputs are encoded via Slepian and Wolf source coders to exploit the correlation between them. At the decoder side, the Slepian and Wolf decoder is used. In the below section, we will derive the asymptotic sum-rate distortion of this source coding problem under high fidelity requirement.

3. ASYMPTOTIC SUM-RATE RESULT

Assuming that the mean square error is used as the fidelity measure, the mean square error distortion for source m is

$$d_m = E[(X_m - \widehat{X}_m)^2]. \tag{1}$$

Theorem: *For the high fidelity region, the sum rate of the above source coding system is approximated by*

$$R_{sum} \approx \frac{1}{2} \log \frac{|K_X|}{|D_d|} + 0,255M, \tag{2}$$

where $|A|$ is the det of matrix A, M is the number of sources, and $D_d = \text{diag}(d_1, d_2, \cdots, d_M)$, the diagonal matrix with mth diagonal element of d_m.

Proof:

To prove the theorem, we use the technique in [8]. Assume that each source is quantized by splitting the range of X_m into an infinite number of interval $I_{n_m} = [g_{n_m}, g_{n_m+1})$. The quantized level is defined as

$$U_m = U_{m_n}, X_m \in I_{n_m}. \tag{3}$$

Note that each encoder can have a different partition depending on its fidelity requirement.

By the quantizing process, the continuous sources are converted into discrete form. The quantized signals, denoted by $(U_1, U_2, \cdots, U_M)$, are correlated. Since the discrete correlated samples are then encoded by the Slepian and Wolf encoder, the sum rate required to recover $U_1, U_2, \cdots, U_M$ with arbitrary small error probability at decoder is, [4],

$$R_{sum} = H(U_1, U_2, \cdots, U_M) = -\sum p_{\mathbf{n}} log p_{\mathbf{n}}, \tag{4}$$

where

$$p_{\mathbf{n}} = Pr[X_1 \in I_{n_1}, X_2 \in I_{n_2}, \cdots, X_M \in I_{n_M}] \tag{5}$$
$$= \int_{I_{n_1}} \cdots \int_{\mathbf{I}_{n_M}} f(x_1, x_2, \cdots, x_M) dx_1 dx_2 \cdots dx_M.$$

$f(x_1, x_2, \cdots, x_M)$ is the joint density function. The corresponding MSE distortion is

$$d_m = \sum_{n_m} \int_{I_{n_m}} f_m(x_m)(x_m - U_{n_m})^2 dx_m, \tag{6}$$

where $f_m(x_m)$ is the marginal distribution of X_m.

We approximate $p_{\mathbf{n}}$ by assuming that the lengths of all the intervals are reasonably small

$$p_{\mathbf{n}} \approx f(g_{n_1}, g_{n_2}, \cdots, g_{n_M}) \prod_{m=1}^{M} (g_{n_m+1} - g_{n_m}). \tag{7}$$

By the same assumption, we can assign U_{n_m} equal to the mid-point of the interval I_{n_m}, the following approximation is obtained

$$U_{n_m} \approx \frac{g_{n_m+1} + g_{n_m}}{2} \tag{8}$$

and

$$d_m \approx \sum_{n_m} f_m(g_{n_m}) \frac{(g_{n_m+1} - g_{n_m})^3}{12}. \tag{9}$$

Define the mesh points g_{n_m} by

$$g_{n_m} = g_m(n_m \delta_m), \tag{10}$$

where $g_m(t)$ is some suitable smooth monotone increasing function [8]. For δ_m small, we can approximate the term

$g_{n_m+1} - g_{n_m}$ as

$$g_{n_m+1} - g_{n_m} \approx \delta_m g_m^{'}(\delta_m n_m), \tag{11}$$

where the prime indicates derivative. Substituting (11) and (7) into (4), the sum rate is approximated by

$$R_{sum} \approx -\sum_{\mathbf{n}} \left\{ f(g_{n_1}, g_{n_2}, \cdots, g_{n_M}) \prod_{m=1}^{M} \delta_m g_m^{'}(\delta_m n_m) \right\} \times \log \left[f(g_{n_1}, g_{n_2}, \cdots, g_{n_M}) \prod_{m=1}^{M} \delta_m g_m^{'}(\delta_m n_m) \right], \tag{12}$$

and the MSE distortion is thus

$$d_m \approx \sum f_m(g_{n_m}(\delta_m n_m)) \frac{(\delta_m g_m^{'}(\delta_m n_m))^3}{12}. \tag{13}$$

For δ_m small, the sums in (12) and (13) are further approximated by integrals

$$\begin{aligned} R_{sum} \approx & -\int f(g_1, g_2, \cdots, g_M) \\ & \times \log f(g_1, g_2, \cdots, g_M) dg_1 dg_2 dg_M \\ & - \sum_{m=1}^{M} \log \delta_m - \sum_{m=1}^{M} \int \log g_m^{'} f(g_m) dg_m \end{aligned} \tag{14}$$

and

$$d_m \approx \frac{\delta_m^2}{12} \int g_m^{'}(x_m) f_m(x_m) dx_m. \tag{15}$$

The problem is to choose $\{g_1(t), g_2(t), \cdots, g_M(t)\}$ so that for fixed $\delta_m, m = 1, 2, \cdots, M$ and $d_m, m = 1, 2, \cdots, M$, the R_{sum} is minimum. Using the Largrange technique, the resulting function $g_m^{'}(t)$ is independent of t. For convenience, we choose $g_m^{'}(t) = 1$ and

$$g_m(t) = t, \forall m = 1, 2, \cdots, M. \tag{16}$$

Inserting (16) into (14) and (15), we have

$$R_{sum} \approx H(X1, X_2, \cdots, X_M) - \sum_{m=1}^{M} \log \delta_m, \tag{17}$$

and

$$d_m \approx \frac{\delta_m^2}{12}. \tag{18}$$

Combining (17) and (18), the resulting approximated sum-rate distortion is

$$\begin{aligned} R_{sum} &\approx \frac{1}{2} \log \left((2\pi e)^M |K_X| \right) - \frac{1}{2} \sum_{m=1}^{M} \log (12 d_m) \\ &\approx \frac{1}{2} \log \frac{|K_X|}{|D_d|} + 0.255M, \end{aligned} \tag{19}$$

where $|A|$ is the det of matrix A and $D_d = \text{diag}(d_1, d_2, \cdots, d_M)$, the diagonal matrix with the mth diagonal element of d_m.

4. GAP TO THEORETICAL BOUNDS

We are now interested in considering how many bits per source the suboptimal system looses in comparison with the optimal outer bound. Let $R_{sum}^o(d_1, d_2, \cdots, d_M)$ be the outer bound sum-rate of the source coding problem. Define the gap per source between the sum-rate of the considered sub-optimal system and the optimal system, denoted as ΔR:

$$\Delta R = \frac{R_{sum}(d_1, d_2, \cdots, d_M) - R_{sum}^o(d_1, d_2, \cdots, d_M)}{M}. \tag{20}$$

Since the source is jointly Gaussian, the outer bound on the sum-rate is equal to [1]

$$R_{sum}^o(d_1, d_2, \cdots, d_M) = \frac{1}{2} \log \frac{|K_X|}{|D_E|}, \tag{21}$$

where $|D_E|$ is achieved by the following convex optimization problem

$$\begin{aligned} |D_E| &= \max_D |D| \\ \text{s.t } & 0 \leq D \leq K_X \\ D(m, m) &\leq d_m, \forall m = 1, 2, \cdots, M, \end{aligned} \tag{22}$$

where D_E is the optimal distortion matrix.

Combining (19) and (21), the gap is thus

$$\Delta R = 0.255 + \frac{1}{2M} \log \frac{|D_E|}{|D_d|}. \tag{23}$$

The explicit result for (23) is not known in general. However, we can have upper bound the gap by using the Hadamard inequality [9] as follow.

$$|D_E| \leq \prod_{m=1}^{M} D_E(m, m) \leq \prod_{m=1}^{M} d_m \tag{24}$$

From (23) and (24), we have

$$\Delta R \leq 0.255 \text{ bits/sample/source}, \tag{25}$$

which means that the penalty for suboptimality is at most 0.255 bits/sample per source.

5. TWO SPECIAL CASES

5.1 Two Gaussian Sources, $M = 2$

Assume we have two Gaussian sources with zero mean ad covariance K_X as

$$K_X = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}. \tag{26}$$

The resulting gap of this special case is stated in the Theorem below.

By evaluating (19) directly for $M = 2$ and covariance matrix in (26), we have the asymptotic sum-rate as

$$R_{sum} = \frac{1}{2} \log \left[\frac{(1 - \rho^2)}{d_1 d_2} \right] + 0.51. \tag{27}$$

On the other hand, the outer sum-rate distortion for two Gaussian source has been previously found as, [7],

$$R_{sum}^o = \frac{1}{2} \log \left[\frac{(1 - \rho^2)}{d_1 d_2} \right]. \tag{28}$$

From (27) and (28), the asymptotic gap is

$$\Delta R = 0.255 \text{ (bits/sample/source)}, \tag{29}$$

which is similar to the penalty of the single source regardless the value of the correlation parameter.

However, in comparison with the inner bound [3], the asymptotical gap is thus

$$\Delta R = 0.255 - \frac{1}{4}\log\frac{\beta}{2}, \tag{30}$$

where

$$\beta = 1 + \sqrt{1 + \frac{4\rho^2 d_1 d_2}{(1-\rho^2)^2}}.$$

5.2 Three Gaussian tree structured sources, $M = 3$

In this subsection, we consider the special case of a three-tree structured source which has been previously studied by Mohammad and et. al [1]. That is, there are three Gaussian sources $\{X_1(t), X_2(t), X_3(t)\}$ which are the sequences of idependent and identical Gaussian random variables, having zero mean and covariance matrix in the following form

$$K_X = \begin{bmatrix} 1 & \rho_{12} & \rho_{13} \\ \rho_{12} & 1 & \rho_{23} \\ \rho_{13} & \rho_{23} & 1 \end{bmatrix}. \tag{31}$$

As proved in [1], the optimal distortion matrix D_E is upper-bounded as

$$D_E = |D^*| \leq d_1 d_2 d_3 (1 - \theta^2(d_1, d_3))(1 - \theta^2(d_2, d_3)), \tag{32}$$

where

$$\theta(d_i, d_j) = \max\left\{0, \frac{\rho_{12} - \sqrt{(1-d_j)(1-d_i)}}{\sqrt{d_j d_k}}\right\}.$$

As a result, the gap is given by

$$\begin{aligned} \Delta R &= \frac{1}{2}\log\frac{|D_E|}{|D_d|} + 0.255 \\ &\leq 0.255 + \frac{1}{6}\log\left[(1 - \theta^2(d_1, d_3))(1 - \theta^2(d_2, d_3))\right] \\ &\leq 0.255, \end{aligned} \tag{33}$$

which is tighter than the universal approximated bound in [1].

6. CONCLUSIONS

In this paper, we address the asymptotic sum-rate distortion region for the suboptimal multiterminal source coding problem of M Gaussian sources for the high fidelity region. We show that the penalty for the suboptimality is at most 0.255 bits/sample per source regardless of the number of sources. This suboptimal bound will be the benchmark for the design of the source coder in medical wireless sensor networks where the complexity is the critical constraint.

7. ACKNOWLEDGMENTS

This work was supported by the MELODY research project funded by the Research Council of Norway under grant NFR 187857.S10.

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