

# Multiple Access Game in Ad-hoc Network

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## ABSTRACT

We study a Modified Multiple Access Game (MMAG) in two approach: repeated game approach and evolutionary game approach. We compute Nash equilibria and Evolutionary Stable Strategy (ESS) in the two cases. We study the delay impact on the performance of the evolutionary MMAG describing competition between mobile terminals over the access to a channel. We discuss about the convergence to the ESS in replicator dynamics and imitate better dynamics with delays.

**Keywords:** multiple access game, imitation, delayed dynamics

## 1. INTRODUCTION

Game theory has been used in several areas of communication theory including flow control [1, 21] and routing [4, 9, 26]. One of the approach studied for the medium access control (MAC) is the non-cooperative approach of the classical game theory. Users are rational and maximize their own payoff by choosing their probability of transmission for each slot. Evolutionary game theory has been proposed by biologists for studying competitive behavior of population instead of finite number of users. Moreover, the notion of equilibrium, which is called an evolutionary stable strategy is stronger than the standard Nash equilibrium as the ESS describes robustness of the equilibrium to mutations in the population strategies. Our aim in this paper is to consider an evolutionary game approach for the competitive MAC problem. Moreover, we study the stability property, in the sense of population dynamics, for this non-cooperative game. The Aloha and the subsequent slotted-Aloha protocols were introduced to improve the utilization of the shared medium by allowing user to send their traffic given some probability of transmission. Because slotted-Aloha exhibits an instability as the number of nodes increases [25], early research focused

on stabilizing the Aloha protocols [15]. For example, in [22], authors discussed the stability of slotted-Aloha with selfish user behavior and perfect information. A non-cooperative game formulation as been proposed in [20].

In this paper, we study a multiple access game in which there is a large population of mobile terminals in ad hoc network with low density [32]. A mobile decide to transmit or not a packet to a receiver when they are within transmission range of each other. Interference occurs as in the ALOHA protocol: if more than one neighbors transmit a packet at the same time then there is a collision. Multiple Access Game introduces the problem of medium access (see [8]). We modify the game in [32] and study the multiple access game in the two following perspectives:

- (i) a regret cost in the multiple access game when no users transmit and study the discounted repeated game equilibria,
- (ii) an evolutionary perspective of this modified multiple access game in which we study delay impact on the convergence to the ESS in two dynamics: replicator dynamics and imitate better dynamics.

The regret cost describe the behavior of mobiles when they are aware of the risk of collision. Risk is an important factor to be taken into consideration while managing revenues. The risk-aware is used in many field areas as security [34], traffic engineering routing [23] and management in autonomous agents [24]. We show that when the regret cost is lower than unit, and the delay is large, the system is not stable in the replicator dynamics and the system oscillates in the imitate better dynamics.

To improve the system performance in ad hoc network, we study the effect of the regret cost for transmission probability. We show that this cost can improve the system performance of ad hoc networks and that an appropriate pricing can be chosen that yields an equilibrium performance that optimizes the system performance.

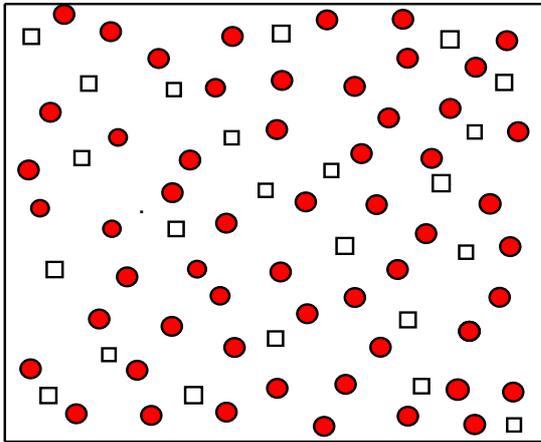
The paper is structured as follows. We first provide in the next section the (modified) multiple access game in noncooperative context. We then study the discounted repeated multiple access game in section 3. After that, we introduce evolutionary MMAG with replicator and imitate better dynamics in section 4 and we discuss about stability conditions and convergence to the ESS. We conclude by numerical investigation of these dynamics in section 5.

## 2. MODIFIED MULTIPLE ACCESS GAME

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We consider a large population of mobile terminals in ad hoc network with low density (see figure 1). We assume that the density of the network is low, so that if a terminal attempts transmission one can neglect the probability of interference from more than one other mobile (called "neighbor"). We assume the mobiles move frequently and they have a



**Figure 1: Population of mobiles in an Ad-Hoc Network.** A square represents a destination, a circle represents a source.

packet to send in each time slot. Interference occurs as in the ALOHA protocol: if more than one neighbors transmit a packet at the same time then there is a collision. Multiple Access Game introduces the problem of medium access (see [8]). The Multiple Access Game is a symmetric nonzero-sum game, the users have to share a common resource, the wireless medium, instead of providing it. Assume that the users use pure strategy. There are two users *Player I* and *Player II* who want to send some packets to their receivers R1 and R2 using a shared medium. Suppose furthermore that *Player I*, *Player II*, R1 and R2 are in the power range of each other, hence their transmissions mutually interfere. Each of the users has two possible strategies: either transmit (*T*) or to stay quiet (*S*). If *Player I* transmits his packet, it incurs a transmission cost of  $K\Delta \in (0,1)$  after a delay  $\tau_T$ . The packet transmission is successful if *Player II* does not transmit (stays quiet) in that given time slot and its delay is  $\tau_S$ , otherwise there is a collision. If there is no collision, *Player I* gets a reward of  $K$  from the successful packet transmission after the delay  $\tau_T$ . When the two players stay quiet, they have a cost  $\kappa K$ . If  $\kappa = 0$  and  $K = 1$ , we have the game described in [8]. The regret cost  $\kappa$  describes the behavior of mobiles when they are aware of the risk of collision.

The interaction is represented in figure 2.

	<i>T</i>	<i>S</i>
<i>Player I</i> receives	<i>T</i> $-\Delta K$	$K - \Delta K$
	<i>S</i> 0	$-\kappa K$

**Figure 2: Modified Multiple Access Game.**  $K$  is a positive parameter.

**Nash Equilibria and Pareto optimality** This matrix game has two pure Nash equilibria (*T*, *S*) and (*S*, *T*) and a

unique mixed Nash equilibrium given by  $(\frac{1-\Delta+\kappa}{1+\kappa}, \frac{\Delta}{1+\kappa})$ . Note that the pure strategies (*T*, *S*) and (*S*, *T*) are also optimal in the Pareto<sup>1</sup> sense. The mixed equilibrium converges the pure strategy *T* when the regret cost is large.

### 3. DISCOUNTED REPEATED MMAG WITH FULL MONITORING

We investigate the (infinitely) repeated game, i.e., the entire game represented by Fig.2 is played infinitely often. The repeated game is assumed to be discounted, i.e., the reward received at time  $t$  is discounted by for some  $\delta^t$ . In general, there are many more Nash equilibria in the repeated game than simply repeating a stage game Nash equilibrium.

**The model** The same static game, called also the stage game, is played an infinite number of times. At the end of each stage, each player is aware of all the actions of all the players at times 1 through  $t-1$ . The overall payoff of player is the discounted sum of its payoff at each stage, for some discount factor  $\delta \in (0,1)$ , normalized by  $1-\delta$ .

**Equilibria** A subgame after a history  $h_t$  is by itself a well-defined repeated game that starts at the time  $t+1$ . A Nash equilibrium is a subgame-perfect Nash equilibrium if for all time  $t$  and for all history the players' strategies constitute a Nash equilibrium in every subgame induced by the history.

Let  $m^i$  the maxmin point of the player  $i$  in the stage game. Note by  $IR$  the set of individually rational feasible payoff region i.e the portion of feasible region that Pareto-dominates the maxmin point. The set  $IR$  is a full dimensional set. Thus, by the discounted folk theorem [13, 14, 31, 10, 11, 7], the limit of the perfect public equilibria set when the users are patient ( $\delta \rightarrow 1$ ) is  $IR$ . Fudenberg, Levine and Takahashi[12] extended the folk theorem when the full dimensionality condition fails.

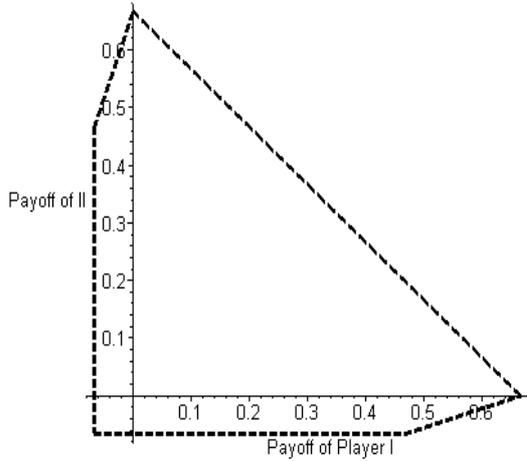
The maxmin point is given by  $m^1 = m^2 = -K \frac{\Delta\kappa}{1+\kappa}$ . The set  $IR$  is represented in Fig.3.

### 4. EVOLUTIONARY APPROACH

**Related works** The evolutionary game framework is used for large populations that have many local interactions, each involving a small number of users. The ESS, first been defined in 1972 by the biologist M. Smith [30], is characterized by a property of robustness against invaders (mutations). More specifically, (i) if an ESS is reached, then the proportions of each population do not change in time. (ii) at ESS, the populations are immune from being invaded by other small populations. The conditions to be an ESS [27, 19] can be related to and interpreted in terms of Nash equilibrium in a matrix game. There is a close relation between the rest points of the replicator equation (more generally in payoff monotonic dynamics [36]) and the Nash equilibria of the related (symmetric) matrix game given by the so called folk theorem of evolutionary game theory [5, 18, 36].

Although ESS has been defined in the context of biological systems, it is highly relevant to engineering as well (see [35]). Bonneau et al. have introduced evolutionary games in the context of unslotted ALOHA in [3]. The authors have identified conditions for the existence of non trivial ESS and

<sup>1</sup>An allocation of payoffs is said Pareto-optimal if the outcome cannot be improved upon without hurting at least one user.



**Figure 3: IR of multiple access game. When  $\kappa > 0$  the perfect Nash equilibria payoff set is larger than the case  $\kappa = 0$ .**

have computed them explicitly for the ALOHA scheme. In [32], the authors consider the multiple access game without cost when the no users transmit and study delay effect in many evolutionary game dynamics with asymmetric delay [33].

## ESS

We assume that the strategy  $T$  has a delay  $\tau$  and the strategy  $S$  no delay. Let  $\mathcal{S} := \{T, S\}$  the set of strategies and  $a_{kj}$  the fitness of subpopulation using  $k$  against the subpopulation  $j$ .

The repartition of the cell can be represented by the probability  $s$  to transmit. The number  $s$  is the proportion of users in the cell using the strategy  $T$ . Then  $sT + (1 - s)S$  is the state of the cell. We denote this state by  $(s, 1 - s)$  or by the parameter  $s$ . Let  $\Delta(\mathcal{S}) := \{(s, 1 - s) \mid 0 \leq s \leq 1\}$  the set of mixed strategies.

Suppose that, initially, the population profile is  $(s, 1 - s) \in \Delta(\mathcal{S})$ . The average payoff in the population is  $u(s, s) = (s, 1 - s)A_{(1-s)}$  where

$$A = K \begin{pmatrix} -\Delta & 1 - \Delta \\ 0 & -\kappa \end{pmatrix}$$

Now suppose that a small group of mutants enters this population playing according to a different profile  $(mut, 1 - mut)$ . If we call  $\epsilon \in (0, 1)$  the size of the subpopulation of mutants after normalization, then the population profile after mutation will be  $\epsilon mut + (1 - \epsilon)s$ . After mutation, the average payoff of non-mutants who are randomly matched to mutants is given by

$$u(s, mut) = (s, 1 - s)A_{(1-mut)},$$

and the average payoff of non-mutants will be given by

$$u(s, \epsilon mut + (1 - \epsilon)s) = \epsilon u(s, mut) + (1 - \epsilon)u(s, s).$$

Analogously, we can construct the average payoff of mutant  $u(mut, \epsilon mut + (1 - \epsilon)s)$ . A proportion of transmitters  $s \in [0, 1]$  is an ESS if for all  $mut \neq s$ , there exists some

$\epsilon_{mut} \in (0, 1)$ , which may depend on  $mut$ , such that for all  $\epsilon \in (0, \epsilon_{mut})$

$$u(s, \epsilon mut + (1 - \epsilon)s) > u(mut, \epsilon mut + (1 - \epsilon)s) \quad (1)$$

That is,  $s$  is ESS if, after mutation, non-mutants are more successful than mutants, in which case mutants cannot invade and will eventually get extinct. The number  $\epsilon_{mut}$  is called invasion barrier [36]. It is the maximum rate of mutants against which  $s$  is resistant. If  $s$  is an ESS then  $s$  is a Nash equilibrium. Equivalently  $s$  is an ESS if and only if it meets the best reply conditions:

$$u(mut, s) \leq u(s, s), \quad \forall mut,$$

$$u(mut, s) = u(s, s) \Rightarrow u(mut, mut) < u(s, mut) \quad \forall mut \neq s$$

We show that the mixed Nash equilibrium given Section 2 is an ESS. The state  $\frac{1-\Delta+\kappa}{1+\kappa}$  (resp.  $\frac{\Delta}{1+\kappa}$ ) represents proportion of individuals which transmit (resp. stay quiet). When the two subpopulation use this strategy, they obtain the same payoff equal  $-K\frac{\Delta}{1+\kappa}$  which is negative. The strategy  $(\frac{1-\Delta+\kappa}{1+\kappa}, \frac{\Delta}{1+\kappa})$  is the unique interior Nash equilibrium. It is the unique symmetric Nash equilibrium. Thus, it is the only candidate to be ESS because symmetric Nash equilibria set contains ESS set. At a mixed equilibrium, the strategies  $T$  and  $S$  must have the same fitness. We check the condition of ESS given in (1). For all  $\xi \neq \frac{1-\Delta+\kappa}{1+\kappa}$

$$\begin{aligned} & \left( \frac{1 - \Delta + \kappa}{1 + \kappa} - \xi, \xi - \frac{1 - \Delta + \kappa}{1 + \kappa} \right) A \begin{pmatrix} \xi \\ 1 - \xi \end{pmatrix} \\ &= \frac{K}{1 + \kappa} ((1 + \kappa)\xi - 1 + \Delta + \kappa)^2 > 0 \end{aligned}$$

We conclude that  $(\frac{1-\Delta+\kappa}{1+\kappa}, \frac{\Delta}{1+\kappa})$  an ESS.

## Replicator Dynamics

Replicator dynamics is one the most studied dynamics in evolutionary game theory. The replicator dynamics has been used for describing the evolution of road traffic congestion in which the fitness is determined by the strategies chosen by all drivers [28]. It has also been studied in the context of the association problem in wireless networks in [29]. We introduce the replicator dynamics which describes the evolution in the population of the various strategies. In the delayed replicator dynamics[33], the share of a strategy in the population grows at a rate equal to the difference between the delayed payoff of that strategy and the average delayed payoff of the population. If  $(\xi(t), 1 - \xi(t))$  denotes de repartition of the population in the multiple access game, then the fitness of the subpopulation using the strategy  $T$  is  $K(-\xi(t - \tau) + 1 - \Delta)$ , and the fitness of the strategy  $S$  is  $-\kappa K(1 - \xi(t))$ . The replicator dynamic equation is given by

$$\dot{\xi}(t) = -K\xi(t)(1 - \xi(t)) [\xi(t - \tau) + \kappa\xi(t) - 1 + \Delta - \kappa] \quad (2)$$

**DEFINITIONS 1.** 1. The state  $\xi^*$  is stationary state (or rest point, see [16]) of the delay differential equation (2) if it is a critical point i.e the right side of (2) is zero at  $\xi^*$ . This condition becomes  $\xi^* \in \{0, 1\}$  or  $f_1(\xi) = f_2(\xi)$

2.  $\xi^*$  is stable if it is a stationary point with the property that for every neighborhood  $\mathcal{V}$  of  $\xi^*$ , there exists a

neighborhood  $\mathcal{U} \subset \mathcal{V}$  with the property that if  $\xi(t) \in \mathcal{U}$  for  $t \in (-\tau, 0)$  then  $\xi(t) \in \mathcal{U}$  for all  $t > 0$ .

3.  $\xi^*$  is asymptotically stable if it is stable and there exists a neighborhood  $\mathcal{W}$  of  $\xi^*$  such that  $\xi(t) \in \mathcal{W}$  for all  $t \in (-\tau, 0)$  implies  $\lim_{t \rightarrow +\infty} \xi(t) = \xi^*$

It is known [2, pp.336],[17, pp.188] or from the Hartman and Grobman theorem adapted to delay differential equation that the steady state  $(\frac{1-\Delta+\kappa}{1+\kappa}, \frac{\Delta}{1+\kappa})$  is asymptotically stable for (2) around the stationary point  $\frac{1-\Delta+\kappa}{1+\kappa}$  if the trivial solution of the linearized version is asymptotically stable.

The trivial solution of the linear delay differential equation

$$\dot{z}(t) = -\frac{K\xi^*(1-\xi^*)}{1+\kappa} [\kappa z(t) + z(t-\tau)], \quad \tau > 0, \quad (3)$$

is asymptotically stable if and only if all roots of the characteristic equation

$$\lambda + \frac{K\xi^*(1-\xi^*)}{1+\kappa} [\kappa + e^{-\tau\lambda}] = 0 \quad (4)$$

have negative real parts. If there exists a roots of (4) with positive real parts, the trivial solution is not asymptotically stable. Note that if  $\lambda = \Re(\lambda) + i\Im(\lambda)$  is solution of the characteristic equation then its conjugate  $\Re(\lambda) - i\Im(\lambda)$  is also solution. The difficulty about (4) is that it is transcendental equation and hence the determination of condition for stability is highly nontrivial.

The following Rouché's theorem on the continuity of the roots of an equation as a function of parameters will be needed in analyzing the characteristic equation (4). A proof of this theorem can be found in Dieudonné [6].

**THEOREM 1 (ROUCHÉ).** *Let  $A$  be an open set in  $\mathbb{C}$ , the set of complex numbers,  $F$  a metric space,  $\phi$  a continuous complex valued function in  $A \times F$ , such that, for each  $\alpha \in F$ ,  $z \mapsto \phi(z, \alpha)$ , is analytic in  $A$ . Let  $B$  be an open set of  $A$ , whose closure  $\bar{B}$  in  $\mathbb{C}$  is compact and contained in  $A$ , and let  $\alpha_0 \in F$  be such that no zero of  $\phi(z, \alpha)$  is on the frontier of  $B$ . Then there exists a neighborhood  $W$  of  $\alpha_0$  in  $F$  such that*

- for any  $\alpha \in W$ ,  $\phi(z, \alpha)$  has no zeros on the frontier of  $B$ ;
- for any  $\alpha \in W$ , the sum of the orders of the zeros of  $\phi(z, \alpha)$  belonging to  $B$  is independent of  $\alpha$

**PROPOSITION 1.** *If  $\kappa \geq 1$  then all roots of the characteristic equation (4) has negative real parts.*

**PROOF.** • Suppose there exists root  $\lambda$  of the characteristic equation (4) with  $\Re(\lambda) > 0$  and  $\kappa > 1$  then  $\Re(\lambda) + K\xi^*(1-\xi^*)\frac{\kappa}{1+\kappa} > K\xi^*(1-\xi^*)\frac{\kappa}{1+\kappa} > \frac{K\xi^*(1-\xi^*)}{1+\kappa} > -\frac{K\xi^*(1-\xi^*)}{1+\kappa} e^{-\tau\Re(\lambda)} \cos(\tau\Im(\lambda))$  a contradiction with (4). Thus all roots of the characteristic equation (4) have real parts strictly negative.

- Suppose that  $\kappa = 1$ .

$$\begin{cases} \Re(\lambda) + K\xi^*(1-\xi^*)\frac{\kappa}{1+\kappa} = \\ = -K\xi^*(1-\xi^*)\frac{\kappa}{1+\kappa} e^{-\tau\Re(\lambda)} \cos(\tau\Im(\lambda)) \\ \Im(\lambda) = K\xi^*(1-\xi^*)\frac{\kappa}{1+\kappa} e^{-\tau\Re(\lambda)} \sin(\tau\Im(\lambda)) \end{cases} \quad (5)$$

If  $\Re(\lambda) > 0$  then  $\Re(\lambda) + K\xi^*(1-\xi^*)\frac{\kappa}{1+\kappa} = -K\xi^*(1-\xi^*)\frac{\kappa}{1+\kappa} e^{-\tau\Re(\lambda)} \cos(\tau\Im(\lambda)) \leq K\xi^*(1-\xi^*)\frac{\kappa}{1+\kappa}$  i.e  $\Re(\lambda) \leq 0$  : impossible. If  $\Re(\lambda) = 0$  then  $\Im(\lambda) = 0$ . The trivial solution is stable.

□

**PROPOSITION 2.** *Suppose that  $\kappa < 1$ . Then all of the characteristic equation (4) has negative real parts if and only if*

$$\tau \leq \frac{(1+\kappa)^2}{K\Delta(1-\Delta+\kappa)} \cos^{-1}(-\kappa) \sqrt{\frac{1+\kappa}{1-\kappa}} \quad (6)$$

Note that when  $\kappa$  goes to 1, the bound

$$\frac{(1+\kappa)^2}{K\Delta(1-\Delta+\kappa)} \cos^{-1}(-\kappa) \sqrt{\frac{1+\kappa}{1-\kappa}}$$

goes to infinity (stability without conditions).

**PROOF.** • Suppose that

$$\tau < \tau_0 := \frac{(1+\kappa)^2}{K\Delta(1-\Delta+\kappa)} \cos^{-1}(-\kappa) \sqrt{\frac{1+\kappa}{1-\kappa}}.$$

When  $\tau = 0$  the system (4) become  $\Im(\lambda) = 0$  and  $\Re(\lambda) = -K\frac{\Delta(1-\Delta+\kappa)}{(1+\kappa)^2} < 0$ .

Let  $b = K\frac{\Delta(1-\Delta+\kappa)}{(1+\kappa)^2}$ ,  $a = \kappa b$ . If (4) has a solution with real part strictly positive, then there must be some  $0 < \tau_1 < \tau_0$  such that (4) has purely imaginary roots  $\pm i\Im(\lambda) \times i$  with  $\Im(\lambda) > 0$  satisfying

$$\begin{cases} a = -b \cos(\tau_1 y) \\ y = b \sin(\tau_1 y) \end{cases} \iff \begin{cases} a = -b \cos(\tau_1 y) \\ y = b \sin(\tau_1 y), y > 0 \end{cases} \\ \iff \begin{cases} y = \sqrt{b^2 - a^2} \\ \tau_1 = \tau_0 \end{cases}$$

We prove that  $\tau_1 = \tau_0$  which is a contradiction. Thus, if  $0 < \tau < \tau_0$  then all solutions of the characteristic equation (4) have negative real parts.

- Suppose that  $\tau > \tau_0$ .

Denote by  $s = \lambda/b$ ,  $r = \tau b > 0$ ,  $r_0 = \tau_0 b$ . The equation (4) can be written as the zeros the function

$$\phi(s, \kappa) := s + \kappa + e^{-rs}$$

We want to prove that all zeros of  $\phi(\cdot, \kappa)$  have negative real parts. The function  $\phi$  is an analytic function in  $s$  and  $\kappa$ . Let

$$\mathcal{O} := \{s, \Re(s) \geq 0, |s| \leq 1\}$$

One has  $r_0 \geq \frac{\pi}{2}$ . Indeed,

$$\arccos(-p) - \frac{\pi}{2} \sqrt{1-p^2} > 0, \quad p \in (0, 1).$$

Thus when  $r > r_0$  and  $r \neq 2k\pi + \frac{\pi}{2}$ ,  $k \in \mathbb{N}$  the function  $\phi(s, 0)$  has no zeros in the boundary of  $\mathcal{O}$ . Thus Rouché's theorem 1 implies that there exists a  $\delta > 0$  such that  $\kappa < \delta \Rightarrow \phi(s, 0)$  and  $\phi(s, \kappa)$  have same sum of the orders of zeros. When  $r > r_0$  the zeros of  $\phi(s, 0)$  is at least 2. Thus,  $\phi(s, \kappa)$  has at least one root with positive real part.

□

## Imitate Better Dynamics

A delayed myopic imitation dynamic has been introduced in [32]. We suppose that users in the population review their strategy and imitate the better's strategy on the time (imitation by dissatisfaction). In a symmetric two player game, the delayed dynamic is given by

$$\dot{x}_k(t) = x_k(t) \left( \sum_{j=1}^n x_j(t) [\rho_{kj}(x(t-\tau_k)) - \rho_{jk}(x(t-\tau_j))] \right) \quad (7)$$

where  $\rho_{kj}(x) = g(f_k(x), f_j(x))$ ,  $k, j = 1, 2$  and  $g(a, b) = 0$  if  $a < b$  and  $g(a, b) = 1$  if  $a > b$ . This equation can be expressed as

$$\dot{\xi}(t) = \xi(t)(1 - \xi(t))h(\xi(t - \tau), \xi(t))$$

where

$$h(\xi(t - \tau), \xi(t)) = \begin{cases} 1 & \text{if } \xi(t - \tau) + \kappa\xi(t) < 1 - \Delta + \kappa \\ 0 & \text{if } \xi(t - \tau) + \kappa\xi(t) = 1 - \Delta + \kappa \\ -1 & \text{if } \xi(t - \tau) + \kappa\xi(t) > 1 - \Delta + \kappa \end{cases}$$

**PROPOSITION 3.** *Suppose that  $\kappa < 1$ . For all  $\tau > 0$ , and for all initial conditions  $\forall t \in (-\tau, 0]$ ,  $\xi(t) = \xi_0 \in (0, \frac{1-\Delta+\kappa}{1+\kappa})$  the solution of imitation equation oscillates around the ESS.*

**PROOF.** Consider the following differential equations

$$\begin{cases} \dot{\xi}(t) = \xi(t)(1 - \xi(t)) & \text{if } \xi(t - \tau) + \kappa\xi(t) < 1 - \Delta + \kappa \\ \dot{\xi}(t) = 0 & \text{if } \xi(t - \tau) + \kappa\xi(t) = 1 - \Delta + \kappa \\ \dot{\xi}(t) = -\xi(t)(1 - \xi(t)) & \text{if } \xi(t - \tau) + \kappa\xi(t) > 1 - \Delta + \kappa \\ \forall t \in (-\tau, 0], \xi(t) = \xi_0 \end{cases}$$

$$(**) \quad \begin{cases} \dot{w}(t) = w(t)(1 - w(t)) \\ w(0) = \xi_0 \end{cases}$$

where  $\xi_0 \in (0, \frac{1-\Delta+\kappa}{1+\kappa})$ .

The solution of (\*\*) is given by  $w(t) = 1 - \frac{1}{1+\chi e^t}$  where  $\chi = \frac{\xi_0}{1-\xi_0}$ . The function  $w$  goes to 1 when  $t$  goes to infinity. The functions  $\xi$  and  $w$  are equal in  $(0, t_1)$  where  $t_1 = t_r + t_*$  and  $t_*$  the first time such that  $w(t) = \xi^*$  (this time exists because  $w$  is a continuous function, starts at  $\xi_0$  lower than  $\xi^*$  and goes to 1),

$$t_1 = \inf\{t > 0, \xi(t - \tau) + \kappa\xi(t) > 1 - \Delta + \kappa\}.$$

We can explicitly compute  $t_*$  and  $t_1$ ,

$$t_* = \log\left(\frac{\xi^*}{\chi(1 - \xi^*)}\right),$$

$$t_1 = \log\left(\frac{\nu + \sqrt{\nu^2 + 4\xi^*(1 - \xi^*)e^{-\tau}}}{2\chi(1 - \xi^*)e^{-\tau}}\right)$$

and  $\nu = -(1 - \xi^*)(1 + e^{-\tau}) + \left(\frac{1+\kappa e^{-\tau}}{1+\kappa}\right)$ . One has  $t_* + \tau > t_1 > t_*$  and  $\xi_1 = \xi(t_1) = w(t_1) > \xi^*$ . Let consider the equation

$$\begin{cases} \dot{v}(t) = -v(t)(1 - v(t)) \\ v(t_1) = \xi_1 \end{cases}$$

$\xi(t) = v(t) = \frac{1}{1 + \frac{e^t}{\chi e^{2t_1}}}$  on  $(t_1, t'_1)$  where

$$t'_1 = \log\left(\chi e^{2t_1} \frac{\alpha + \sqrt{\alpha^2 + 4(1 - \xi^*)\xi^*e^{-\tau}}}{2e^{-\tau}\xi^*}\right)$$

and  $\alpha = -\xi^*(1 + e^{-\tau}) + \left(\frac{1+\kappa e^{-\tau}}{1+\kappa}\right)$ .  $\xi(t'_1) = v(t'_1)$  is lower than  $\xi^*$ .  $\square$

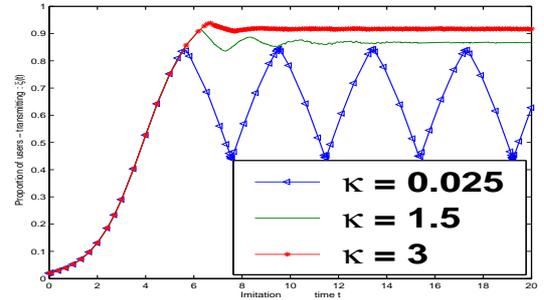
**REMARK 1.** *Suppose that that  $\kappa < 1$ . For all  $\tau > 0$ , and for all initial conditions  $\xi(t) = \xi_0 \in (0, 1)$  on  $(-\tau, 0)$ , the solution of imitation equation oscillates around the ESS.*

**PROOF.** Same arguments as in proposition 3.  $\square$

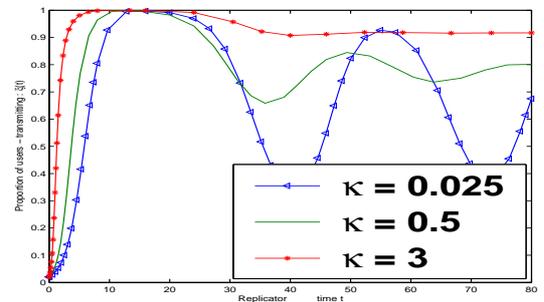
## 5. NUMERICAL INVESTIGATION: IMPACT OF THE REGRET COST AND THE DELAY ON THE CONVERGENCE TO THE ESS

**Impact of the cost  $\kappa$**  The ESS depends on the parameter  $\kappa$ . When  $\kappa$  is large, the ESS point is closed to  $(1, 0)$ : the pure strategy *transmit*. Our first numerical experiment studies the convergence of these dynamics for the case of the unit growth parameter  $K$  and a small delay  $\tau = 0.02$  as a function of  $\kappa$ : we check the speed of convergence and the stability of the dynamics as a function of the cost  $\kappa$ . The state  $2/3$  is a stationary point for these parameters, for which  $2/3$  of the population choose to transmit. We took  $\Delta = 1/3$ .

Figure 4 (resp. Figure 5) represents the trajectories of the population using the strategy  $T$  in the imitation dynamic (resp. replicator dynamic) when the initial condition is  $\xi(t) = 0.02 \in (-\tau, 0)$ .



**Figure 4: Effect of  $\kappa$  on stability of imitation dynamic.  $\tau = 1$**



**Figure 5: Effect of  $\kappa$  on stability of replicator dynamic when  $\tau = 8$**

**Impact of the delay** The ESS does not depends on the delay but has a big influence on the stability of the system. Our second numerical experiment studies the convergence of these dynamics for the case of the unit growth parameter  $K$  as a function of delay: we check the speed of convergence

and the stability of the dynamics as a function of the delay  $\tau$ . The state  $2/3$  is a stationary point for these parameters, for which  $2/3$  of the population choose to transmit. We took  $\Delta = 1/3$ ,  $\kappa = 0.002$ . The resulting trajectories of the population using the strategy  $T$  is represented as a function of time.

We evaluate the stability varying the delay  $\tau$  between 0.02 and 15 time units in the replicator dynamic in figure 6 and figure 7. For  $\tau = 0.02$ , we have stability but the convergence speed is slow. The other extreme is illustrated for  $\tau = 15$  which the trajectory oscillates rapidly and the amplitude is seen to be greater than  $2/3$ . The system is unstable.

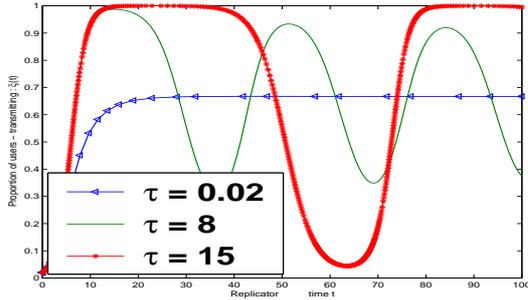


Figure 6: Effect of  $\tau$  on stability of replicator dynamic when  $\kappa = 0.002$

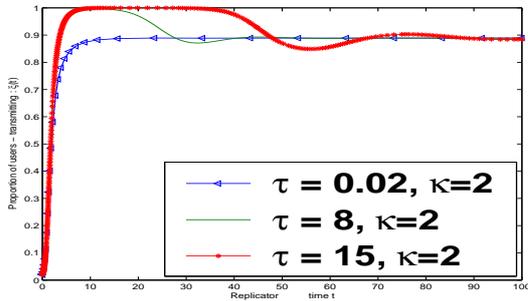


Figure 7: Replicator dynamic.  $\kappa = 2$ . The system is always stable.

Figure 8 represents the trajectories of the population using the strategy  $T$  in the imitation dynamic in which the system is unstable for all  $\tau > 0$ . The amplitude of oscillation growth with the delay.

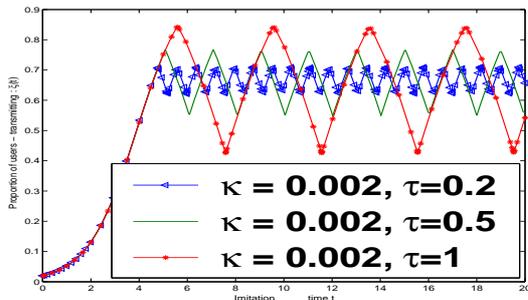


Figure 8: Imitation dynamic. Impact of  $\tau$ .

## 6. CONCLUDING REMARKS

The game studied in this paper, considers the population of nodes that have many local interactions between two nodes. Hence, the probability of success of a node is given by  $\xi^*(1 - \xi^*)$ . It is easy to see that the probability of success is maximum when  $\xi = 1/2$ . We can investigate the impact of regret cost on the equilibrium ESS and show how this cost can be used to enforce the population to converge ESS  $\xi^* = 1/2$ .

Now, we can easily show that the population converge to the ESS  $\xi^* = 1/2$  if the regret cost  $\kappa^* = 2\Delta - 1$  and  $\Delta > 1/2$  (see figure 9). When  $\Delta < 1/2$ , the cost  $\kappa^* = 0$  gives best performances and the population converges to the ESS  $\xi^* = 1 - \Delta$ .

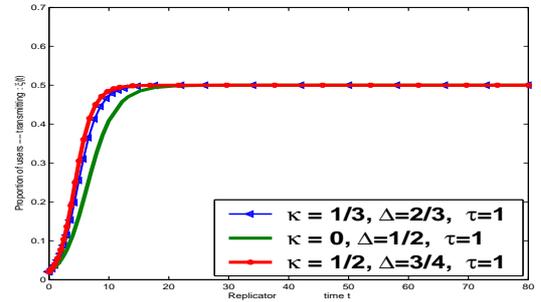


Figure 9: Replicator Dynamic. Optimal probability of success when  $\kappa = 1/3, 0, 1/2$ .

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