End-to-End Link Power Control in Optical Networks Using Nash Bargaining Solution

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ABSTRACT

An efficient and intelligent resource allocation mechanism is the heart of any communication networks. Based on previous work on non-cooperative game approach and direct centralized optimization, this paper addresses the issue of efficiency and fairness in optical network power control. We use Nash bargaining solution (NBS) to achieve a fair and efficient solution for optical network power control at the end-to-end optical link level. We study different formulations based on Nash bargaining model and characterize their solutions.

Keywords

Centralized optimization, Game Theory, Optical Networks, Nash Bargaining Solution, Power Control

1. INTRODUCTION

Recent technological advances have enabled a new generation of Optical Wavelength-Division Multiplexed (WDM) communication networks. Devices such as Optical Add/Drop MUXes (OADM), optical cross connects (OXC) and dynamic gain equalizer (DGE) have provided essential building blocks for smart optical networks [1]. With advent of these new technologies, current networks are evolving towards dynamic networks, able to respond to changes in traffic and requirements. A static network management mechanism can no longer service such networks. Therefore, intelligent network management and control systems need to be part of future network design. Complex in their own structure, networks need control on different levels. The first level is an optical device level control, where smart feedback algorithms are used to reduce noise and stabilize the device. Examples have been seen in [31] and [16] where control principles are applied to study EDFA and SOA, respectively. The next level of management is on the link level,where we need to optimize the quality of transmission and reduce nonlinear physical effects. We can formulate optimization-based models for optical networks, specifically related to channel optical signal-to-noise ratio (OSNR) optimization.Such models have been seen in the case of wireless networks in [29, 28]. However, the unique physical structure of optical networks imposes different challenges on modeling and solution concepts in optical networks. Our study of link-level control is an initial step towards the control of large-scale complex networks.

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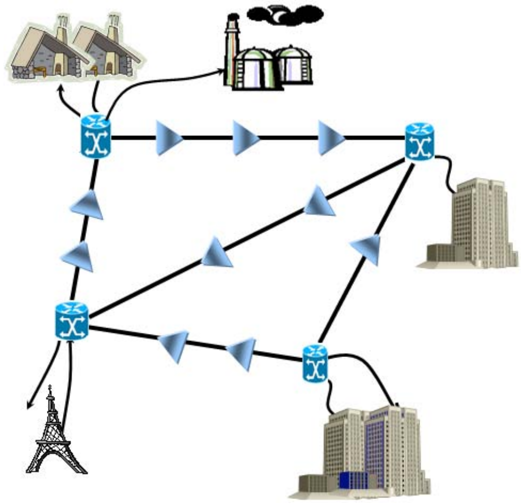

Figure 1: A Small-scale Optical Networks

The third level is the network level, where problems of interests are optimal routing and congestion control. These problems are on a higher level and they have been well studied in a general network setting such as in [33]. The last but not least is the system level control. This level of research encapsulates optical network as a dynamical system as seen in [22]. Interesting problems are usually on the robustness and stability of large scale networks.

Our focus here is on the control and management of optical networks at the link level. Investigated in [23, 24], the problem has been solved by two prevalently used optimization-based approaches: the central cost and non-cooperative game approach. The goals and models from the two approaches are inherently different. Central cost approach gives a model to satisfy the target OSNR performances with minimum total power consumption. The model is centralized in nature but an algorithm can be built in a decentralized manner. It is important to note that the central cost approach in [24] yields a solution with no concept of fairness. It can happen that the solution can favor one particular user for the social minimum of power consumption. On the other hand, non-cooperative game approach is a naturally distributed model which sees channel users as competitors. Each user optimizes her own utility to achieve the best possible OSNR. The solution from this approach is Nash equilibrium. It is well-known in literature [7, 25] that Nash equilibrium may not be efficient. Therefore, fairness and efficiency of solutions become our major interest. As an alternative, we use Nash bargaining solutions to achieve these goals by optimizing a product form of utilities. Such approach has been taken in wireless networks as in [32], [10] and [8]. However, it is the first time to be used to solve the power control problem in optical networks.

This paper is organized as the following. In section 2, we review a network OSNR model and give a brief introduction to central cost and non-cooperative game approaches. In section 3 , we review the concept of fairness and Nash Bargaining Solution (NBS). In section 4, we formulate two optimization-based models for the power control problem in optical networks and characterize their solutions. Section 5 will point out future direction of this research work and section 6 will conclude the paper.

2. CENTRAL COST APPROACH AND NON-COOPERATIVE GAME APPROACH

The network power management is regarded as the Achilles' heel for optical networks, in which an intelligent network resource allocation mechanism is needed to achieve desired network performance. This is commonly measured by bit error rate (BER), which is closely related to OSNR. Central cost and game approaches are two existing schemes used to design power control algorithms in optical networks. Both schemes are based on a static OSNR optimization and iterative algorithms are derived from the optimal solution. In this section, we first review OSNR optical network model and then review the concepts of two optimization-based approaches.

2.1 Review of Optical Network Model

Consider a WDM network with a set of optical links $\mathcal{L} = \{1, 2, .., L\}$ connecting the optical nodes (as in Figure 1), where channel add/drop is realized. A set $\mathcal{N} = \{1, 2, ..., N\}$ of channels are transmitted, corresponding to a set of multiplexed wavelengths. Illustrated in Figure 2, a link l has K_l cascaded optically amplified spans. Let N_l be the set of channels transmitted over link l. For a channel $i \in \mathcal{N}$, we denote by $\mathcal{R}_i$ its optical path, or collection of links, from source (Tx) to destination (Rx). Let u_i be the ith channel input optical power (at Tx), and $\mathbf{p} = [p_1, ..., p_N]^T$ the vector of all channels' input powers. Let s_i be the ith channel output power (at Rx), and n_i the optical noise power in the ith channel bandwidth at Rx. The ith channel optical OSNR is defined as $OSNR_i = \frac{s_i}{n_i}$. In [24], it is assumed that the dispersion and nonlinearity effects are considered to be limited, the ASE noise accumulation is the dominant impairment in the model. This assumption simplifies the OSNR expression, and thus the OSNR for the ith channel is given as

$$OSNR_i = \frac{p_i}{n_{0,i} + \sum_{j \in \mathcal{N}} \Gamma_{i,j} p_j}, i \in \mathcal{N} \quad (1)$$

where $\mathbf{\Gamma}$ is the full $n \times n$ system matrix which characterizes the coupling between channels. $n_{0,i}$ denotes the ith channel noise power at the transmitter. System matrix $\mathbf{\Gamma}$ encapsulates the basic physics present in optical fiber transmission and implements an abstraction from a network to an input-output system. This approach has been used in [30] for the wireless case to model CDMA uplink communication. Different from the system matrix used in wireless case, the matrix $\mathbf{\Gamma}$ given in (2) is commonly asymmetric and is more complicatedly dependent on parameters such as spontaneous emission noise, wavelength-dependent gain, and the path channels take.

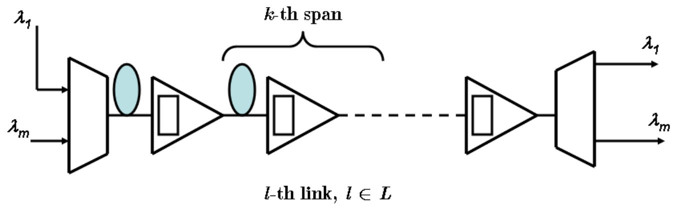

Figure 2: A Typical Optical Link in WDM Optical Networks

$$\Gamma_{i,j} = \sum_{i \in \mathcal{R}_i} \sum_{k=1}^{K_l} \frac{G^k_{l,j}}{G^k_{l,i}} \left(\prod_{q=1}^{l-1} \frac{\mathbf{T}_{q,j}}{\mathbf{T}_{q,i}} \right) \frac{ASE_{l,k,i}}{P_{0,l}}, \forall j \in \mathcal{N}_l. \quad (2)$$

where $G_{l,k,i}$ is the wavelength dependent gain at kth span in lth link for channel i; $\mathbf{T}_{l,i} = \prod_{q=1}^{K_l} G_{l,k,i} L_{l,k}$ with $L_{l,k}$ being the wavelength independent loss at kth span in lth link; $ASE_{l,k,i}$ is the wavelength dependent spontaneous emission noise accumulated across cascaded amplifiers; $P_{0,l}$ is the output power at each span.

It is also shown in [24] that the OSNR model can be further extended to include crosstalk terms due to WDM components at the optical nodes (OADM or OXC), such as optical filters, demultiplexers, add/drop modules, routers or switches [1].

2.2 Central Cost Approach

Similar to the SIR optimization problem in the wireless communication networks [29, 28], OSNR optimization achieves the target OSNR predefined by each channel user by allowing the minimum transmission power. Let $\gamma_i, i \in \mathcal{N}$ be the target OSNR for each channel. By setting the OSNR requirement as a constraint, we can arrive at the following central cost optimization problem (CCP):

$$\begin{array}{ll} (CCP) & \min_{\mathbf{p}} \sum_{i \in \mathcal{N}} p_i \\ \text{subject to} & OSNR_i \geq \gamma_i \;\; \forall i \in \mathcal{N}. \end{array} \quad (3)$$

Under certain conditions, it has been shown in [24] that the feasible set of CCP is nonempty and the optimal solution is achievable at the boundary of the feasible set.

The formulated optimization problem can be extended to incorporate more constraints such as

$$p_{i,\min} \leq p_i \leq p_{i,\max}, \quad (4)$$

where $p_{i,\min}$ is minimum threshold power required for transmission for channel i and $p_{i,\max}$ is maximum power channel i can attain. In the central cost approach, power p_i are the parameters to be minimized and the objective function is linearly separable. In addition, the constraints are linearly coupled. These nice characteristics in central cost approach leads to a relatively simple optimization problem.

2.3 Non-cooperative Game Approach

Non-cooperative game theory is a branch of microeconomic theory dealing with multi-agent interactions in a non-cooperative manner [20],[21]. It can naturally model the coupling between channels in terms of OSNR, provided that cooperation between channels is impractical. In a large-scale and dynamic network, this assumption can be easily justified as the centralized information is difficult to obtain.

Consider a game defined by a triplet $\langle \mathcal{N}, (A_i), (J_i) \rangle$. $\mathcal{N}$ is the index set of players or channels; A_i is the strategy set $\{p_i \mid p_i \in [p_{i,\min}, p_{i,\max}]\}$; and, J_i is the payoff function. In [23], J_i is defined as

$$J_i(p_i, p_{-i}) = \alpha_i p_i - \beta_i \ln\left(1 + a_i \frac{p_i}{X_{-i}}\right), \quad (5)$$

where α_i, β_i are channel specific parameters, that quantify the willingness to pay the price and the desire to maximize its OSNR, respectively, a_i is a channel specific parameter, X_{-i} is defined as $X_{-i} = \sum_{j \neq i} \Gamma_{i,j} p_j + n_{0,i}$. This specific choice of utility function is non-separable, nonlinear and coupled. However, J_i is strictly convex in p_i and takes a specially designed form such that its first-order derivative takes a linear form with respect to $\mathbf{p}$.

The solution from the game approach is usually characterized by Nash equilibrium (NE), which is defined in the context of optical networks as $\mathbf{p}^*$ such that

$$J_i(p^*) \leq \inf_{p_i \in [p_{i,\max}, p_{i,\max}]} J_i(p_i, p^*_{-i}), \forall i \quad (6)$$

Provided that $\sum_{j \neq i} \Gamma_{i,j} \leq a_i$,the resulting NE solution is given in a closed form by

$$\overline{\mathbf{\Gamma}} \mathbf{p}^* = \overline{\mathbf{b}}, \quad (7)$$

where $\overline{\Gamma}_{i,j} = a_i$, for $j = i$; $\overline{\Gamma}_{i,j} = \Gamma_{i,j}$, for $j \neq i$ and $\overline{b}_i = \frac{a_i b_i}{\alpha_i} - n_{0,i}$.

3. NASH BARGAINING MODEL

Non-cooperative game theory in section 2.3 leads to a distributed algorithm that converges to the analytical Nash equilibrium. However, the choice of utility function and derivation of the solution doesn't take the issues of efficiency and fairness into account. In fact, Nash equilibrium may be undesirably inefficient due to the fact that each agent is behaving selfishly to optimize their own utility. The social optimum isn't achieved in this game setting. On the other hand, central cost approach in section 3.1 doesn't have the concept of fairness in its solution. It is well-known in the literature of bandwidth allocation that this criterion can lead to situations in which the allocation is null for one or more users [6]. One way to improve is to adopt a new solution concept, i.e., Nash Bargaining solution. Its inherent property of being proportionally fair and Pareto optimal is much desirable for an efficient distributed algorithm. As a summary, Table 1 compares the properties of solutions from different approaches.

3.1 Concept of Fairness

Recently a growing literature [6, 25, 26, 12, 32, 2] is concerning the efficiency and fairness of the solutions. The solutions we obtained in section 2 do not embody those concepts from their models. This has brought us to investigate such problems in the power control problem in optical networks.

To study the concept of fairness, we need to first define fairness criteria. There are multiple proposals in the fairness criteria in current literature, namely, max-min fairness, proportional fairness, and generalized $(\mathbf{w},\alpha)$ fairness, etc. Max-min fairness has been pervasively used in the context of rate control and it can be equivalently interpreted as the following statement: An allocation is max-min fair if and only if each source has a bottleneck. Proportional fairness is a relatively new concept, first defined in [15]. It is mathematically defined as follows.

DEFINITION 3.1. *(Proportional fairness) An allocation* $\mathbf{p}^* \in X$*, a convex compact subset of* $\mathcal{R}^N$*, is* proportionally fair *if it maximizes*

$$\max_{\mathbf{p} \in X} \sum_{i \in \{1,..,N\}} \ln(p_i).$$

The definition can be interpreted as follows. If a point $\mathbf{p}$ is proportionally fair, any deviation from this point will result in a non-positive sum of percentage change of each user's utility. The definition of the proportional fairness results in the following property [17].

PROPOSITION 3.1. *If* $\mathbf{p}^* \in X \subseteq \mathcal{R}^N$ *is proportionally fair, then*

$$\sum_{i \in \{1,..,N\}} \frac{p'_i - p^*_i}{p^*_i} \leq 0, \forall \mathbf{p}' \neq \mathbf{p}^*.$$

In [18], another solution concept is defined in the context of bandwidth allocation by minimizing the transfer time, the inverse of the bandwidth, i.e., $\min_{\mathbf{p} \in X} \sum_{i \in 1,..,N} \frac{1}{p_i}$. With the presence of these fairness criteria, Mo and Walrand recently showed in [19] that these criteria can be generalized into a parametric $(\mathbf{w},\alpha)$ fairness criteria. In the case when $\alpha = 0$, the solution is a global optimization over the sum of the utility functions. As $\alpha \to 1$, the problem becomes a Nash bargaining solution and yields proportional fairness in the solution. As $\alpha \to \infty$, the solution corresponds to the Max-Min fairness solution.

DEFINITION 3.2. *[$(\mathbf{w},\alpha)$ Fairness][Mo and Walrand, [19]] A solution* $\mathbf{p}$ *is called* $(w, \alpha)-$*fair if it solves*

$$\max_{\mathbf{p} \in X} \frac{1}{1-\alpha} \sum_{i \in \{1,..,N\}} w_i p_i^{1-\alpha}, \forall \alpha \geq 0, \alpha \neq 1$$

and solves

$$\max_{\mathbf{p} \in X} \prod_{i \in \{1,..,N\}} p_i, \ \textit{when } \alpha = 1.$$

Similarly,the definition of $(\mathbf{w},\alpha)$ fairness criteria gives the property in Proposition 3.2.

PROPOSITION 3.2. *Let* $w = (w_1, ..w_N)$ *and* α *be positive numbers. A vector of rates* $\mathbf{p}^*$ *is* (α, w)*-proportionally fair,if and only if it is feasible and for any other feasible vector* $\mathbf{p}'$ *such that* $\sum_{i \in \{1,...,N\}} w_i \frac{p'_i - p^*_i}{p^{*\alpha}_i} \leq 0$

The above criteria are prevalently used in rate allocation; consequently, the utility is simply the allocation, a separable linear function of the allocated resource. To apply the fairness criteria into our power control problem, we need to generalize them in terms of a function of $\mathbf{p}$. The following are the extended definitions.

Table 1: Efficiency and Fairness Comparisons

Methods	Efficiency	Fairness
Central Cost Approach	Efficient	No concept of fairness
Non-cooperative Game Approach	Inefficient	Fair competition
Nash Bargaining Solution	Efficient	Proportional Fairness

DEFINITION 3.3. *An allocation $\mathbf{p}^*$ is utility-based proportional fair if it solves the problem*

$$\max_{\mathbf{p}\in X} \sum_{i\in\{1,..,N\}} \ln(f_i(p_i)).$$

PROPOSITION 3.3. *(Utility-based proportional fairness) Suppose each user has a utility function $f_i(\cdot) : X \to \mathcal{R}$, showing its preference over her own allocation. Assume that f_i is continuously differentiable. The allocation $\mathbf{p}^*$ is utility-based proportionally fair if it satisfies the following property:*

$$\sum_{i\in\{1,...,N\}} w_i(p_i^*)\frac{p_i' - p_i^*}{p_i^*} \leq 0 \ \forall p_i' \neq p_i^*, \tag{8}$$

where $w_i(p_i^*) = v_i(p_i^*)p_i^*$ can be regarded as the weights, and $v_i(p_i^*) = \frac{f_i'(p_i^*)}{f_i(p_i^*)}$. $w_i(p_i^*)$.

PROOF. Every $\mathbf{p}^*$ that satisfies Definition 3.3 should satisfy the optimality condition in variational form, i.e.,:

$$\sum_{i\in\{1,...,N\}} \frac{f_i'(p_i^*)}{f_i(p_i^*)}(p_i' - p_i^*) \leq 0$$

for all $\mathbf{p}' \in X$. The result will follow by defining appropriately $w_i(p_i^*)$. □

LEMMA 3.4. *Suppose $v_i(p^*)$ defined in Proposition (3.3) is affine, separable, and of the form $v_i(p_i^*) = v_a p_i^* + v_b; v_a, v_b \in \mathcal{R}^+$. If an allocation $\mathbf{p}^*$ satisfies Proposition 3.3 , then one of the following statements holds.*

1. $\sum_i v_i(p^*)p_i^* \geq \frac{4}{3}\sum_i v_i(p_i)p_i$, $\forall p_i$
2. $\frac{\sum_i v_i(p_i^*)p_i^*}{\max_{\mathbf{p}} \sum_i v_i(p_i)p_i} \geq \frac{4}{3}$

PROOF. It is easy to observe that proportional fairness is defined in a variational form and we can apply the results of price of anarchy from [25, 5]. Starting with inequality (8), we have

$$\sum_i v_i(p_i^*)p_i^* \geq \sum_i v_i(p_i^*)p_i'$$

$$= \sum_i v_i(p_i')p_i' + \sum_i (v_i(p_i^*) - v_i(p_i'))p_i'. \tag{9}$$

When $v_i(p_i^*) - v_i(p_i') \leq 0$, a simple sufficient condition for (9) to hold is to have $\sum_i v_i(p_i^*)p_i^* \geq \sum_i v_i(p_i')p_i'$. Therefore, we only need to worry about the case when $v_i(p_i^*) \geq v_i(p_i')$. Using the same argument from [5], due to the fact that $v_i(\cdot)$ is an affine and separable function of the form $v_i(p^*) = v_a p_i^* + v_b, v_a, v_b \in \mathcal{R}^+$, we can observe that the area formed by $(v_i(p_i^*) - v_i(p_i'))p_i'$ is always less than or equal to 1/4 of the area formed by $v_i(p_i^*)p_i^*$, for every i. Thus, we have

$$\sum_i (v_i(p_i^*) - v_i(p_i'))p_i' \leq \frac{1}{4}\sum_i v_i(p_i^*)p_i^*.$$

Therefore, $\sum_i v_i(p^*)p_i^* \geq \frac{4}{3}\sum_i v_i(p_i')p_i' \ \forall p_i'$. Equivalently, taking the maximum of the right-hand side, we obtain

$$\frac{\sum_i v_i(p_i^*)p_i^*}{\max_{\mathbf{p}} \sum_i v_i(p_i)p_i} \geq \frac{4}{3}.$$

□

Utility based proportional fairness can be seen as a weighted proportional fairness criteria, whose weights depend on the solution. Proposition 3.3 gives a sufficient condition on proportional fairness. With the knowledge of the form of function v_i, 3.4 gives a necessary condition. This result can be extended for a more general form of v_i. Different fairness criteria results in a different point in the Pareto set. However, since Nash equilibrium is not always in the Pareto set, the fairness of Nash equilibrium needs to be otherwise defined.

3.2 Nash Bargaining Solution

Different from a central optimization approach and non-cooperative game approach, Nash Bargaining Solution (NBS) is a point with the property of proportional fairness and Pareto Efficiency. NBS is based on Nash's axiomatic model of bargaining. The axiom of Pareto efficiency inherently guarantees the efficiency of the solution. In optical networks, we can analogously relate the situation of negotiation among users for limited resources to the bargaining situation. We are interested in the equilibrium of such negotiation rather than the transient process of bargaining, which is commonly modeled in extensive game form by the Rubinstein Bargaining model [21].

Let's review the results from Nash Bargaining Theory [21, 32]. Suppose there are N individuals. Each agent $i(i \in \{1...N\})$ has an utility function $f_i(\cdot)$ and a minimal utility requirement f_i^0, where $f_i(\cdot)$ is defined on a set $X \subset \mathcal{R}^N$ and is assumed to be upper-bounded. X is the set of feasible strategies and is assumed to be a convex closed and non-empty set. The set $U \subset \mathcal{R}^N$ is defined as $U = \{\mathbf{u} \in \mathcal{R}^N \mid \exists \mathbf{p} \in X, \mathbf{u} = (f_1(\mathbf{p}), ..., f_N(\mathbf{p}))\}$ and it is assumed to be non-empty, convex, and closed. Vector $\mathbf{u}^0$ is thus defined as $\mathbf{u}^0 = (f_1^0, ..., f_N^0)$.

The set $U^0 \in U$ is a subset of U in which agents achieve more than their minimum requirements $\mathbf{u}^0$. $U^0 = \{u \in U \mid \mathbf{u}_i \geq \mathbf{u}_i^0, \forall i \in \{1, ..., N\}\}$. Similarly, we define $X_0 = \{X_0 \subset X \mid f_i(\mathbf{p}) \geq \mathbf{u}_i^0, \forall i \in \{1, ..., N\}\}$ as the strategy set that contribute to the utilities in set U^0.

The mapping of f and the notions of defined sets are illustrated in Figure 3.

DEFINITION 3.4. *a Nash bargaining solution (NBS) $\mathbf{u}^*$ is given by $S(U, \mathbf{u}^0)$, where $S : \mathcal{G} \to \mathcal{R}^N$ is a mapping that satifies:*

1. *(FEA) Feasibility, i.e.,$S(U, \mathbf{u}^0) \in U^0$*
2. *(PAR) Pareto efficiency, i.e., $S(U, \mathbf{u}^0)$ is Pareto optimal.*

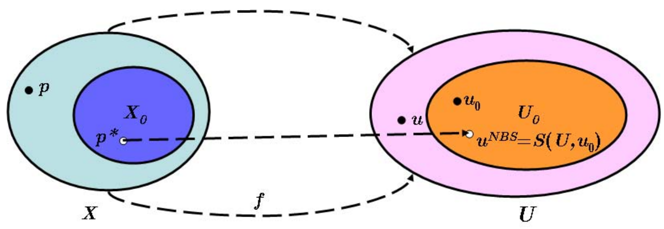

Figure 3: Illustration of Bargaining Solution.

3. *(LIN) Linearity if* $\phi : \mathcal{R}^N \to \mathcal{R}^N, \phi(\mathbf{u}) = \mathbf{u}'$ *with* $\mathbf{u}'_i = a_i\mathbf{u}_i + b_i, a_i > 0, i = 1, ..., N$ *then* $S(\phi(\mathbf{u}), \phi(\mathbf{u}^0)) = \phi(S(U, \mathbf{u}^0))$.
4. *(IIA) Property of irrelevant alternatives if* $V \subset U, (V, \mathbf{u}^0) \in \mathcal{G}$*, and* $S(U, \mathbf{u}^0) \in V$*, then* $S(U, \mathbf{u}^0) = S(V, \mathbf{u}^0)$
5. *(SYM) Property of Symmetry if* U *is symmetric with respect to a subset* $J \subseteq \{1, ..., N\}$ *of indices; i.e., if* $\mathbf{u} \in U$ *and* $i, j \in J$*, then if* $\mathbf{u}^0_i = \mathbf{u}^0_j$*, then* $S_i(U, \mathbf{u}^0) = S_j(U, \mathbf{u}^0)$.

The properties of LIN, IIA and SYM are the axioms of fairness. The linearity property implies that the bargaining solution is scale invariant, i.e., the bargaining solution remains the same if new performance objectives are affine functions of the old one. The irrelevant alternative property states that the bargaining point is not affected by enlarging the domain if the agreement can be found in the feasible set. The symmetry property implies that users of the same initial points and objectives will realize the same performance.

The following Theorem 3.5 in [32] is based on the Nash solution to the Nash axiomatic bargaining games. It is worth pointing out that Nash's four axioms uniquely define a bargaining solution. It can be shown that none of the axioms are superfluous. For each axiom, a solution that satisfies the remaining three axioms and is different from Nash's. When the axiom axiom is generalized it can be shown that NBS can also be extended to asymmetric Nash bargaining solution (ANBS) [21]. To provide a complete picture of the bargaining games, we have noticed that in the economics literature, the third axiom in which solution is assumed to have the IIA property is quite debatable. In addition, Kalai-Smorodinsky solution (KSS) replaces this assumption with monotonicity in [13].

THEOREM 3.5. *(Yaicke, Mazumdar and Rosenberg, [32]) Let the utility functions* $f_i(\cdot) : X \to \mathcal{R}$ *be concave, upper-bounded and defined on a convex and compact set* $X \in \mathcal{R}^N$*. Let* J *be the set of users able to achieve a performance strictly superior to their initial performance. Assume* $\{f_j\}_{j\in J}$ *are injective. Then, there exists a unique Nash Bargaining Solution (NBS)* $\mathbf{p}$ *that satisfies the NBS properties and is a unique solution to the problem of*

$$\max \prod_{j\in J}(f_j(\mathbf{p}) - u^0_j), \mathbf{p} \in X_0. \tag{10}$$

Equivalently, it is a solution to

$$\max \sum_{j\in J} \ln(f_j(\mathbf{p}) - u^0_j), \mathbf{p} \in X_0. \tag{11}$$

4. NBS IN POWER CONTROL OF OPTICAL NETWORKS

To illustrate the Nash bargaining solution (NBS) concept in the context of power control of optical networks, let's consider a problem setup: N channels compete for their optimal OSNR in the fiber transmission. Each channel $i(i \in \{1...N\})$ has an OSNR measure function $OSNR_i(\cdot)$ and a required initial OSNR given by γ^0_i at the transmitter, which guarantees the minimum quality of transmission. Measure function $OSNR_i(\cdot) : \mathcal{R}^N_+ \to \mathcal{R}_+$ is defined in (1). Clearly, function $OSNR_i(\cdot)$ is a nonseparable function in terms of vector $\mathbf{p}$. To be more general, the utility function of each channel is defined as a function of OSNR measurement,i.e., $f_i(\cdot) = h_i(OSNR_i(\cdot))$. $f_i(\cdot)$ is desirable to be upper bounded and convex in p_i. Provided that $\gamma_i \neq 0$ and $p_i \neq 0$, one choice of the utility functions $f_i(\cdot)$ is given by

$$f_i(\cdot) = \frac{\beta_i OSNR_i}{1 - \Gamma_{i,i}OSNR_i} = \frac{p_i}{\delta_i X_{-i}}, \tag{12}$$

where $\delta_i = 1/\beta_i$. Similarly, the initial utility requirement will be given by $\mathbf{u}^0 = (f_1(\gamma^0_1), ..., f_N(\gamma^0_N))$.

Before applying Theorem 3.5, we need to show that $\ln f_i(\cdot)$ is concave in $\mathbf{p}$.

PROPOSITION 4.1. *Suppose* $p_i \in [p_{\min}, p_{\max}]$*, where* $p_{\min}, p_{\max} \in \mathcal{R}^+$*, and the noise term* $n_{0,i}$ *negligible. Function* $\ln f_i(\cdot) : X \to \mathcal{R}$ *is concave in* $\mathbf{p}$*, defined on a compact, convex set* $X \subseteq R^N$*. if* $\delta_i \leq \frac{p_{\min}}{p_{\max}} \leq 1$.

PROOF. Let $\mathbf{p}^1$ and $\mathbf{p}^2$ be vectors in convex compact feasible set X, which can be obtained from the constraints.

Let $\mathbf{p}^0 = \lambda\mathbf{p}^1 + (1 - \lambda)\mathbf{p}^2$. We need to show

$$\ln f_i(\lambda\mathbf{p}^1 + (1-\lambda)\mathbf{p}^2) \geq \lambda \ln f_i(\mathbf{p}^1) + (1-\lambda) \ln f_i(\mathbf{p}^2), \ \forall\lambda \in [0, 1]. \tag{13}$$

Using (12), we have

$\ln f_i(\lambda\mathbf{p}^1 + (1 - \lambda)\mathbf{p}^2) =$

$$\ln\left(\frac{\lambda\mathbf{p}^1_i + (1 - \lambda)\mathbf{p}^2_i}{\delta_i \sum_{j\neq i} \Gamma_{i,j}(\lambda\mathbf{p}^1_j + (1 - \lambda)\mathbf{p}^2_j)}\right). \tag{14}$$

and

$\lambda \ln f_i(\mathbf{p}^1) + (1 - \lambda) \ln f_i(\mathbf{p}^2) =$

$$\ln\left(\frac{(\mathbf{p}^1_i)^\lambda(\mathbf{p}^2_i)^{1-\lambda}}{(\sum_{j\neq i} \Gamma_{ij}\mathbf{p}^1_j)^\lambda(\sum_{j\neq i} \Gamma_{ij}\mathbf{p}^2_j)^{(1-\lambda)}}\right). \tag{15}$$

From Holder's inequality [27], we know that

$$(\mathbf{p}^1_i)^\lambda(\mathbf{p}^2_i)^{1-\lambda} \leq \lambda\mathbf{p}^1_i + (1 - \lambda)\mathbf{p}^2_i,$$

since $\mathbf{p}^1_i, \mathbf{p}^2_i \geq 0$. Similarly, using Holder's inequality, we have

$$\delta_i \sum_{j\neq i} \Gamma_{i,j}(\lambda\mathbf{p}^1_j + (1 - \lambda)\mathbf{p}^2_j)$$

$$= \delta_i \left(\lambda \sum_{j\neq i} \Gamma_{i,j}\mathbf{p}^1_j + (1 - \lambda) \sum_{j\neq i} \Gamma_{i,j}\mathbf{p}^2_j\right)$$

$$\geq \delta_i \left(\left(\sum_{j\neq i} \Gamma_{i,j}\mathbf{p}^1_j\right)^\lambda \left(\sum_{j\neq i} \Gamma_{i,j}\mathbf{p}^2_j\right)^{1-\lambda}\right) \tag{16}$$

Suppose the denominator in (15) is larger than the one in (14). Using inequality from (16), we arrive at the necessary condition in (17) that gives rise to (13).

$$\left(\sum_{j\neq i}\Gamma_{ij}\mathbf{p}_j^1\right)^{\lambda}\left(\sum_{j\neq i}\Gamma_{ij}\mathbf{p}_j^2\right)^{(1-\lambda)} \geq \delta_i\sum_{j\neq i}\Gamma_{i,j}(\lambda\mathbf{p}_j^1+(1-\lambda)\mathbf{p}_j^2) \quad (17)$$

$$\geq \delta_i\left(\left(\sum_{j\neq i}\Gamma_{i,j}\mathbf{p}_j^1\right)^{\lambda}\left(\sum_{j\neq i}\Gamma_{i,j}\mathbf{p}_j^2\right)^{1-\lambda}\right) \quad (18)$$

Therefore, it is necessary that $\delta_i \leq 1$, i.e., $\beta_i \geq 1$.

To ensure the assumption that the denominator in (15) larger than the one in (14) holds, we need to impose a sufficient condition such that

$$\min_{\mathbf{p}^1,\mathbf{p}^2}\left(\sum_{j\neq i}\Gamma_{ij}\mathbf{p}_j^1\right)^{\lambda}\left(\sum_{j\neq i}\Gamma_{ij}\mathbf{p}_j^2\right)^{(1-\lambda)} \geq \max_{\mathbf{p}^1,\mathbf{p}^2}\delta_i\sum_{j\neq i}\Gamma_{i,j}(\lambda\mathbf{p}_j^1+(1-\lambda)\mathbf{p}_j^2). \quad (19)$$

Since $p_i \in [p_{\max}, p_{\min}]$, the minimum and maximum in (19) are achieved at the boundary, and thus for $0 \leq \lambda \leq 1$, we need to have

$$\delta_i \leq \frac{\sum_{j\neq i}\Gamma_{i,j}p_{\min}}{\sum_{j\neq i}\Gamma_{i,j}p_{\max}} = \frac{p_{\min}}{p_{\max}}.$$

It is satisfied and naturally follows that it also satisfies the necessary condition that $\delta_i \leq 1$. □

By Theorem 3.5, the bargaining solution is uniquely solved by the following maximization problem (OP):

$$(OP)\max\prod_{i\in I}(f_i(\mathbf{p})-u_i^0)\ \ \mathbf{p}\in X_0, \quad (20)$$

where X_0 denotes the set of power vector $\mathbf{p}$ that can achieve beyond the minimum OSNR target, i.e.,$X_0 = \{\mathbf{p} \in X : f_i(\mathbf{u}) \geq f_i^0 \text{ for some } i\}$. An equivalent problem to (OP) is (OP') given by

$$(OP')\max\sum_{i\in I}\ln(f_i(\mathbf{p})-u_i^0)\ \ \mathbf{p}\in X_0. \quad (21)$$

The constraints are:

$$\begin{array}{ll} p_{i,min} \leq p_i \leq p_{i,max} & (C2) \\ \sum_{i\in J} p_i \leq C & (C3) \end{array} \quad (22)$$

Suppose constraint (C2) gives a feasible set $W_1 = \{\mathbf{p} \mid p_{i,min} \leq p_i \leq p_{i,max}\}$ and constraint (C3) gives a feasible set $W_2 = \{\mathbf{p} \mid \sum_{i\in J} p_i \leq C\}$. Therefore, $X = X_W = W_1 \bigcap W_2$.

Due to the high coupling and nonlinearity in utility function (12), it is challenging to find a closed form solution. Instead, we can use change of variable to take advantage of the resulting linearity to characterize the bargaining solution. It will eventually give the following proposition.

PROPOSITION 4.2. *Suppose conditions in Proposition 4.1 holds and* $\mathbf{u}^0 = 0$. *In addition,* $p_{i,min}$ *and* $p_{i,max}$ *are determined sufficiently small and large such that they will not behave as active constraints, then a necessary condition for solving the optimality in OP is to consistently solve the following systems of equations for optimal solution* $\mathbf{p}^*$*:*

$$\mathbf{v} = \mathbf{\Gamma}^0\mathbf{w} + \nu. \quad (23)$$

Denote $r_i = \frac{1}{w_i}$ *and* $D_\delta = diag\{\delta_1, ..., \delta_N\}$*, then*

$$\mathbf{r} = \mathbf{D}_\delta\mathbf{\Gamma}^0\mathbf{p}. \quad (24)$$

In addition, we have

$$\mathbf{p}^\mathbf{T}\mathbf{v} = N, \mathbf{w}^\mathbf{T}\mathbf{r} = N. \quad (25)$$

where $\frac{1}{\delta_i X_{-i}} = w_i$, $\frac{1}{p_i} = v_i$*, and* ν *solves the slackness condition* $\nu(\mathbf{1}^T\mathbf{p} - C) = 0$.

PROOF. Suppose $p_{max,i}$ are chosen to be large enough to be inactive constraints. To solve the above convex optimization problem (21), we form the Lagrangian $\mathcal{L}(\mathbf{p}, \nu)$ as

$$\mathcal{L}(\mathbf{p},\nu) = \sum_i \ln\left(\frac{p_i}{\delta_i X_{-i}}\right) + \nu^T(\mathbf{1}^T\mathbf{p} - C).$$

Using the KKT first-order necessary condition, we have

$$\nabla_{p_i}\mathcal{L}(\mathbf{p},\nu) = 0.$$

that is,

$$\frac{1}{p_i} = \frac{\Gamma_{1i}}{\delta_1 X_{-1}} + ... + \frac{\Gamma_{i-1,i}}{\delta_{(i-1)}X_{-(i-1)}} + ... + \frac{\Gamma_{i+1,i}}{\delta_{(i+1)}X_{-(i+1)}} +$$

$$... + \frac{\Gamma_{Ni}}{\delta_N X_{-N}} + \nu. \forall i.$$

For example, for index $i = 1$ it yields

$$\frac{1}{p_1} = \frac{\Gamma_{21}}{\delta_2 X_{-2}} + \frac{\Gamma_{31}}{\delta_3 X_{-3}} + ... + \frac{\Gamma_{N1}}{\delta_N X_{-N}} + \nu$$

Let's denote $\frac{1}{p_i} = v_i$ and $\frac{1}{\delta_i X_{-i}} = w_i$. We will have in matrix form

$$\mathbf{v} = \mathbf{\Gamma}^0\mathbf{w} + \nu, \quad (26)$$

where

$$\mathbf{\Gamma}^0 = \begin{bmatrix} 0 & \Gamma_{21} & \dots & \Gamma_{N1} \\ \Gamma_{12} & 0 & \dots & \Gamma_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \Gamma_{1N} & \Gamma_{2N} & \dots & 0 \end{bmatrix},$$

denote $r_i = \frac{1}{w_i}$ and $D_\delta = diag\{\delta_1, ..., \delta_N\}$, then

$$\mathbf{r} = \mathbf{D}_\delta\mathbf{\Gamma}^0\mathbf{p}. \quad (27)$$

In addition, we have

$$\mathbf{p}^\mathbf{T}\mathbf{v} = N, \mathbf{w}^\mathbf{T}\mathbf{r} = N. \quad (28)$$

Therefore, a necessary condition for an optimal solution is that it should simultaneously satisfy (26), (27), and (28) for some ν. □

PROPOSITION 4.3. *A necessary condition for optimal solution* $\mathbf{p}^*$ *is to satisfy*

$$\mathcal{A}(\widetilde{\mathbf{p}})(\widehat{\mathbf{D}}_\delta\widetilde{\mathbf{p}} + \widehat{\mathbf{1}}) = \mathbf{n},$$

where $\mathbf{n} = [N, N]^T$, $\widetilde{\mathbf{p}} = [\mathbf{w}^T, \mathbf{p}^T]^T$, $\mathcal{A}(\widetilde{\mathbf{p}}) = \begin{bmatrix} \mathbf{p}^T & 0 \\ 0 & \mathbf{w}^T \end{bmatrix}$, $\widehat{\mathbf{D}}_\delta = \begin{bmatrix} \mathbf{\Gamma}^0 & 0 \\ 0 & \mathbf{D}_\delta\mathbf{\Gamma}^0 \end{bmatrix}$, $\hat{\mathbf{1}} = \begin{bmatrix} \nu \\ 0 \end{bmatrix}$

PROOF. From Equations (26),(27), (28),the necessary conditions can be formulated into

$$\begin{bmatrix} \mathbf{v} \\ \mathbf{r} \end{bmatrix} = \begin{bmatrix} \mathbf{\Gamma}^0 & 0 \\ 0 & \mathbf{D}_\delta \mathbf{\Gamma}^0 \end{bmatrix} \begin{bmatrix} \mathbf{w} \\ \mathbf{p} \end{bmatrix} + \begin{bmatrix} \nu \\ 0 \end{bmatrix}; \qquad (29)$$

$$\begin{bmatrix} \mathbf{p}^T & 0 \\ 0 & \mathbf{w}^T \end{bmatrix} \begin{bmatrix} \mathbf{v} \\ \mathbf{r} \end{bmatrix} = \begin{bmatrix} N \\ N \end{bmatrix}; \qquad (30)$$

The result will follow by substituting Equation (30) into Equation (29), which completes the proof. □

4.1 Alternative Formulation

Finding a closed form solution for OSNR-related objective functions is quite challenging. Even if the linear transformation can simplify the nonlinear KKT conditions, the bilinear matrix equality in Proposition 4.3 is still difficult to solve. In addition, due to the inseparability and coupling in objective functions, it is also difficult to decentralize the optimization problem to achieve a distributed algorithm. However, we can find an alternative way to formulate the problem, similar to the method that has been adopted in the central cost approach. The problem will be transformed into a much simpler optimization problem (AP) with target OSNRs in the constraints

$$\begin{array}{lll} (AP) & \max_{\mathbf{p}} \prod_{i\in J}(f_i(p_i) - f_i(p_{max,i})) & \\ \text{subject to} & & \\ & OSNR_i(\mathbf{p}) \geq \gamma_i^0 & (C1) \\ & p_{min,i} \leq p_i \leq p_{max,i} & (C2) \\ & \sum_{i\in J} p_i \leq C & (C3) \end{array} \qquad (31)$$

and its equivalent form (AP').

$$\begin{array}{lll} (AP') & \max_{\mathbf{p}} \sum_{i\in J} g_i(p_i) & \\ \text{subject to} & & \\ & OSNR_i(\mathbf{p}) \geq \gamma_i^0 & (C1) \\ & p_{min,i} \leq p_i \leq p_{max,i} & (C2) \\ & \sum_{i\in J} p_i \leq C & (C3) \end{array} \qquad (32)$$

where $g_i(\cdot) : \mathcal{R} \to \mathcal{R}$ and is defined as $g_i(p_i) = \sum_{i\in J} \ln(f_i(p_i) - f_i(p_{max,i}))$.

In the foregoing, $f_i(p_i)$ is the utility function of each channel. It can be chosen to satisfy certain desirable properties, such as upper-boundedness and concavity. As in [9], an exponential function can be chosen. The constraints (C1) and (C2) can be transformed into a set of linear constraints given by $\widehat{\mathbf{\Gamma}}\mathbf{p} \geq \mathbf{b}$ and $p_{min,i} \leq p_i \leq p_{max,i}$, where

$$\widehat{\mathbf{\Gamma}} = \begin{bmatrix} 1 - \Gamma_{11}\gamma_1^0 & -\Gamma_{12}\gamma_1^0 & \dots & -\Gamma_{1N}\gamma_1^0 \\ -\Gamma_{21}\gamma_2^0 & 1 - \Gamma_{22}\gamma_2^0 & \dots & -\Gamma_{2N}\gamma_2^0 \\ \vdots & \vdots & \ddots & \vdots \\ -\Gamma_{N1}\gamma_N^0 & -\Gamma_{N2}\gamma_N^0 & \dots & 1 - \Gamma_{NN}\gamma_N^0 \end{bmatrix},$$

$$\mathbf{b} = \begin{bmatrix} \gamma_1^0 n_{0,1} \\ \gamma_2^0 n_{0,2} \\ \vdots \\ \gamma_N^0 n_{0,N} \end{bmatrix}.$$

Let $\mathbf{D} = diag\{\gamma_1^0, ..., \gamma_N^0\}$, then $\widehat{\mathbf{\Gamma}} = \mathbf{I}_N - \mathbf{D\Gamma}$,$F_1 = \{\mathbf{p} \in \mathcal{R}^N \mid OSNR_i(\mathbf{p}) \geq \gamma_i^0\}$, $F_2 = \{\mathbf{p} \in \mathcal{R}^N \mid p_{min,i} \leq p_i \leq p_{max,i}\}$ and $F_3 = \{\mathbf{p} \in \mathcal{R}^N \mid \sum_{i\in J} p_i \leq C\}$.

Using similar arguments as in [24] and with reference to [11],[3], we can show the following Proposition 4.4.

PROPOSITION 4.4. *Let $\rho(\cdot)$ denote the spectral radius of a matrix. There exists $\mathbf{p} > 0$ such that $\widehat{\mathbf{\Gamma}}\mathbf{p} \geq \mathbf{b}$ if one of the following statements holds*
(1) $\widehat{\mathbf{\Gamma}}$ is diagonal dominant.
(2) $\rho(\mathbf{D\Gamma}) < 1$
(3) $\widehat{\mathbf{\Gamma}}^{-1} = \sum_{k=0}^{\infty}(\mathbf{I} - \mathbf{D\Gamma})^k$ exists and positive componentwise

Proposition 4.4 gives a sufficient condition on the feasibility of constraints (C1) and (C2), or the non-emptiness of sets F_1 and F_2. We next propose a sufficient condition for set F_3, or capacity constraint (C3).

PROPOSITION 4.5. *Assume that $\widehat{\mathbf{\Gamma}}$ is diagonally dominant and thus non-singular. In addition, $p_{i,\min}$ and $p_{i,\max}$ are small and large enough. Let κ be the condition number of $\widehat{\mathbf{\Gamma}}$. If $C \geq \frac{\kappa \max_i \gamma_i^0 n_{0,i}\sqrt{N}}{\sqrt{\rho(\widehat{\mathbf{\Gamma}}^{\mathbf{T}}\widehat{\mathbf{\Gamma}})}}$, then $C \geq \mathbf{1}^T\widehat{\mathbf{\Gamma}}^{-1}\mathbf{b}$. Furthermore, if equivalent conditions in Proposition 4.4 hold, then the feasible set $X = X_F = F_1 \cap F_2 \cap F_3$ is nonempty.*

PROOF. Using the definition of condition number $\kappa = \|\widehat{\mathbf{\Gamma}}\|\|\widehat{\mathbf{\Gamma}}^{-1}\|$ and matrix norm inequality $\|\widehat{\mathbf{\Gamma}}_{N\times N}\|_2 \leq \sqrt{N}\|\widehat{\mathbf{\Gamma}}_{N\times N}\|_\infty$ [11],we can obtain an upper bound on $\|\mathbf{1}^T\widehat{\mathbf{\Gamma}}^{-1}\mathbf{b}\|_\infty$.

$$\|\mathbf{1}^T\widehat{\mathbf{\Gamma}}^{-1}\mathbf{b}\|_\infty \leq \|\mathbf{1}^T\widehat{\mathbf{\Gamma}}^{-1}\|_\infty\|\mathbf{b}\|_\infty \leq \frac{\kappa}{\|\widehat{\mathbf{\Gamma}}\|_\infty}\|\mathbf{b}\|_\infty \leq$$

$$\frac{\kappa \max_i \mathbf{b}_i}{\|\widehat{\mathbf{\Gamma}}\|_\infty} \leq \frac{\kappa \max_i \gamma_i^0 n_{0,i}\sqrt{N}}{\|\widehat{\mathbf{\Gamma}}\|_2} = \frac{\kappa \max_i \gamma_i^0 n_{0,i}\sqrt{N}}{\sqrt{\rho(\widehat{\mathbf{\Gamma}}^{\mathbf{T}}\widehat{\mathbf{\Gamma}})}}.$$

From the first and last inequality, it follows that $C \geq \mathbf{1}^T\widehat{\mathbf{\Gamma}}^{-1}\mathbf{b}$ if capacity is chosen such that $C \geq \frac{\kappa \max_i \gamma_i^0 n_{0,i}\sqrt{m}}{\sqrt{\rho(\widehat{\mathbf{\Gamma}}^{\mathbf{T}}\widehat{\mathbf{\Gamma}})}}$. Furthermore, $\widehat{\mathbf{\Gamma}}$ is an M-matrix [3],[11], which has the monotone property, i.e., $\widehat{\mathbf{\Gamma}}\mathbf{p} \geq \mathbf{b} > 0$ implies $\mathbf{p} \geq 0$. Therefore, under the assumption that conditions in Proposition 4.4 hold, letting $C \geq \mathbf{1}^T\mathbf{p} \geq \mathbf{1}^T\widehat{\mathbf{\Gamma}}^{-1}\mathbf{b}$ ensures that $F_1 \cap F_3$ is nonempty. □

Capacity is dependent on the size of the matrix, i.e., the number of users.When OSNR targets are uniform,i.e. $\gamma_i = \gamma$, and $\widehat{\mathbf{\Gamma}}$ is symmetric, then C needs to satisfy $C \geq \frac{\kappa\gamma\sqrt{m}}{\lambda_{\max}}$,where $\lambda_{\max} = \rho(\widehat{\mathbf{\Gamma}})$. On the other hand, given a capacity, we can use the inequality to estimate the number of channels that we shall admit.

Based on Proposition 4.4 and 4.5, the feasible set X_F can be ensured to be compact, nonempty, and convex. Thus, (AP') is a convex program if we choose $g_i(p_i)$ to be a convex function in $p_i, \forall i$.

PROPOSITION 4.6. *The system problem (AP') can be decomposed into two sub-problems: User's problem (UP) and Network problem (NP), which are defined as follows.*

$$\begin{array}{lll} \textit{USER Problem (UP):} & \max_{p_i} g_i\left(\frac{w_i}{\lambda_i}\right) - w_i & \\ \textit{subject to} & \lambda_i p_{min,i} \leq w_i \leq \lambda_i p_{max,i} & (CD1) \end{array} \qquad (33)$$

$$\begin{array}{lll} \textit{NETWORK Problem (NP):} & \max_{\mathbf{p}} \sum_i w_i \ln(p_i) & \\ \textit{subject to} & \widetilde{\mathbf{\Gamma}}\mathbf{p} \geq \widetilde{\mathbf{b}}, & (CD2) \end{array} \qquad (34)$$

where $\widetilde{\mathbf{\Gamma}}^T = [\widehat{\mathbf{\Gamma}}^T, -\mathbf{1}^T]$ *and* $\widetilde{\mathbf{b}}^T = [\mathbf{b}^T, -C]$*, and* $p_i = w_i/\lambda_i$*.* w_i *and* λ_i *can be regarded as the user payment per unit of time and charge per unit of power, respectively [15].*

PROOF. Without losing generality, assume $p_{i,min} = 0$ to save the heavy notation. The proof will be similar otherwise. Use the idea from [14], we can form a Lagrangian from (AP') as

$$\mathcal{L}(\mathbf{p}, \xi, \theta) = \sum_i g_i(p_i) + \xi^T(-\widetilde{\mathbf{\Gamma}}\mathbf{p} + \widetilde{\mathbf{b}}) + \theta^T(\mathbf{p} - \mathbf{p_{max}}),$$

where $\xi \in \mathcal{R}^{(N+1)}, \theta \in \mathcal{R}^N, (\mathbf{p_{max}})_i = p_{max,i}$. Rearranging the terms, we can obtain

$$\mathcal{L}(\mathbf{p}, \xi, \theta) = \sum_i g_i(p_i) + (\theta^T - \xi^T\widetilde{\mathbf{\Gamma}})\mathbf{p} + \xi^T\widetilde{\mathbf{b}} - \theta^T\mathbf{p}_{\max}.$$

Use KKT necessary condition for optimality.

$$\nabla_{p_i}\mathcal{L} = g_i'(p_i) - (\widetilde{\mathbf{\Gamma}}^T\xi)_i + \theta_i = 0. \forall i \tag{35}$$

We can also form Lagrangian from (NP) and (UP) as well.

$$\mathcal{L}_{UP}(\mathbf{p}, \alpha, \gamma) = g_i\left(\frac{w_i}{\lambda_i}\right) - w_i + \gamma_i(w_i - \lambda_i p_{i,max}),$$

hence, a necessary condition is given by

$$\nabla_{w_i}\mathcal{L}_{UP} = \frac{g_i'(p_i)}{\lambda_i} - 1 + \gamma_i = 0.$$

Therefore, equivalently,

$$g_i'(p_i) = \lambda_i - \gamma_i\lambda_i. \tag{36}$$

As for (NP),

$$\mathcal{L}_{NP}(\mathbf{p}, \beta) = \sum_i w_i \ln(p_i) + \beta^T(-\widetilde{\mathbf{\Gamma}}\mathbf{p} + \widetilde{\mathbf{b}}),$$

hence the necessary condition gives

$$\nabla_{p_i}\mathcal{L}_{NP} = \frac{w_i}{p_i} + (-\tilde{\mathbf{\Gamma}}^T\beta)_i = 0 \tag{37}$$

Since $\lambda_i = w_i/p_i$, use (37) and substitute into (36), and we obtain

$$g_i'(p_i) - (\tilde{\mathbf{\Gamma}}^T\beta)_i + \gamma_i\lambda_i = 0, \forall i \tag{38}$$

Comparing to (35), it can be easily observed that the first-order KKT conditions are the same when $\xi = \beta$ and $\theta_i = \gamma_i\lambda_i, \forall i$. Therefore, since both (UP) and (NP) are convex problems with linear constraints, the first-order KKT condition is both necessary and sufficient. Thus, they form an equivalent problem with (AP'). □

Such decomposition provides a framework to decentralize our algorithm in power control. Users optimize their own utility by submitting an amount to pay and the network collects these payments and determines the allocation to each user. The network problem also maintains the property of proportional fairness.

4.2 Exponential Utility Function

In this section, we particularly choose $f_i(p_i) = -e^{p_i}$, an exponential function defined on the interval $[p_{min,i}, p_{max,i}]$, as in [9]. Directly following system model (AP), the system problem (EP) will become

$$\begin{array}{lll} (EP) & \max_{\mathbf{p}} \prod_{i\in J}(e^{p_{max,i}} - e^{p_i}) & \\ \text{subject to} & & \\ & OSNR_i(\mathbf{p}) \geq \gamma_i^0 & \text{(C1)} \\ & p_{min,i} \leq p_i \leq p_{max,i} & \text{(C2)} \\ & \sum_{i\in J} p_i \leq C & \text{(C3)} \end{array} \tag{39}$$

Due to the fact that the chosen objective function is separable and the constraints are linearly coupled, the optimization problem (EP) can thus be decomposed into user's problem and network problem as appeared in Proposition 4.6. By Thereom 3.5, system problem (EP) is equivalent to system problem (EP') as follows.

$$\begin{array}{lll} (EP') & \max_{\mathbf{p}} \sum_{i\in J} \ln(e^{p_{max,i}} - e^{p_i}) & \\ \text{subject to} & & \\ & OSNR_i(\mathbf{p}) \geq \gamma_i^0 & \text{(C1)} \\ & p_{min,i} \leq p_i \leq p_{max,i} & \text{(C2)} \\ & \sum_{i\in J} p_i \leq C & \text{(C3)} \end{array} \tag{40}$$

By Proposition 4.6, system problem (EP') can be decomposed into the user problem (EUP) and network problem (ENP) as follows.

$$\begin{array}{lll} (EUP) & \max_{w_i} \ln(e^{p_{max,i}} - e^{w_i/\lambda_i}) - w_i & \\ \text{subject to} & \lambda_i p_{min,i} \leq w_i \leq \lambda_i p_{max,i} & \text{(CD1)} \end{array} \tag{41}$$

$$\begin{array}{lll} (ENP) & \max_{\mathbf{p}} \sum_i w_i \ln(p_i) & \\ \text{subject to} & \widetilde{\mathbf{\Gamma}}\mathbf{p} \geq \widetilde{\mathbf{b}} & \text{(CD2)} \end{array} \tag{42}$$

5. FUTURE WORK

In this paper, we proposed Nash bargaining model for the power control problem in optical networks. Its inherent properties of proportional fairness and efficiency are attractive for our applications. We formulated two different optimization problems. We find it a challenge to give a closed form solution for the direct OSNR-based optimization. However, it is possible to develop numerical techniques such as in [2] to find an iterative and feedback algorithm to attain the optimal solution. On the other hand, the indirect power minimization approach with target OSNR in constraints gives a relatively simpler optimization problem. We should be able to derive distributed and iterative algorithms by using Proposition 4.6 as a starting point.

The fairness criteria that we used for Nash bargaining model in this paper is proportional fairness. We can generalize NBS by solving a parametric class of problems for different fairness criteria in Definition 3.2. With the assumption of separability in utility function, we observe that the general optimization problem falls into a class of problem of monotropic programming in [4]. With this tool, we may further characterize the generalized NBS and apply duality theory to achieve a hierarchical decomposition of the problem.

6. CONCLUSION

Noncooperative game and central cost approaches are two commonly used tools for network power control. However, Nash equilibrium may not guarantee efficiency; and central optimization may yield an unfair solution. We used the concepts from Nash bargaining model and its inherent properties to achieve an efficient and proportionally fair solution. We formulated two optimization problems in the context of power control in optical networks and characterize the solutions in each model. This theoretical work will lead us to find a distributed, iterative and feedback algorithm to be implemented in optical network.

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