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A jamming power control game with unknown user's communication metric

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Abstract

We consider a jamming problem in which a jammer aims to degrade a user's communication in which the user might differ in applied applications or communication purposes. Such differences are reflected by different communication metrics used by the user. Specifically, signal-to-interference-plus-noise ratio (SINR) is used as a metric to reflect regular data transmission purposes. Meanwhile, as another metric, latency, modeled by the inverse SINR, is used to reflect emergency communication purposes. We consider the most difficult scenario for the jammer where it does not know which application (metric) the user employs. The problem is formulated as a Bayesian game. Equilibrium is found in closed form, and the dependence of equilibrium on network parameters is illustrated.

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1. Introduction

The wireless medium's open-access nature exposes wireless networks to potential hostile interference, which can lead to communication disruptions. These issues typically involve multiple agents, such as a user and a jammer, each with distinct objectives. Consequently, game theory has been extensively utilized to analyze problems related to hostile interference, like jamming [1]. In such jamming problems throughput [2–4], SINR [5–11] and latency [12, 13] are considered to be typical user communication metrics. Throughout all of the above papers, the user's communication is considered homogeneous in the sense that there is a single metric being considered in its communication. It is important to highlight that the communication metric has a direct influence on the performance of the user's application. The papers mentioned above focus on jamming issues in scenarios where the user is using a single purpose or application, which may not always align with real-world jamming situations. Given the open nature of wireless networks, users may have varying communication needs or applications, leading to the necessity of addressing different communication metrics such as SINR and latency to cater to regular and emergency communication requirements. The goal of this paper is to study such a jamming problem where the jammer could have only limited statistical information given by a priori probabilities, about which purpose or application, and, so, metric, the user implements, which reflects a more practical and dangerous case for such agent acting as a jammer. To the best knowledge of the authors, such a type of uncertainty has not been studied in the literature. In the paper, the jamming problem with such type of uncertainty is formulated and solved as Bayesian game in Nash and Stackelberg equilibrium framework. The dependence of equilibrium on network parameters and the a priori probabilities is illustrated as well as the disadvantage the jammer could meet is illustrated for the user acting as a follower to model the user's ability to quickly learn the jammer's behaviour.

2. Communication model

Let us consider a network with two agents, namely, a *user* and an *adversary*. The adversary is a *jammer*, who intends to degrade the user's communication with a receiver through interference. The communication occurs on a single carrier and the channel is assumed to be flat fading. The resource for the user to control, i.e.,

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its strategy, is its transmission power P, with $P \in \mathbb{R}_+$, meanwhile, the resource for the jammer to control, i.e., jammer's strategy, is its jamming power J, with $J \in \mathbb{R}_+$. Then,

(a) the SINR of the transmitted signal is

$$\mathrm{SINR}(P,J) \triangleq \frac{hP}{N+gJ}, \quad (1)$$

where h is the source-destination fading channel gain, g is the jammer-destination channel gain (or interference channel gain) and N is the background noise variance;

(b) the (proxy) latency of the transmitted signal can be modeled by the inverse SINR (please, see [12]), i.e.,

$$L(P,J) \triangleq \frac{1}{\mathrm{SINR}(P,J)}. \quad (2)$$

It might be worth noting that SINR and negative latency might be included in a uniform scale of user's communication utilities, namely, α-fairness utilities with $\alpha = 0$ and $\alpha = 2$, respectively [14].

It is a common assumption that in order to enhance communication performance, users employ a relevant metric based on the purpose or application. Additionally, we acknowledge that the jammer may possess only limited statistical knowledge regarding the specific application and metric employed by the user. This scenario presents a more practical and perilous situation for the agent acting as a jammer. To model such an incomplete information case we associate a user's type with the implemented metric. Specifically, we refer to the user implementing SINR or latency metric as S-type user or L-type user, respectively. Denote by $P_S \in \mathbb{R}_+$ and $P_L \in \mathbb{R}_+$ strategies of the S-type user and L-type user, respectively. The jammer does not know exactly the user's type but only knows that with a priori probabilities α and $1-\alpha$ the user is S-type and L-type user, respectively.

The payoff to the S-type user is given as the difference between the SINR and transmission cost

$$v_S(P_S,J) = \mathrm{SINR}(P_S,J) - C_T P_S, \quad (3)$$

where C_T is transmission cost per power unit.

The payoff to the L-type user is given as the negative of the sum of latency and transmission cost

$$v_L(P_L,J) = -\lambda L(P_L,J) - C_T P_L, \quad (4)$$

where λ is a conversion coefficient.

The jammer wants to decrease the expected SINR of S-type user as well as involved jamming power cost, and also increase the expected latency of L-type user. Hence, the expected payoff to the jammer is

$$v_J(P_S,P_L,J) = (1-\alpha)\lambda L(P_L,J) - \alpha \mathrm{SINR}(P_S,J) - C_J J, \quad (5)$$

where C_J is jamming cost per jamming power unit.

Thus, such user's communication, where the user might implement different communication metrics and the jammer does not know which of them occurs, might be considered as heterogeneous communication by metrics. It is worth noting that, in [15], a multi-access communication heterogeneous network with selfish users was considered where although users might differ in communication metrics, each user applies a fixed metric that is known to all other users.

Each user's type, as well as the jammer, wants to maximize its own payoff. Thus, we look for (Bayesian) Nash equilibrium [16], i.e., for a triple of strategies (P_S, P_L, J) such that each of them is the best response to the others:

$$P_S = \mathrm{BR}_S(J) \triangleq \mathrm{argmax}\{v_S(\tilde{P}_S,J) : \tilde{P}_S \ge 0\}, \quad (6)$$
$$P_L = \mathrm{BR}_L(J) \triangleq \mathrm{argmax}\{v_L(\tilde{P}_L,J) : \tilde{P}_L \ge 0\}, \quad (7)$$
$$J = \mathrm{BR}_J(P_S,P_L) \triangleq \mathrm{argmax}\{v_J(P_S,P_L,\tilde{J}) : \tilde{J} \ge 0\}. \quad (8)$$

Denote this Nash (Bayesian) game by $\Gamma^{\mathcal{N}}$. Further we will apply a constructive approach to find the equilibrium and prove its uniqueness via solving the best response equations (6)-(8). First we solve them in closed form in the following proposition.

Proposition 1. **(a)** *For a fixed jammer's strategy J the best response P_S of S-type user is given by:*

$$P_S = BR_S(J) = \begin{cases} = 0, & \dfrac{h}{N+gJ} < C_T, \\ \in \mathbb{R}_+, & \dfrac{h}{N+gJ} = C_T, \\ = \infty, & \dfrac{h}{N+gJ} > C_T. \end{cases} \quad (9)$$

(b) *For a fixed jammer's strategy J the best response P_L of L-type user is:*

$$P_L = BR_L(J) = \sqrt{\frac{\lambda(N+gJ)}{hC_T}}. \quad (10)$$

(c) *For fixed S-type and L-type user' strategies P_S and P_L, respectively, the best response $J = BR_J(P_S,P_L)$ of the jammer is given by:*

(c-i) *if*

$$\frac{(1-\alpha)\lambda g}{hP_L} + \frac{\alpha h g P_S}{N^2} \le C_J \quad (11)$$

then $J = 0$,

(c-ii) *if*

$$\frac{(1-\alpha)\lambda g}{hP_L} < C_J < \frac{(1-\alpha)\lambda g}{hP_L} + \frac{\alpha h g P_S}{N^2} \quad (12)$$

then J is the unique positive root of the equation

$$\frac{(1-\alpha)\lambda g}{hP_L} + \frac{\alpha h g P_S}{(N+gJ)^2} = C_J, \quad (13)$$

(c-iii) *if*

$$C_J \le \frac{(1-\alpha)\lambda g}{hP_L} \quad (14)$$

then $J = \infty$.

Proof: Please find in Appendix.

In the following theorem we find Nash equilibrium in closed form.

Theorem 1. *In game $\Gamma^{\mathcal{N}}$, Nash equilibrium (P_S, P_L, J) is unique except only for the case (b) below, where a continuum of S-type user's strategies might arise. Specifically,*

(a) *if $h/N < C_T$ then*

$$P_S = 0, \quad (15)$$

$$P_L = \max\left\{\frac{(1-\alpha)\lambda g}{hC_J}, \sqrt{\lambda\frac{N}{hC_T}}\right\}, \quad (16)$$

$$J = \max\left\{\frac{(1-\alpha)^2\lambda g C_T}{hC_J^2} - \frac{N}{g}, 0\right\}, \quad (17)$$

(b) *if $h/N = C_T$ then multiple S-type user equilibrium strategies might arise. Specifically, besides (P_S, P_L, J) given by (15)-(17), $(P_S, \sqrt{\lambda N/(hC_T)}, 0)$ also is an equilibrium with any P_S such that*

$$0 \le P_S \le \max\left\{\frac{N^2}{\alpha h g}\left(C_J - (1-\alpha)g\sqrt{\frac{\lambda C_T}{hN}}\right), 0\right\}, \quad (18)$$

(c) *if $C_T < h/N$ then*

$$P_S = \max\left\{\frac{1}{\alpha g C_T^2}\left(hC_J - (1-\alpha)\sqrt{\lambda} g C_T\right), 0\right\}, \quad (19)$$

$$P_L = \max\left\{\frac{\sqrt{\lambda}}{C_T}, \frac{(1-\alpha)\lambda g}{hC_J}\right\}, \quad (20)$$

$$J = \max\left\{\frac{h}{gC_T} - \frac{N}{g}, \frac{(1-\alpha)^2 g\lambda C_T}{hC_J^2} - \frac{N}{g}\right\}. \quad (21)$$

Proof: Please find in appendix.

Note that, for a small transmission cost, i.e., in case (a), maximum, in (16) and (17), are achieved at the same entries. The S-type user is inactive, i.e., $P_S = 0$, meanwhile the L-type user is active, i.e., $P_L > 0$, for any set of network parameters. Its strategy is sensitive to the a prior probability that S-type user occurs while the jammer is active, i.e., $J > 0$. For a large transmission cost, i.e., in case (c), maximum, in (19)-(21), are achieved at the same entries. The L-type user as well as the jammer are active for any set of network parameters. Jammer's strategy is not sensitive to a prior probability that S-type user occurs while such probability is large. Meanwhile L-type user and S-type user differ by their sensitivity to the a priori probability, namely, one of them is sensitive while the other is not.

3. Smart L-type user

Motivated by the interpretation that the user focusing on a reduction in latency (specifically, L-type user), has to react quickly compared with one focusing just on communication, we model such a situation by a Stackelberg game denoted by $\Gamma^{\mathcal{S}}$ with L-type user as a follower, and for which the S-type user and the jammer are leaders. The Stackelberg equilibrium for such a scenario can be found as the solution of a two-level optimization problem:

In the first step of Stackelberg game $\Gamma^{\mathcal{S}}$, L-type user implements its best response $\mathrm{BR}_L(J)$ given by (10).

In the second step of Stackelberg game $\Gamma^{\mathcal{S}}$, taking into account such the L-type user's behavior, the jammer wants to maximize its payoff given as follows:

$$V_J(P_S, J) = v_J(P_S, \mathrm{BR}_L(J), J), \quad (22)$$

and S-type user wants to maximize its payoff $v_S(P_S, J)$ given (3). Thus, in the second step of the Stackelberg game $\Gamma^{\mathcal{S}}$, the S-type user and the jammer play a (Nash) sub-game between each other. Denote this sub-game by $\underline{\Gamma}^{\mathcal{N}}$, and let $(\underline{P}_S, \underline{J})$ be an Nash equilibrium in this sub-game, i.e.,

$$\underline{P}_S = \mathrm{argmax}\{v_S(\tilde{P}_S, \underline{J}) : \tilde{P}_S \in \mathbb{R}_+\}, \quad (23)$$

$$\underline{J} = \underline{\mathrm{BR}}_J(\underline{P}_S) \triangleq \mathrm{argmax}\{V_J(\underline{P}_S, \tilde{J}) : \tilde{J} \in \mathbb{R}_+\}. \quad (24)$$

Then $(\underline{P}_S, \mathrm{BR}_L(\underline{J}), \underline{J})$ is Stackelberg equilibrium in game $\Gamma^{\mathcal{S}}$.

Theorem 2. *In Stackelberg game $\Gamma^{\mathcal{S}}$, equilibrium $(\underline{P}_S, BR_L(\underline{J}), \underline{J})$ is unique except for the case (b) below, where a continuum of S-type user strategies might arise. Specifically,*

(a) *if $h/N < C_T$ then*

$$\underline{P}_S = 0, \quad (25)$$

$$\underline{J} = \max\left\{\frac{(1-\alpha)^2\lambda g C_T}{4hC_J^2} - \frac{N}{g}, 0\right\}, \quad (26)$$

(b) *if $h/N = C_T$ then multiple S-type user equilibrium strategies might arise. Specifically, besides $(\underline{P}_S, \underline{J})$ uniquely given by (26), also $\underline{P}_S$ can be such that such that*

$$0 \le \underline{P}_S \le \max\left\{\frac{N^2}{\alpha h g}\left(C_J - \frac{(1-\alpha)g}{2}\sqrt{\frac{\lambda C_T}{hN}}\right), 0\right\}, \quad (27)$$

meanwhile jammer's equilibrium strategy uniquely given as follows:

$$\underline{J} = 0, \quad (28)$$

(c) *if $C_T < h/N$ then*

$$\underline{P}_S = \max\left\{\frac{1}{\alpha g C_T^2}\left(hC_J - \frac{(1-\alpha)\sqrt{\lambda} g C_T}{2}\right), 0\right\}, \quad (29)$$

$$\underline{J} = \max\left\{\frac{h}{gC_T} - \frac{N}{g}, \frac{(1-\alpha)^2 g\lambda C_T}{4hC_J^2} - \frac{N}{g}\right\}. \quad (30)$$

Proof: Please find in appendix.

Note that, for large transmission cost, i.e., in case (c), maximum, in (29) and (30), is archived at the same entries.

4. Discussion of the results

In this section we show how Nash and Stackelberg equilibrium strategies derived in Theorem 1 and Theorem 2 have an impact on the communication latency and SINR, and the disadvantage the jammer could meet if the user acts as a follower. Let us illustrate it via an example with transmission and jamming costs $(C_T, C_J) = (1, 2)$, the background noise variance $N = 1$, fading gains $h = 2$ and $g = 3$ or 5, and conversion coefficient $\lambda = 10$. Thus, $C_T = 1 < 2 = h/N$, i.e., cases (c) of Theorem 1 and Theorem 2 hold. Fig. 1[1] illustrates that jammer and L-type user are active, i.e., $J > 0$ and $P_L > 0$, for any network parameters. Meanwhile the S-type user is inactive, i.e., $P_S = 0$, for small a priori probability α that S-type user occurs. The jammer's strategy is not sensitive to the a priori probability, meanwhile L-type user and S-type user share their sensitivity in the sense that for small a priori probability only L-type user strategy is sensitive to that probability, meanwhile S-type user strategy is sensitive otherwise.

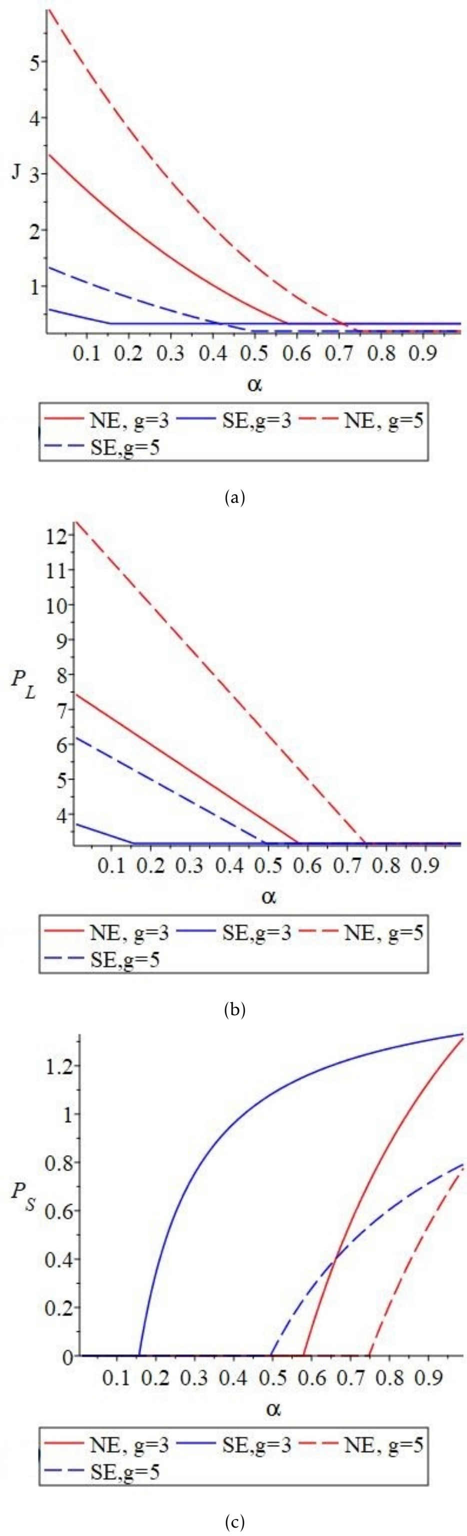
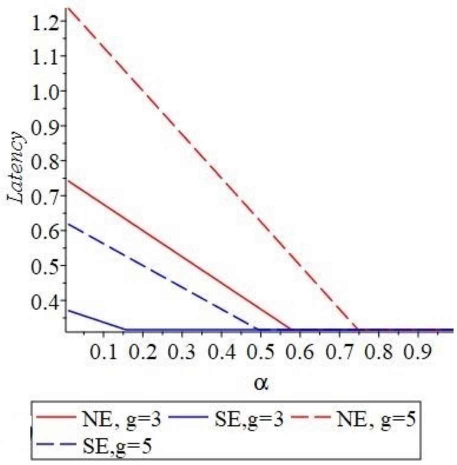
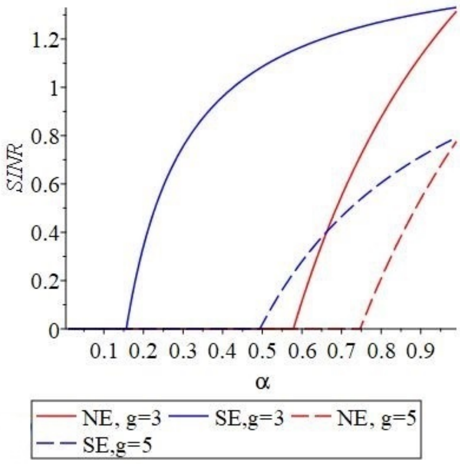


Figure 1. (a) Jammer's strategy, (b) L-type user's strategy and (c) S-type user's strategy as functions on probability α.

[1]Here we use abbreviation SE and NE for Stackelberg and Nash equilibrium, respectively.



(a)



(b)

Figure 2. (a) Latency of L-type user's strategy and (b) SINR of S-type user's strategy as functions on probability α.

Fig. 2 illustrates that an increase in jamming fading gain leads to an increase in latency of L-type user and a decrease in SINR of S-type user, respectively. An increase of a priori probability α that S-type user occurs leads to a decrease in latency and an increase in SINR for both user's types. Also, the user's latency in Stackelberg game is smaller than in Nash equilibrium. Meanwhile, the user's SINR in Stackelberg game is greater than in Nash equilibrium. This shows that modelling a user acting as a follower in Stackelberg game allows one to model its ability to quickly learn the jammer's strategy and accordingly adjust its own strategy to achieve a gain in the implemented communication metric associated with its communication purposes, in other words, to model the disadvantage the jammer could meet if the user could have such ability to quickly learning.

5. Conclusions

A jamming power control problem of user's communication in which the jammer has limited statistical information (a priori probabilities) about what communication purpose the user has or what application it implements, has been modeled by a Bayesian game. Communication metrics have been used to model the communication purposes of the user. Specifically, the SINR metric has been used to reflect regular data transmission purposes, while latency, modeled by the inverse SINR, has been used to reflect emergency communication purposes. Equilibrium strategies are derived in closed form for the Nash and Stackelberg game frameworks. The dependence of equilibrium on network parameters as well as on the a priori probability of the user's communication purposes has been investigated and illustrated.

Appendix A. Proof of Proposition 1

By (3), we have that

$$v_S(P_S, J) = \left(\frac{h}{N + gJ} - C_T\right)P_S \tag{A.1}$$

and (a) follows. By (4), we have that

$$\frac{\partial v_L(P_L, J)}{\partial L} = \frac{\lambda(N + gJ)}{hP_L^2} - C_T. \tag{A.2}$$

Thus, $\partial v_L(P_L, J)/\partial L$ is decreasing from infinity for $P_L \downarrow 0$ to $-C_T$ for $P_L \uparrow \infty$. Thus, the best response P_L is given as the unique root of

$$\frac{\lambda(N + gJh)}{P_L^2} = C_T, \tag{A.3}$$

and (b) follows.

By (5), we have that

$$\frac{\partial v_J(P_S, P_L, J)}{\partial J} = \frac{(1-\alpha)\lambda g}{hP_L} + \frac{\alpha hgP_S}{(N + gJ)^2} - C_J. \tag{A.4}$$

Thus, $\partial v_J(P_S, P_L, J)/\partial L$ is decreasing with respect to J.

Then, (c) follows by straightforward substituting the boundary values $J = 0$ and $J = \infty$ into (A.4). ■

Appendix B. Proof of Theorem 1

Let (P_S, P_L, J) be a Nash equilibrium. First we prove that

$$P_S \neq \infty, P_L \neq \infty \text{ and } J \neq \infty. \tag{B.1}$$

Let $J = \infty$. Then, by (10), $P_L = \infty$, and, so, by (14), $C_J = 0$. This contradiction implies that $J \neq \infty$. Similarly assumption $P_L = \infty$, by (10), leads to $J = \infty$, which cannot hold. Thus, $P_L \neq \infty$. Finally, let $P_S = \infty$. Then

(11) cannot hold. Thus, $J > 0$. Then, by (13), $J = \infty$. This contradiction completes the proof of (B.1).

Assume that $h/N < C_T$. Then $h/(N+gJ) < C_T$ for all $J \geq 0$. Thus, by (9), $P_S = 0$. We have separately to consider two cases: (i) $J = 0$ and (ii) $J > 0$.

(i) Let $J = 0$, which is the second entry in the max-function (17). Substituting $J = 0$ into (10) implies that

$$P_L = \sqrt{\frac{\lambda N}{hC_T}}, \tag{B.2}$$

which is the second entry in the max-function (16). Substituting this P_L into (11) implies that

$$\sqrt{\frac{\lambda N}{hC_T}} \geq \frac{(1-\alpha)\lambda g}{hC_J}. \tag{B.3}$$

(ii) Let $J > 0$. Substituting $P_S = 0$ into (13) implies that

$$P_L = \frac{(1-\alpha)\lambda g}{hC_J} \tag{B.4}$$

which is the first entry in max-function (16). Substituting there P_S and P_L into (13) implies that J given by the first entry in the max-function (17). Substituting these P_L and P_S also into (12) implies that

$$\sqrt{\frac{\lambda N}{hC_T}} < \frac{(1-\alpha)\lambda g}{hC_J}. \tag{B.5}$$

0 Finally, straightforward calculation shows that (B.3) holds if and only if second entries of max-functions (16) and (17) are greater than corresponding their second entries, and (a) follows.

Assume that $h/N = C_T$. Then besides (P_S, P_L, J) uniquely given by (15)- (17), a continuum of Nash equilibria might arise with $J = 0$. Specifically, since $h/N = C_T$ and $J = 0$, by (9), P_S can be any of them. Also, substituting $J = 0$ into (10) implies (B.2). Substituting (B.2) into (11) implies

$$\frac{(1-\alpha)\lambda g}{h}\sqrt{\frac{hC_T}{\lambda N}} + \frac{\alpha hgP_S}{N^2} \leq C_J. \tag{B.6}$$

Solving (B.6) on P_S implies (18), and (b) follows.

Assume that $C_T < h/N$. Then we have to consider separately two cases: (I) $P_S = 0$ and (II) $P_S > 0$.

(I) Let $P_S = 0$, which is the second entry in max-function (19). Then, by (9), we have that

$$\frac{h}{N+gJ} \leq C_T. \tag{B.7}$$

Since $C_T < h/N$, (B.7) implies that $J > 0$, i.e., (13) holds. Substituting $P_S = 0$ into (13) and solving with respect to P_L implies $P_L = (1-\alpha)\lambda g/(hC_J)$, which is the second entry in the max-function (20). Substituting this P_L into (10) implies that J has to be given by the second entry in the max-function (21). Substituting this J into (B.7) implies

$$hC_J \leq (1-\alpha)\sqrt{\lambda}gC_T. \tag{B.8}$$

(II) Let $P_S > 0$. Then, by (9), we have that

$$\frac{h}{N+gJ} = C_T. \tag{B.9}$$

Solving (B.9) on J implies that J has to be given by the first entry in the max-function (21). Substituting this J into (10) implies that P_L has to be given by the first entry in the max-function (20). Substituting both these P_L and J into (13) and solving the obtained equation with respect to P_S implies that P_S has to be given by the first entry in the max-function (19). Moreover, such P_S is positive if and only if $hC_J > (1-\alpha)\sqrt{\lambda}gC_T$.

Finally, a straightforward calculation shows that (B.8) holds if and only if the second entries of max-functions (19)-(21) are greater than their corresponding first entries, and (c) follows. ∎

Appendix C. Proof of Theorem 2

By (6) and (23), for a fixed jammer's strategy J the best response P_S of S-type user is given by (9). By (5), (10) and (22), we have that

$$\frac{\partial V_J(P_S, J)}{\partial J} = \frac{(1-\alpha)g\sqrt{C_T\lambda}}{2\sqrt{h}\sqrt{N+gJ}} + \frac{\alpha hgP_S}{(N+gJ)^2} - C_J. \tag{C.1}$$

Thus, $\partial V_J(P_S, J)/\partial L$ is decreasing on J, and tending to $-C_T$ for $J \uparrow \infty$. Thus, for a fixed P_S, the jammer's best response $J = \underline{\mathrm{BR}}_J(P_S)$ is $J = 0$ if $\partial V_J(P_S, 0)/\partial L \leq 0$ and it is the unique positive root of equation $\partial V_J(P_S, J)/\partial J = 0$ otherwise. Then following the proof of Theorem 1 the result can be established straightforward by solving both these best response equations jointly. ∎

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