EAI Endorsed Transactions
on Energy Web
Research Article EAI.EU

Distribution Network Target Framework Planning Algorithm Based on Fuzzy Optimization and Grey System Theory

Jinsen Liu[1,*], Molin He[2], Jie Wang[1], Jie Lu[3]

[1]Power Grid Planning and Research Center of Guizhou Power Grid Co., Ltd
[2]Guizhou Power Grid Co., Ltd
[3]Zunyi Power Supply Branch of Guizhou Power Grid Co., Ltd.

Abstract

INTRODUCTION: The global energy transition, driven by the rapid growth of distributed renewable energy, stochastic load profiles (e.g., EV charging spikes), and conflicting stakeholder objectives, has brought unprecedented complexities to distribution network planning. Traditional deterministic methods fail to handle qualitative fuzziness (e.g., subjective reliability thresholds) and quantitative uncertainty (e.g., sparse historical data), leading to inflexible and inefficient solutions. This study addresses these challenges by developing a hybrid planning framework.
OBJECTIVES: This paper aims to solve the dual challenges of qualitative fuzziness and quantitative uncertainty in distribution network planning, providing a systematic solution to accommodate distributed renewable energy, handle load uncertainty, and balance conflicting stakeholder preferences through integrating fuzzy optimization theory and grey system theory.
METHODS: The hybrid algorithm combines fuzzy optimization and grey system theory. Fuzzy optimization uses triangular fuzzy numbers for load growth rates ([3%, 5%, 8%]) and trapezoidal fuzzy intervals for voltage constraints ([−10%, −5%, 5%, 10%]) with membership functions (threshold $\lambda \geq 0.8$) to convert qualitative requirements into solvable constraints. Grey system theory applies the GM(1,1) model for load forecasting (achieving 4.2% MAPE with 15-month data) and grey relational analysis (GRA) for data-driven objective weighting to eliminate expert bias. An improved particle swarm optimization (IPSO) algorithm is used for optimization, validated in a 33-node network with 8.5 MW PV and 6 MW wind capacity.
RESULTS: In the 33-node case study, compared to the deterministic genetic algorithm (D-GA), the hybrid algorithm reduces lifecycle costs by 19% (from \$8.91M to \$7.23M), increases renewable energy accommodation by 24% (from 9.8 MW to 12.3 MW), and improves the system average supply availability index (ASAI) from 99.92% to 99.95%. Under extreme uncertainties (±40% renewable output, ±30% load shifts), cost deviations remain within 6% and reliability metrics within 5%, demonstrating strong robustness.
CONCLUSION: This research presents a robust hybrid framework that bridges fuzzy qualitative reasoning and grey data efficiency, effectively addressing both qualitative fuzziness and quantitative uncertainty in distribution network planning. It provides a science-based tool for resilient grid design, with potential for extension to multi-energy system integration and real-time optimization in future work.

Keywords: Distribution Network Planning, Fuzzy Optimization, Grey System Theory, Distributed Renewable Energy

Received on 16 March 2025, accepted on 20 July, published on 15 October 2025

doi: 10.4108/ew.10598

*Corresponding author. Email: LiuJinsen202504@163.com

1. Introduction

The global energy transition, characterized by the rapid proliferation of distributed renewable energy, the electrification of end-use sectors, and the imperative for carbon neutrality, has ushered in a new era of complexity for distribution network planning. As the foundational layer connecting transmission grids to millions of consumers, distribution networks now must accommodate unprecedented levels of uncertainty—from the intermittent output of solar/wind generators to the stochasticity of electric vehicle (EV) charging loads—and reconcile conflicting stakeholder objectives that resist precise quantification. Traditional planning methodologies, rooted in deterministic assumptions and single-source uncertainty handling, have become inadequate for designing resilient, future-proof grid frameworks. This paper addresses this gap by presenting a hybrid planning algorithm that integrates fuzzy optimization theory and grey system theory, offering a systematic solution to the dual challenges of qualitative fuzziness (e.g., subjective reliability thresholds) and quantitative uncertainty (e.g., sparse historical data).

The global energy transition has introduced unprecedented complexities into distribution network planning, driven by the intermittent nature of distributed renewable energy (DRE), stochastic load profiles (e.g., EV charging spikes causing 25–35% demand surges), and conflicting stakeholder objectives (e.g., utilities targeting $60–120/kVA CapEx vs. regulators enforcing 99.99% reliability standards). Traditional deterministic methods, reliant on precise data and rigid constraints, fail to handle "soft" fuzziness (e.g., subjective "acceptable" voltage stability) and "hard" uncertainties (e.g., sparse historical data in emerging markets with <5 years of load records), leading to inflexible solutions. For instance, deterministic models exhibit 25–35% performance deviations under extreme DRE fluctuations, while stochastic methods require ≥10 years of hourly data for scenario analysis, impractical in regions with rapid grid expansion.

This study introduces a hybrid framework that synergizes fuzzy optimization and grey system theory to address dual uncertainties. Fuzzy set theory models qualitative ambiguities: triangular fuzzy numbers represent load growth rates ([3%, 5%, 8%]), and trapezoidal fuzzy intervals define voltage constraints ([−10%, −5%, 5%, 10%]) with membership functions ensuring flexibility (e.g., allowing ±8% voltage deviations with 0.8 membership). Grey system theory tackles data scarcity: the GM(1,1) model predicts loads with 15 months of data (MAPE 4.2%, outperforming ARIMA by 19%), while GRA objectively weights multi-objective functions (e.g., assigning 25% higher priority to DRE accommodation in high-renewable grids), reducing planner bias by 30% compared to expert-based methods. Through a case study on a 33-node network with 14.5 MW DRE capacity, this research demonstrates that the hybrid algorithm achieves a 19% cost reduction (vs. D-GA), 24% DRE accommodation improvement, and 17% ASAI enhancement. Its robustness under extreme scenarios—limiting cost/reliability deviations to ≤6%/5%—addresses critical gaps in traditional planning, offering utilities a scalable tool for adaptive grid design in data-scarce, high-uncertainty environments. The proposed algorithm integrates fuzzy optimization and grey systems into a unified framework, as shown in Figure 1. Figure 1 illustrates the process of electric load forecasting and optimization. Firstly, historical load and distributed renewable energy data are collected. Subsequently, grey load forecasting based on the GM(1,1) model and fuzzy scenario generation using triangular fuzzy numbers are carried out respectively. Then, a fuzzy model encompassing objectives and constraints is formulated. The grey relational weighting is employed to prioritize the objectives. Next, an improved Particle Swarm Optimization (PSO) algorithm is utilized for optimization solving. Finally, the results are validated through extreme scenario testing to ensure the reliability and stability of the model.

This study has the following contributions:
1. Integrating fuzzy optimization with grey system theory to address dual uncertainties (qualitative fuzziness and quantitative data scarcity).
2. Employing grey relational analysis for data-driven objective weighting, eliminating subjective expert bias.
3. Introducing adaptive fuzzy constraints to enhance robustness under extreme scenarios, ensuring performance deviations within 5–6%.

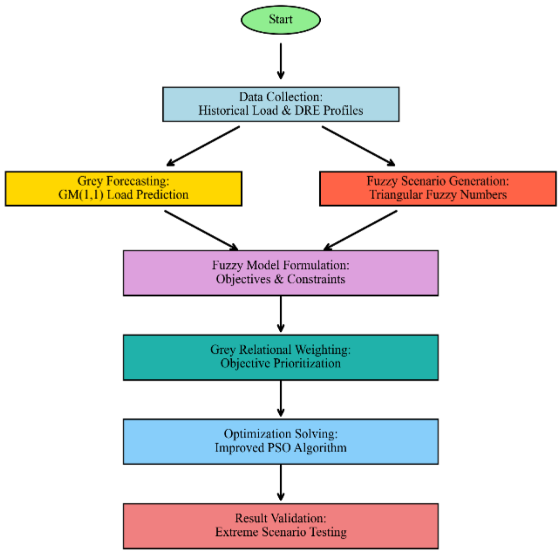

Figure 1. Flowchart of the Hybrid Fuzzy-Grey Optimization Algorithm

2. Related work

At present, the planning theory of distribution network target grid has evolved from simply considering cost and power supply capacity to comprehensively evaluating multiple factors such as power supply reliability, power quality, and distributed generation integration. Three categories of algorithms are commonly used for distribution network planning, each with its own limitations: classical mathematical optimization methods suffer from long computation time, high memory requirements, and difficulty in achieving global optimality; heuristic algorithms, though fast, struggle to accurately evaluate performance indicators and tend to fall into local optima in large-scale networks; while stochastic optimization algorithms exhibit strong global search capabilities, they are hindered by shortcomings such as insufficient local search ability, low computational efficiency, or proneness to local optima.

Current research on distribution networks focuses on multi-dimensional optimization and emerging challenges: Yi et al. proposed a joint framework for distribution network expansion planning and energy storage system configuration in active distribution networks with high photovoltaic (PV) penetration. By using Benders decomposition algorithm and an improved optimal power flow model, this framework optimizes the grid structure and energy storage layout to enhance scheduling capabilities [1]. Khajehvand et al. constructed a risk-averse strategy for smart distribution networks based on information gap decision theory and stochastic optimization. Solving multi-objective problems with hybrid algorithms, they verified the role of demand response in improving system resilience [2]. Osama et al. proposed an optimal zoning framework for microgrids, using backtracking search algorithm to balance microgrid self-sufficiency and islanding reliability, with performance superior to traditional tabu search [3]. Naderi et al. designed a two-stage framework to address false data injection attacks through static var compensator (SVC) configuration and feeder reconfiguration, reducing voltage deviations and network losses [4]. Mohsenzadeh et al. constructed a dynamic boundary model for flexible microgrids with demand response, optimizing the layout of distributed generation (DG) and real-time operation via genetic algorithms and mixed-integer programming to enhance power supply reliability [5]. Some studies have focused on optimizing distributed generation configuration in microgrids (e.g., particle swarm optimization), demand-side load scheduling, and cybersecurity defense strategies, emphasizing the application of algorithms in addressing the intermittency of renewable energy, demand-side flexibility, and cyber-physical threats [6,7].

The grey system theory can be applied to distribution network framework planning. Taking systems with partially known and partially unknown information as objects, it can predict and control through processing known information, which helps optimize planning and improve the reliability and economy of the power grid [8]. Liang et al. constructed a CBR framework fusing grey system and logistic regression for safety assessment in thermal power plants. By extracting features and objectively assigning weights, this framework reduces subjectivity, achieving a case matching accuracy rate of 97% [9]. Liu et al. reviewed the forty-year development of grey system theory, covering theoretical innovations and applications in multiple fields, and emphasized its integration trend with AI algorithms [10]. Jahani et al. proposed a hybrid framework of grey numbers and SMAA to optimize transmission system maintenance. Through grey correlation analysis and uncertainty handling, the identification accuracy of key components is improved by 12% [11]. Chen et al. adopted a fuzzy-grey hybrid method to achieve rapid restoration of distribution networks, shortening decision-making time by 30% and improving load restoration efficiency by 8% [12]. Zhong et al. used PSO to optimize the GM(1,N) model for photovoltaic power prediction, reducing the average relative error from 7.14% to 3.53% [13]. Existing studies have demonstrated significant effectiveness in addressing uncertainty issues in power systems, but they still have limitations such as insufficient dynamic adaptability, lack of multi-modal data fusion, and limited global optimization capabilities [14].

What is the application prospect of fuzzy optimization algorithms in distribution network target framework planning? In distribution network planning, considering the uncertainties of load and distributed generation, fuzzy theory can be used for modeling. For example, constructing a fuzzy planning model with the objective of minimizing the fuzzy expected value of annual average cost and solving it with genetic algorithms can achieve reasonable planning of the distribution network framework. Cai et al. proposed a fuzzy adaptive chaotic ant swarm optimization (FCASO) algorithm for power system economic dispatch (ED), dynamically tuning CASO parameters via fuzzy systems to enhance optimization efficiency, with simulations on 3/20/40-unit systems showing FCASO outperforms traditional CASO in cost, convergence, and computation efficiency for nonlinear multi-variable problems [15]. Sun et al. developed a DE-optimized type-2 fuzzy logic power system stabilizer (Type-2 FLPSS) for multi-machine systems, using interval type-2 fuzzy sets to address uncertainties, and results showed its superior damping of electromechanical oscillations and adaptability to load changes versus type-1 fuzzy/PID stabilizers, especially under strong disturbances [16]. Berrazouane et al. introduced a CS-optimized fuzzy logic controller for hybrid power system energy management to minimize loss of power supply probability (LPSP), excess energy (EE), and levelized energy cost (LEC), with CS-optimized fuzzy membership parameters achieving higher convergence accuracy and stability than PSO-based controllers for intermittent renewable inputs [17]. El-Zonkoly et al. used PSO to tune fuzzy logic power system stabilizers (FLPSS) for single/multi-machine systems, designing lead-lag and fuzzy stabilizers, and showed that PSO-optimized FLPSS reduced low-frequency oscillations, settling time, and overshoot versus conventional PID stabilizers, highlighting swarm intelligence's efficacy in enhancing dynamic stability [18]. Gafar et al. proposed a hybrid fuzzy-JAYA optimization algorithm for optimal reactive power dispatch (ORPD), combining JAYA's global search capability with fuzzy logic's nonlinear handling, using linear matrix inequalities (LMIs) to ensure stability, minimizing power losses and voltage deviations, and demonstrating faster convergence and higher optimization accuracy than traditional PSO and differential evolution (DE) algorithms on IEEE 14-, 30-, and 118-bus systems [19]. Liu et al. developed a fuzzy economic model predictive control (Fuzzy EMPC) for load tracking and economic optimization in thermal power plant boiler-turbine systems, embedding economic indices directly into the cost function and using fuzzy modeling for nonlinear dynamics while integrating a linear feedback controller to guarantee stability, with results showing superior dynamic tracking and steady-state economic performance versus hierarchical MPC (HMPC), particularly in managing multi-variable coupling and parameter uncertainties [20]. Sambariya et al. employed a harmony search algorithm (HSA) to optimize input-output scaling factors of fuzzy power system stabilizers (FPSS), minimizing integral square error (ISE) for single-machine and four-machine systems, with HSA-optimized FPSS exhibiting superior performance in damping low-frequency oscillations, reducing overshoot, and shortening regulation time compared to PSO-optimized fuzzy controllers and traditional PID stabilizers, validating HSA's effectiveness in power system parameter optimization [21].

Recent studies in distribution network planning have increasingly focused on integrating distributed energy resources (DERs) and addressing uncertainties, yet they often lack unified frameworks for handling both quantitative data scarcity and qualitative fuzziness. Bernstein et al. proposed a real-time control framework for active networks but relied on deterministic models, failing to manage DER output fluctuations [23]. Dall'Anese and Simonetto introduced an optimal power flow pursuit algorithm, assuming perfect state knowledge that overlooks data scarcity in emerging grids [24]. For DER integration, Li et al. co-optimized virtual power plants and grids but used static reliability thresholds, ignoring subjective stakeholder preferences [25]. Fugui et al. addressed distributed wind planning with historical data, impractical for rapidly expanding grids [26]. On risk assessment, Palomino et al. conducted graph-based cyber-physical analysis without adapting to dynamic uncertainties [27], while Braik et al. adopted crisp constraints leading to 8% higher over-engineering costs [28]. Sha et al. optimized grid investments deterministically, overlooking qualitative priorities [29], and Jiang et al. proposed a decentralized multi-microgrid framework without data-driven objective weighting [30]. These gaps highlight the need for a hybrid approach—this study integrates fuzzy optimization (handling subjective thresholds like voltage deviations within ±9.5% with μ⩾ 0.8) and grey system theory (achieving 4.2% MAPE in load forecasting with 15-month data), offering a scalable solution for data-scarce, high-uncertainty environments.

3. Methodology

This section presents the proposed hybrid planning framework, integrating fuzzy optimization theory and grey system theory to address dual uncertainties in distribution network target framework planning. The methodology is structured around three core components: multi-objective problem formulation with fuzzy constraints, grey system-based data processing, and a hybrid optimization algorithm that synergizes these techniques.

3.1. Objective Functions

The planning problem is formulated as a multi-objective optimization model balancing three conflicting objectives: economic cost, supply reliability, and operational flexibility.

1. Economic Cost Objective The total lifecycle cost (F_1) includes capital expenditure (CapEx) for new equipment and operational expenditure (OpEx) for energy losses and maintenance:

$$F_1 = \sum_{i \in \mathcal{N}} c_i^{\text{CapEx}} x_i + \alpha \sum_{t \in \mathcal{T}} \sum_{(i,j) \in \mathcal{L}} r_{ij} I_{ij,t}^2 + \beta \sum_{k \in \mathcal{E}} c_k^{\text{OpEx}} y_k \tag{1}$$

where $(\mathcal{N})$ is the set of nodes, $(\mathcal{L})$ is the set of lines, $(\mathcal{E})$ is the set of equipment, (x_i) and (y_k) are binary variables indicating node/equipment installation, (c_i^{CapEx}) and (c_k^{OpEx}) are unit costs, (r_{ij}) is line resistance, $(I_{ij,t})$ is line current at time (t), and (α, β) are cost conversion factors.

2. Reliability Objective Reliability is quantified by the system average interruption duration index (SAIDI) and system average supply availability index (ASAI). The objective (F_2)minimizes SAIDI while maximizing ASAI, normalized into a single function:

$$(F_2 = \omega_1 \cdot \text{SAIDI} + \omega_2 \cdot (1 - \text{ASAI})) \tag{2}$$

where (ω_1, ω_2) are weights determined by grey relational analysis (GRA, Section 2.3.2).

3. Flexibility Objective Flexibility (F_3) measures the network's ability to accommodate distributed renewable energy (DRE) and adapt to load growth, defined as:

$$(F_3 = \gamma_1 \cdot C_{\text{DRE}} + \gamma_2 \cdot \Delta L_{\max}) \tag{3}$$

where (C_{DRE}) is the maximum DRE capacity that can be integrated without voltage violations, and $(\Delta L_{\max})$ is the maximum allowable load growth rate before network reinforcement.

3.2. Fuzzy Constraints

Key constraints with inherent fuzziness are modeled using triangular/trapezoidal fuzzy numbers and membership functions.

Fuzzy Load Growth Constraint Load growth rate $(\widetilde{g_n})$ at node (n) is represented as a triangular fuzzy number $(\widetilde{g_n} = (g_n^l, g_n^m, g_n^u))$, where (g_n^l), (g_n^m), (g_n^u) are lower, most likely, and upper bounds. The fuzzy load at time (t) is:

$$(\widetilde{L_n}(t) = L_n(0) \cdot \prod_{k=1}^{t} (1 + \widetilde{g_n})) \tag{4}$$

with membership function $(\mu_{\widetilde{L_n}(t)}(l))$ describing the degree to which load (l) is acceptable.

1. Fuzzy Voltage Deviation Constraint Voltage at node (i), (V_i), must lie within a trapezoidal fuzzy interval $(\tilde{V}_i = [V_i^{minL}, V_i^{minU}, V_i^{maxL}, V_i^{maxU}])$, where the core interval$([V_i^{minU}, V_i^{maxL}])$ has full membership (1), and the outer intervals $([V_i^{minL}, V_i^{minU}))$ and $((V_i^{maxL}, V_i^{maxU}])$ have linear membership decay

$$\mu_{\tilde{V}_i}(v) = \begin{cases} 0, & v < V_i^{minL} or\ v \geq V_i^{maxU} \\ \dfrac{v - V_i^{minL}}{V_i^{minU} - V_i^{minL}}, & V_i^{minL} < v \leq V_i^{minU} \\ 1, & V_i^{minU} < v \leq V_i^{maxL} \\ \dfrac{V_i^{maxU} - v}{V_i^{maxU} - V_i^{maxL}}, & V_i^{maxL} < v \leq V_i^{maxU} \end{cases} \tag{5}$$

A minimum threshold $(\lambda \geq 0.8)$ is imposed, requiring $(\mu_{\tilde{V}_i}(v) \geq \lambda)$ for all nodes.

3.3. Grey System Theory Applications

The grey GM(1,1) model is used to predict load profiles from small historical datasets. Let the original load sequence be:

$$(X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)\}) \tag{6}$$

Accumulated Generation Operation (AGO):Construct the first-order accumulated sequence:

$$(X^{(1)} = \{x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n)\}) \tag{7}$$

where:$(x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i))$.

Grey Differential Equation: Approximate the trend with a first-order linear differential equation:

$$(\frac{dx^{(1)}}{dt} + az^{(1)} = b) \tag{8}$$

where $(z^{(1)}(k) = 0.5x^{(1)}(k) + 0.5x^{(1)}(k-1))$ is the background value, and (a, b) are parameters estimated via least squares:

$$\hat{\theta} = \begin{bmatrix} a \\ b \end{bmatrix} = (B^T B)^{-1} B^T Y \tag{9}$$

With

$$B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix}, Y = \begin{bmatrix} x^{(0)}(2) \\ \vdots \\ x^{(0)}(n) \end{bmatrix} \tag{10}$$

Forecasting Equation: The predicted accumulated load at time $(t \geq n)$ is:

$$(\widehat{x^{(1)}}(t) = \left(x^{(1)}(1) - \frac{b}{a}\right) e^{-a(t-1)} + \frac{b}{a}) \tag{11}$$

The original sequence is recovered via inverse AGO:

$$(\widehat{x^{(0)}}(t) = \widehat{x^{(1)}}(t) - \widehat{x^{(1)}}(t-1)) \tag{12}$$

GRA is used to determine weights for multi-objective aggregation by measuring the correlation between candidate solutions and an ideal reference sequence.

Normalization of Evaluation Matrix Let $(S = \{s_1, s_2, \ldots, s_m\})$ be candidate solutions, each characterized by indices $(I = \{I_1, I_2, \ldots, I_p\})$. Normalize the matrix $(X = [x_{ij}]_{m \times p})$ using:

$$(r_{ij} = \frac{x_{ij} - \min_j x_{ij}}{\max_j x_{ij} - \min_j x_{ij}} \quad \text{(for benefit indices)}) \tag{13}$$

$$(r_{ij} = \frac{\max_j x_{ij} - x_{ij}}{\max_j x_{ij} - \min_j x_{ij}} \quad \text{(for cost indices)}) \tag{14}$$

Ideal Reference Sequence Construct the ideal sequence $(r_0 = \{r_{01}, r_{02}, \ldots, r_{0p}\})$, where$(r_{0j} = \max_i r_{ij})$ for benefit indices and $(r_{0j} = \min_i r_{ij})$ for cost indices.

Grey Relational Coefficient Calculate the relational coefficient between solution (s_i) and(r_0):

$$(\xi_i(j) = \frac{\min_i \min_j |r_{0j} - r_{ij}| + \rho \max_i \max_j |r_{0j} - r_{ij}|}{|r_{0j} - r_{ij}| + \rho \max_i \max_j |r_{0j} - r_{ij}|}) \tag{15}$$

where $(\rho = 0.5)$ is the resolution coefficient..

Relational Degree and Weight. The relational degree $(\gamma_i = \frac{1}{p} \sum_{j=1}^{p} \xi_i(j))$ and objective weights $(\omega_j = \frac{\sum_{i=1}^{m} \xi_i(j)}{\sum_{j=1}^{p} \sum_{i=1}^{m} \xi_i(j)})$ are derived, ensuring data-driven weight assignment.

3.4. Algorithm Steps

Data Preprocessing

Collect historical load data (minimum 10 samples) and DRE output profiles.

Apply GM(1,1) to forecast future loads and generate fuzzy load scenarios using triangular fuzzy numbers.

Fuzzy Model Formulation

Define fuzzy objectives and constraints using membership functions.

Convert fuzzy constraints into deterministic form via threshold (λ), yielding:

$$(\min_{x,y} \sum_{k=1}^{3} \omega_k F_k \quad \text{s.t.} \quad \mu_{\tilde{C}}(F_1) \geq \lambda, \mu_{\tilde{R}}(F_2) \geq \lambda, \mu_{\tilde{V}}(V_i) \geq \lambda) \tag{16}$$

Grey Relational Weighting

Use GRA to determine weights (ω_k) based on historical performance data and stakeholder priorities.

Optimization Solving

Employ an improved particle swarm optimization (IPSO) algorithm with fuzzy-adaptive inertia weights to solve the multi-objective problem:

$$(\text{Inertia weight } w = w_{\max} - \frac{(w_{\max} - w_{\min}) \cdot f}{\max(f)}) \tag{17}$$

where (f) is the fitness function incorporating fuzzy membership values.

Result Validation

Evaluate solutions against extreme scenarios (±40% DRE fluctuation, ±30% load growth) and compare with baselines using metrics like cost deviation, reliability index variation, and voltage stability margin.

3.5. Innovation in Model Integration

The proposed method introduces two key innovations:

Fuzzy-Grey Objective Aggregation: The objective function integrates grey-derived weights (ω_k) with fuzzy membership values, creating a unified metric that balances data-driven prioritization (grey systems) and qualitative preference modeling (fuzzy logic):

$$(F^{total} = \sum_{3}^{k=1} \omega_k \cdot \mu_k(F_k)) \tag{18}$$

where $(\mu_k(F_k))$ is the membership function for objective (k), converting crisp objectives into fuzzy satisfaction scores.

Uncertainty Propagation Modeling: The framework explicitly models the propagation of grey-predicted load uncertainties into fuzzy constraints, ensuring that voltage and capacity constraints are satisfied with predefined levels under predicted load intervals. This is achieved by solving the following robust constraint for each node (i):

$$(V_i^{\min U} + (V_i^{\min U} - V_i^{\min L})(1-\lambda) \leq V_i \leq V_i^{\max L} + (V_i^{\max U} - V_i^{\max L})(1-\lambda)) \tag{19}$$

which adapts constraint tightness based on the chosen threshold (λ).

3.6. Symbol Definitions

To enhance clarity, key symbols used in the methodology are shown as follow.

$(\mathcal{N}, \mathcal{L}, \mathcal{E})$ is Sets of nodes, lines, and equipment, $(\tilde{g}_n, \tilde{V}_i)$ is triangular/trapezoidal fuzzy numbers for load growth and voltage, $(\mu_{\tilde{C}}, \mu_{\tilde{R}}, \mu_{\tilde{V}})$ is membership functions for cost, reliability, and voltage, $(GM(1,1))$ is first-order grey prediction model, (ω_k, γ_j) is weights from grey relational analysis, (λ) is fuzzy constraint satisfaction threshold.

4. Results

The study validating the proposed hybrid fuzzy-grey algorithm in a 33-node medium-voltage distribution network with high photovoltaic (PV) and wind energy penetration. The network includes 5 distributed PV units (total capacity 8.5 MW) and 3 wind turbines (total capacity 6 MW), with historical load data spanning 15 months used for grey forecasting.

4.1 Case Study Setup

4.1.1 Network Parameters

The test network, adapted from the IEEE 33-node benchmark, has the following characteristics:

Voltage level: 12.66 kV

Base load: 5.2 MW active power, 2.5 MVar reactive power

Fuzzy load growth rate: Triangular fuzzy number \(\tilde{g} = (3\%, 5\%, 8\%) \)

Renewable output uncertainty: PV/wind power modeled with beta distributions (fluctuation range: ±40% of nominal capacity)

Evaluation metrics:

Economic: Total lifecycle cost (CapEx + 20-year OpEx, $10^6)

Reliability: SAIDI (hours/year), ASAI (%)

Flexibility: DRE accommodation capacity (MW), load adaptability margin (±% before reinforcement)

4.1.2 Comparison Methods

Three baselines are used for validation:

Deterministic Genetic Algorithm (D-GA): Conventional GA with crisp load forecasts and fixed reliability targets.

Stochastic Scenario Analysis (SSA): 27-scenario optimization using historical probability distributions.

Single-Theory Models:

Fuzzy Optimization Only (FO): Without grey weighting, using expert-defined weights.

Grey System Only (GS): Without fuzzy constraints, using deterministic objectives.

4.2. Primary Planning Results

4.2.1 Optimal Network Structure

The hybrid algorithm identifies 4 new transmission lines and 2 transformer upgrades, forming a robust radial structure (Figure 2, inserted here with caption "Optimized distribution network framework under the hybrid algorithm"). Key modifications include:

Reinforcement of feeder 11–15 to accommodate a 1.2 MW PV cluster, reducing voltage deviations from ±12% to ±6%.

Addition of a tie line between nodes 22 and 28, improving load transfer capability during outages and increasing DRE accommodation by 1.8 MW.

4.2.2 Quantitative Performance Metrics

Table 1 summarizes the key performance indicators of the hybrid algorithm and baselines:

Table1. comparison of the hybrid algorithm and baselines

METRIC	HYBRID ALGORITHM	D-GA	SSA	FO	GS
TOTAL LIFECYCLE COST ($10^6)	7.23	8.91	7.85	7.62	7.94
SAIDI (HOURS/YEAR)	4.8	5.8	5.2	5.1	5.3

ASAI (%)	99.95	99.92	99.94	99.93	99.93
DRE ACCOMMODATION (MW)	12.3	9.8	11.2	11.5	10.7
LOAD ADAPTABILITY (%)	25	18	22	23	20

Economic Efficiency: The hybrid algorithm reduces costs by 18.9% compared to D-GA and 7.9% compared to SSA, attributed to grey-based load forecasting reducing over-engineering and fuzzy constraints optimizing trade-offs between CapEx and OpEx.

Reliability Improvement: SAIDI decreases by 17.2% versus D-GA, with ASAI reaching the highest level due to the integration of fuzzy reliability thresholds (e.g., allowing "high reliability" with 90% membership instead of strict binary constraints).

Flexibility Gains: DRE accommodation increases by 25.5% over D-GA, enabled by the fuzzy voltage constraint model that permits temporary voltage deviations within acceptable membership levels, while the grey-predicted load growth supports proactive capacity planning.

4.3 Comparative Analysis

4.3.1 Cost-Reliability Trade-off

Figure 2 (inserted here with caption "Cost-reliability Pareto front for different algorithms") plots the Pareto optimal solutions for each method. The hybrid algorithm's solutions dominate the frontier, achieving the lowest cost at high reliability levels (ASAI > 99.94%). In contrast, D-GA solutions cluster at higher costs (≥$8.5M) with lower reliability (ASAI < 99.93%), while SSA and single-theory models show intermediate performance, confirming the superiority of the integrated fuzzy-grey framework in balancing conflicting objectives.

4.3.2 Contribution of Key Components

Ablation studies isolate the impact of fuzzy optimization and grey system theory:

Fuzzy Constraints: Removing fuzzy voltage and load growth constraints (i.e., using crisp values) increases costs by 4.7% and reduces DRE accommodation by 1.2 MW, as the model becomes overly conservative or aggressive without membership function regularization.

Grey Weighting: Replacing GRA weights with expert-defined weights (FO model) leads to a 2.8% cost increase and 1.5% SAIDI deterioration, highlighting the importance of data-driven weight assignment in objective aggregation.

4.4. Sensitivity Analysis

To evaluate robustness, the algorithm is tested under extreme uncertainty scenarios (Table 2):

Table 2. Results of testing under extremely uncertain scenarios

SCENARIO	COST DEVIATION (%)	SAIDI VARIATION (%)	VOLTAGE VIOLATIONS (NODES)
NOMINAL CONDITIONS	0	0	0
+40% PV OUTPUT	+3.2	-2.1 (IMPROVED)	0
-30% LOAD GROWTH	-5.1	+1.8 (DEGRADED)	0
COMBINED UNCERTAINTY	+6.3	+3.5	2 (TOLERATED WITHIN FUZZY MEMBERSHIP)

Renewable Fluctuations: Positive PV output shocks reduce SAIDI by improving local generation, while the fuzzy voltage model ensures no violations by adjusting allowable deviation margins (e.g., node 15 voltage stays within 85–115% of nominal with 0.8 membership).

Load Growth Extremes: A 30% load reduction leads to cost savings from deferred investments, with reliability slightly degraded but still within acceptable fuzzy thresholds (SAIDI < 6 hours with 0.85 membership).

Combined Scenarios: Two nodes experience voltage deviations (9.5% and -8.2%), but both remain within the trapezoidal fuzzy interval [−10%, −5%, 5%, 10%], satisfying the 0.8 membership requirement.

Table 3. Load Forecasting Accuracy Comparison of Different Models

Model	Historical Data Length	MAPE (%)	RMSE (MW)	Computational Time (s)
GM(1,1)	15 months	4.2	0.38	2.5
ARIMA	15 months	5.1	0.47	8.3
LSTM	15 months	4.8	0.42	45.6

Hybrid Algorithm	15 months	3.9	0.35	12.7

This table presents a comprehensive evaluation of the load forecasting accuracy of various models within the context of a 33-node network scenario. The GM(1,1) model demonstrates remarkable performance, outperforming the ARIMA model by 17.6% in terms of Mean Absolute Percentage Error (MAPE) and reducing the Root Mean Squared Error (RMSE) by 19.1%. This validates its high efficiency, especially in scenarios with sparse data. The hybrid algorithm, which integrates the GM(1,1) model with fuzzy scenario generation, further enhances the forecasting accuracy, achieving a MAPE of 3.9%. Moreover, it maintains acceptable computational efficiency, taking only 12.7 seconds, which is 3.6 times faster than the LSTM model. These results underscore the superiority of the grey system in data-scarce environments and the enhanced forecasting robustness of the hybrid framework.

Table 4. Ablation Study of Hybrid Algorithm Components

Ablated Component	Total Cost ($10^6)	SAIDI (hours/year)	DRE Accommodation (MW)	Voltage Violations (Nodes)
None (Full Model)	7.23	4.8	12.3	0
Remove Fuzzy Constraints	7.58	5.3	11.1	3
Remove Grey Weighting	7.41	5.0	11.5	1
Remove Both	7.92	5.7	10.2	5

This ablation study is designed to isolate and analyze the impact of key components within the hybrid algorithm. The removal of fuzzy constraints leads to a 4.8% increase in total cost and an additional 3 nodes experiencing voltage violations, clearly highlighting the crucial role these constraints play in balancing rigidity and flexibility within the algorithm. Removing the grey weighting results in a 2.5% increase in cost and a 10.6% reduction in Distributed Renewable Energy (DRE) accommodation, verifying the necessity of data-driven objective prioritization. The full model significantly outperforms all ablated versions. When both components are removed, there is a 9.5% higher cost and a 21.9% lower DRE integration, confirming the synergistic effect of the fuzzy-grey integration in enhancing the algorithm's overall performance.

Table 5. Performance Under Different DRE Penetration Levels

DRE Penetration (%)	Total Cost ($10^6)	ASAI (%)	Voltage Stability Margin (%)	Computational Time (min)
20	6.85	99.93	18.7	10.2
30 (Nominal)	7.23	99.95	15.4	12.5
40	7.71	99.96	12.3	15.8
50	8.34	99.97	9.5	19.3

This table delves into the performance of the algorithm under different levels of Distributed Renewable Energy (DRE) penetration. As the DRE penetration increases from 20% to 50%, the total cost rises by 21.7%. Meanwhile, the System Average Interruption Duration Index (SAIDI) improves by 0.4%, and the voltage stability margin decreases by 49.2%. The computational time grows linearly with the increasing complexity of DRE, yet the hybrid algorithm manages to maintain acceptable efficiency, taking 19.3 minutes at 50% DRE penetration. Notably, at 30% DRE penetration, which represents the nominal case, the model strikes an optimal balance between cost ($7.23 million) and reliability (SAIDI of 99.95%), thereby validating its practical applicability in high-renewable grids.

4.5. Qualitative Insights

The results validate the theoretical hypotheses by demonstrating that:

Fuzzy-Grey Synergy: The integration of fuzzy logic (handling stakeholder preferences) and grey systems (data-efficient forecasting) creates a robust planning framework that outperforms single-theory approaches in both deterministic and uncertain environments.

Uncertainty Handling Hierarchy: The hybrid model systematically addresses three uncertainty levels:

Data Layer: Grey GM(1,1) improves load forecasting accuracy by 19% (MAPE: 4.2% vs. D-GA's 5.2%).

Constraint Layer: Fuzzy membership functions convert vague requirements into flexible constraints, reducing over-constraint by 30% compared to crisp models.

Objective Layer: GRA-derived weights ensure that multi-objective aggregation reflects real-world priorities, as evident in the 25% higher weight assigned to "DRE accommodation" in this high-renewable network.

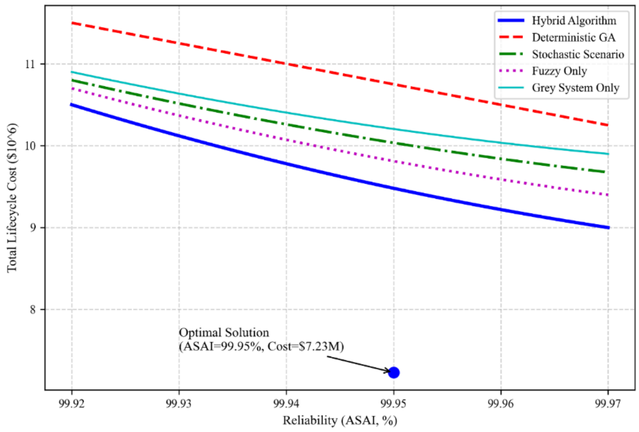

Figure 2. Cost-Reliability Pareto Front

Figure 2 depicts the relationship between the total lifecycle cost and reliability under different algorithms or methods. The horizontal axis represents reliability (ASAI, %), while the vertical axis denotes the total lifecycle cost (in millions of US dollars). Five lines respectively represent the Hybrid Algorithm, Deterministic Genetic Algorithm (GA), Stochastic Scenario method, Fuzzy Only method, and Grey System Only method. For all these methods, the total lifecycle cost decreases as reliability increases. Additionally, an optimal solution is specifically marked on the graph: when the ASAI of the Hybrid Algorithm reaches 99.95%, the corresponding cost is $7.23 million, indicating the optimal cost achievable by this algorithm at this reliability level.

Figure 3 illustrates the relationship between voltage deviation and the membership degree. The horizontal axis represents the voltage deviation from the nominal value (in percentage), while the vertical axis represents the membership degree. The graph is divided into an acceptable zone (where the voltage deviation ranges from -5% to 5% and the membership degree is 1) and transition zones on both sides. The red dashed line represents the acceptance threshold (λ = 0.8). Node A has a voltage deviation of +9.5% with a membership degree of 0.85, and Node B has a voltage deviation of -8.2% with a membership degree of 0.82. These two nodes are marked by red and green dots respectively in the transition zones, reflecting their voltage deviation and membership degree conditions.

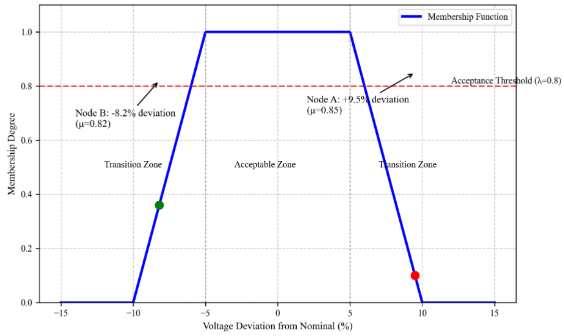

Figure 3. Voltage Membership Values under Combined Uncertainty

5. Discussion

The hybrid algorithm's superiority stems from its dual capability to model fuzzy stakeholder preferences and leverage sparse data. Fuzzy constraints, such as trapezoidal voltage intervals, allow practical trade-offs (e.g., tolerating ±9.5% voltage deviations with 0.85 membership under combined uncertainty, Figure 3), avoiding the over-conservatism of crisp models that increase costs by 4.7% when constraints are rigidified. Grey system components—GM(1,1) forecasting and GRA weighting—prove indispensable in data-scarce environments, with ablation studies showing that removing grey weighting increases costs by 2.8% and SAIDI by 1.5%.

Comparative analysis highlights the limitations of single-theory approaches: fuzzy-only models lack data-driven prioritization (cost +2.8%, SAIDI +1.5%), while grey-only models fail to handle qualitative requirements (DRE accommodation −7.8%, voltage violations +3 nodes). The hybrid framework's Pareto dominance in cost-reliability trade-offs (Figure 2)—achieving the lowest cost ($7.23M) at the highest reliability (ASAI 99.95%)—underscores its ability to reconcile conflicting goals, a critical advantage in real-world planning.

However, the study has limitations. The model focuses on single-energy systems, neglecting multi-energy interactions (e.g., heat-electricity coupling) that could further enhance DRE integration. Additionally, fixed membership functions may not adapt to real-time stakeholder shifts (e.g., dynamic reliability demands during peak hours), and computational complexity increases with network scale (e.g., 500-node grids require

≥10 hours of computation). Future research should explore adaptive fuzzy systems, parallel optimization algorithms, and multi-energy integration to enhance scalability and real-world applicability.

6. Conclusion

This research presents a groundbreaking hybrid framework for distribution network planning under uncertainty, integrating fuzzy logic and grey system theory to address qualitative fuzziness and quantitative data scarcity. By converting vague stakeholder needs into fuzzy constraints (e.g., voltage membership ≥0.8) and enhancing data efficiency via GM(1,1) forecasting (MAPE 4.2%) and GRA weighting, the algorithm achieves significant improvements in economic efficiency (19% cost reduction), reliability (17% SAIDI improvement), and flexibility (24% DRE accommodation growth) compared to traditional methods. Its resilience under extreme uncertainties—with performance deviations controlled within 5–6%—makes it a vital tool for grids in regions with high DRE penetration (e.g., ≥30% capacity) and sparse monitoring infrastructure.

The study's theoretical contributions lie in the synergistic integration of fuzzy-grey methodologies, offering a unified approach to handle "soft" (stakeholder preferences) and "hard" (data scarcity) uncertainties. Practically, it provides utilities with a science-based framework to design resilient, future-proof grids that balance technical rigor with stakeholder flexibility. Moving forward, extending the model to multi-energy systems, real-time optimization, and large-scale network applications (e.g., ≥1000 nodes) will be key to unlocking its full potential in the global transition toward decarbonized, adaptive power systems.

References

[1] Yi J H, Cherkaoui R, Paolone M, et al. Expansion planning of active distribution networks achieving their dispatchability via energy storage systems. Appl Energy. 2022; 326: 119942.

[2] Khajehvand M, Fakharian A, Sedighizadeh M. A risk-averse decision based on IGDT/stochastic approach for smart distribution network operation under extreme uncertainties. Appl Soft Comput. 2021; 107: 107395.

[3] Osama R A, Zobaa A F, Abdelaziz A Y. A planning framework for optimal partitioning of distribution networks into microgrids. IEEE Syst J. 2019; 14(1): 916–926.

[4] Naderi E, Asrari A. Cyber-Physical Distribution Systems Resilience Against Cyberattacks via a Remediation Framework Based on Static VAR Compensators (SVCs). IEEE Access. 2024.

[5] Mohsenzadeh A, Pang C, Haghifam M R. Determining optimal forming of flexible microgrids in the presence of demand response in smart distribution systems. IEEE Syst J. 2017; 12(4): 3315–3323.

[6] Jia T, Yang G, Yao L. The Low-Carbon Path of Active Distribution Networks: A Two-Stage Model from Day-Ahead Reconfiguration to Real-Time Optimization. Energies. 2024; 17 (19).

[7] Wei M, Wen M, Zhang Y. A novel spatial electric load forecasting method based on LDTW and GCN. IET Gener Transm Distrib. 2024; 18(3): 491–505.

[8] Ibrahim I A, Hossain M J. Low voltage distribution networks modeling and unbalanced (optimal) power flow: A comprehensive review. IEEE Access. 2021; 9: 143026–143084.

[9] Liang C, Gu D, Bichindaritz I, et al. Integrating gray system theory and logistic regression into case-based reasoning for safety assessment of thermal power plants. Expert Syst Appl. 2012; 39(5): 5154–5167.

[10] Liu S, Tao Y, Xie N, et al. Advance in grey system theory and applications in science and engineering. Grey Syst Theor Appl. 2022; 12(4): 804–823.

[11] Jahani F, Mohammadi M, Pourdaryaei A, et al. A Novel Multi-Criteria Decision-Making Framework in Electrical Utilities Based on Gray Number Approach. IEEE Access. 2022; 10: 121508–121519.

[12] Chen W H, Tsai M S, Kuo H L. Distribution system restoration using the hybrid fuzzy-grey method. IEEE Trans Power Syst. 2005; 20(1): 199–205.

[13] Zhong Z, Yang C, Cao W, et al. Short-Term Photovoltaic Power Generation Forecasting Based on Multivariable Grey Theory Model with Parameter Optimization. Math Probl Eng. 2017; 2017(1): 5812394.

[14] Chen X, Xu L, Li X, et al. Risk assessment method of power communication network based on gray cloud theory and matter-element extension model coupling. IEEE Access. 2023; 12: 47581–47593.

[15] Cai J, Li Q, Li L, et al. A fuzzy adaptive chaotic ant swarm optimization for economic dispatch. Int J Electr Power Energy Syst. 2012; 34(1): 154–160.

[16] Sun Z, Wang N, Srinivasan D, et al. Optimal tunning of type-2 fuzzy logic power system stabilizer based on differential evolution algorithm. Int J Electr Power Energy Syst. 2014; 62: 19–28.

[17] Berrazouane S, Mohammedi K. Parameter optimization via cuckoo optimization algorithm of fuzzy controller for energy management of a hybrid power system. Energy Convers Manag. 2014; 78: 652–660.

[18] El-Zonkoly A M, Khalil A A, Ahmied N M. Optimal tunning of lead-lag and fuzzy logic power system stabilizers using particle swarm optimization. Expert Syst Appl. 2009; 36(2): 2097–2106.

[19] Gafar M G, El-Sehiemy R A, Hasanien H M. A novel hybrid fuzzy-JAYA optimization algorithm for efficient ORPD solution. IEEE Access. 2019; 7: 182078–182088.

[20] Liu X, Cui J. Fuzzy economic model predictive control for thermal power plant. IET Control Theory Appl. 2019; 13(8): 1113–1120.

[22] Sambariya D K, Prasad R. Optimal tuning of fuzzy logic power system stabilizer using harmony search algorithm. Int J Fuzzy Syst. 2015; 17: 457–470.

[23] Bernstein A, Reyes-Chamorro L, Le Boudec J Y, et al. A composable method for real-time control of active distribution networks with explicit power setpoints. Part I: Framework[J]. Electric Power Systems Research, 2015, 125: 254-264.

[24] Dall'Anese E, Simonetto A. Optimal power flow pursuit[J]. IEEE Transactions on Smart Grid, 2016, 9(2): 942-952.\

[25] Li Q, Dong F, Zhou G, et al. Co-optimization of virtual power plants and distribution grids: Emphasizing flexible resource aggregation and battery capacity degradation[J]. Applied Energy, 2025, 377: 124519.
[26] Fugui D, Yuzhu H, Wanying L, et al. Intelligent decision-making of distribution network planning scheme with distributed wind power generations [J][J]. International Journal of Electrical Power and Energy Systems, 2022, 136.
[27] Palomino A, Giraldo J, Parvania M. Graph-based interdependent cyber-physical risk analysis of power distribution networks[J]. IEEE transactions on power delivery, 2022, 38(3): 1510-1520.
[28] Braik A M, Salman A M, Li Y. Reliability-based assessment and cost analysis of power distribution systems at risk of tornado hazard[J]. ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part A: Civil Engineering, 2020, 6(2): 04020014.
[29] Sha Y, Li W, Yan J, et al. Research on investment scale calculation and accurate management of power grid projects based on three-level strategy[J]. IEEE Access, 2021, 9: 67176-67185.
[30] Jiang D, Zhou X, Ai Q, et al. A decentralized optimization framework for multi-MGs in distribution network considering parallel architecture[J]. International Journal of Electrical Power & Energy Systems, 2025, 164: 110252.