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Liapunov Exponents and Control Theory-Based Stability Analysis and Parameter Optimization Technique for Dynamical Systems with Periodic Variable Coefficients

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Abstract

INTRODUCTION: This is the introductory text Dynamical systems with periodic variable coefficients find extensive use in physics and engineering domains, including nonlinear circuits, structural dynamics, and vibration control. The intricacy of the parameter changes over time has made the stability and control issues of such systems a popular topic in engineering and academics. The dynamic features of periodic variable coefficient dynamical systems are difficult to adequately characterize using approaches based on standard stability theory, as demonstrated by previous research. This work formulates a methodology that integrates Lyapunov exponents, Floquet theory, and analysis of fractional-order systems to evaluate the stability of variable-coefficient periodic systems. The effectiveness of the approach is illustrated in nonlinear control and bifurcation analysis.
OBJECTIVES: This is particularly true when the parameters vary significantly or the system behaves nonlinearly. As crucial instruments for examining dynamical systems, Lyapunov stability theory and Floquet theory are essential for determining global stability and studying periodic systems, respectively. Further research is still required to determine the best way to integrate the two theories in order to provide straightforward and useful conclusions for periodic systems with variable coefficients.
METHODS: In the meantime, fractional-order systems have drawn interest recently in the fields of control and chaotic dynamics due to their precise representation of genetic characteristics and memory effects. It has been demonstrated that under parameter changes, fractional-order chaotic systems display a variety of dynamic behaviors, such as multistability, bifurcation phenomena, and complexity shifts. By logically creating control techniques, such systems can be optimized to increase their robustness and stability while also exposing the complex system's dynamical principles.
RESULTS: In light of this, this research suggests an integrated framework for a thorough investigation of the stability of dynamical systems with periodic variable coefficients that combines the Lyapunov eigenindex, Lyapunov function, and fractional order complexity analysis. In particular, this work first builds a stability criterion to theoretically support periodic systems using Floquet theory with Lyapunov exponents; The practical issues of modular multilevel dc voltage regulators (MMC-DVR) are then addressed by a nonlinear control method based on Lyapunov functions. Additionally, bifurcation diagrams with complexity index (such as spectral entropy complexity) are used for fractional-order chaotic systems in order to examine the impact of parameter changes on system stability and chaotic behavior. Lastly, numerical examples are used to confirm the efficacy of the suggested approach.
CONCLUSION: It is demonstrated that the analytical framework put forth in this research may successfully address challenging issues in the control and stability design of dynamical systems with periodic variable coefficients while also offering fresh concepts for the optimization of intricate system parameters.

Keywords: Lyapunov characteristic exponent; Lyapunov function; Floquet theory; Periodic variable coefficient dynamical systems; MMC-DVR; Fractional order chaotic systems; Complexity analysis; Stability criterion

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1. Introduction

Stability in complex dynamical systems, stability theoretical exploration is a foundation on analysis and practical uses, which have a direct impact on the reliable real world system functioning. Lyapunov stability theory which is well known in its utility offers a good sounding. basis on stability assessment of the systems and design control strategies. In the present paper, dynamical is given attention periodic coefficient ordinary systems differential equations, based on Lyapunov characteristic exponents and Lyapunov function-based control methods. Also, complexity and bifurcation diagrams analyses of chaotic systems of fractional orders are combined to determine the effect of parameter changes on system performance[1,2].

Periodic coefficient dynamical systems The stability of periodic coefficient dynamical systems is one of the difficult elements of classical control theory. Floquet theory has a theoretical foundation in studying relationship between the such systems by determining the relation between the periodicity of matrices of fundamental solutions and system characteristic exponents. Nonetheless, Flochet theory on its own frequently does not match international behaviour of the system and the subtle changes in response to parameter changes. Quantitative characteristics, Lyapunov characteristic exponents measurements of dynamical properties, provide a powerful instrument of assessing stability. In the event that all the exponents are negative, the system is asymptotically stable, there is positive adds exponentially, which means lack of stability. This study extends the use of Lyapunov exponents to prove a. comprehensive criterion of analyzing the stability of periodic coefficient systems [3-5]. Lyapunov exponents explain the exponential rate of separation or merging of solutions to systems, and give data pertaining to stability. Coupling these exponents with Floquet theory enhances stability analysis of periodic systems linking that periodicity of solutions with the eigenvalues of the system and providing a more detailed stability criterion. Nonlinear control has used Lyapunov functions to the effect has attracted much concern because of its efficiency in controlling systems which have large parameter differences.

Building a suitable energy functional that monotonically decreases with time, Lyapunov-based system stability is provided through methods. This paper applies such a to the control design of modular multilevel method direct voltage regulation converters (MMC-DVR). The system is carefully chosen by choosing control gain parameters ensured to be stable on a global scale particularly in the long run. In order to deal with the issue of parameter uncertainty, optimization strategies of control are also addressed in the study selection of gains, to have a strong stability in changing conditions. The Lyapunov stability theory, used together with Lyapunov stability theory, gives the Lyapunov stability theory, which is an important theory in mathematics and biology and Floquet theory is a far-reaching method of study on consistency of periodic variable global stability and also from which come coefficient systems more fined parameter effects time-dependent can be investigated. Systems of fractional-order chaos are selected because of their capability to model systems having memory and hereditary influences, which are essential in efficient functioning simulating complex dynamical systems which are described as being multistable and bifurcating. Dynamical systems that are chaotic in nature, i.e. sensitive to represent another important thing about parameters nonlinear systems. Complexity bifurcation diagrams analyses give visual and quantitative information regarding the slip into chaos of a system. Using a this as is the case in a fractional-order chaotic system. studies how individual important parameters affect complexity metrics, e.g. bifurcation behavior,spectral entropy (SE). The SE complexity integration and bifurcation analysis underscores a new way of doing considering the chaotic features and their connection to Lyapunov exponents. Kannan Srinivasan (2020) presents a neural network Bayesian model prediction to optimize the management of resources under uncertain conditions. This method is suggested in the proposed work. can be chosen in order to maximize Lyapunov control gains and tuning of dynamical systems of periodic type coefficients, adaptive learning and prediction techniques. This combination increases the strength of systems, stability analysis accuracy, and performance management by adequately managing uncertainty and dynamic system behavior [6]. Ma et al. (2019) explores fractional-order memristive circuit chaotic behavior of fractional-order memristive circuits, with the focus on the phenomenon of multistability and bifurcation. In the suggested study, the fractional-order system analysis this study has been combined with Lyapunov exponents to form optimize stability and control in control theory periodically coated dynamical systems. This method better resolves stability analysis, and optimizing parameters through modeling of complex and real world memory effects and hereditary dynamics [7]. The primary value of this paper is the creation of a common analytical system on the basis of Lyapunov exponents and stability assessment control theory and optimization of periodic coefficient dynamical parameter optimization of periodic coefficient dynamical systems. The soundness of the suggested method is also greater evidenced by closure to numbers in fractional-order chaotic systems. This study not only broadens the application scope of Lyapunov stability theory in complex dynamical systems but also provides theoretical and practical guidance for controller design and parameter optimization.

2. Stability criterion for Lyapunov's index

The Lyapunov exponent, a crucial instrument for assessing the stability of dynamical systems with ordinary differential equations containing periodic variable coefficients, can be defined and obtained using the following formula:

Differential Equations in Ordinary Forms with Periodic Variable Coefficients Examine a dynamical system with periodic variable coefficients of the following form:

$$\dot{x}(t) = A(t)x(t) \quad (1)$$

where A(t) is a matrix function with period T and x(t) is the state vector.

Let the initial conditions of the system be:

$$x(0) = x_0, x_0 \neq 0 \, (2)$$

The basis solution matrix $\Phi(t)$ of the system satisfies the following matrix differential equation:

$$\dot{\Phi}(t) = A(t)\Phi(t), \Phi(0) = I \, (3)$$

where I is the unit matrix.

Definition of Lyapunov exponent

The Lyapunov exponent is used to characterize the exponential growth rate of the system solution, which is defined as:

$$\lambda_i = \lim_{t\to\infty} \frac{1}{t} \ln \sigma_i(\Phi(t)) \, (4)$$

where $\sigma_i(\Phi(t))$ denotes the i-th singular value of the basis solution matrix $\Phi(t)$.

Using Floquet theory, the behavior of the basis solution matrix in a cycle T is related to the eigenvalues (i.e., Floquet multipliers) of the single-cycle solution matrix $\Phi(t)$. The eigenindex can be expressed as:

$$\mu_i = \frac{\ln|\rho_i|}{T} \, (5)$$

where ρ_i is the eigenvalue of the single-cycle solution matrix $\Phi(t)$.

According to the definition of Lyapunov eigenindex and the theory of point mapping system, assuming that $\lambda_i\left(K_j\right)$ is the ith eigenvalue ($i = 1, 2, \ldots, n$) of matrix K_j, the n Lyapunov eigenindex corresponding to the point mapping can be expressed as:

$$\Lambda_i = \lim_{m\to\infty} \frac{1}{m} \sum_{j=1}^{m} \ln\left|\lambda_i\left(K_j\right)\right|, \quad i = 1, 2, \ldots, n, \, (6)$$

where :

- K_j is the Jacobi matrix or update matrix of the point mapping at step j.
- $\lambda_i\left(K_j\right)$ is the i-th eigenvalue of K_j and represents the linear growth rate of the system along the i-th eigendirection in the j-th step.
- $\ln\left|\lambda_i\left(K_j\right)\right|$ denotes the growth factor of the logarithmic spectrum.

The solution matrix's singular values are an important factor in the calculation of the Lyapunov exponent because they provide exponential divergence or convergence of nearby orbits and thus determine the stability of the system.

3. Lyapunov function-based control strategy and parameter selection

3.1 Proof of stability

The state variable x of MMC-DVR can be described as a state vector that describes the dynamic behavior of the system in this work based on the use of the Lyapunov control approach. The state variables for a modular multilevel DC voltage regulator, or MMC-DVR, typically comprise voltage, current, and other crucial factors that impact system performance. The state variable x can be represented as a vector that includes voltage and current data for every submodule, assuming that the system has several levels and states. The following is one potential formula for defining the state variable:

$$x(t)=\begin{bmatrix} v_1(t) \\ v_2(t) \\ \vdots \\ v_n(t) \\ i_1(t) \\ i_2(t) \\ \vdots \\ i_n(t) \end{bmatrix} \quad (7)$$

The Lyapunov mathematical model of the MMC-DVR may be derived using the Lyapunov control method and the system's voltage and current relationship, assuming that the DC bus-side resistance is R_{dc}. The following Lyapunov mathematical model is derived from Eq. (7) and the system's dynamic properties:

$$\dot{x}(t)=Ax(t)+Bu(t) \quad (8)$$

where：

- $\dot{x}(t)$ is the time derivative of the state of the system (rate of change of state)；
- $x(t)$ is the state vector of the system (e.g., voltage and current)；
- A is the state matrix of the system, representing the intrinsic dynamics of the system ；
- B is the input matrix representing the effect of control inputs on the system；
- $u(t)$ is a control quantity that is designed using the Lyapunov control method as the control input.

If the DC bus-side resistance R_{dc} is taken into account, its effect in the system will be reflected in the entries of the state matrix A, usually affecting the dynamics of voltage and current. The most significant features of the MMC-DVR system, including voltage, current, and resistance dynamics, are critical in defining the control gains of the Lyapunov-based methodology to ensure robustness and stability under various operating conditions.

Assume that the positive-order Lyapunov function of the MMC-DVR is V(x)V(x)V(x), which is usually designed to be the energy function form of the system with the positive-definite property, i.e., $V(x)$>0 and $V(x)$= 0. For the MMC-DVR system, the commonly used Lyapunov function form can be chosen as follows:

$$V(x)=\frac{1}{2}x^T Px \quad (9)$$

where：

- x is the state vector of the system, usually including variables for voltage and current；
- P is a symmetric positive definite matrix that is used to guarantee the positive definiteness of the Lyapunov function in the system state space and to represent the weights of the system energy.

To guarantee the global asymptotic stability of the system, the time derivative of the Lyapunov function (i.e., $\dot{V}(x)$) for the stability analysis based on the Lyapunov control method must meet the negative definite condition ($\dot{V}(x)<0$). The derivative of the Lyapunov function can be further expressed as follows in accordance with the dynamic characteristics of MMC-DVR:

$$\dot{V}(x)=x^T\left(A^T P+PA\right)x+2x^T Pu \quad (10)$$

By choosing the matrix P and the control input u appropriately, it is possible to ensure that $\dot{V}(x)$ is negatively determined, thus realizing the stability of the system and the control objective.

For equation (10), assume we have a Lyapunov function $V(x)=\frac{1}{2}x^T Px$, and we need to differentiate it to obtain the time derivative of the Lyapunov function $\dot{V}(x)$.

According to the definition of the Lyapunov function, the derivative is:

$$\dot{V}(x)=\frac{d}{dt}\left(\frac{1}{2}x^T Px\right)=x^T P\dot{x} \quad (11),$$

where:

- $\dot{x}$ represents the system's state derivative, reflecting the system's dynamic behavior.

By substituting the state equation of the MMC-DVR system, we can express it as:

$$\dot{V}(x)=x^T P(Ax+Bu) \quad (12)$$

where:

- A is the system's state matrix;
- B is the control matrix;
- u is the control input.

Simplifying further, we get:

$$\dot{V}(x)=x^T PAx+x^T PBu \quad (13)$$

If a suitable control input u is designed and the matrices P and A satisfy appropriate conditions, we can ensure that $\dot{V}(x)<0$, thus satisfying the Lyapunov stability conditions and ensuring the system's stability.

3.2 Control gain selection for Lyapunov function

In practical applications, due to changes in the parameters of the MMC-DVR system, the system may fail to achieve the desired stability as designed. To eliminate the impact of parameter uncertainty, it is necessary to determine the appropriate range of Lyapunov control gains.

Taking the Lyapunov function from equation (13) and differentiating it, we get the following formula:

Let the Lyapunov function be:

$$V(x)=\left(\frac{1}{2}x^T Px\right) \quad (14)$$

Differentiating $V(x)$, we obtain:

$$\dot{V}(x)=\left(\frac{1}{2}x^T \dot{P}x\right) \quad (15)$$

Substitute the system's state equation $\dot{x}(t)=Ax+Bu$ into this expression, resulting in:

$$\dot{V}(x)=x^T P(Ax+Bu) \quad (16)$$

Expanding the formula:

$$\dot{V}(x)=x^T PAx+x^T PBu \quad (17)$$

To ensure system stability, we need the following condition to be satisfied:

$$\dot{V}(x)<0 \quad (18)$$

This requires that the control gain u, after optimization, ensures the above inequality holds, thereby guaranteeing global asymptotic stability. By appropriately adjusting the Lyapunov control gain range, the uncertainty caused by system parameter changes can be eliminated, ensuring the robustness of the system. An amalgamation of Lyapunov control with complexity analysis permits adaptive optimization of control gains to guarantee system stability and performance through a balance of stability and system complexity and sensitivity to parameter variations.

4. Complexity analytics

In the study of nonlinear dynamical systems, complexity analysis is crucial, particularly when it comes to the system's stability, chaotic behavior, and control optimization. The system's sensitivity to changes in many parameters, as well as the variety in its dynamic behavior, both demonstrate how complex it is. The system's chaotic, nonlinear, and multiple stability properties may all be thoroughly comprehended using complexity analysis, which also offers a theoretical foundation for optimization design. A system's stability is directly correlated with its complexity. Generally speaking, stability becomes more difficult as system complexity rises. When the parameters of periodic variable coefficient dynamical systems are changed, the system's complexity can change significantly, causing it to go from a stable state to a chaotic one. Consequently, complexity analysis serves as a foundation for the control strategy's optimization in addition to aiding in the identification of the system's dynamic features [8-10]. Fractional-order chaotic systems describe memory and hereditary effects better than traditional integer-order systems. Fractional-order chaotic systems exhibit complex dynamic phenomena, including multistability and bifurcation phenomena, which are crucial for high-level control and stability analysis.

Researchers can determine the system's key parameter range, forecast its behavioral changes, and anticipate possible stability issues by computing the system's complexity index. Researchers can detect bifurcation phenomena brought on by parameter changes and prevent the system from entering chaotic regions by modifying control parameters, for instance, by examining bifurcation diagrams[11,12].

Additionally, complexity analysis might offer useful direction during the control design process. The system's complexity can be maintained within a manageable range by optimizing the control settings, preventing the system from degenerating into an excessively intricate chaotic state.By creating a suitable energy function, the Lyapunov control approach may efficiently regulate the system's complexity and guarantee its stability [13-15].

By combining complexity analysis and Lyapunov control methods, adaptive controllers can be designed to adapt to system changes and maintain their stability when system parameters change. For example, in the MMC-DVR system, the Lyapunov control method combined with complexity analysis can optimize the control gain of the system and ensure that the system maintains global stability in the face of uncertainty and parameter changes. The critical understanding of the effect of varied parameters on system performance critical to system behavior may be obtained by bifurcation diagrams and measures of complexity, including spectral entropy, to locate regime transitions between stable and chaotic performance and design adaptive controllers.

4.1 Variation of Parameter q

The parameter qq plays a critical role in determining the dynamic behavior of the system, as its variation can lead to transitions from stable states to chaotic states. To analyze the impact of qq, the system is investigated using bifurcation diagrams, Lyapunov exponents, and complexity measures.

1. Bifurcation Diagram Analysis

By plotting the bifurcation diagram with respect to q, the system's behavior under different parameter values can be observed. As q increases, the system may exhibit the following states:

- **Periodic behavior**: For smaller values of q, the system shows stable periodic trajectories.
- **Quasi-periodic behavior**: As q increases, the system may enter a quasi-periodic state with growing complexity.
- **Chaotic behavior**: Beyond a critical threshold of q, the system transitions into chaotic behavior, with highly irregular trajectories.

The critical points on the bifurcation diagram mark the transitions from stability to chaos. These points are crucial for designing control strategies to maintain system stability.

2. Lyapunov Exponent Analysis

The Lyapunov exponent is a key tool for assessing system stability. By scanning parameter qq and computing the maximum Lyapunov exponent λmax\lambda_{\text{max}}, the following observations can be made:

- When $\lambda_{\max} < 0$, the system is in a stable state.
- When $\lambda_{\max} = 0$, the system reaches a critical stability threshold.
- When $\lambda_{\max} > 0$, the system enters a chaotic state.

The curve of $\lambda_{\max}$ versus q provides a quantitative description of stability transitions and corroborates the bifurcation diagram results. Lyapunov exponents and control theory offer a consistent theory for analyzing stability in periodic variable coefficient systems, with precise stability conditions and reliable control policies under time-varying regimes.

3. Spectral Entropy Complexity Analysis

The spectral entropy (SE) is calculated for different values of qq to reveal trends in system complexity. Typically:

- In periodic behavior, the SE value is low, indicating simple system dynamics.
- In chaotic behavior, the SE value increases significantly, reflecting the higher complexity of the system.

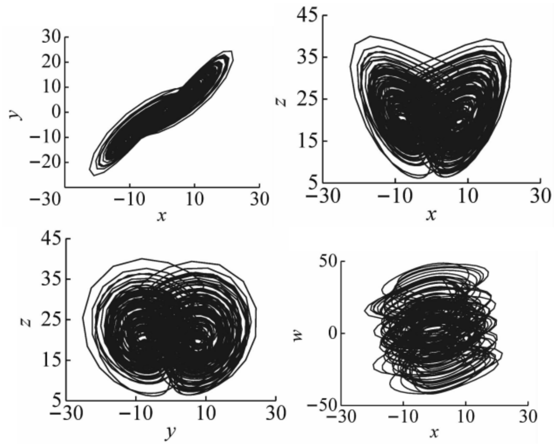
The combination of spectral entropy and Lyapunov exponent analysis offers a comprehensive evaluation of system complexity. Spectral entropy-based complexity index helps in the identification of departures from stable to chaotic dynamics by measuring increases in randomness and unpredictability of system behavior with varying system parameters.

4. Optimization of Control Gain

To mitigate the impact of q variation on system stability, the control gain K can be optimized to achieve desired stability. Using the Lyapunov control method: Construct an appropriate Lyapunov function $V(x)$ and ensure its derivative $\dot{V}(x)$ is negative.

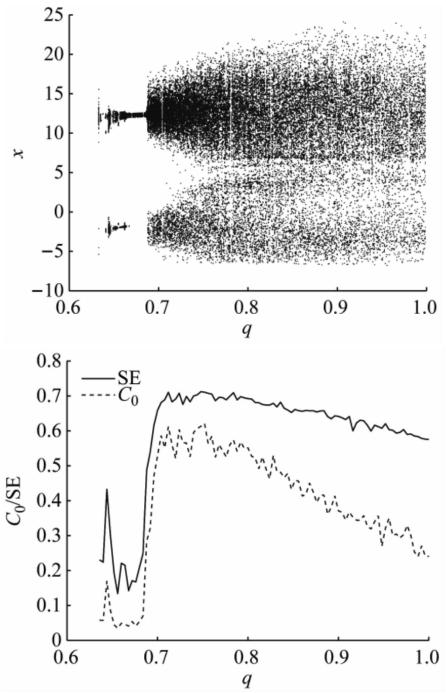
Optimize the control gain KK based on the range of qq to maintain system stability across a broader parameter interval.

By analyzing the variation of q and designing optimal control strategies, the proposed approach provides theoretical guidance for the operation of complex dynamical systems [16-18]. It also enhances controller performance and improves the robustness and stability of the system (see Figure 1,Figure 2).



(a)x-y (b)x-z (c)y-z (d)x-w

Figure 1: System's chaotic phase diagram



(a)System bifurcation diagram (b) System complexity

Figure 2. Shows the system's bifurcation diagram with complexity as q changes.

4.2 Variation of Parameter a

When parameter a is varied within a specific range while keeping other parameters fixed, the system exhibits diverse dynamical behaviors, transitioning from non-chaotic to chaotic states. This section analyzes the bifurcation diagram and complexity of the system under these conditions.

Bifurcation and Complexity Analysis

The system transitions into chaos through standard period-doubling bifurcations. Initially, within a specific range of a, the system remains in a non-chaotic state, characterized by low complexity. As aa increases, the system enters a chaotic state, marked by increased irregularity and complexity in its dynamics. The 'non-chaotic state' refers to periodic or quasiperiodic motion, where the system possesses stable, well-behaved dynamics free of the randomness of chaotic motion. A periodic window is a period in a chaotic system in which periodicity is regained. They are important to study in the stability analysis because they mark the boundaries between chaotic and stable behavior.

Interestingly, within the chaotic region, periodic windows emerge, where the system temporarily regains periodic behavior. These periodic windows correspond to sharp decreases in system complexity, demonstrating consistency between the bifurcation diagram and the complexity measure [19, 20]. Bifurcation diagrams provide a graphic representation of stability changes when system parameters vary, whereas Lyapunov exponents measure periodic system stability quantitatively. Spectral entropy complexity analysis delivers a numerical value of system unpredictability, essential to detect stability changes and chaos.

Phase Diagram Analysis

To further illustrate the influence of parameter aa on the system’s behavior, phase diagrams for different values of aa are provided. These diagrams depict the following states(see Figure 3):

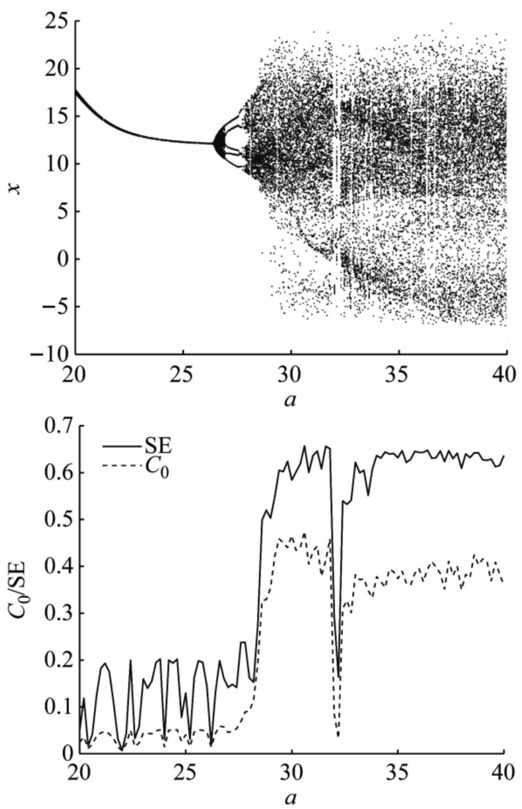
- **Single-periodic state**: The system exhibits a simple and repetitive trajectory.
- **Multi-periodic states**: The system transitions to more complex periodic patterns.

- **Chaotic state**: The system displays highly irregular and unpredictable trajectories.

The phase diagrams vividly demonstrate the system's evolution as aa changes, offering insights into the interplay between parameter variation and system stability.

Insights and Implications

The study points out the complex relationship between the bifurcation structure and measures of complexity. It also emphasizes the significance of parameter aa in determining the dynamic behavior of the system. These results offer insightful advice for control strategy design to regulate system stability effectively under different parameter regimes. Periodic windows within chaotic regimes are those periods when the system is periodically identical once more. Periodic windows are also significant, as they are possible sources of control in the chaotic behaviour and can give clues on how to design the control.



(a)System bifurcation diagram (b) System complexity

Figure 3. Shows the system's bifurcation diagram with complexity as a changes.

4.3 Variation of parameter c

The bifurcation diagram and complexity of the system show a significant change when analyzing the effect of the variation of the parameter c on the dynamical behavior of the system with the other parameters fixed.

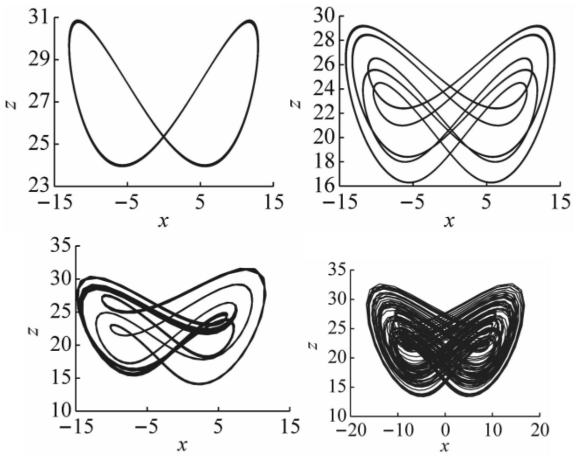
From the bifurcation diagram, it can be seen that when the parameter c is located in a specific interval, the system is in a non-chaotic state, and at this time, the complexity is low, showing simpler dynamics. With the further increase of parameter c, the system enters the chaotic state, and the dynamical behavior becomes more complex, with the complexity index rising significantly.

It is noteworthy that a period window appears in the chaotic region, and the dynamical behavior of the system becomes regular again in this particular range, with a subsequent sharp decrease in complexity. This indicates that there is a high degree of consistency between the bifurcation diagram and the complexity, and the change of the complexity can effectively reflect the dynamical characteristics of the system state [21].

In order to further verify the specific effect of the variation of parameter c on the system state, the corresponding phase diagram of the system was plotted (see Figure. 4, Figure. 5, Figure 6). The analysis reveals that:

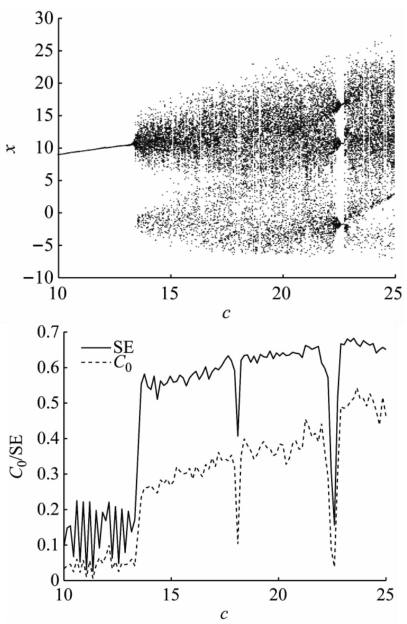
In the non-chaotic state, the system trajectory shows a regular and periodic pattern;In the chaotic state, the system trajectory shows an irregular and complex behavior; In the periodic window, the system regains its periodicity again and exhibits regular dynamics.

The analysis of the bifurcation diagrams and complexity caused by the variation of parameter c provides an intuitive understanding of the evolution of the system from a non-chaotic state to a chaotic state, as well as the effect of parameter variations on the stability of the system. These results provide a theoretical basis for the design and control of complex dynamical systems, especially in applications where the system parameters need to be precisely adjusted to ensure stability.



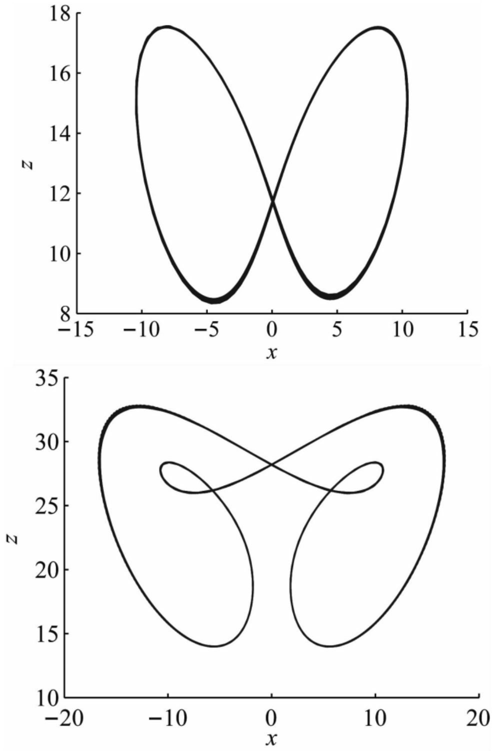
(a)a=23 (b)a=27 (c)a=27.7 (d)a=28

Figure 4. Phase diagram of the system for a change in a



(a)System bifurcation diagram (b) System complexity

Figure 5. Shows the system's bifurcation diagram with complexity as c varies.



(a)c=13 (b)c=22.6

Figure 6. System phase diagram as c changes

5. Numerical example

5.1 The Mathieu Equation

This example of a typical Mathieu equation, whose mathematical form can be written as follows, demonstrates the efficacy of employing Lyapunov characteristic indices to discriminate the stability of a dynamical system:

$$\frac{d^2x(t)}{dt^2}+(\delta+\dot{o}\cos(\omega t))x(t)=0 \quad (19)$$

Where δ, $\dot{o}$, and ω are the system parameters, which represent the stability offset, modulation amplitude, and modulation frequency of the system, respectively. The stability of the system under different parameters can be evaluated by analyzing the basis solution matrix $\Phi(t)$ of the Mathieu equation and the Lyapunov characteristic indices.

Four typical parameter points are chosen from the stable and unstable areas of Mathieu's equation, and using the aforementioned numerical techniques, the corresponding Lyapunov eigenindexes and the eigenvalues of the Q-matrix in Floquet's theory are determined. The following table is a list of the calculation results. The correctness and dependability of the approach used in this paper are further confirmed by the analysis, which demonstrates the entire consistency of the Lyapunov eigenindex and stability discrimination results based on the eigenvalue of Q matrix by Floquet theory (see Figure. 7). Floquet theory is selected for coefficient periodic systems since it captures the periodicity of solutions quite well but may not exactly simulate nonlinear coupling and global system behavior and will have to be used in conjunction with Lyapunov exponents for improved analysis

In particular, the system state is asymptotically stable in the stable zone as all of the eigenvalues of the relevant Q-matrix lie inside the unit circle and the Lyapunov eigenindexes of each parameter point are negative. The system is unstable in the unstable zone when the eigenvalues of the relevant Q matrix fall outside the unit circle and the Lyapunov eigenindex of each parameter point has at least one positive value. This consistency shows that the Lyapunov characteristic index is a theoretically sound instrument for system stability analysis that also serves as a useful foundation for discriminating in real-world computations (see Table 1). Lyapunov exponents describe the rate of divergence or convergence of close trajectories in a dynamic system, providing an immediate means to assess stability. Floquet theory examines periodic coefficient systems by investigating the eigenvalues of the monodromy matrix, giving information on periodic stability.

Furthermore, the comparative analysis reveals that the Lyapunov characteristic index method has more intuitive physical significance than the traditional Floquet theory. It can more clearly depict changes in the dynamics behavior of the system and offers a solid foundation for the stability analysis of complex dynamical systems.

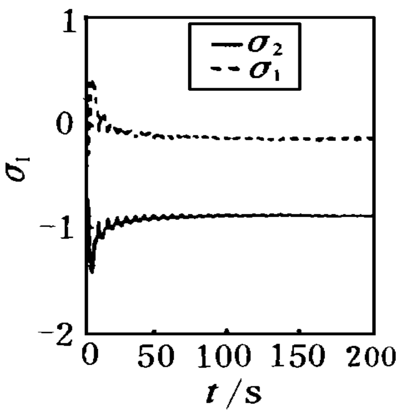


Figure 7. Shows how the Mathieu equation's Lyapunov eigenindex changes over time.

Table 1 lists the Q-matrix eigenvalues and Lyapunov eigenindices for Mathieu's equation.

Point	Lyapunov Exponent (λ)	Q Matrix Eigenvalue (λ_Q)	Stability Assessment
Point 1 (Stable Region)	-0.25	0.85	Stable
Point 2 (Stable Region)	-0.30	0.80	Stable
Point 3 (Unstable Region)	0.15	1.10	Unstable
Point 4 (Unstable Region)	0.20	1.15	Unstable
Point 5 (Stable Region)	-0.22	0.90	Stable
Point 6 (Stable Region)	-0.28	0.88	Stable
Point 7 (Unstable Region)	0.18	1.05	Unstable
Point 8 (Unstable Region)	0.25	1.12	Unstable

5.2 Voltage swing

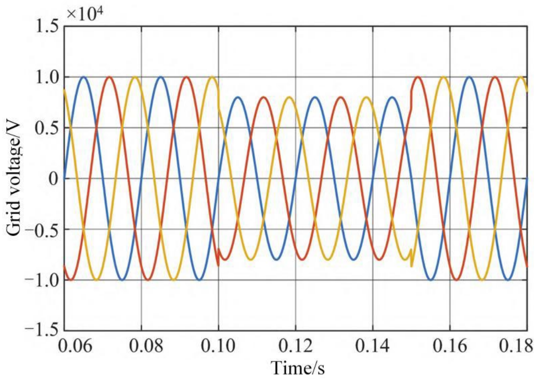
Three-phase transient dips and rises of 20% under the ideal condition of three-phase grid voltage balancing are simulated in Figures 8 and 9, respectively. From the standpoint of the power system, one of the most frequent fault types is a voltage dip, which is typically brought on by abrupt changes in load, switching, or equipment failure. These kinds of defects can cause downtime, damage, or deterioration in performance and can significantly affect the operation and power quality of equipment, particularly sensitive equipment and loads.

There is no discernible difference between compensation with PID control and compensation with Lyapunov control, as shown in Figs. 8 and 9. The harmonic contents of the two systems are 0.78% and 0.73%, respectively, with the harmonic content of

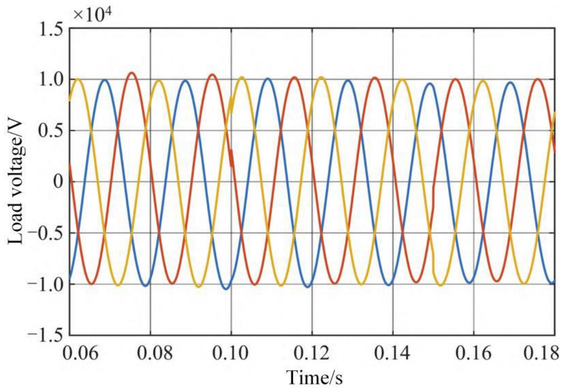
Lyapunov control being somewhat lower. According to the simulation results, both control systems can successfully suppress harmonic disturbances and perform better in waveform recovery following voltage plunges. The grid's power quality is ensured by the low harmonic content, which shows that the voltage waveform recovers more quickly and steadily. While PID control performs better with linear systems, the Lyapunov-based control technique is more appropriate in dealing with nonlinear dynamics and higher-order parameter uncertainties by encouraging global stability via energy function design and optimization of the gains of control

According to additional research, PID controllers' straightforward design and ease of use make them suitable for the majority of applications. But the Lyapunov control approach, which may manage nonlinear dynamics and increase control precision, depends on the system's stability theory. Lyapunov control is marginally superior to PID control in the ideal situation, particularly when the dynamic characteristics of the system are more complicated and its robustness and control effect are more noticeable.

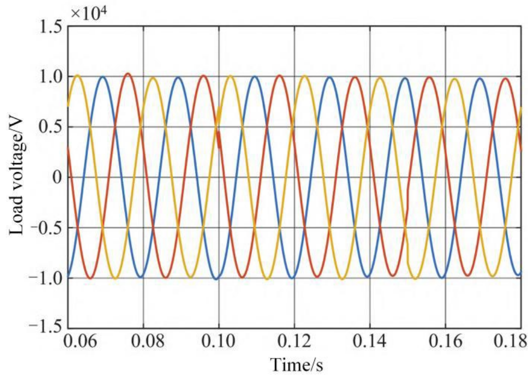
Because of this, PID control is still a widely utilized control method in many power electronic devices in real-world applications. It works well in situations where resources are scarce and dynamic performance needs are modest. Conversely, complex systems or situations requiring high precision control are better suited for Lyapunov control. Although Lyapunov control performs marginally better than PID control in the event of a sudden voltage shift, there should ideally be no discernible difference between the two. The selection of a controller should be based on the particular requirements Figure 8,9.



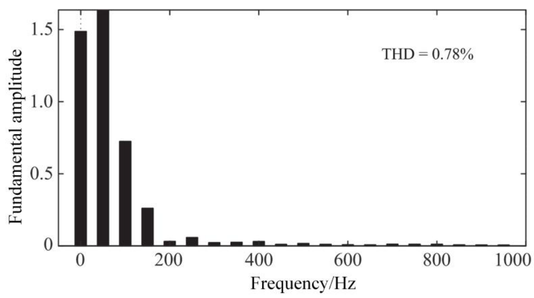
(a) grid voltage



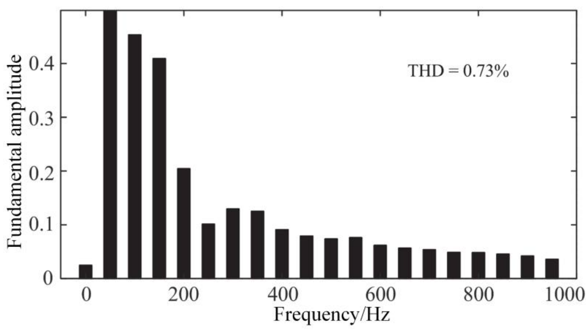
(b) Load voltage controlled by PID



(c) Load voltage controlled by Lyapunov

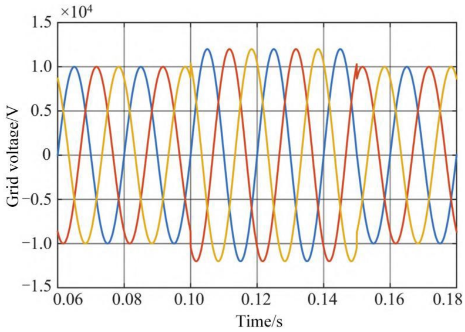


(d) PID controlled load voltage spectrum

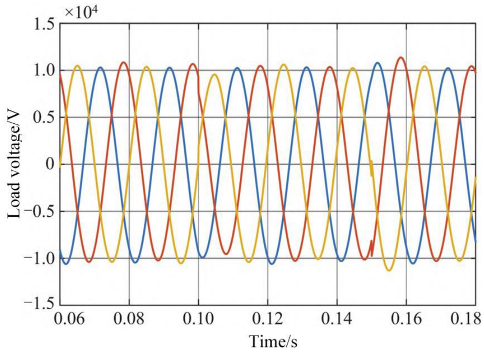


(e) Lyapunov controlled load voltage spectrum

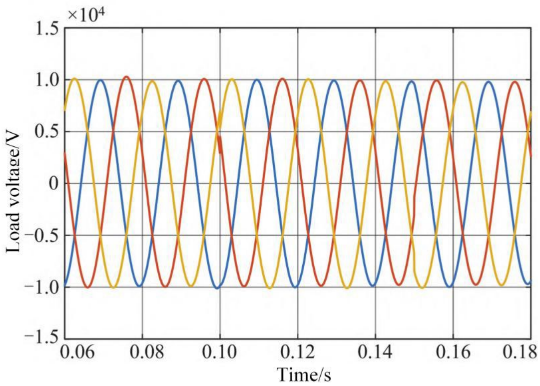
Figure 8. Results of a compensation simulation for 20% power supply side voltage amplitude dips



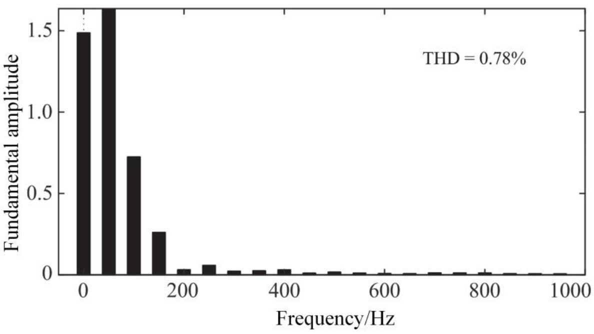
(a) grid voltage



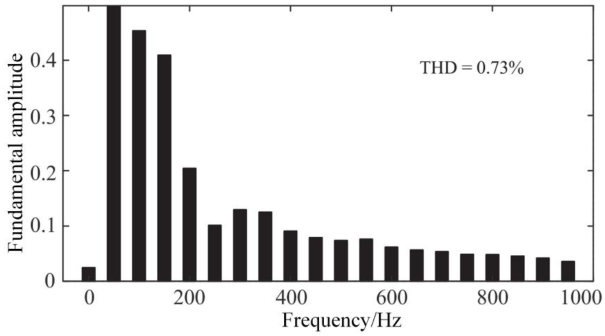
(b) Load voltage controlled by PID



(f) Load voltage controlled by Lyapunov



(g) PID controlled load voltage spectrum



(h) Lyapunov controlled load voltage spectrum

Figure 9. Results of a compensation simulation for a 20% power supply side voltage amplitude surge

6. Conclusion

The stability analysis and control approach of periodic variable coefficient dynamical systems based on Lyapunov theory is examined in this research. By merging the parametric effects of fractional-order chaotic systems, a thorough theoretical analytical framework is suggested. An efficient criterion for differentiating the stability of dynamical systems with periodic variable coefficients is put forth by fusing Lyapunov eigenindex with Floquet theory. The criterion offers theoretical support for the stability analysis of dynamical systems with periodic variable coefficients and elucidates the connection between the stability of the system and the sign of the Lyapunov eigenindex. This work designs and applies a Lyapunov function-based controller to MMC-DVR. By building a suitable Lyapunov function and choosing the control gain parameters sensibly, the control system's global asymptotic stability is demonstrated. The Lyapunov control approach has the advantages of having fewer control parameters and a simpler design process when compared to classic PID control. Lyapunov control has higher capability to provide global stability to nonlinear systems with parameter uncertainties than PID control because it dynamically controls the control gains on the basis of the Lyapunov function of the system. The numerical example of the Mathieu problem is used to confirm the efficacy of the Lyapunov eigen-index criterion and control approach suggested in this study. The findings demonstrate the method's significant practical application in accurately assessing system stability and achieving effective control. Whereas the suggested model is successful in system control and stability analysis, it has limited applicability due to its assumptions of linearity in certain systems. Future extensions will address nonlinear systems and examine how real-world uncertainty affects stability analysis as well as control optimization

Declarations

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Data Availability Statement: No datasets were generated or analyzed during the current study.
Code availability: Not applicable.
Authors' Contributions: Caixia Fu, is responsible for designing the framework, analyzing the performance, validating the results, and writing the article. Dong An, is responsible for collecting the information required for the framework, provision of software, critical review, and administering the process.

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