Data science analysis of Vassiliev invariants and knot similarity based on distributed machine learning

Chenggang Huo[1,*]

[1]School of Mathematics and Statistics, Suzhou University, Suzhou 234000, China

Abstract

INTRODUCTION: Knot theory has a long history, and as a branch of topology, it has received extensive attention. At present, the scientific analysis of data based on the similarity of Vassiliev invariants and knots under machine learning technology is the focus of the mathematical community. However, at present, there are some difficulties in the research work on the similarity of Vassiliev invariants and knots, including the low accuracy of the analysis of the characteristics of knots, the long analysis time and the high memory resources occupied in the analysis process. These difficulties not only delay the progress of Vassiliev invariants research, but also slow down the speed of knot similarity research.
OBJECTIVES: However, with the acceleration of the intelligent process, various intelligent technologies have been applied in the research of mathematics, biology and physics, providing excellent help for the research of many disciplines. Therefore, machine learning technology could be used to carry out new research on Vassiliev invariants and knot similarity.
METHODS: Traditional knot analysis technology was combined with machine learning technology to find a more efficient and stable way of exploring Vassiliev invariants and knot similarity. his paper proposed a research method of data scientific analysis based on Vassiliev invariants and knot similarity under machine learning technology. Its purpose was to combine traditional knot research methods with machine learning technology to improve the efficiency of knot research. The algorithm proposed in this paper was the knot Vassiliev invariant analysis algorithm based on machine learning, which could use the intelligent and efficient analysis algorithm of machine learning technology to process the data of complex knots. This algorithm has improved the accuracy of the analysis of knot characteristics and reduced the analysis time and the memory consumption at runtime.
RESULTS: By testing the similarity between the Vassiliev invariant based on machine learning and the knot, the results showed that the analysis accuracy of the traditional Vassiliev invariant computing technology for the chiral characteristics, the number of intersections and the number of knots in the knot image was 84.25%, 83.27% and 85.56% respectively. The accuracy of knot Vassiliev invariant analysis algorithm based on machine learning for these indicators was 91.87%, 92.66% and 92.12% respectively. Obviously, the knot Vassiliev invariant analysis algorithm based on machine learning was superior to the traditional knot computing technology, and its analysis results were more excellent.
CONCLUSION: In general, the research topic proposed in this paper has been proved to be of practical value. This research result proved that machine learning technology could play an excellent role in the current knot research, which correspondingly expanded the research direction of Vassiliev invariants and knot similarity.

Keywords: Vassiliev Variants, Knot Similarity, Machine Learning, Knot Vassiliev Invariant Analysis Algorithm, Data Science Analysis.

Received on 27 February 2023, accepted on 27 May 2023, published on 29 June 2023

doi: 10.4108/eetsis.3088

* Corresponding author. Email: huochenggang_abc@163.com

1. Introduction

In recent decades, knot theory has developed rapidly and has become a hot field in many disciplines, such as group theory, topology, spatial geometry, etc. In addition, the application of knot theory is also extensive, and has been rapidly expanded in multiple life and production scenes. At this stage, more and more scholars have devoted themselves to the study of knot theory. Among them, the research on the similarity of Vassiliev invariants and knots is a key research topic. Therefore, the research direction of this topic is to use machine learning technology to further explore the similarity between Vassiliev invariants and knots. This topic not only improves the accuracy of the analysis of knot characteristics, but also helps expand the application direction of machine learning, and also plays an important role in the intelligent research of knot theory. At present, the academic research on the related topics of knot theory is quite fruitful, which can better research and explore the knot theory. However, with the in-depth study of knot theory, the traditional knot research technology appears to be inadequate. Therefore, this topic proposes to combine machine learning technology with traditional knot research technology to seek a new breakthrough in knot research. However, the research on knot similarity and Vassiliev invariants in the context of machine learning is still at the initial stage, which needs to be continued.

Since knot theory was put forward, its academic theory and application fields have been rapidly expanded. Many scholars have conducted scientific research on knot theory. In the research, Cheng Zhiyun introduced a new virtual node invariant to find a breakthrough in the virtual knot theory. He proved that this transcendental function invariant extended several polynomial invariants of virtual nodes, such as torsion polynomial, affine exponential polynomial and zero polynomial. In addition, he also pointed out several application directions of this new invariant [1]. Budney Ryan explored the Abelian group of knots and concluded that the component mapping of knots was a finite type knot. He calculated and converged to the page where the total degree of the spectral sequence of the knot component was zero. He provided evidence for the conjecture of the universal finite invariant of the knot [2]. Kim Se-Goo defined the Upsilon interval of knot invariants, and induced homomorphisms from smooth knot uniform groups to piecewise linear function groups. Here, he defined a set of related quadratic invariants, and assigned piecewise linear functions to the nodes of each invariant. In addition, he gave the factors for the disappearance of Upsilon and provided an example for the detection of knot quadratic invariants [3]. Jeong Myeong-Ju studied the correlation properties of Vassiliev invariants. He pointed out that if there was a separate association between two nodes, there was a gap in the second coefficient of Conway polynomial of the two nodes. He expanded the research results of virtual knots by using the Vassiliev invariants of virtual knots derived from the polynomials of virtual knots [4]. The above research topics have studied knot theory from multiple directions, which has a certain guiding and enlightening effect on this paper and provides effective help for the research work of this topic. However, their research direction does not link the research of knot theory with machine learning. Therefore, more relevant literature should be consulted.

Nowadays, with the rapid development of mathematics and intelligent technology, the two disciplines are constantly merging. Intelligent technology has also been applied to many interdisciplinary research topics, which greatly facilitates the research of various basic disciplines. Therefore, machine learning technology can be applied to the research of knot theory, which provides a more intelligent operation for the research of knot theory. At present, many scholars have integrated machine learning technology and knot theory. Nelson Sam extended the concept of double quantum to knot theory and used machine learning technology to study and define the invariants of directional singular links and pseudo links. He also introduced the research results of knot invariants and polynomials and considered the Alexander coloring of pseudo links. In addition, he defined many properties of Alexander polynomials of directional singular links and pseudo links, and calculated invariants of all pseudo nodes with multiple intersections and all knot graphs with multiple classical intersections [5]. Ekholm Tobias studied the enhancement optimization of knot contact, from which the group ring and the outer subgroup of the knot could be inferred. He determined the isotropy of a knot by using machine learning technology. The enhancement optimization included contact homology related to the union of the conformal torus of the knot and the disjoint cotangent fiber ball, as well as the product on the filtering part of the homology. In addition, his results proved a new holomorphic curve and pointed out that the isomorphic class of conformal torus was a complete knot invariant [6]. Obviously, the research of the above scholars has provided a good direction for the research of this topic. They have made good use of machine learning technology to study knot theory. However, their research is still lack of practicality and feasibility. Therefore, this article aims at this deficiency and conducts more detailed and practical testing research in this study.

In mathematical research, a knot can be regarded as a simple closed curve in three-dimensional Euclidean space. Knot theory is not only applied in traditional mathematical theory, but also related to string theory and quantum field theory in theoretical physics. Therefore, the exploration and development of knot theory has received full academic attention. However, the analysis and research methods of knot theory at this stage are no longer suitable for the current academic environment. Therefore, the traditional knot analysis technology should be improved and optimized and combined with cutting-edge machine learning technology. Compared with the traditional knot research and analysis technology, the analysis process of knot data can be accelerated through machine learning. It can effectively improve the accuracy of knot analysis and achieve more intelligent and efficient knot data analysis operation.

2. Knot Theory

2.1 Definition and Development of Knot Theory

In the field of modern mathematics, knot theory is the main component of topology theory in this century [7-8]. Topology is the study of continuous changes in geometry. Similarly, knot theory is to study the laws and characteristics of circles and knots when continuous changes occur, and its research direction is similar to topology. Because of the intuitive and interesting connection between knots and links, knot theory, as a very attractive branch of modern topological structure, is increasingly important. In general, the basic knot theory can be understood as the basic research of the realization problem from one topological space to another. The basic knot theory is generally expressed by the plane projection of its nodes. The definition of the basic knot theory has experienced a long evolution and development and has constantly formed a new concept. With the development and improvement of image mapping technology and knot theory, the concept of virtual intersection has been introduced into some special graphic transformations. Under this concept, the equivalence of graphics is called basic knot. In addition, the research field of knot theory is very broad, which has many branches.

(1) Concept of classical knot theory

At present, knot theory has become a major branch of mathematics. Knot theory has been developed for a long time, which is abstracted from the well-known knotting of ropes and gradually becomes the research object of mathematics [9-10]. In order to study knotty, mathematicians have introduced the concept of the number of circles between closed curves. In addition, a function integral calculation method of low dimensional topological interaction is proposed, which opens up the research field of quantum field theory. Later, the theory of knots is unexpectedly merged with the most mysterious string theory. Under the guidance of string theory, people have found many interesting phenomena about knots, which determine the development of knots to a certain extent. In addition, due to the discovery of knot Jones polynomials, knot theory has once again become a major branch of mathematics. Another obvious discovery is that people have found the connection between knot polynomial and quantum group equation, which also promotes scholars to explain Jones polynomial from another aspect. In addition, some mathematicians have accidentally found that the hyperbolic quantity of hyperbolic node complementary space has a very strange relationship with the asymptotic property of Newton polynomial quantity. The discovery of this relationship provides some inspiration for the research of mathematical theory and chord theory.

(2) Concept of virtual knot theory

Virtual knot theory is an extension of classical node theory. Since this theory is proposed, it has attracted extensive attention and become a key research topic of topology [11]. In recent years, the virtual knot theory has achieved fruitful research results. Many problems and influences in the application of virtual knot theory have entered the forefront of New York research. At present, some properties of virtual knot algebra are the same as Jones knot polynomials. Knot polynomials are derived from virtual clusters to the representation of virtual knot algebra. Then, the special virtual knots are constructed, and the commutative group of their core group is proved by calculation. They are not equivalent to each other, nor are they classical nodes. In addition, some properties of these basis groups are studied. It is concluded that they are linear. The upper limit of the number of virtual knot nodes is obtained by deformation calculation method. Then, the concept of standard virtual link point with virtual data is introduced, and the upper bound search algorithm of this virtual knot is obtained through the deformation calculation method. Finally, the lower bound of the number of virtual knot nodes is obtained through this algorithm.

(3) Concept of welded knot theory

The welded knot theory also evolved from the classical knot theory. After years of development and exploration in the academic community, the welded knot theory has been fully improved [12]. As a kind of knot theory, it is defined and studied by graph definition, which allows the first kind of knot transformation. As for the welded knot, it can be explained and expressed through topology, and an algorithm for a banded torus knot of the knot chart can be given in this way. The algorithm does not depend on the selection of the graph of the knot, and it is a mapping from the set of welded knots to the banded torus knots. In addition, when a welded knot is untwisted, all the number of knots cannot be ignored. With the continuous study of the welded knot, the progress on its simple invariants and Jones polynomials is fast. However, at present, the performance of Jones polynomial on the welded knot is not stable. Therefore, so far, there is no good and effective welded knot multiple invariants, which leads to the slow development of this theory.

Among them, the branch theory of knot theory is shown in Figure 1.

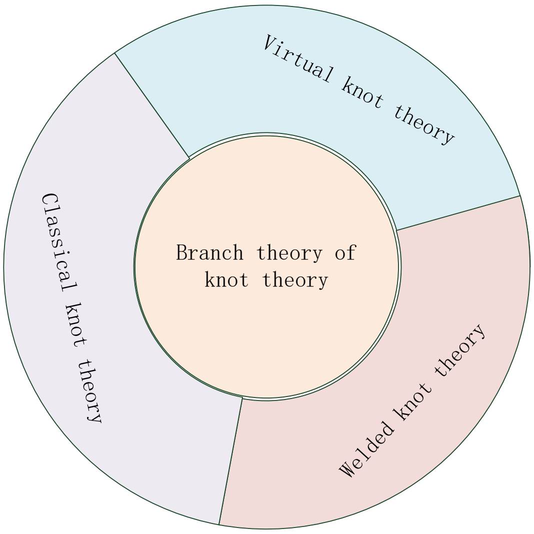


Figure 1. Branch theory diagram of knot theory

2.2 Basic Concepts and Characteristics of Knot Theory

Knot theory is a theoretical discipline with a long history, but it has always maintained a strong vitality [13-14]. First, the basic concept of knots is simple. Knot is a simple closed curve in 3D space, which is uninterrupted, disjoint and closed. In addition, the spatial image composed of simple closed curves with finite intersection points is called a link. Each closed curve in the link is called a branch of the chain. Therefore, it can be said that a node is actually a branch. Ordinary disjoint nodes on the plane form a chain, which is called a trivial link. Knots and links are graphics in three-dimensional space. However, in order to display and study them conveniently, they are usually mapped to a two-dimensional plane, and the image on the two-dimensional plane is also the projection of the knot. The diagram should satisfy three characteristics. First, there are a limited number of overlapping points. Second, each overlapping point has multi-directional overlap. Finally, on the overlapping point, the projection of the upper and lower lines has the property of crossing each other. In addition, the research on knots is very complex, and its main characteristics include: chirality problem, intersection transformation and knot number, which are very important in knot theory.

(1) Chiral characteristics of knots

In nature, many physical, chemical and even biological phenomena are related to chiral characteristics, so chiral characteristics cannot be ignored. In academia, the chiral problem is also called the mirror image problem. It is supposed that there is a knot or link, and the image of the knot in the mirror is its mirror image. The mirror is still a knot or link. If the projection of the original knot is given, its mirror knot can be easily drawn. In the first step, the original knot image is projected. The second step is to invert and exchange the upper and lower lines at all intersections of the knot. At this time, the image of the original knot is obtained. In addition, if the original knot does not have the same trace as the mirror knot, it is said that the original knot has chiral characteristics. On the contrary, the original knot is chirality free. However, intuitively, it is generally difficult to distinguish whether a knot has chiral characteristics. After appearance of the Jones polynomials, chiral characteristics of the knots can be clearly identified by this method.

(2) Intersection transformation of knots

Another important feature of knots is the intersection transformation. There are many intersections in knots. These intersections have the function of distinguishing and distinguishing knots and are an important parameter in knot images. For an intersection point in a knot graph, there are upper arc and lower arc curves at that point. It is supposed that the position relationship between the upper and lower arcs is transformed, that is, the upper and lower arcs are exchanged, which is called the intersection transformation of knots.

(3) Knot number

It is supposed that there is a knot, and then there is a projection graph of the knot, on which all the results of the knot can be resolved by mapping n cross points, thus obtaining a trivial result. For other projections of the knot, if the intersection transformation performed less than n times still fails to yield trivial results, then n times are regarded as the number of knots solved. For the whole knot image, the number of knots is also a constant.

Among them, the main characteristics of knot theory are shown in Figure 2.

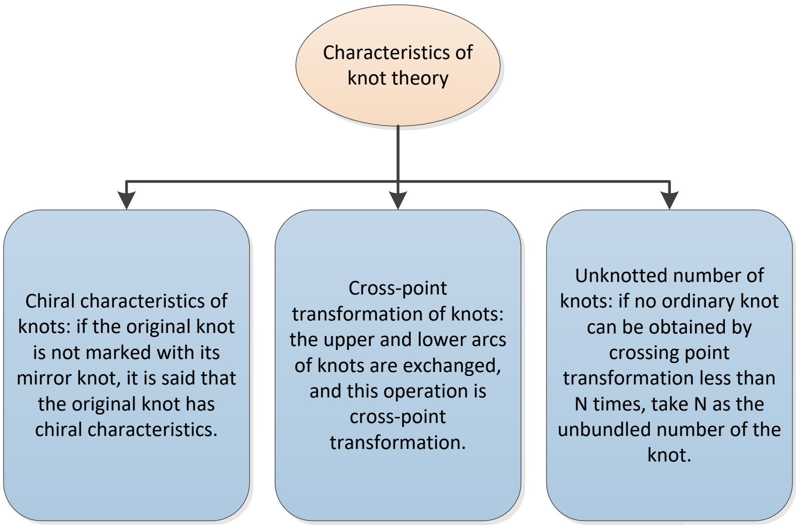


Figure 2. Main characteristics of knot theory

2.3 Polynomials in Knot Theory

Since the proposition of knot theory in the last century, knot polynomials have attracted mathematicians' research [15]. After years of research, many polynomials have been proposed for the study of knots, including Alexander polynomials, Jones polynomials and Kauffman polynomials. The properties of the above knot polynomials are different from each other. Specifically, these polynomials have special disjoint relations. In addition, the calculation of polynomial invariants is simplified, and the research method of knot similarity is optimized and improved through its special connection relationship. These main knot polynomials are described below.

(1) Alexander polynomial

Alexander polynomial is the first polynomial knot invariant proposed by the mathematical world. As early as 1928, mathematicians first proposed Alexander polynomial. This polynomial is called a sudden progress in knot research. Since then, many knot polynomial invariants have been extended. In addition, Alexander polynomial has been improved a lot through optimization of low dimension extension and disjoint relation, which makes the calculation of invariant of the polynomial faster and simpler. The proposition of this polynomial points out a new direction in the study of knot invariants, which plays an important role in the study of polynomial knot invariants. In addition, for Alexander polynomials, it has three characteristics. First, the same trace invariance of knots. Second, the polynomial has a strict disjoint relation. Third, the number of knots to be solved is 1.

(2) Jones polynomial

A major breakthrough in the theory of knot polynomials is the discovery of Jones polynomials. Jones polynomial is the second knot polynomial in academia, which is the optimization and improvement of Alexander polynomial. Generally speaking, Alexander polynomials cannot distinguish the chiral properties of knots, while Jones polynomials can. Jones polynomials are discovered in the course of studying finite dimensional von Neumann algebras. The proposed polynomial reveals the similarity of knots and links knot invariants with knot similarity. Later, after more research, Jones polynomials also have the split relationship of knot combination. Jones polynomial also means that in a knot, if the knot and its projection meet the same trace property, the corresponding polynomials are equal.

(3) Kauffman polynomial

Kauffman polynomials are different from other polynomials. In the knot satisfying the polynomial, each projection graph of Kauffman polynomials has a polynomial. In addition, Kauffman polynomial is invariant under the transformation of knot intersection, that is, the polynomial is a stable quantity of the knot. In the projection diagram of knots, it also has a good disjunctive relationship like polynomials, that is, the equations connecting different knot polynomials. This equation is conducive to the fast calculation of polynomial invariants. At the same time, Kauffman polynomial points out that the number of knot intersections is an important indicator to judge the complexity of a knot. Under such conditions, when the multinomial number of knots with multiple intersections is calculated, it can be converted into the polynomial number of knots with fewer intersections. In this way, the calculation of knot intersection polynomial is greatly convenient.

The main knot polynomials are shown in Figure 3.

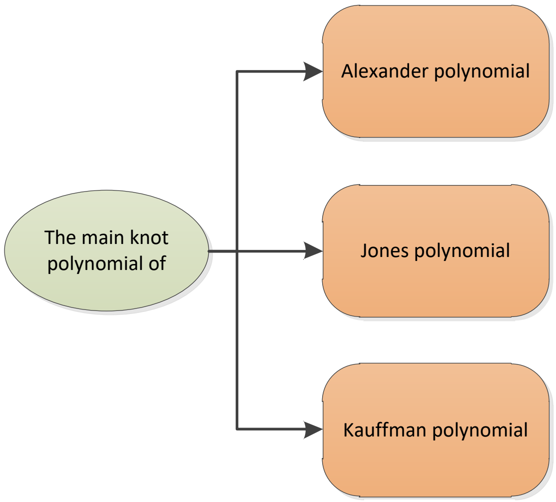


Figure 3. Major knot polynomials

3. Machine Learning and Knot Theory

3.1 Overview of Machine Learning Technology

With the continuous advancement of intelligence, machine learning has entered into a remarkable era and become the core of artificial intelligence [16]. It automatically analyses and obtains models from data, enabling it to make predictions about unknown data. Machine learning is an interdisciplinary field that combines statistics, system identification, approximation theory, neural networks, optimization theory, computer science, brain science, and many other fields. Its fundamental goal is to simulate or recreate human learning behaviour to acquire new knowledge or skills, to reorganize existing knowledge structures and continuously improve its own performance. From many aspects, machine learning technology delves deep into intelligent computing analysis technology, effectively improving the speed and resource consumption of intelligent computing, which plays a critical role in the intelligent progress of society.

In contrast to traditional machine learning, modern machine learning mostly improves system performance by using data as a basis. Data-based machine learning is one of the key methodologies of today's intelligent technology. By discovering rules from observed data, machine learning can predict future or unobservable data. The development of machine learning can be classified into three stages.

(1) In the first stage (1956 - 1960), the main research objects were various highly adaptive machine learning systems. Its main research direction was to enhance the function of the system activities by continuously improving the control parameters of the system, including the application of certain other technologies. Later, due to the emergence and development of computer technology, the practical application of machine learning has gradually become possible. The exploration of this period formed this new pattern recognition science and produced two major computer knowledge technologies: judgment function method and evolutionary learning technology. However, these perceptual cognitive technologies, which are separated from knowledge, have great limitations. Whether it is neural simulation, evolutionary method or judgment function method, the knowledge achievements obtained are very limited and far from reaching the goal of human computer knowledge technology.

(2) The second stage (1970-1980) is called the computer cooling stage. The research objects in this period mainly used logical digital forms to replace the internal representation of computers, imitating human concept learning programs. Computers can express ideas in abstract form, thus providing various hypotheses for learning theory. The typical examples of this period are structured learning systems and inductive knowledge systems based on logic. Although this kind of knowledge system has been realized, it only allows one theory and has no practicability. In addition, because the theoretical defects do not achieve the expected results, the development of neural networks is going to the low tide.

(3) The third stage (since 1990) is called the intelligent rejuvenation period. People have shifted from learning one concept to learning multiple concepts, learning different learning strategies and learning methods. Machine learning emerged as a tool to “break through the bottleneck of knowledge engineering”. At this stage, humans find themselves immersed in the ocean of data, and machine learning has shifted from using experience to using data to improve productivity. At present, the demand for machine learning is more and more urgent. Machine learning process is usually based on large-scale knowledge base to achieve knowledge-based learning process. Fortunately, at this stage, the learning system has been integrated with various applications and has made significant progress in promoting machine learning.

To sum up, the classic machine learning process is the process of marking unknown data with the algorithm of existing data and the style of new data. First, the new data needs to include training sets and samples. By selecting an appropriate machine learning algorithm, the learning data is trained in the model, and a new sample set is marked with the model. The process of machine learning is shown in Figure 4.

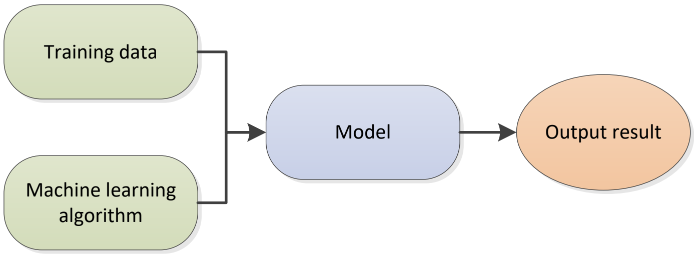


Figure 4. Process diagram of machine learning

3.2 Machine Learning Technology and Data Science

Machine learning technology is closely intertwined with the field of data science analysis [17], which is an exciting area because it involves the collection, storage, and processing of vast amounts of information, providing a level of knowledge that is hard to find in other disciplines. Data science refers to the research field related to data systems and data processing, with the aim of maintaining data and inferring its significance. Data scientists use various tools, intelligent applications, theories, and algorithms to comprehend data. With data growing exponentially across all organizations and systems globally, it is challenging to control and store. Data science focuses on data modelling and data warehousing. The development of data is tracked, and information is extracted through data science software to guide business processes and achieve specific goals.

Data scientific analysis is also a type of intelligent technology that can process voluminous data and information, and analyse the current situation, model future trends, and extract insights. The resulting data analysis report provides an understanding of specific transactional data trends, which can then serve as a basis for devising action plans. Through machine learning, data science and technology can be optimized to have several desirable characteristics:

(1) Causal prediction: By relying on machine learning technology, data analysts use this model to model and predict businesses. As long as relevant data is input, more accurate trends and results can be predicted. Machine learning is an efficient and good tool for causal prediction.

(2) Regulatory analysis: By relying on machine learning technology to distract the big data in the enterprise, it can help the enterprise's operation supervision and assist the supervision process of various dimensions and departments of the enterprise. This technology provides reliable basis and help for enterprises and makes outstanding contributions to the development and supervision of enterprises.

(3) Data science and technology: In machine learning, the participation of external software and applications is indispensable. By using SQL, Python, java and other data driven applications, it can help to model and optimize machine learning. In addition, these technologies also contribute to the development of statistical analysis, data science and technology and data visualization technology.

3.3 Knot Vassiliev Invariant Algorithm Based on Machine Learning

Knot invariants whose value range is Abelian group can be obtained through the following harness relations:

$$u(K_D) = u(K_+)+u(K_-) \quad (1)$$

Among them, K_D is a knot image with nodes. K_+ and K_- are images of mutually mirrored knots.

Let u be the knot invariant of the range in the Abelian group. If there is $u(K_D) = 0$ for any knot with more than n singular points, this invariant is called n-order Vassiliev invariant.

In addition, when the knot is exchanged, the value of the intersection in the knot image remains unchanged, namely:

$$u(K^n) = u\overline{(K^n)} \quad (2)$$

Among them, K^n is the original knot image with n singular points, and $\overline{(K^n)}$ is its mirror knot image.

Let u and w be two knot invariants, and then their product is:

$$u * w(K) = u(K) * w(K) \quad (3)$$

In addition, let u and w be the Vassiliev invariants of knot N, and K be the knot with i singular points, then:

$$(u * w)^i(K) = \sum_{N\subseteq\{1,\dots,i\}} u^N (K_N) * w^{\overline{N}}(K_{\overline{N}}) \quad (4)$$

When u and w are knot invariants of order m and n respectively. It has the following characteristics:

$$u^m = 0 \quad (5)$$

In addition:

$$w^n = 0 \quad (6)$$

Among them, there are the following properties:

$$(u * w)^{(m+n)} = 0 \quad (7)$$

The above equation explains the relationship between knot coefficients and Vassiliev invariants, which is of great significance to the study of knot similarity. This algorithm simplifies the calculation of Vassiliev invariants and makes an important contribution to the research of topology and knotting.

4. Implementation and Testing of Vassiliev Invariants and Knot Similarity Based on Machine Learning

4.1 Test of Knot Vassiliev Invariant Algorithm Based on Machine Learning

This paper proposes a knot Vassiliev invariant analysis algorithm based on machine learning. After that, the performance of the algorithm needs to be tested to verify the effectiveness and feasibility of the algorithm. Therefore, this test tests the traditional knot Vassiliev invariants calculation method and knot Vassiliev invariants analysis algorithm technology based on machine learning respectively through comparison of comparative tests.

In the test, in order to eliminate the interference of external factors, this test uses the same computing processing equipment, and simultaneously calculates the knot Vassiliev invariant for the same knot image. The test is to record the corresponding performance data and make statistics on the following performance indicators. The test results are shown in Table 1.

(1) Definition of precision: In the calculation of knot Vassiliev invariants, it refers to the correct proportion of positive samples predicted. Therefore, the definition of precision is the proportion of the results guessed correctly by the model in all results. The more guessed correctly, the higher the score.

(2) Definition of recall: When calculating the knot Vassiliev invariant, it refers to the correct proportion of the samples predicted to be positive samples. In all positive classes, the model is checked to see how much it can find correctly. The more people are found, the higher the recall rate.

(3) Definition of F1 score: This value represents the accuracy and robustness of the algorithm. The maximum value is 1 and the minimum value is 0. The higher the F1 score, the better the performance of the algorithm, and the better the comprehensive performance of knot Vassiliev invariant analysis.

Table 1. Performance comparison of different knot Vassiliev invariants calculation methods

	Traditional knotted Vassiliev invariant calculation method	Machine learning-based knotted Vassiliev invariant analysis algorithm
Precision	86.7%	93.2%
Recall	85.4%	92.1%
F1 Score	0.862	0.932

It can be seen from the test results in Table 1 that the accuracy rate, recall rate and F1 score of the traditional knot Vassiliev invariant calculation method are 86.7%, 85.4% and 0.862 respectively. The accuracy, recall and F1 score of knot Vassiliev invariant analysis algorithm based on machine learning are 93.2%, 92.1% and 0.932 respectively. Obviously, the performance indicators of knot Vassiliev invariant analysis algorithm based on machine learning are better than those of traditional knot Vassiliev invariant calculation methods. The above data shows that the knot Vassiliev invariant analysis algorithm based on machine learning has higher analysis accuracy, higher recall and better comprehensive performance. To sum up, the knot Vassiliev invariant analysis algorithm based on machine learning is better than the traditional knot Vassiliev invariant calculation technology.

4.2 Test Experiment of Vassiliev Invariants and Knot Similarity

(1) Investigation direction

After completing the performance test of the algorithm, it is also necessary to test and analyse the Vassiliev invariants and knot similarity in the actual environment. Through this test, the performance of the knot Vassiliev invariant analysis algorithm based on machine learning in real environments can be explored.

(2) Investigation content

This survey is mainly divided into three levels. One is to test and count the accuracy of the knot Vassiliev invariant analysis algorithm based on machine learning in its analysis of various characteristics in the knot image during operation. The second is to test the running performance of the knot Vassiliev invariant analysis algorithm based on machine learning at runtime. Finally, a questionnaire survey is conducted to count the opinions of knot researchers on the algorithm. Through the investigation and test of the above three dimensions, the purpose is to study the optimization degree of this algorithm compared with the traditional knot analysis and calculation technology, and to comprehensively understand the theoretical and practical significance of the research in this article.

(3) Investigation methods

In this investigation and test, scientific comparative test methods are adopted. Through this scientific experimental method, the preciseness and effectiveness of the test investigation are ensured.

(4) Investigation findings

In the process of investigation, first, the accuracy of the traditional knot Vassiliev invariant computing technology and knot Vassiliev invariant analysis algorithm based on machine learning in their analysis of various characteristics in knot images is tested and counted. Among them, the chiral characteristics, the number of intersections and knots in the knot image are analysed and calculated. The characteristics of these knots are very important in the similarity analysis of knots. Therefore, the accuracy of prediction and analysis of these characteristics is more important. The result is shown in Figure 5.

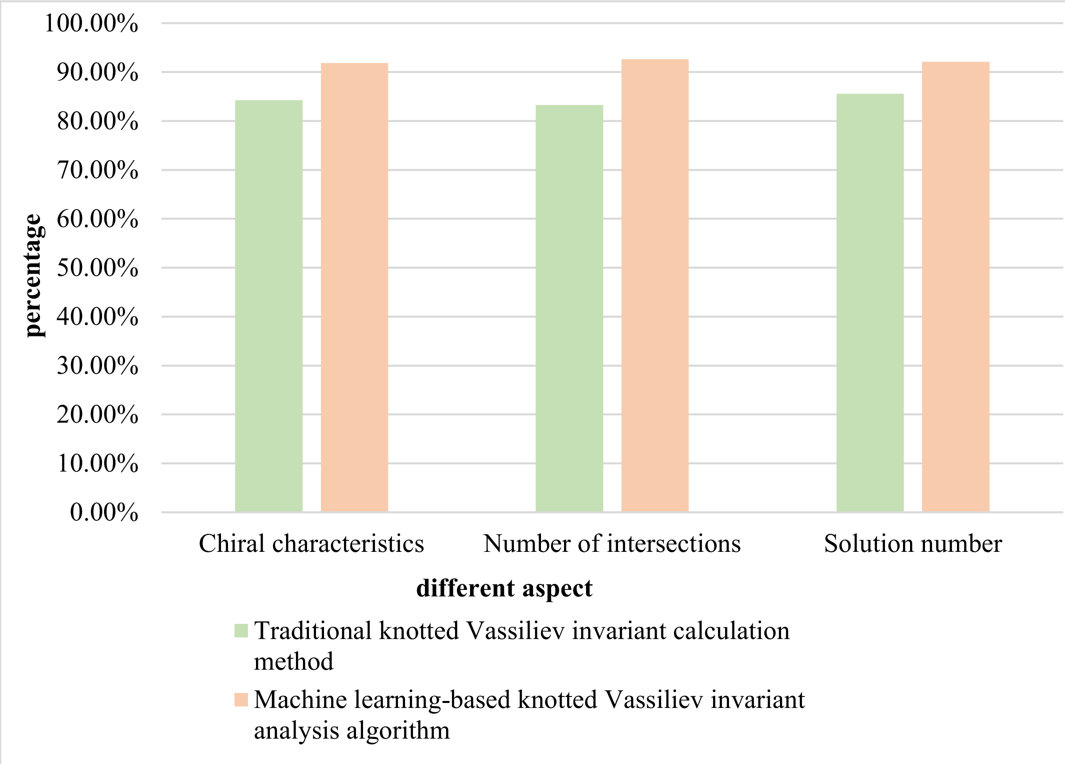


Figure 5. Analysis accuracy of each characteristic in knot image by two technologies

It can be seen from Figure 5 that during the operation of the two algorithms, the accuracy of the traditional knot Vassiliev invariant calculation technology in analysing the chiral characteristics, the number of intersections and the number of knots in the knot image is 84.25%, 83.27% and 85.56% respectively. The accuracy of knot Vassiliev invariant analysis algorithm based on machine learning for these indicators is 91.87%, 92.66% and 92.12% respectively. It can be seen that the knot Vassiliev invariant analysis algorithm based on machine learning has a higher analysis accuracy rate for each characteristic in the knot image. This shows that the analysis algorithm is more accurate and efficient in the process of operation.

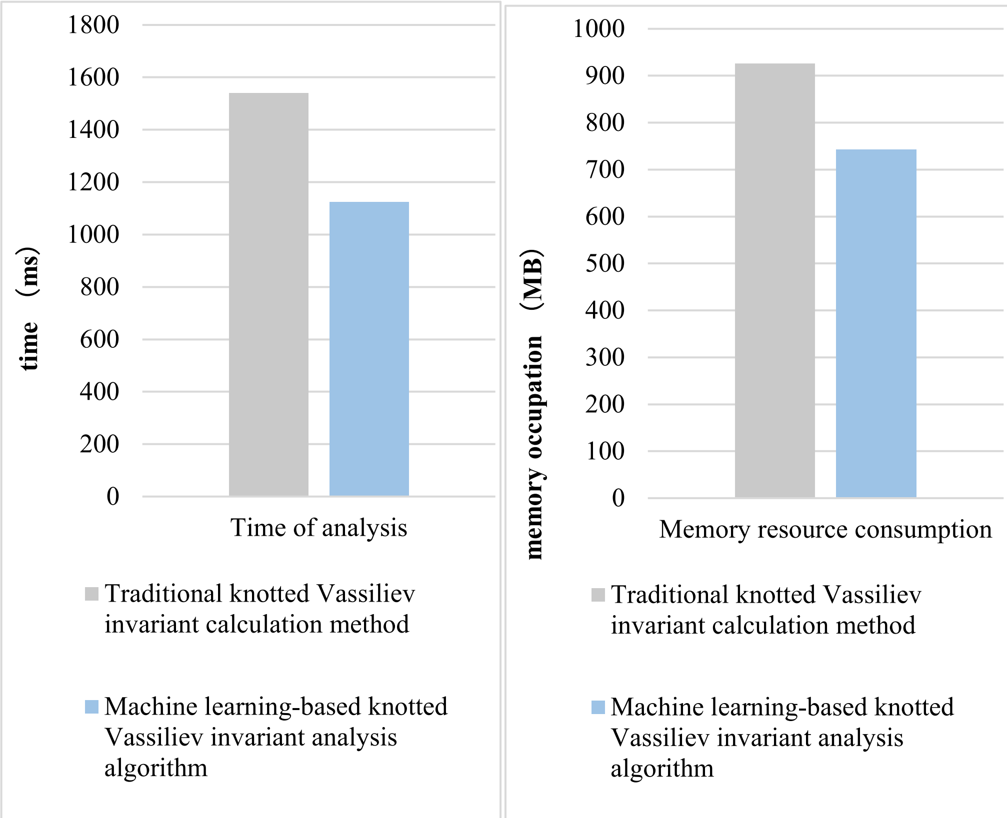
Next, the second performance test is carried out. It is also necessary to test the running performance of the knot Vassiliev invariant analysis algorithm based on machine learning at runtime and compare it with the traditional knot Vassiliev invariant calculation technology. Among them, the selected operation performance indicators are shown below, and the detection results are shown in Figure 6.

The performance indicators of these algorithms are explained as follows:

Analysis time: When analysing the data of knot images, different algorithms consume different time, and the analysis time has a greater impact on the analysis experience of knot images. The analysis time should be as short as possible.

Memory resource consumption: When running the knot image analysis algorithm, the computing processing equipment occupies and consumes memory. For the algorithm, it is better if the memory resource is smaller.

Among them, Figure 6 (a) is a comparison chart of analysis time, and Figure 6 (b) is a comparison chart of memory resource consumption.



(a) Comparison chart of analysis time (b) Comparison chart of memory resource consumption

Figure 6. Comparison diagram of operation performance indexes of two technologies

According to Figure 6, the analysis time of traditional knot Vassiliev invariant computing technology and knot Vassiliev invariant analysis technology based on machine learning is 1541ms and 1124ms respectively. In addition, their respective memory resource consumption is 926MB and 742MB respectively. The results show that the knot Vassiliev invariant analysis algorithm based on machine learning has shorter analysis time and can perform knot analysis more quickly. In addition, its memory resources are smaller, and its performance is better. To sum up, the knot Vassiliev invariant analysis algorithm proposed in this paper based on machine learning has better analysis time and memory resource consumption than traditional computing methods.

After that, a questionnaire survey is also required. Many researchers in the field of knots are investigated by questionnaire, and their questionnaire selection data are counted to investigate their views on the effect of knot Vassiliev invariant analysis algorithm based on machine learning. The survey results are shown in Figure 7.

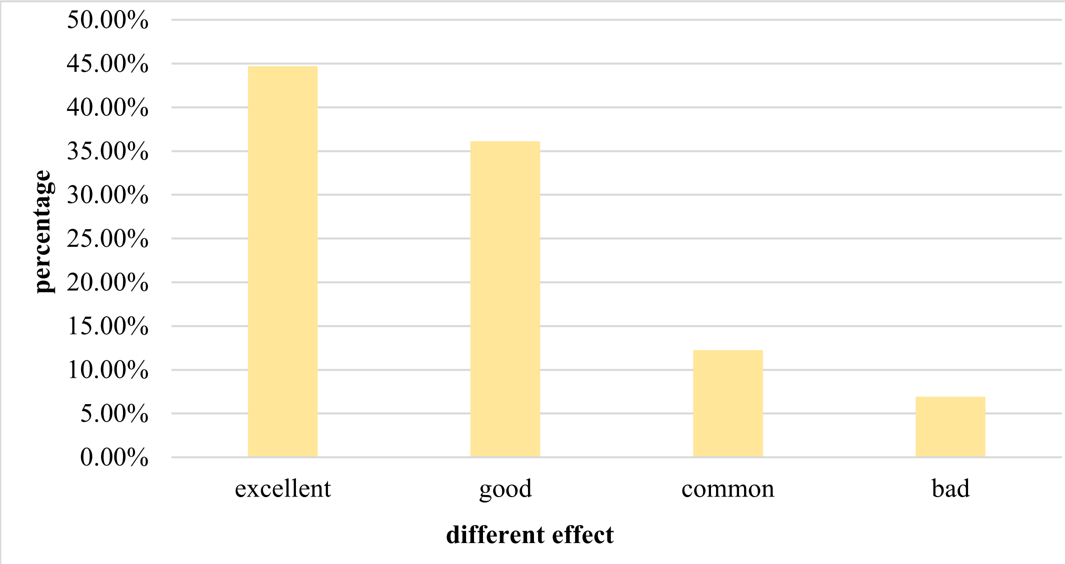


Figure 7. Survey of knotting researchers' views on the effect of algorithms

According to the results in Figure 7, 44.71% of researchers believe that the algorithm has excellent results. 36.12% of the researchers believe that the algorithm has a good effect. Only 6.91% of researchers believe that the algorithm is not effective. Obviously, many researchers have expressed positive views on the knot Vassiliev invariant analysis algorithm based on machine learning, which shows that the algorithm has been recognized by most knot research experts.

Finally, a further questionnaire survey is needed to investigate the specific role of the knot Vassiliev invariant analysis algorithm based on machine learning in knot research. Several researchers in the field of knots are investigated and their opinions are counted to evaluate the effect of the knot Vassiliev invariant analysis algorithm based on machine learning in knot research. The survey results are shown in Figure 8.

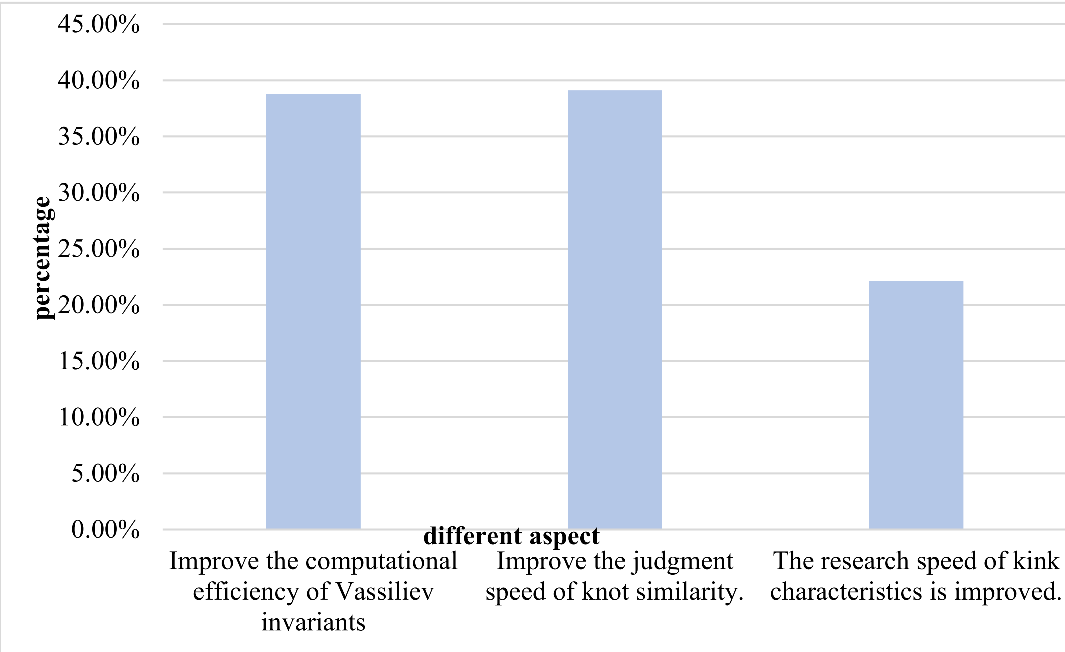


Figure 8. Survey of knotting researchers on the specific role of algorithms

From the survey results in Figure 8, it can be seen that 38.75% and 39.11% of researchers believe that the algorithm can improve the calculation efficiency of Vassiliev invariants and the judgment speed of knot similarity, accounting for a large number of people. In addition, 22.14% of researchers believe that the algorithm can improve the research speed of knot characteristics. Through the above questionnaire results, the role of this algorithm in the field of knot research can be clearly understood.

From the above all-round test results, it can be effectively explained that the knot Vassiliev invariant analysis algorithm based on machine learning is more accurate in analysing the characteristics of knot images, and its analysis speed and memory resource occupation have also been significantly optimized. This also shows that machine learning technology has played an obvious role in the field of knot similarity and Vassiliev invariant analysis. To sum up, the research on the data scientific analysis of Vassiliev invariants and knot similarity based on machine learning technology has important practical and reference value.

5. Conclusions

At present, knot theory is developing and deepening, and has become one of the most active mathematical research directions. The application of knot theory is very important, at the same time, its theoretical research is also very difficult. The machine learning technology proposed in this paper was used to study knot similarity and Vassiliev invariants. Based on effective test data, it was proved that the effect was better in the real research environment. This research believed that after using the knot Vassiliev invariant analysis algorithm based on machine learning, it could effectively improve the analysis accuracy of various characteristics of knots. It could also improve the analysis speed and reduce the memory resource consumption of analysis. Through excellent machine learning algorithms, the research work of knots could be more effectively faced. However, the research method of knot similarity and Vassiliev invariants based on machine learning has many advantages. However, this technology also has shortcomings. Because machine learning technology is required, a powerful processor is required to conduct complex modelling and analysis of input knot data. Therefore, in order to use the machine learning algorithm to study knots, it is necessary to ensure that the processor has strong processing power. In a word, this topic has applied machine learning technology to the research of Vassiliev invariants and knot similarity, thus optimizing and improving the traditional knot research method, and its effect is remarkable. In addition, the research results of this topic also expand the application field of machine learning.

Data availability statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

Funding

The author(s) received no financial support for the research, authorship, and/or publication of this article.

References

[1] Cheng, Zhiyun. "A transcendental function invariant of virtual knots." Journal of the Mathematical Society of Japan 69.4 (2017): 1583-1599.

[2] Budney, Ryan. "Embedding calculus knot invariants are of finite type." Algebraic & Geometric Topology 17.3 (2017): 1701-1742.

[3] Kim, Se-Goo, and Charles Livingston. "Secondary upsilon invariants of knots." The Quarterly Journal of Mathematics 69.3 (2018): 799-813.

[4] Jeong, Myeong-Ju, and Chan-Young Park. "Delta Moves and Arrow Polynomials of Virtual Knots." Kyungpook Mathematical Journal 58.1 (2018): 183-202.

[5] Nelson, Sam, Natsumi Oyamaguchi, and Radmila Sazdanovic. "Psyquandles, singular knots and pseudoknots." Tokyo Journal of Mathematics 42.2 (2019): 405-429.

[6] Ekholm, Tobias, Lenhard Ng, and Vivek Shende. "A complete knot invariant from contact homology." Inventiones mathematicae 211.3 (2018): 1149-1200.

[7] Aistleitner, Christoph, and Bence Borda. "Quantum invariants of hyperbolic knots and extreme values of trigonometric products." Mathematische Zeitschrift 302.2 (2022): 759-782.

[8] Etnyre, John, David Vela-Vick, and Rumen Zarev. "Sutured Floer homology and invariants of Legendrian and transverse knots." Geometry & Topology 21.3 (2017): 1469-1582.

[9] Ito, Tetsuya. "Applications of the Casson-Walker invariant to the knot complement and the cosmetic crossing conjectures." Geometriae Dedicata 216.6 (2022): 1-15.

[10] Bettin, Sandro, and Sary Drappeau. "Modularity and value distribution of quantum invariants of hyperbolic knots." Mathematische Annalen 382.3 (2022): 1631-1679.

[11] Bar-Natan, Dror, and Roland van der Veen. "A polynomial time knot polynomial." Proceedings of the American Mathematical Society 147.1 (2019): 377-397.

[12] Baldwin, John, and David Vela-Vick. "A note on the knot Floer homology of fibered knots." Algebraic & geometric topology 18.6 (2018): 3669-3690.

[13] Bar-Natan, Dror, and Zsuzsanna Dancso. "Finite type invariants of w-knotted objects II: tangles, foams and the Kashiwara–Vergne problem." Mathematische Annalen 367.3 (2017): 1517-1586.

[14] Korablev, F. G. "Quasoids in knot theory." Proceedings of the Steklov Institute of Mathematics 303.1 (2018): 156-165.

[15] Jeong, Myeong-Ju, Chan-Young Park, and Maeng Sang Park. "Polynomials and homotopy of virtual knot diagrams." Kyungpook Mathematical Journal 57.1 (2017): 145-161.

[16] Douglas, Michael R. "Machine learning as a tool in theoretical science." Nature Reviews Physics 4.3 (2022): 145-146.

[17] Shang, Chao, and Fengqi You. "Data analytics and machine learning for smart process manufacturing: Recent advances and perspectives in the big data era." Engineering 5.6 (2019): 1010-1016.

Chenggang Huo was born in Dezhou, Shandong, P.R. China, in 1980. He received the master's degree from Liaoning Normal University, P.R. China. Now, he works in School of mathematics and statistics, Suzhou University.

Email: huochenggang_abc @163.com