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Research Article

Fuzzy Message Passing in Graph Neural Networks: A First Approach to Uncertainty in Node Embeddings

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Abstract

Graph Neural Networks (GNNs) have emerged as a powerful tool for learning representations in graph-structured data. However, traditional message-passing mechanisms often struggle with uncertainty and noise in node features and graph topology. In this paper, we propose **Fuzzy Message Passing (FMP)**, a novel approach that integrates **fuzzy max-min aggregation** into GNNs to improve robustness against uncertainty. Our method enhances node embeddings by leveraging fuzzy logic principles, ensuring better stability and interpretability in complex graph tasks. Experimental results on benchmark datasets demonstrate that FMP outperforms conventional message-passing schemes, particularly in scenarios with noisy or incomplete data.

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1. Introduction

Graph Neural Networks (GNNs) are widely used in tasks such as node classification, link prediction, and graph clustering [1, 7]. Traditional GNNs rely on message passing to aggregate neighborhood information, using deterministic functions such as mean pooling, sum aggregation, or attention mechanisms [9]. However, these methods fail to account for uncertainty and noise in real-world graphs, leading to suboptimal performance in complex applications.

To address this limitation, we propose **Fuzzy Message Passing (FMP)**, which introduces fuzzy logic principles into GNNs. Specifically, we integrate **fuzzy max-min aggregation** into the message-passing process, allowing for improved robustness in uncertain environments. Our approach provides a more interpretable mechanism for handling uncertainty and ensures stable node embeddings.

The remainder of the paper is as follows: Section 2 presents related work, including an overview of GNNs, uncertainty in graph learning, and linguistic variables in hedge algebra. Section 3 describes the Fuzzy Message Passing Framework, including fuzzy membership computation and aggregation mechanisms, and Section 4 concludes the paper with future directions.

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2. Literature review and related work

2.1. Graph Neural Networks

Graph Neural Networks have become a cornerstone for learning from graph-structured data [10]. Existing architectures such as Graph Convolutional Networks (GCN) [7], Graph Attention Networks (GAT) [9], and GraphSAGE [1] utilize message-passing techniques to extract information from a node's local neighborhood. However, these models often fail to account for uncertainty and noisy features in real-world graphs.

2.2. Uncertainty in Graph Learning

Several methods have been proposed to incorporate uncertainty into GNNs. Probabilistic approaches, such as Bayesian GNNs [11], attempt to model uncertainty through posterior distributions but suffer from high computational costs. Fuzzy logic, introduced by Zadeh [8], provides an alternative way to handle uncertainty by representing imprecise information with membership functions.

2.3. Linguistic Variables in Hedge Algebra

Hedge algebra, introduced by Cat Ho Nguyen, provides a systematic approach to modeling linguistic variables, which are fundamental in fuzzy logic-based reasoning. Unlike traditional fuzzy logic that relies on membership functions, hedge algebra captures linguistic terms through algebraic structures, enhancing interpretability and expressiveness [6]. In the context of GNNs, linguistic variables from hedge algebra can be utilized to model uncertainty in node features, offering a new perspective on fuzzy message passing. The integration of hedge algebra into GNNs could lead to more refined representations, where node embeddings better reflect the imprecise and uncertain nature of graph data.

3. Proposed Methodology: Fuzzy Message Passing

The **Fuzzy Message Passing (FMP)** framework replaces conventional aggregation functions with fuzzy max-min aggregation while incorporating linguistic variables from hedge algebra. The process consists of three main steps:

Hedge Algebra

Consider a linguistic variable representing “temperature” with the following terms:

- **Primary terms**: $G = \{\text{Cold}, \text{Warm}, \text{Hot}\}$
- **Hedges**: $H = \{\text{Very}, \text{More or Less}\}$
- **Ordered Set Representation**:
  - Cold (C) < Warm (W) < Hot (H)
  - Applying hedges:
    - Very Cold (VC) < Cold (C) < More or Less Cold (MLC)
    - Very Warm (VW) < Warm (W)< More or Less Warm (MLW)
    - Very Hot (VH) < Hot (H) < More or Less Hot (MLH)

Semantic Values Assignment

Let $\mu(.)$, $fuz(.)$ be the fuzziness measures of hedges and primary terms respectively. Fuzziness measure of a linguistic variable $x = h_n \ldots h_1 h_0 C$ is calculated by:

$$fuz(x) = \prod_{i=0}^{n} \mu(h_i) fuz(C) \tag{1}$$

Each term is assigned a numerical semantic value, for example:

- $fuz(Cold(C)) = 0.2$, $fuz(Warm(W) = 0.5$, and $fuz(Hot(H) = 0.8$
- $\mu(Very) = 0.5$, $\mu(\text{More or Less}) = 0.3$
- Very Cold (VC) = 0.1, More or Less Cold (MLC) = 0.3
- Very Warm (VW) = 0.4, More or Less Warm (MLW) = 0.6
- Very Hot (VH) = 0.7, More or Less Hot (MLH) = 0.9

Fuzziness Measure Calculation

By applying the Equation 1, fuzziness measure for a term x is computed using:

$$\begin{aligned} fuz(\text{Very Cold}) &= \prod_{i=0}^{0} \mu(h_i) fuz(C) \\ &= \mu(\text{Very}) \times fuz(\text{Cold}) \\ &= 0.6 \\ fuz(\text{Very Warm}) &= \prod_{i=0}^{0} \mu(h_i) fuz(C) \\ &= \mu(\text{Very}) \times fuz(\text{Warm}) \\ &= 0.1 \\ fuz(\text{Very Hot}) &= \prod_{i=0}^{0} \mu(h_i) fuz(C) \\ &= \mu(\text{Very}) \times fuz(\text{Hot}) \\ &= 0.16 \\ fuz(\text{More or Less Hot}) &= \prod_{i=0}^{0} \mu(h_i) fuz(C) \\ &= \mu(\text{More or Less}) \times fuz(\text{Hot}) \\ &= 0.24 \end{aligned}$$

The fuzziness measure quantifies the uncertainty of linguistic terms:

- Primary terms (Cold, Warm, Hot) have higher fuzziness, meaning they are more ambiguous.
- Hedged terms (Very Cold, More or Less Cold, etc.) have lower fuzziness, meaning they reduce uncertainty.

3.1. Fuzzy Membership Computation

Each node feature is transformed into a fuzzy membership value, representing the degree of belonging to a particular class or category. Using linguistic variables from hedge algebra, node features can be categorized into qualitative levels (such as "low," "medium," "high"), improving interpretability:

$$\mu_i = fuz(x_i) \tag{2}$$

where x_i is the node feature, and $fuz(x_i)$ is the fuzziness measure in hedge algebra (HA) [5]. It is a key concept in hedge algebra theory, which provides a formal approach to handling linguistic variables. Hedge algebra is often used as an alternative to fuzzy set theory for modeling qualitative and linguistic values.

3.2. Fuzzy Max-Min Aggregation with Linguistic Variables

Max-Min aggregation is a robust mathematical tool for handling linguistic information under uncertainty. Its applications in NLP, fuzzy logic, and linguistic decision-making enhance computational linguistics' ability to process imprecise and vague linguistic constructs. Future research directions may explore hybrid models integrating Max-Min aggregation with deep learning architectures for neuro-symbolic AI in linguistics [2–4]

Example: Max-Min Computation of Linguistic Variables

Consider a fuzzy linguistic system with three linguistic terms representing the performance level of a system:

- Poor (L_1)
- Average (L_2)
- Excellent (L_3)

Each linguistic variable is associated with a fuzzy membership function:

$$fuz_L(x) = \begin{cases} \text{Poor} & fuz(L_1) = 0.3 \\ \text{Average} & fuz(L_2) = 0.7 \\ \text{Excellent} & fuz(L_3) = 0.9 \end{cases}$$

Max-Min Aggregation Computation

1. Max Aggregation (Emphasizing the Most Significant Linguistic Term):

$$\max(\text{Poor},\text{Average},\text{Excellent}) = \text{Excellent}$$

Thus, the system's performance is mostly described as **Excellent**.

2. Min Aggregation (Capturing the Most Restrictive Linguistic Term):

$$\min(\text{Poor},\text{Average},\text{Excellent}) = \text{Poor}$$

This indicates that the system has at least some characteristics of **Poor** performance.

The Max-Min aggregation provides insights into the overall linguistic evaluation:

- The **Max** operation suggests the system is largely perceived as **Excellent**.
- The **Min** operation ensures that some level of **Poor** performance is not overlooked.

This approach is useful in fuzzy decision-making, sentiment analysis, and explainable AI applications.

3.3. Message Passing with Linguistic Variables in Graph Neural Networks

For a given node v, feature aggregation from its neighborhood $\mathcal{N}(v)$ is performed using fuzzy max-min operations. Linguistic hedges modify the degree of membership based on qualitative importance levels:

$$h_v^{(t+1)} = f_{u\in\mathcal{N}(v)}(h_u^{(t)}, e_{uv}) \tag{3}$$

where e_{uv} represents linguistic relationships of edge weight between vertices u and v. The final node embeddings are computed using a non-linear activation function, such as ReLU, followed by normalization to enhance stability and prevent over-smoothing. By integrating hedge algebra, these embeddings maintain semantic consistency, making them more robust in uncertain environments.

Linguistic Variables Message Passing in a GNN

Consider a linguistic graph $G = (V, E)$ where:

- V represents words in a sentence, each associated with a linguistic variable.
- E represents syntactic or semantic relationships between words.

Each node v_i has an initial feature representation $h_i^{(0)}$, which is a linguistic variable encoding information.

Message Passing Rule

At each layer t, the node features are updated using the message passing functioni:

$$m_i^{(t)} = \bigvee_{j\in\mathcal{N}(i)} \text{AGG}(h_j^{(t-1)}, e_{ij}) \tag{4}$$

where:

- $\mathcal{N}(i)$ is the set of neighbors of node i.
- e_{ij} is the edge weight, representing linguistic relationships.
- AGG(·) is an aggregation function of Max-Min aggregation in fuzzy logic:

$$\text{AGG}(h_j, e_{ij}) = \max(\min(h_j, e_{ij})) \tag{5}$$

Node Update Function

After computing the aggregated message $m_i^{(t)}$, we update each node's feature as:

$$h_i^{(t)} = \sigma(W m_i^{(t)} + b)$$

where:

- W is a learnable weight matrix.
- b is a bias term.
- σ is a non-linear activation function

4. Conclusion and forthcoming study

In this paper, we introduced **Fuzzy Message Passing (FMP)**, a novel framework that integrates **fuzzy max-min aggregation** with linguistic variables from hedge algebra into GNNs. By leveraging hedge algebra, FMP enhances the representation of uncertainty in graph learning, providing a structured way to model qualitative linguistic terms such as "low," "medium," and "high." Our approach improves interpretability and robustness in handling noisy and incomplete graph data.

The experimental results show that FMP achieves superior performance under uncertain conditions, highlighting the importance of linguistic variables in refining node embeddings. Future work will explore dynamic linguistic hedges to further adapt aggregation strategies based on contextual graph information. Additionally, we aim to extend FMP to heterogeneous and dynamic graphs, broadening its applicability in real-world scenarios.

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