

# Research on Green Supply Chain Game of Joint Decision-making of Carbon Emission Reduction Effort and Inventory

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**Abstract:** Greenhouse gas emissions have caused serious environmental problems, which are gradually affecting social and economic development. This paper takes the environmental perspective into supply chain decision-making and researches the two-level green supply chain game problem of joint decision-making of carbon emission reduction effort and inventory. First, a centralized supply chain decision-making model is built, proving that carbon emission reduction effort can increase the optimal order quantity of the centralized supply chain system, and the increase of demand variability will reduce the optimal profit of the centralized supply chain system. Secondly, in the manufacturer-led Stackelberg game model, the equilibrium solution and its existence conditions are obtained by using the backward induction method, and proving that the manufacturer's carbon emission reduction is beneficial to itself. Finally, calculating numerical examples to verify the above results by Matlab.

**Keywords:** Green supply chain; Demand variability; Dynamic games; Carbon reduction effort; Inventory decisions

## 1 Introduction

Global warming is one of the serious problems facing human society today, and the emission of carbon dioxide and other greenhouse gases is considered to be the main cause of global warming [1]. According to the State Council's "14th Five-Year Plan" for energy saving and emission reduction, carbon reduction actions should be promoted in key industries such as steel and iron. To protect the environment, the manufacturer in the supply chain can increase investment to produce green products. At the same time, more and more consumers are showing a preference for green products [2] and are willing to pay a higher price for green products than ordinary products [3]. In addition, demand variability causes a great challenge to matching supply and demand, so it is important to explore the green supply chain game problem of joint decision-making of carbon emission reduction effort and inventory under stochastic demand.

Several scholars have explored the supply chain decision problem of the manufacturer's carbon reduction effort under deterministic demand. Zhang et al.[4] researched the influence of consumers' low-carbon consumption awareness on supply chain decisions by comparing the optimal decision of the supply chain system under three models. Benjaafar et al. [5] researched the decision problem of supply chain enterprises under different carbon regulatory policies. Zhang et al. [6] studied a two-level supply chain in which the manufacturer reduces carbon emissions and the retailer advertises under deterministic demand. However, most of the

literature have only considered the issue of carbon reduction effort decisions under deterministic demand. In this paper, we research the joint decision-making of inventory and carbon reduction effort of a two-level green supply chain system under stochastic demand and analyze the impact of demand variability on the supply chain system.

In addition, several scholars have researched the impact of market demand variability on supply chain systems. Song [7] researched the impact of stochastic lead time on the system. Gerchak and Mossman [8] researched the effect of demand variability on optimal inventory levels and optimal costs through the mean-preserving transformation. Yu et al. [9] proposed a demand function whose demand depends on the information-gathering effort and built three models, which show that retailer's information-gathering benefits manufacturers.

The main research contributions of this paper include the following two aspects: (i) Extending the deterministic demand function proposed by Cui et al.[10] , proposing a stochastic demand function, and obtaining the optimal decision and the optimal profit of the centralized supply chain system; (ii) Under the manufacturer-led Stackelberg model, giving the equilibrium solution and the optimal profit of the manufacturer and retailer and proving that the increase of the manufacturer's carbon emission reduction effort will increase its wholesale price.

## 2 Centralized supply chain system

Consider an inventory system that produces a single cycle of a single product, with no inventory in the inventory system until the sales cycle begins. Assuming that people have a preference for green products, the supply chain integrator can increase market demand by taking carbon reduction effort in a stochastic market demand environment. The level of carbon reduction effort is denoted as  $\tau$ ,  $\tau \geq 0$ , so that the market demand  $D(\tau)$  is a function of the level of carbon reduction effort  $\tau$  and a stochastic factor  $X$ ,  $D(\tau)$  is given by equation (1),

$$D(\tau) = d(\tau) + X_{\alpha} \quad (1)$$

where  $d(\tau)$  is an increasing function of  $\tau$ ,  $X_{\alpha} = \alpha X + (1-\alpha)\mu$ ,  $X$  is a random variable defined on the interval  $[\underline{\ell}, \bar{\ell}]$  and obeying a general probability distribution with mean  $\mu$ . The cumulative and inverse distribution functions of the random variable  $X$  are  $F(\cdot)$  and  $F^{-1}(\cdot)$ , the probability density function is  $f(\cdot)$ , and the  $F(\cdot)$  is strictly monotonically increasing.

The carbon emission reduction cost of the supply chain integrator is  $\varphi(\tau)$ , which is a strictly increasing convex function of  $\tau$ , i.e.  $\varphi'(\tau) > 0$  and  $\varphi''(\tau) > 0$ . When the sales cycle begins, the supply chain integrator decides to order products at a unit price  $c$ , and the order quantity is denoted as  $q$ . When  $q$  is greater than the realization of the market demand, the remaining products are subject to price reduction at unit price  $v$ . When  $q$  is less than the realization of market demand, no out-of-stock penalty is considered. Assuming that the delivery lead time for the product is zero and that no fixed ordering costs are considered. The market retail price of each product is  $p$ ,  $p > c > v$ .

The objective of a centralized supply chain system is to determine the level of carbon reduction effort  $\tau$  and the order quantity  $q$  of the product that maximizes its expected profit, i.e. equation (2),

$$\max_{\tau \geq 0, q \geq 0} \pi_c(\tau, q) = E[\Pi(q, D(\tau))], \quad (2)$$

here,  $\Pi(q, D(\tau))$  is given by equation (3),

$$\Pi(q, D(\tau)) = p \min(q, D(\tau)) + v(q - D(\tau))^+ - cq - \varphi(\tau), \quad (3)$$

and  $(x)^+ = \max\{x, 0\}$ . In equation (3), the first term is sales revenue, the second term is sales surplus, the third term is ordering cost, and the last term is carbon reduction cost.

**Remark 1.** In particular, when  $d'(\tau) > 0$  and  $\alpha = 0$ , problem (2) is a centralized supply chain system model considering carbon reduction effort under deterministic demand.

Let the optimal carbon reduction effort level of the centralized supply chain system is  $\tau^c$ , the optimal order quantity is  $q^c$ , and the optimal profit is  $\pi_c(\tau^c, q^c)$  and  $\rho = (p - c)/(p - v)$ ,  $0 < \rho < 1$ . The following Theorem 1 gives the relevant propositions of the centralized supply chain system.

**Theorem 1.** Consider the centralized supply chain system of equation (2),

(i) If  $\varphi''(\tau) > 0$  and  $d''(\tau) \leq 0$  hold for any  $\tau \geq 0$ , then the expected profit of the centralized supply chain system  $\pi_c(\tau, q)$  is a joint concave function of  $(\tau, q)$  and the optimal solution  $(\tau^c, q^c)$  exists and are given by equation (4) and (5),

$$(p - c)d'(\tau^c) - \varphi'(\tau^c) = 0, \quad (4)$$

$$q^c = \alpha F^{-1}(\rho) + d(\tau^c) + (1 - \alpha)\mu, \quad (5)$$

here,  $A(\tau, q)$  is given by equation (6),

$$A(\tau, q) = [q - d(\tau) - (1 - \alpha)\mu] / \alpha, \quad (6)$$

(ii) The optimal profit  $\pi_c(\tau^c, q^c)$  of the centralized supply chain system is given by equation (7)

$$\pi_c(\tau^c, q^c) = (p - v)[\alpha GL_X(\rho) + \rho(d(\tau^c) + (1 - \alpha)\mu)] - \varphi(\tau^c), \quad (7)$$

here,  $GL_X(\rho)$  is given by equation (8),

$$GL_X(u) = \int_{\xi}^{F^{-1}(u)} (u - F(x)) dx + u\xi, \gamma \in [0, 1] \quad (8)$$

(iii) When demand variability  $\alpha$  is given, the optimal order quantity  $q^c$  of the centralized supply chain system increases as the level of carbon reduction effort  $\tau$  increases; when  $\tau$  is given, the optimal order quantity  $q^c$  increases as the mean value of demand increases.

(iv) When the level of carbon reduction effort  $\tau$  is given, changes in demand variability  $\alpha$  cause changes in the optimal order quantity. When  $0 < \rho \leq F(\mu)$ , the increase of demand variability  $\alpha$  will reduce the optimal order quantity  $q^c$  of low-profit products [11]; when  $F(\mu) < \rho < 1$ , the increase of demand variability  $\alpha$  will increase the optimal order quantity  $q^c$  of high-profit products [11].

(v) When the demand variability  $\alpha$  increases, the optimal expected profit of the centralized supply chain system decreases.

**Proof:** (i) Equation (2) can be rewritten as equation (9),

$$\pi_c(\tau, q) = (p - c)q - \alpha(p - v) \int_{\xi}^{A(\tau, q)} F(x) dx - \varphi(\tau) \quad (9)$$

Given  $\tau$ , from equation (9), we can obtain  $\partial\pi_c(\tau,q)/\partial q=p-c-(p-v)F(A(\tau,q))$ ,  $\partial^2\pi_c(\tau,q)/\partial q^2=-(p-v)f(A(\tau,q))/a$ . Given  $q$ , we obtain  $\partial\pi_c(\tau,q)/\partial\tau=(p-v)F(A(\tau,q))d'(\tau)-\varphi'(\tau)$ ,  $\partial^2\pi_c(\tau,q)/\partial\tau^2=(p-v)F(A(\tau,q))d''(\tau)-(p-v)f(A(\tau,q))[d'(\tau)]^2/\alpha-\varphi''(\tau)$ ,  $\partial^2\pi_c(\tau,q)/\partial q\partial\tau=(p-v)f(A(\tau,q))d'(\tau)/a$ . When  $\varphi''(\tau)>0$  and  $d''(\tau)\leq 0$  holds for any  $\tau\geq 0$ , the hessian matrix is negative definite,  $\pi_c(\tau,q)$  is a joint concave function of  $(\tau,q)$  and the optimal solution  $(\tau^c,q^c)$  exists. Let  $\partial\pi_c(\tau,q)/\partial q=0, \partial\pi_c(\tau,q)/\partial\tau=0$ , we can obtain the optimal solution  $(\tau^c,q^c)$ . (ii) Replace  $(\tau^c,q^c)$  in equations (4) and (5) into equation (9), and we obtain the optimal profit of the centralized system. (iii) It can be directly obtained from equation (4). (iv) From equation (4),  $\partial q^c/\partial\alpha=F^{-1}(\rho)-\mu$ . When  $F^{-1}(\rho)>\mu$ , the  $q^c$  is a monotonically increasing function of  $\alpha$ ; when  $F^{-1}(\rho)\leq\mu$ , the  $q^c$  is a monotonically decreasing function of  $\alpha$ . (v) From equation (9), we obtain  $\partial\pi_c(\tau^c,q^c)/\partial\alpha=(p-v)(GL_X(\rho)-\rho\mu)$ , note that  $L_X(\rho)=GL_X(\rho)/\rho, 0<\rho<1$ , we obtain  $\partial L_X(\rho)/\partial\rho=\int_{\underline{\ell}}^{F^{-1}(\rho)} F(x)dx/\rho^2>0$ ,  $L_X(\rho)$  is a monotonically increasing function of  $\rho$  and  $\underline{\ell}<L_X(\rho)<\mu$  holds for any  $0<\rho<1$ , so  $GL_X(\rho)<\rho\mu$  holds for any  $0<\rho<1$ .  $\square$

**Remark 2.** From Theorem 1(iii), the optimal order quantity increases when the carbon emissions of a product decrease, because people prefer environmental products. Theorem 1(iii) shows that when the level of carbon reduction effort and demand variability is given, the optimal order quantity is influenced by the average demand for both high and low-profit products [11], and the higher the average market demand, the higher the optimal order quantity.

### 3 Dynamic decentralized supply chain systems

This section analyses dynamic game model of the decentralized supply chain. In the Stackelberg decentralized decision-making process, due to their different power positions, assuming the manufacturer is the leader who first decides the wholesale price  $w$  of the product to the retailer and the level of carbon reduction effort  $\tau$ , and the retailer is the follower who decides the order quantity  $q$  and both of them are risk-neutral. The rest of the assumptions are the same as in Section 2,  $p>w>c>v$ .

In the first stage, the manufacturer decides the wholesale price  $w$  and the level of carbon reduction effort  $\tau$  that maximizes its profits, its objective function is given by equation (10),

$$\max_{w>c, \tau\geq 0} \pi_m(w, \tau, q)=(w-c)q-\varphi(\tau) \quad (10)$$

In the second stage, the retailer decides on an order quantity  $q$  that maximizes its expected profit, its objective function is given by equation (11),

$$\max_{q\geq 0} \pi_r(w, \tau, q)=E[\Pi_r(q, w, D(\tau))] \quad (11)$$

here,  $\Pi_r(q, w, D(\tau))$  is given by equation (12),

$$\Pi_r(q, w, D(\tau))=p\min(q, D(\tau))+v(q-D(\tau))^+-wq \quad (12)$$

In the manufacturer-led Stackelberg game model, the manufacturer's optimal wholesale price is  $w^*$ , the optimal level of carbon reduction effort is  $\tau^*$ , the retailer's optimal order quantity is  $q^*$ , the manufacturer's optimal profit is  $\pi_m(w^*, \tau^*, q^*)$  and the retailer's optimal profit is  $\pi_r(w^*, \tau^*, q^*)$ . Proposition 1 gives the equilibrium solution  $(w^*, \tau^*, q^*)$  in the Stackelberg game model and the optimal profit of the manufacturer and the retailer.

**Proposition 1.** Consider the manufacturer-led Stackelberg game model,

(i) If  $f(x)$  is a log-concave function,  $d''(\tau) < 0$  and equation

$$2 + \left( \underline{\ell} + (d(\tau) + (1-\alpha)\mu) / \alpha \right) \frac{f'(\underline{\ell})}{f(\underline{\ell})} \geq \frac{(d'(\tau))^2}{(-d''(\tau))(\underline{\ell} + (d(\tau) + (1-\alpha)\mu) / \alpha)} \quad (13)$$

holds for any  $\tau \geq 0$ , then  $(w^*, \tau^*, q^*)$  exists and given by equation (14), (15) and (16),

$$w^* = p - (p-v)F(A(\tau^*, q^*)) \quad (14)$$

$$\frac{(p-v)d'(\tau^*)q^*}{\alpha} f(A(\tau^*, q^*)) - \phi'(\tau^*) = 0 \quad (15)$$

$$p - c - (p-v)F(A(\tau^*, q^*)) - \frac{\phi'(\tau^*)}{d'(\tau^*)} = 0 \quad (16)$$

(ii) The manufacturer's optimal profit  $\pi_m(w^*, \tau^*, q^*)$  is given by equation (17),

$$\pi_m(w^*, \tau^*, q^*) = (p-v)(q^*)^2 f(A(\tau^*, q^*)) / \alpha - \phi(\tau^*) \quad (17)$$

(iii) The retailer's optimal profit  $\pi_r(w^*, \tau^*, q^*)$  is given by equation (18),

$$\pi_r(w^*, \tau^*, q^*) = (p-v)[\alpha GL_X(\rho_0) + \rho_0(d(\tau^*) + (1-\alpha)\mu)] - \phi(\tau^*) \quad (18)$$

here  $\rho_0 = (p-w)/(p-v)$  and  $GL_X(u)$  is given by equation (8).

**Proof:** (i) The Stackelberg dynamic game model is solved using inverse induction. In the second stage of the decision process, equation (11) can be rewritten as equation (19),

$$\pi_r(w, \tau, q) = (p-w)q - \alpha(p-v) \int_{\underline{\ell}}^{A(\tau, q)} F(x) dx \quad (19)$$

given  $\tau$ , from equation (19), we obtain  $\partial \pi_r(w, \tau, q) / \partial q = p - w - (p-v)F(A(\tau, q))$ ,  $\partial^2 \pi_r(w, \tau, q) / \partial q^2 = -(p-v)f(A(\tau, q)) / \alpha < 0$ ,  $\pi_r(w, \tau, q)$  is a strictly concave function of  $q$ , then the optimal order quantity for the retailer exists and is unique, let  $\partial \pi_r(w, \tau, q) / \partial q = 0$ , we can obtain equation (20),

$$w(\tau, q) = p - (p-v)F(A(\tau, q)) . \quad (20)$$

Next solve the first stage of the problem, from equation (20),  $F(x)$  is a strictly monotonic increasing function of  $x$ , so the problem of solving for  $w$  is transformed into an optimization problem for  $q$ , the manufacturer's profit function can be rewritten as equation (21),

$$\pi_m(w(\tau, q), \tau, q) = (p-c - (p-v)F(A(\tau, q)))q - \phi(\tau), \quad (21)$$

given  $q$ , from equation (21), we obtain  $\partial \pi_m(w(\tau, q), \tau, q) / \partial \tau = (p-v)q d'(\tau) f(A(\tau, q)) / \alpha - \phi'(\tau)$ ,  $\partial^2 \pi_m(w(\tau, q), \tau, q) / \partial \tau^2 = -(p-v)q((d'(\tau))^2 f'(A(\tau, q)) / \alpha - d''(\tau) f(A(\tau, q)) / \alpha + \phi''(\tau))$ ; given  $\tau$ , from equation (21),  $\pi_m(w(\tau, q), \tau, q) / \partial q = p - c - (p-v)F(A(\tau, q)) - (p-v)q f(A(\tau, q)) / \alpha$ ,  $\partial^2 \pi_m(w(\tau, q), \tau, q) / \partial q^2 = -(p-v)(q f'(A(\tau, q)) / \alpha + 2f(A(\tau, q))) / \alpha$ , and  $\partial^2 \pi_m(w(\tau, q), \tau, q) / \partial \tau \partial q = (p-v)d'(\tau)(q f'(A(\tau, q)) / \alpha + f(A(\tau, q))) / \alpha$ , where  $A(\tau, q)$  is given by equation (6). When  $f(x)$  is a log-concave function,  $f'(x) \geq 0$ ,  $d''(\tau) < 0$ , and equation (13) holds for any  $\tau \geq 0$ ,  $\pi_m(w(\tau, q), \tau, q)$  is a joint concave function of  $\tau$  and  $q$  and the  $(w^*, \tau^*, q^*)$  exists, let  $\partial \pi_m(w(\tau, q), \tau, q) / \partial \tau = 0$ ,  $\partial \pi_m(w(\tau, q), \tau, q) / \partial q = 0$ , we obtain equilibrium solution. (ii) Substituting equations (14), (15), and (16) in (21), we obtain the manufacturer's optimal profit. (iii) Substituting equations (14), (15), and (16) in (19), we obtain the retailer's optimal profit.  $\square$

**Remark 3.** From equation (14), When manufacturers make more effort to reduce carbon emissions, the wholesale price of manufacturers will also increase, which means that it is beneficial for manufacturers to reduce carbon emissions.

## 4 Numerical examples

By using Matlab software, the results of the previous study are verified. The results are as follows. Example 1 below gives the effect of demand variability  $\alpha$  on the centralized supply chain system when demand follows a uniform distribution.

**Example 1.** Suppose  $p = 10$ ,  $v = 2$ , and  $c$  is taken to be 4 and 8 respectively, i.e. the inventory service level is 0.75 (high-profit products [11]) and 0.25 (low-profit products [11]) respectively. Assume that the random variable  $X$  obeys a uniform distribution defined in the interval  $[-1, 1]$  with mean 0, and its cumulative distribution function and probability density function are denoted as  $F(x) = (x + 1)/2$  and  $f(x) = 1/2$ , respectively,  $x \in [-1, 1]$ . Let the cost function of carbon reduction effort  $\varphi(\tau) = k\tau^2/2$  [12],  $k$  is the cost coefficient of carbon reduction, take  $k = 0.3$ . Let  $d(\tau) = a + \eta\tau$  [10],  $a$  is the potential market demand,  $\eta$  is the low carbon preference coefficient of consumers, take  $\eta = 0.2$  and  $a = 2$ . Table 1 gives the impact on the centralized supply chain system when  $\alpha$  is taken to different values under the uniform distribution.

**Table 1.** Optimal decision-making and optimal profit of a centralized supply chain system with different values of  $\alpha$  under the uniform distribution

$\alpha$	$\rho = 0.75$			$\rho = 0.25$		
	$\tau^c$	$q^c$	$\pi_c(\tau^c, q^c)$	$\tau^c$	$q^c$	$\pi_c(\tau^c, q^c)$
0.10	4.0000	2.8500	14.2500	1.3333	2.2167	4.1167
0.30	4.0000	2.9500	13.9500	1.3333	2.1167	3.8167
0.50	4.0000	3.0500	13.6500	1.3333	2.0167	3.5167
0.70	4.0000	3.1500	13.3500	1.3333	1.9167	3.2167
0.90	4.0000	3.2500	13.0500	1.3333	1.8167	2.9167

**Remark 4.** From Table 1,  $\alpha$  indicates the demand variability, when demand variability increases, the optimal order quantity for low-profit products[11] decreases, and the optimal order quantity for high-profit products increases; the optimal carbon reduction effort does not change with changes in demand variability, which means that demand changes do not affect the incentives of supply chain integrators to reduce carbon emissions. In addition, demand variability is detrimental to supply chain profit and we can increase supply chain profits by taking some measures to reduce demand variability.

## 5 Conclusions

This paper extends the deterministic demand function proposed by Cui et al.[10], proposing a stochastic demand function, and researching the green supply chain game problem of joint decision-making of carbon emission reduction effort and inventory. A model of the centralized supply chain system and a dynamic decentralized system model is built, and equilibrium solutions and its existence conditions in the different models are obtained. The main findings are: (i) In the centralized supply chain system, an increase in the level of carbon emission

reduction effort increases the optimal order quantity; when the level of carbon emission reduction effort remains constant, the optimal order quantity is positively related to the mean value of demand; when demand variability increases, the optimal order quantity of low-profit products decreases, the optimal order quantity of high-profit products increases; the optimal profit of the centralized supply chain system decreases with the increase of demand variability. The optimal profit of the centralized supply chain system under deterministic demand is always greater than the optimal profit under stochastic demand. (ii) In the manufacturer-led Stackelberg game model, when the manufacturer makes more effort to reduce carbon emissions, the manufacturer's wholesale price will also increase.

The issues worthy of further research include: Consider the problem of joint decision-making when manufacturers make carbon reduction effort and retailers make green sales effort, and explore how the optimal profit of the supply chain changes.

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