# An effective method for calculating crossed span distance of overhead line based on three-dimensional inclination algorithm 

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#### Abstract

In order to safely operate the electric transmission lines, to monitoring the crossed span distance of the overhead lines is one of the key measures. Since the current measurement methods of crossed span distance of overhead line has high cost, low accuracy, and traditional measurement methods has low efficiency, the monitoring model of overhead line in threedimensional space is constructed, and based on oblique parabola equation. The algorithm for detecting the crossed span distance based on three-dimensional inclination under the windless and windy conditions is studied, which are applied to the on-line monitoring system. Through actual measurement, the accuracy and practicability of the three-dimensional model and algorithm are verified which provides reliable algorithm support for traverse crossing distance measurement and on-line monitoring system, and improves work efficiency.


Keywords: overhead line, oblique parabola, three dimensional dip.
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## 1. Introduction

In order to safely operate the electric transmission lines, to monitoring the crossed span distance of the overhead lines is one of the key measures. Due to the influence of environment and its own running state, the crossed span distance of the overhead lines will change. Discharge, alternating flashover and tripping accidents may occur when the distance is too short [1-4]. In recent years, as the number of overhead lines has increased, the problems caused by crossed span distance have become more apparent. Therefore, the periodic measurement of the crossing distance of overhead line plays a vital role in the operation and maintenance of transmission lines. At present, the
research on the crossed span distance monitoring of overhead lines at world mainly includes temperature monitoring [5], tension monitoring [6], image monitoring [7] and Unmanned Aerial Vehicle (UAV) monitoring [8], but they have not been widely used due to the high cost, inconvenient installation or high requirements for equipment. At present, the research on the crossed span distance monitoring of overhead lines at world mainly include temperature monitoring, tension monitoring, image monitoring and Unmanned Aerial Vehicle (UAV) monitoring, but they have not been widely used due to the high cost, inconvenient installation or high requirements for equipment. At present, the method of manual measurement is still adopted, such as the theodolite [9], rangefinder [10$11]$ and total station [12-13], which requires the surveyor to
carry the instrument to the site for measurement. These methods are accurate, but the work is heavy, the operation is tedious, and the measurement data cannot be stored and analysed in real time. Therefore, it is urgently needed to study the monitoring method of the crossed span distance of overhead lines, which is easy and cheap to operate, and can meet the requirements of accuracy.

Therefore, this paper studies three-dimensional model and algorithm of the crossed span distance of overhead lines, calculates the crossed span distance of overhead lines under windless and windy conditions according to the change of three-dimensional inclination of the lines, and applies it to the online monitoring system to realize the fast and accurate measurement of the crossing distance of overhead lines.

## 2. Monitoring models and algorithm of line's crossed span distance

### 2.1. Static oblique parabolic equation

The overhead line is suspended between two towers for power transmission. There are three equations to indicate the suspension status: the catenary equation, the oblique parabolic equation and the parabolic equation [15]. As for the catenary equation, it provides the highest accuracy [16], but the calculation is too complicated. On the other hand, the parabolic equation's deviation is too high to accept. In order to calculate the crossed span distance of the overhead line accurately, the authors adopted the oblique parabolic equation to set up a three-dimensional model.

Figure 1 shows the oblique parabolic model of the overhead line, and the equation of the oblique parabolic is:

$$
\begin{equation*}
y(x)=\frac{x^{2}}{2 M \cos \theta} . \tag{1}
\end{equation*}
$$

Where $x$ is $x$-axis coordinates of any point on the curve According to Figure $1, \theta=\tan ^{-1}-$, where $h$ and $l$ are the height difference, and the distance between two towers, respectively, and $M=-$, where $\sigma$ and $g$ are horizontal stress at the lowest point of the line, and gravity specific load.


Figure 1. The diagrammatic sketch of oblique parabolic

For the convenience of calculation, the coordinate origin $(\mathrm{O})$ is moved to the low suspension point (A), so that the equation of the oblique parabolic [17] becomes:

$$
\begin{equation*}
\mathrm{y}(\mathrm{x})=\frac{\left(x-l_{O A}\right)^{2}}{2 M \cos \theta}-h_{O A} . \tag{2}
\end{equation*}
$$

Where is the vertical distance between the lowest point of the line $(\mathrm{O})$ and the suspension point $(\mathrm{A})$,

As shown in Figure $1, l_{O A}+l_{O B}=l, \mathrm{y}\left(l_{O B}\right)-\mathrm{y}\left(-l_{O A}=h\right.$, combining them with equation (1), a new equation is derived:

$$
\left\{\begin{array}{c}
l_{O A}=\frac{l}{2}-M \sin \theta  \tag{3}\\
l_{O B}=\frac{l}{2}-M \cos \theta \\
\mathrm{y}_{1}^{\prime}\left(-l_{O A}\right)=\frac{l_{O A}}{M \cos \theta}=\tan \beta
\end{array} .\right.
$$

If A is set as the low suspension point and $\beta$ is set as A's vertical inclination angle, $M$ and $l_{O A}$ can be derived from equation (3):

$$
\left\{\begin{array}{c}
M=\frac{l}{2(\sin \theta+\cos \theta \tan \beta)}  \tag{4}\\
l_{O A}=\frac{l}{2}-\frac{l \cos \theta}{2(\sin \theta+\cos \theta \tan \beta)}
\end{array}\right.
$$

The oblique parabolic equation is derived by substituting equation (2) with equation (4).

### 2.2 Three-dimensional model and algorithm of crossed span distance under windless condition

## The crossed span distance between the static line

 and the objectWhen the line crosses the object, the crossed span distance is the difference between the line's altitude and the object's height. The height of the object is supposed as $\mathrm{H}_{0}$, the crossed span distance between the line and the object can be derived as below from equation (2):

$$
\begin{equation*}
G(\mathrm{x})=h_{1}+\frac{\left(x-l_{O A}\right)^{2}}{2 M \cos \theta}-\mathrm{h}_{O A}-H_{0} \tag{5}
\end{equation*}
$$

The crossed span distance of two static lines

The following space model shows the crossed span of two static lines, as illustrated in Figure 2.


Figure 2.The diagrammatic sketch of crossed span under windless condition

The arc $A_{2} P_{2} B_{2}$ passes below $A_{1} P_{1} B_{1}, \mathrm{P}_{1}$ and $\mathrm{P}_{2}$ are the crossing points, $\mathrm{P}^{\prime}$ is the projection of $P_{1} P_{2}$ to the ground, $|\mathrm{P} 1 \mathrm{P} 2|$ is the crossing span distance. $A_{1}^{\prime} B_{1}^{\prime}$ and $A_{2}^{\prime} B_{2}^{\prime}$ are the projections of $A_{1} P_{1} B_{1}$ and $A_{2} P_{2} B_{2}$, respectively, with an angle of $\varphi$. Taking A as the origin and establishing a threedimensional Cartesian coordinate system, $A_{I} P_{l} B_{I}$ can be calculated as:

$$
\left\{\begin{array}{c}
\mathrm{z}=\frac{1}{2 M_{1} \cos \theta_{1}}\left(x-l_{O A 1}\right)^{2}-\mathrm{h}_{O A 1}  \tag{6}\\
y=0
\end{array}\right.
$$

$\mathrm{A}_{2}$ is set as coordination $\left(\mathrm{x}_{2}, \mathrm{y}_{2}, \mathrm{z}_{2}\right)$, then the equation of $A_{2} P_{2} B_{2}$ can be translated by $A_{l} P_{l} B_{l}$ around the $z$-axis (rotation angle $\varphi$ ), the equation is:

$$
\begin{equation*}
\mathrm{z}=\frac{\left[\left(x-x_{2}\right) \cos \varphi-\left(y-y_{2}\right) \sin \varphi-l_{O A 2}\right]^{2}}{2 M_{2} \cos \theta_{2}}-\mathrm{h}_{O A 2} . \tag{7}
\end{equation*}
$$

The horizontal plane coordinate of the projection point $\mathrm{P}^{\prime}$ is $\left(y_{2} \cot \varphi+\quad\right)$. Substituting equations (6) and (7) one can obtain the z-axis coordinate and further calculate the crossover distance:

$$
\begin{align*}
\left|P_{1} P_{2}\right|= & \left\lvert\, \frac{\left(y_{2} \cot \varphi+x_{2}-l_{O A 1}\right)^{2}}{2 M_{1} \cos \theta_{1}}-\frac{\left[y_{2}(\operatorname{cotacos} \alpha+\sin \alpha)-l_{O A 2}\right]^{2}}{2 M_{2} \cos \theta_{2}}-\right. \\
& \left(\mathrm{h}_{O A 1}-\mathrm{h}_{O A 2}\right)-z_{2} \mid . \tag{8}
\end{align*}
$$

In equation (8), $M$ and $M$ are load ratios; $\theta$ and $\theta$ are elevation angles; and are vertical distances from the lowest point to the lowest suspension point; The and from the lowest point to the lowest point of suspension point, can be obtained from equations (2) and (4).

### 2.3 Three-dimensional model and algorithm of crossed span distance under wind condition

The crossed span distance between the swinging line and the object
Line swinging is one of the main reasons for line breakage and tower tilting [18]. Under the action of wind, the line swings in a certain range. The three-dimensional model of the line under wind condition is shown in Figure 3.


Figure 3.The diagrammatic sketch of the line under wind condition

Curve APB is the state when the line is swinging, the dotted line AP"B is the projection of APB in the vertical plane (xoz plane), which can be approximated as the static state curve of the line, and $\mathrm{A}^{\prime} \mathrm{P}^{\prime} \mathrm{B}^{\prime}$ is the projection of the line APB on the horizontal plane. If setting it as a parabolic contour, $\alpha$ is the horizontal dip (also called the yaw angle), based on force analysis:

$$
\begin{equation*}
\tan \alpha=\frac{M_{h}}{M_{v}} \tag{9}
\end{equation*}
$$

Where $M_{h}, M_{v}$ are the vertical and horizontal load ratios, respectively, $M_{v} \approx M, M_{h}=M_{v} \tan \alpha$, the horizontal tilt angle $\alpha$ can be measured by the tilt sensor.

When the wind drifts, the expression of line APB is:

$$
\left\{\begin{array}{l}
\mathrm{z}=\frac{1}{2 M_{v} \cos \theta}\left(x-l_{O A}\right)^{2}-h_{O A}  \tag{10}\\
y=\frac{1}{2 M_{h} \cos \theta}\left[\left(x-\frac{l}{2}\right)^{2}-\frac{l^{2}}{4}\right]
\end{array} .\right.
$$

At this time, the height of the line to the ground is only related to the projection $A E$ ' $B$ of the line $A E B$ in the vertical plane, so the distance between any point of the line and the fixed object is:

$$
\begin{equation*}
H(x)=h_{1}+\frac{\left(x-l_{O A}\right)^{2}}{2 M_{v} \cos \theta}-h_{O A}-\mathrm{H}_{0} \tag{11}
\end{equation*}
$$

## The crossed span distance of two swinging lines

The crossed span distance model under the swinging state is shown in Figure 4. At this time, the projections of the two crossed lines on the horizontal plane are $\mathrm{A}_{1}^{\prime} \mathrm{P}^{\prime} \mathrm{B}_{1}^{\prime}$ and respectively. The expression of line $A_{1} P_{1} B_{1}$ is obtained from equation (10):

$$
\left\{\begin{array}{c}
\mathrm{z}=\frac{1}{2 M_{1 v} \cos \theta_{1}}\left(x-\mathrm{l}_{O A 1}\right)^{2}-\mathrm{h}_{O A 1}  \tag{12}\\
y=\frac{1}{2 M_{1 h} \cos \theta_{1}}\left[\left(x-\frac{l_{1}}{2}\right)^{2}-\frac{l_{1}^{2}}{4}\right]
\end{array} .\right.
$$

The equation (12) is transformed by polar coordinates and inverse transformation to obtain the expression of line $\mathrm{A}_{2} \mathrm{P}_{2} \mathrm{~B}_{2}$ :
$\left\{\begin{array}{c}\mathrm{z}=\frac{1}{2 M_{2 v} \cos \theta_{2}}\left(x \cos \varphi-y \sin \varphi-\mathrm{l}_{O A 2}\right)^{2}+z_{2}-\mathrm{h}_{O A 2} \\ y \cos \varphi+x \sin \varphi=b+\left[\frac{x \cos \varphi-y \sin \varphi-a}{2 M_{2 h} \cos \theta_{2}}-c\right]^{2}-c\end{array}\right.$.

Where $a=x_{2} \cos \varphi-y_{2} \sin \varphi, b=x_{2} \sin \varphi-y_{2} \cos \varphi$, and $c=\frac{l_{2}^{2}}{8 M_{2 h} \cos \theta_{2}}$.


Figure 4.The diagrammatic sketch of crossed span under wind condition

According to the Newton iteration method, from the second equation in equations (12) and (13), the coordinates ( $x^{\prime}, y^{\prime}$ ) of $\mathrm{P}^{\prime}$ can be obtained and taken into the first equation, then the $z$ value can be obtained. At last, the crossed span distance of two power transmission lines under the wind deflection is calculated: $\left|\mathrm{P}_{1} \mathrm{P}_{2}\right|=\left|\mathrm{z}_{1}-\mathrm{z}_{2}\right|$, where $z_{1}$ and $z_{2}$ are the $z$-axis coordinates of the points $P_{1}$ and $P_{2}$, respectively.

## 3. Model and algorithm application

The model and algorithm have been applied to the real-time monitoring system of overhead traverse crossing distance, and have been tested and verified on a number of 220 kv lines of State Grid Fushun Power Supply Company. The artificial measurement values in the windless and windy states were compared with the measured values of the system, as shown in Table 1. This measurement result is close to the measurement result of the traditional method (total station), and the measurement error under windy condition is slightly larger than that under windless condition.

Table 1.Comparison between manual and system measurements

| Monitorin <br> g point | Crossing objects | State | Vertical <br> inclination <br> $\beta\left(^{\circ}\right)$ | Horizontal <br> inclination <br> $\alpha\left(^{\circ}\right)$ | Manual <br> measured <br> values $(\mathrm{m})$ | System <br> measured <br> values $(\mathrm{m})$ | Discrepancy <br> $(\%)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Motorway | Windless | 15 | 0 | 6.52 | 6.45 | 1.07 |
| 1 | Motorway | Windy | 15 | 10 | 7.18 | 7.03 | 2.09 |
| 2 | Railway | Windless | 10 | 0 | 5.75 | 5.69 | 1.04 |
| 2 | Railway | Windy | 10 | 8 | 6.34 | 6.22 | 1.89 |
| 3 | House | Windless | 8 | 0 | 7.55 | 7.48 | 0.93 |


| 3 | House | Windy | 8 | 12 | 7.89 | 7.75 | 1.77 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4 | Highway | Windless | 12 | 0 | 4.97 | 4.9 | 1.41 |
| 4 | Highway | Windy | 12 | 9 | 5.23 | 5.13 | 1.91 |
| 5 | Overhead line | Windless | 18 | 0 | 8.51 | 8.43 | 0.94 |
| 5 | Overhead line | Windy | 18 | 10 | 8.90 | 8.75 | 1.69 |

In order to more intuitively represent the state of the lines, Matlab tool was used to simulate the state of the two crossing lines at monitoring point 5 in real time. In line 1 , the span between two towers is 300 m , height difference between two towers is 10.7 m , the distance from the intersection to the low suspension point is 88 m , horizontal inclination Angle is 10 degrees, and vertical inclination Angle is 18 degrees. In line 2, the span between two towers is 250 m , height difference between two towers is 6.2 m , the distance from the intersection to the low suspension point is 115 m , the horizontal inclination Angle is 8 degrees, and vertical inclination Angle is 15 degrees. The intersection angle of the two lines is 86.5 degrees. According to the above three-dimensional model, the curves of overhead lines at windless and windy condition are drawn, as shown in Figure 5 and Figure 6.


Figure 5.Simulation under windless condition


Figure 6.Simulation under wind condition

The dynamic change of lines can be seen intuitively through the graph. According to the span, height difference, the distance from the intersection to the low suspension
point, the horizontal inclination Angle, the vertical inclination Angle, etc., the crossed span distance is calculated, and the state of overhead lines is monitored in real time. The electric power department only needs to collect the inclination data through the real-time monitoring device, and obtain the real-time status data of the wire by using the three-dimensional model and algorithm, and conduct circuit inspection and maintenance based on the data, so as to ensure the safe operation of the line. It is a very effective monitoring method.

## 4. Conclusion

In this paper, the crossed span distance of the overhead lines under the windless and wind bias conditions was studied and a three-dimensional of the overhead line's crossed span model which was applied to the online monitoring system, was established. The result showed that the monitoring system applied the model could replace the traditional manual measurement and the crossed span distance of the overhead lines was calculated more timely and accurately so as to reduce the manual measurement workload greatly. The power department can carry out inspection and maintenance of the line according to the monitoring result, which decreases the operation cost significantly and ensures the safe and stable operation of the transmission lines. As a result, the model provides the technical support for the construction of the smart grid.

## References

[1] Xinbang Sun. (2008) Accident caused by intersecting line discharge. Rural electrician (3): 36.
[2] Daowen Yang. (2013) Analysis of a 220 kV line and a 10 kV crossing discharge trip. Journal of suzhou university 28(5): 81-82.
[3] Shuyun Liu. (2011) Cause analysis of intersecting and intersecting discharges of 110 kV line and 35 kV line. Proceedings of 2011 academic annual conference of gansu electrical engineering society.
[4] Zhiye Du. (2016) Selection of lead for double-return dc transmission line with mixed pressure in the same tower. High voltage technology 42 (8): 2605-2611.
[5] Zhenneng Liang. (2015) Application of total station in the measurement of high-voltage transmission line transmission range and sag.Chinese high-tech enterprises 2(19): 60-61.
[6] Hongjian Qian. (2012) Measurement and calculation of cross span distance under special topography, new technology and product of China(24): 43-44.
[7] G.Sohn,Y and Jwa. (2012) Automatic powerline scene classification and reconstruction using airborne LIDAR data.ISPRSAnnalsof Photogrammetry, Remote Sensing and Spatial Information Sciences I-3(1).
[8] M. Bleier. (2017) Low-Cost 3D Laser Scanning in Air or Water Using Self-calibrating Structured Light ISPRS -International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences, XLII-2/W3.
[9] Zhuopeng Chen. (2016) Research on the application of total station in high-voltage transmission line transmission range and sag measurement. Scientific and technological innovation and application, (29): 199.
[10] Dapeng Zheng. (2014) Application of airborne laser radar data in transmission line final survey and selection, China electric power industry (technical edition) (9): 7-10.
[11] Nie Song. (2016) A new sag calculation method based on the temperature of overhead transmission line. Journal of North China Electric Power University 40(6): 27-32.
[12] Kongsen Wang. (2011) Sag Online Monitoring System Based on Measurement of Axial Tension Transmission Lines.East China Electric Power 39(3): 339-343.
[13] Siya Chen. (2011) Application of Digital Image Processing and Photogrammetric Technology to Sag Measuring Method. High Voltage Engineering 37(4): 904-909.
[14] Xiang Li and yi Du. (2014) A System and Method to Measure Sag Height Based on Unmanned Aerial Vehicle: China, 201310635510.1.
[15] Yutan Song and LiyingZhao. (2017 )A Calculation Method of the Height to the Ground Based on Transmission Line Sag Base on Inclination. Computer Knowledge and Technology 13(5): 206-207.
[16] Xingyang Zhou. (2015) 3-D simulation and safe distance verification of transmission line sag based on catenary formula. Journal of electrical engineering 10(12): 44-49.
[17] Ruiling Liu. (2002) Power Engineering High Voltage Transmission Line Design Manual, 2nd ed. (Beijing: China Electric Power Press).
[18] Siming Chen and BinYan. (2014) Study on Threedimensional Model Algorithm for Transmission Line Sag Base on Inclination. Computer Applications and Software 31(3): 82-85.

