

Research on the Generation Mechanism of Product Adoption Network under Weak Ties

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Abstract: Under weak ties, consumer randomly interacts with the purchaser on the e-commerce platform or the online review community to acquire information about the product, which promotes the product diffusion. Based on the heterogeneous online reviews' information level of purchaser and consumer surplus of potential consumer, this paper analyzes the probability that they connect each other to generate a new edge in the network under weak ties, and proposes the generation mechanism of product adoption network. Then we analyze statistical characteristics of degree distribution, clustering coefficient and average path length of product adoption network. The results show that online reviews' information level of purchaser and consumer surplus of potential consumer significantly influence the generation of product adoption network under weak ties, and the network has scale-free property and small-world property. The clustering coefficient of the network tends to be stable with the increase of the network size, and the average path length increases with the increase of the network size.

Key words: weak ties; product diffusion; product adoption network; network structure characteristic

1 Introduction

Social relationships between people can be categorized into strong and weak ties. Strong ties are characterized by frequent and close interactions between an individual and other individuals in the individual's direct network of relationships, while weak ties are characterized by weak or random interactions between an individual and individuals outside the individual's direct network. In social relationship networks, sometimes weak ties affect the effectiveness of interactions between consumers more than strong ties [1]. The convenience of the Internet and social networks has led to an increasing tendency for consumers to utilize weak ties to interact for product information.

Studying the effect of consumer social networks topology on product diffusion has been an important research theme in recent years [2]. Many scholars have studied the formation mechanism of social networks directly from the perspective of network generation, and the more classical network generation mechanisms include small-world networks [3], scale-free networks [4], and scientists' co-authorship networks [5]. However, such studies generally do not distinguish between strong and weak ties. The study of product diffusion in social networks is less concerned with the formation mechanism of product adoption network under weak ties, while the literature studying the formation mechanism of social networks focuses on social

networks in general or certain real networks (e.g. knowledge networks [6]), and seldom considers the characteristics of individual consumers' attributes to study the generation mechanism of product adoption networks. Therefore, based on the characteristics of consumer interaction behavior under weak ties, the study of the generation mechanism of product adoption network and its structural characteristics can help enterprises better control the diffusion of products in order to implement more targeted marketing strategies.

2 Generation mechanism of product adoption network

2.1 Description of product adoption network

A product adoption network consists of a finite but sufficient number of nodes, where each node V represents a consumer who has purchased a product, and the connecting edges E between the nodes represent the channels through which the two nodes interact to obtain information, $\Gamma = (V, E)$ denoting the product adoption network. Assume that the product adoption network is an undirected network and there are no heavy edges or self-loops in the network.

At the initial time, the network has N interconnected purchaser nodes, which constitute a regular network. In a unit time, each potential consumer, influenced by his/her own consumer surplus and the information level of the purchaser's review, interacts with the purchaser with a certain probability to obtain product information, and after purchasing the product, forms a new edge and joins the existing product adoption network. As new potential consumers continue to interact with the existing nodes in the network, obtaining product information and making purchase decisions to join the network, the size of the network continues to grow, and the structure of the network continues to change until all M potential consumers in the market purchase the product, and product diffusion ends. Eventually the node size of the entire product adoption network is $N + M$.

2.2 Consumer heterogeneity

For those who have already made a purchase, the level of information of purchasers' review under weak ties is bound to influence the choice of the target of potential consumers' interactions, which in turn affects their decision to purchase the product. For any purchaser i in the product adoption network, assume that his/her information level is x_i , obeys a certain probability distribution, and the probability density function is $\rho(x_i)$, which satisfies $\rho(x_i) > 0$, $\int_0^{\infty} \rho(x) dx = 1$, and the information level of the purchaser remains unchanged during the product diffusion cycle.

To simplify the analysis, it is assumed that the product price remains constant over the product diffusion cycle. For any potential consumer, assume that the consumer's consumer surplus is y_j , $y_j = f(v_j - p_0)$, where v_j is the consumer's reservation price of the product, obeys a certain probability distribution, and p_0 remains unchanged throughout the diffusion cycle, is a

constant. The probability density function is y_j denoted as $\rho(y_j)$, and satisfies $\rho(y) \geq 0$, $\int_0^{\infty} \rho(y) dy = 1$.

2.3 Generation mechanism

A product adoption network is composed of interconnected nodes of consumers who have purchased a product, and the connected edges of the network represent channels of interactive communication between individual consumers [7-9]. Potential consumers under weak ties randomly interact with purchasers who post reviews to obtain information, and tend to interact with purchasers with higher information levels. When the product price is constant, the higher the consumer surplus of a potential consumer is after interacting with a purchaser with a certain level of information, the more likely he or she is to buy the product. Therefore, the information level of purchasers' comments and the consumer surplus of potential consumers under weak ties jointly affect the probability of connectivity between the two, which in turn affects the diffusion of the product and the generation of the product adoption network. Considering the independent correlation between the information level of the purchased individuals and the consumer surplus of the potential consumers, the connection probability of the newly added potential consumer node with the purchased individual nodes in the network can be expressed as: $f(x_i, y_j) = f_1(x_i) f_2(y_j - p_0)$, where $f_1(x_i)$, $f_2(y_j - p_0)$ respectively, denote the probability distribution functions of the information level of the purchased individual nodes' reviews and the consumer surplus of the potential consumer nodes.

The evolution mechanism of the product adoption network is as follows: at the initial moment, there are N purchaser nodes in the product adoption network, and the new potential consumer node will be connected with W purchaser nodes in the network, generating W new edges, in order to simplify the analysis, it is assumed that the number of consumers who choose to interact with each consumer is fixed, and the probability that the new purchaser node with consumer surplus y_j and the purchaser node with the information level x_i will be connected to generate a new edge is $f(x_i, y_j)$, until all M potential consumers join the network. The product diffusion ends. Therefore, when the relevant variables are determined, the generative model of the product adoption network can be specified.

3. Analysis and simulation

In this section, the validity of the generation mechanism of product adoption network under weak ties will be verified, and three important network features of product adoption network will be theoretically analyzed and simulated.

Parameters and initial conditions are set: the number of nodes in the product adoption network is 200, and there are five purchaser nodes in the network at the initial moment that have been purchased and are connected to each other. Consumer reservation price setting in reference [10], assuming that the reservation price of potential consumers obeys a truncated normal distribution with a mean of 6 and variance of 0.3, and the information level of online reviews of purchasers already purchased obeys a truncated normal distribution with a mean of 9 and variance of 0.5.

3.1 Product adoption network

Fig. 1 is a graph of the product adoption network under weak ties generated after all 200 consumers adopt the product according to the aforementioned evolutionary rule by using Pajek software. It shows that the heterogeneous information level of purchaser's reviews and the consumption surplus of potential consumers under weak ties jointly affect the connection probability of both, which in turn affects the generation of the product adoption network. Intuitively, consumers communicate information with each other by interconnecting with each other, but the number of nodes communicating with each other may vary, with a small number of nodes having more nodes connected to them than most of the other nodes, i.e., there is a difference in the degree value of the nodes. By varying the values of the relevant parameters, the obtained product adoption network graphs are similar in their structural features, indicating that the heterogeneity of the reservation price of potential consumers and the level of information of the reviews of the purchasers who have already purchased them are the key factors in the mechanism of generating the product adoption network under the weak ties.

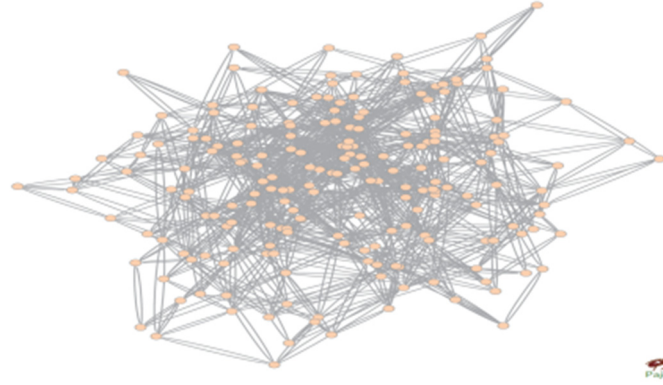


Fig. 1 Products adoption network with information levels and reservation price obeying truncated normal distributions

3.2 Degree and Degree Distribution

Firstly, we use the mean-field theory to approximate analysis the degree distribution of the product adoption network. Assume that time is continuous and the degree of a node can be any non-negative real number. At the initial moment, there are N purchaser nodes in the network. At each unit time step, a new purchaser node j with consumer surplus y in the process of selecting a new edge, the probability that a node with an information level x is selected is roughly $wf(x, y)[1 - f(x, y)]^{w-1} \approx wf(x, y)$. The node degree value of a consumer node

approximately satisfies the following dynamics equation: $\frac{\partial k_i(x, t)}{\partial t} = wf(x, y)$. Then, We can

calculate the degree distribution expression of product adoption network, which is

$$P(k) \approx \int_0^{\infty} \frac{k - w}{wtf(x, y)} \rho(x) dx \quad .$$

Then, the degree distribution graph of the product adoption network as Fig. 2. As can be seen from Fig. 2, most of the points with larger node degrees in the product adoption network are concentrated at the tail end of the curve, corresponding to relatively small probability values. Combined with the stratified graph of consumer nodes in Fig. 2, a small number of individual consumers have larger node degrees. However, the percentage of this group is relatively small, which is more in line with reality. Among the purchasers who provide online reviews, the number of purchasers who provide detailed reviews and have more interactions with them accounts for a relatively small percentage of the total number of reviews, and the other consumers in weak ties obviously tend to interact with this group of consumers, while most of the other purchasers have fewer interactions with them. Thus, the network is characterized as scale-free.

In order to further verify the scale-free feature described in Fig. 2, the power law distribution index of the node degree distribution of the generated product adoption network is calculated by fitting, and the power law index of the degree distribution of the product adoption network in Fig. 2 is approximated to be 2.73, which is in line with the range of values of the power law distribution of the common scale-free network. The validity of the product adoption network generation model is further verified.

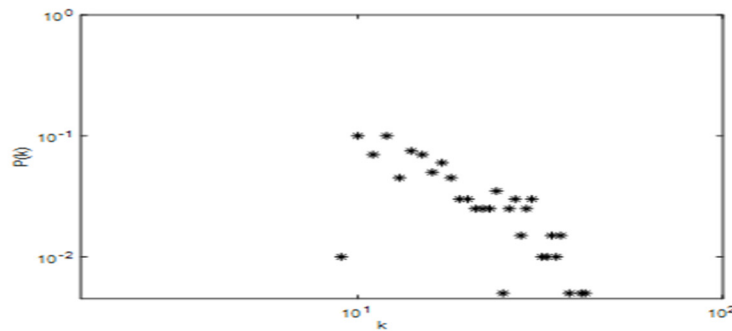


Fig. 2 degree distribution of product adoption network

3.3 Cluster coefficient

Fig. 3 shows the relationship between the clustering coefficient of the product adoption network and the network node size under weak ties calculated by the simulation. As a whole, the clustering coefficient of the network shows a decreasing trend, but the decrease is greater in the early stage. This is because, in the early stage of product diffusion, the node size of the network and the degree value of the nodes are small, and the new nodes and the number of connected edges make the clustering coefficient change of the network nodes change more compared with the change of the node size, and thus the clustering coefficient of the network decreases faster in the early stage. In the later stage, as the number of nodes in the network has been purchased by the buyer increases and there is a preference connection between the nodes, the degree value of some nodes increases, making the clustering coefficient of the nodes change more and more small compared to the change in node size, so the clustering coefficient of the product adoption network declines relatively smoothly in the later stage. When the number of nodes in the

network takes a fairly large value, the clustering coefficient of the network will tend to a stable value.

In order to further verify the above conclusions, a larger node size is selected respectively, and the node size is set to 500, 1000, and repeated for 20 times, and the simulation graph is basically similar to Fig. 3, which again verifies the conclusions.

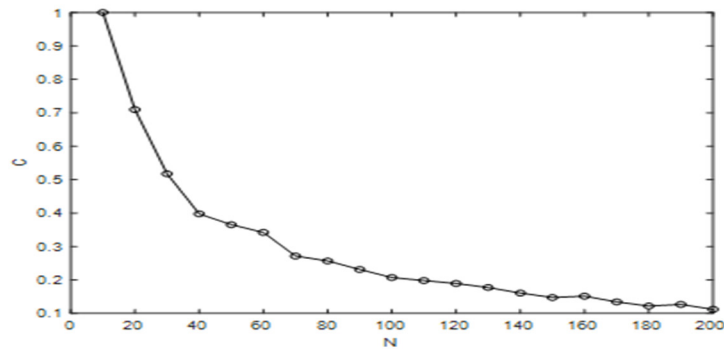


Fig. 3 cluster coefficient and network node size relationship diagram

3.4 Average path length

Fig. 4 is a graph of the relationship between the average path length and node size of the product adoption network, which is calculated by simulation. From Fig. 4, it can be observed that overall the average path length of the product adoption network is small, which is an important characteristic of small-world networks. This also indicates that product adoption network is relatively efficient in terms of information flow. For enterprises, the average path length can be further shortened to enhance the efficiency of information communication by improving the visibility and positivity of information communication among consumers. Moreover, the average path length of the product adoption network grows more and more slowly as the size of the nodes increases. As in the case of calculating the clustering coefficient, the node size is also changed, and several simulations are carried out to verify the conclusions again.

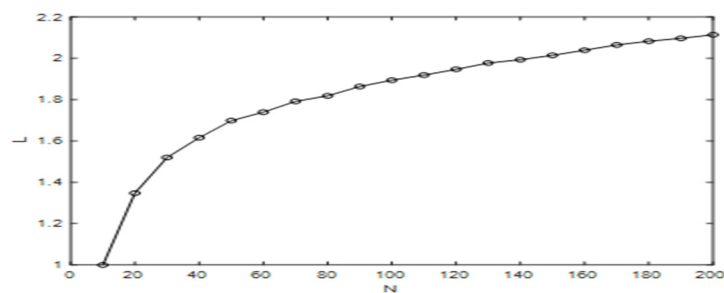


Fig. 4 diagram of the relationship between average path length and node size

4 Conclusion

This paper analyzes the probability of generating new edges by connecting new potential consumers with purchasers in the network under weak ties, proposes the generation mechanism of the product adoption network under weak ties, and verifies the validity of the generation mechanism of the product adoption network through simulation analysis. At the same time, the theoretical analysis and simulation calculate the degree distribution, clustering coefficient and average path length of the generated product adoption network. The conclusions show that the information level of online reviews of heterogeneous purchasers and the consumer surplus of potential consumers significantly affect the generation of the product adoption network under weak ties, and the generated product adoption network has scale-free and small-world characteristics, and the node degree of the product adoption network obeys a power law distribution, with a very small portion of nodes possessing a large node degree value. The clustering coefficient of the product adoption network gradually decreases and tends to stabilize with the increase of the network size, and the average path length increases with the increase of the network size, and also eventually tends to stabilize. If enterprises can combine the characteristics of product adoption network structure, distinguish important network nodes and connecting edges, and adopt targeted marketing strategies, it is possible to better promote product sales.

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