

Multi-objective Robust Optimization for Home Health Care Routing and Scheduling Problem

Yibin Li¹, Qian Du^{2*}

{liyibin@chd.edu.cn¹, 18080712876@163.com^{2*}}

School of Economics and Management, Chang'an University, Xi'an, Shaanxi 710064, China

Abstract. To address the uncertainty surrounding travel and service times in home health care routing and scheduling problem, a budget uncertainty set was employed from the perspective of Robust Optimization. Considering time window and skill-level constraints, a multi-objective mixed-integer programming model was formulated with the objectives of minimizing costs, maximizing robustness and maximizing patient satisfaction. NSGA-III was proposed to obtain the Pareto non-dominated solution set for this model. Research findings indicate that, compared with the exact model, the robust optimization model can indeed improve the robustness of the solution, and the lower the risk preference of the manager, the higher the robustness of the solution, however, this comes at the trade-off of higher total costs and diminished patient satisfaction. The number of caregivers significantly influences scheduling outcomes, with a constant number of caregivers showing a positive correlation between total costs and patient satisfaction, and a negative correlation between robustness and patient satisfaction.

Keywords: home health care; robust optimization; multi-objective; route planning; NSGA-III algorithm

1 Introduction

With the increasing aging population, the demand for Home Health Care (HHC) services is on the rise. In practical operation, managers need to plan routes for caregiver based on patient information, known as the Home Health Care Routing and Scheduling Problem (HHCRSP)^[1], which forms the foundation for cost control and the provision of high-quality services. Traditional HHCRSP is typically characterized as an exact model^[2]. However, in practice, travel and service times are influenced by factors such as traffic, weather, and the actual condition of the patients^[3]. Consequently, the incorporation of uncertainties in travel and service times has emerged as a central focus in contemporary HHCRSP research.

Currently, common approaches for handling uncertainty problems mainly include stochastic programming and robust optimization^[4]. Among these, stochastic programming relies on historical data and distribution assumptions. However, it is challenging for enterprises to accurately estimate the probability distribution of these parameters in practice. In contrast, robust optimization methods typically use deterministic bounded sets (referred to as uncertainty sets) to describe the fluctuations of uncertain parameters, making them more suitable for addressing uncertainties with a lack of distribution information. Existing research in handling uncertain parameters in HHCRSP has experimented with various uncertainty sets.

Shi et al.^[3] and Shahnejat-Bushehri et al.^[5] have utilized budget uncertainty sets, while Shiri et al.^[6] have established a multi-stage HHCRSP robust optimization model based on scenario-based uncertainty sets. Hosseinpour-Sarkarizi et al.^[7] have separately employed box uncertainty sets, polyhedral uncertainty sets, and ellipsoidal uncertainty sets, and have compared the results of these three uncertainty sets.

The establishment of a robust model aims to enhance the robustness of the solution; however, this improvement also implies additional resource investment^[8]. Therefore, managers expect to strike a balance between costs and robustness. Among the uncertainty sets used by the aforementioned scholars, the budget uncertainty set^[9] can well meet this requirement. It allows managers to flexibly adjust the conservatism and optimality of solutions according to disturbance coefficients and risk preference coefficients. Notably, Shi et al.^[3] and Shahnejat-Bushehri et al.^[5] have also employed budget uncertainty sets in their research, both their studies consider the total costs and the penalty of time window violation as objective function, and treating them as a single objective function optimized through a weighted-sum approach. However, this method is not conducive to observing the complex relationship between multiple objective functions, and the determination of weights is often difficult and subjective^[10, 11]. In summary, within the robust studies of HHCRSP utilizing budget uncertainty sets, scholars predominantly focused on the single-objective model. However, a comprehensive exploration of the multi-objective robust model for HHCRSP and its corresponding Pareto non-inferior solution set has been notably absent in the existing literature.

Considering patient satisfaction is crucial for maintaining long-term caregiver-patient relationships. Due to the uncertainties within HHCRSP, delays often occur between the scheduled start times of patient services and their expected time windows. This not only leads to the inability to provide time-sensitive medical services, such as medication administration or the supply of medical equipment^[1], but can also result in continual delays or interruptions in subsequent services along the route. Consequently, scholars have regarded this aspect as a crucial metric for assessing patient satisfaction^[10, 12, 13]. However, the studies conducted by the aforementioned scholars have overlooked the varying degrees of sensitivity to time among different patients, as well as the differences in patient tolerance for both early and delayed service starts. Additionally, related research has indicated that patient satisfaction is influenced by factors such as the skill level of caregiver^[14] and their familiarity with the patients^[15].

Based on these considerations, this paper employs a budget uncertainty set to describe the uncertain travel and service times in HHCRSP, taking into account time window and skill-level constraints. It establishes a multi-objective HHCRSP robust optimization model that aims to minimize operational costs, maximize robustness, and maximize patient satisfaction. Concerning patient satisfaction, it comprehensively assesses service time satisfaction, the skill level of caregiver, and their familiarity with the patients, while considering variations in patients' sensitivity to time and their differing tolerance for early and delayed service starts. The NSGA-III algorithm is utilized to solve the model and obtain the Pareto non-dominated solution set.

2 Mathematical model

2.1 Problem Description

Let N represent the set of all points, $N = \{0\} \cup N_c$, $N_c = \{1, 2, \dots, n\}$ is the set of patients, 0 represent the HHC center. $K = \{1, 2, \dots, m\}$ is the set of caregivers, Then a directed Euclidean graph is given $G = (V, A)$, A represents the set of arcs, $A = \{(i, j, k) : i \in V, j \in V, k \in K, i \neq j\}$, $V = N$. There is an HHC center, and caregivers depart from the HHC center and return within HHC center opening hours. Each patient can only be served by one caregiver and can only be served once. every caregiver k have a skill level q_k , meanwhile every patient i 's service context p also have a skill level q_{ip} , The requirement is that the caregiver's skill level should be at least equal to the skill level necessary for the patient. The travel time and cost between node i and node j are expressed as t_{ij} and c_{ij} , respectively. The service time of caregiver k for patient i is expressed as t_{ik} .

The patient's expected time window is $[e_i, l_i]$, There are tolerable time lengths A and B , which means that the patient can accept the early start of service with time length A and the late start of service with time length B . If the caregiver arrives earlier than e_i , they need to wait to start the service, while $e_i' = e_i - A$, but there will be no waiting cost. If arrives late than l_i , then the patient cannot be served, $l_i' = l_i + B$. Since the patient's tolerance for early start of service is usually greater than that of late start of service, it is set $A > B$. At the same time, due to the different sensitivity of patients to time, the time sensitivity of patients to early service is set to obey the normal distribution function $X_1 \sim N(u_1, \sigma_1^2)$, while $u_1 = A$, the time sensitivity of patients to delayed service initiation follows a normal distribution function $X_2 \sim N(u_2, \sigma_2^2)$, $u_2 = B$.

As for biological sample collection, such as urine or blood samples et al., sc_i is used to describe that if a patient needs the caregiver to collect the biological samples and transfer it to the HHC center, then it is 1, otherwise it is 0. In this paper, considering uncertainties in the pathway and the limited time for sample processing, we opt the caregiver to collect samples and return directly to the HHC Center for conservative considerations. and the residence time at HHC Center is t_{hhc} . If the caregiver has other patients not served, they can depart from the HHC center again to the next patient. The model assumes that special circumstances such as road traffic accidents or natural disasters are not taken into account.

decision variable:

x_{ijk} , if caregiver k travels from node i to node j , it is 1, otherwise it is 0.

y_{ik} , if caregiver k serves the patient i , it is 1, otherwise it is 0.

s_{ik} , the beginning service time of caregiver k to the patient i .

2.2 Objective function

2.2.1 Cost

The total costs include travel costs and labor costs, and labor costs include fixed costs and service costs. cf_k is the fixed cost of caregiver. ϕ_{qk} is the service cost per unit time for caregivers of different skill levels.

$$\text{cost} = \sum_{k \in K} \sum_{i \in N} \sum_{j \in N} c_{ij} x_{ijk} + \sum_{k \in K} \sum_{j \in N_c} cf_k x_{0jk} + \sum_{k \in K} \sum_{i, j \in N} \phi_{qk} t_{ik} x_{ijk} \quad (1)$$

2.2.2 Robustness

Since HHCRSP is generally regarded as a Vehicle Routing Problem (VRP)^[16], therefore, we adopt the best robustness metric proposed by Zhang Qian et al.^[17] at vehicle routing problems in logistics distribution to measure the robustness of our model.

$$\text{robustness} = \sum_{i \in N} \sum_{j \in N} \sum_{k \in K} x_{jik} \frac{l_i' - (s_{jk} + t_{jk} + t_{ji})}{t_{ik} + t_{ji}} \quad (2)$$

2.2.3 Patient satisfaction

We use a linear continuous function to describe the relationship between the patient satisfaction and the actual start time of service. Since patients prefer high-skill level and relatively familiar caregiver^[18], the skill level satisfaction is represented by the ratio of the actual skill level difference and maximum skill level difference E between patient and caregiver. In terms of familiarity, the initial familiarity level for caregivers serving patients is set to ν . With an increasing number of service instances d_{ik} , the incremental satisfaction gain will gradually decrease. Therefore, patient satisfaction is shown as follows. Figure 1 is the schematic diagram of patient satisfaction with service time. ($S(t)$ is patient satisfaction of service time)

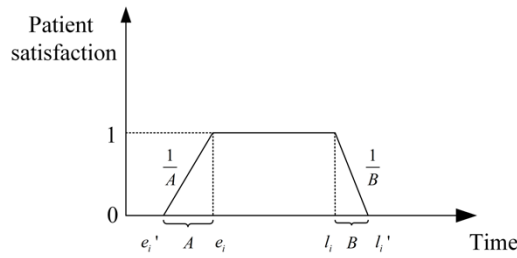


Fig. 1. Schematic Diagram of Service Time Satisfaction

$$\text{patient Satisfaction} = \sum_{i \in N_c} (S(t)_i + \frac{(q_k - q_{ip})}{E} + (1 - (\nu)^{d_{ik}})) \quad (3)$$

$$S(t)_i = \begin{pmatrix} 0, & s_{ik} \leq e_i' \\ \frac{1}{A}(s_{ik} - e_i'), & e_i' < s_{ik} \leq e_i \\ 1, & e_i < s_{ik} \leq l_i \\ \frac{1}{B}(l_i' - s_{ik}), & l_i < s_{ik} \leq l_i' \\ 0, & s_{ik} > l_i' \end{pmatrix} \quad (4)$$

2.3 Constraint condition

$$\sum_{k \in K} \sum_{j \in N} x_{ijk} = 1, \forall i \in N_c, \quad (5)$$

$$y_{jk} = \sum_{i \in N} x_{ijk}, \forall j \in N_c, \forall k \in K \quad (6)$$

$$\sum_{k \in K} y_{ik} = 1, \forall i \in N_c \quad (7)$$

$$\sum_{i \in N} x_{ihk} - \sum_{j \in N} x_{hjk} = 0, \forall h \in N_c, k \in K, \quad (8)$$

$$\sum_{i \in N_c} x_{i0k} - \sum_{j \in N_c} x_{0jk} = 0, \forall k \in K, \quad (9)$$

$$q_{ip} \leq \sum_{k \in K} y_{ik} q_k, \forall i \in N_c, \quad (10)$$

$$e_0 \leq s_{0k} \leq l_0, \forall k \in K \quad (11)$$

$$e_i' \leq s_{ik} \leq l_i', \forall i \in N, k \in K \quad (12)$$

$$s_{ik} + t_{ik} + t_{ij} - M(1 - x_{ijk}) \leq s_{jk}, \forall i, j \in N, k \in K \quad (13)$$

$$x_{ijk} \in \{0,1\}, y_{ik} \in \{0,1\}, \forall i, j \in N, k \in K. \quad (14)$$

Constraints (5) ensure that each patient can only be served once. Constraints (6) represents the relationship between x_{ijk} and y_{jk} . Constraints (7) mean that each patient can only be served by one caregiver. Constraints (8) denote that the caregiver leaves the patient after completing the service. Constraint (9) specifies that the caregiver's departures from and returns to the HHC center should be equal. Constraint (10) specifies that the caregiver's skill level should not be lower than the required skill level for the patient's service. Constraints (11) indicates that caregiver must start and end work within the time window of HHC center. Constraints (12) indicates that the actual service time of caregiver should be within the patient's tolerable time window. Constraints (13) expresses the relationship of actual arrival time between caregiver to the patients, while M is a very large number. Constraints (14) means that decision variables are binary.

2.4 Uncertain travel and service times

2.4.1 Uncertain travel time

Given an upper and lower bound on travel time, expressed as a continuous number of intervals $[\underline{t}_{ij}, \bar{t}_{ij}]$, $\underline{t}_{ij} \leq t_{ij} \leq \bar{t}_{ij}$. Let \hat{t}_{ij} be the maximum deviation in the travel time of caregiver, and $\bar{t}_{ij} = \underline{t}_{ij} + \hat{t}_{ij}$. There is a disturbed coefficient $\sigma_{ij} \in [0, 1]$ represents the extent to which the travel time of the caregiver from i to j deviates from the lower bound, when $\sigma_{ij} = 0$, indicates that the travel time is lower bound \underline{t}_{ij} , when $\sigma_{ij} = 1$, means that the travel time is upper bounded.

$$\sigma_{ij} = \frac{t_{ij} - \underline{t}_{ij}}{\hat{t}_{ij}} \quad (15)$$

There is a manager's risk preference coefficient Ω , used to regulate the level of conservatism in the travel time within robust model. The subsequent constraints are then introduced:

$$\sum_{x_{ijk}=1} \sigma_{ij} = \sum_{x_{ijk}=1} \frac{t_{ij} - \underline{t}_{ij}}{\hat{t}_{ij}} \leq \Omega \quad (16)$$

Then we can assume that the travel time belongs to an uncertainty set U_1 .

$$U_1 = \{t_{ij} \mid t_{ij} = \underline{t}_{ij} + \sigma_{ij} \hat{t}_{ij}, 0 \leq \sigma_{ij} \leq 1, \sum_{x_{ijk}=1} \sigma_{ij} \leq \Omega\} \quad (17)$$

2.4.2 Uncertain service time

Given an upper and lower bound on service time $[\underline{t}_{ik}, \bar{t}_{ik}]$, $\underline{t}_{ik} \leq t_{ik} \leq \bar{t}_{ik}$. \hat{t}_{ik} is the maximum deviation in service time for caregiver, $\bar{t}_{ik} = \underline{t}_{ik} + \hat{t}_{ik}$. The disturbed coefficient $\lambda_{ik} \in [0, 1]$, indicates the extent to which the caregiver's service time for the patient deviates from the lower threshold. t_p Indicates the service time corresponding to each service content, where $\lambda_{ik} = 0$, indicates that the patient's service time takes a lower bound \underline{t}_{ik} , $t_{ik} = t_p$. When $\lambda_{ik} = 1$, Indicates that the patient's service time takes an upper bound \bar{t}_{ik} .

$$\lambda_{ik} = \frac{t_{ik} - \underline{t}_{ik}}{\hat{t}_{ik}} \quad (18)$$

Similarly, there is a manager's risk preference coefficient Γ , used to regulate the level of conservatism in the service time within robust model.

$$\sum_{x_{ijk}=1} \lambda_{ik} = \sum_{x_{ijk}=1} \frac{t_{ik} - \underline{t}_{ik}}{\hat{t}_{ik}} \leq \Gamma \quad (19)$$

We can assume that the service time belongs to an uncertainty set:

$$U_2 = \{t_{ik} \mid t_{ik} = \underline{t}_{ik} + \lambda_{ik} \hat{t}_{ik}, 0 \leq \lambda_{ik} \leq 1, \sum_{x_{ijk}=1} \lambda_{ik} \leq \Gamma\} \quad (20)$$

3 Solution method

Considering the model's status as an NP-hard problem, a multi-objective algorithm can effectively tackle the issue. NSGA-III, proposed by Deb and Jain^[19] as an extension of the NSGA-II algorithm, replaces the crowding degree in NSGA-II with associative reference points. This adjustment ensures better preservation of high convergence and diversity while dealing with multi-objective optimization problems. NSGA-III is particularly suited for high-dimensional objective optimization problems featuring three or more objective functions.

We generate the initial solution based on the principles of time window constraints and skill level constraints, and encode it using real numbers. The crossover, mutation, and local search in the algorithm are inspired by the approach used by Li Yanfeng et al^[18]. In terms of constraint handling, this paper employs the direct elimination method to remove solutions that do not meet the constraints of time window and skill level et al.

4 Experiment

In this section, a small-scale example is designed and programmed using MatlabR2020a software platform. All experiments are conducted on a computer with an Intel(R) Core (TM)i5-7200U CPU @ 2.50GHz.

4.1 Introduction to the instances

This section is based on the generation method of the standard Vehicle Routing Problem with Time Window (VRPTW) example proposed by Solomon^[20]. In a two-dimensional plane $[0,100^2]$, random patient nodes are generated, with the coordinate of the HHC center being $(50,50)$. A numerical example involving 15 patients and 6 caregivers was constructed. The skill levels are divided into three categories: high, medium, normal, each accounting for 1/3 of the total number of caregivers. The skill level of caregiver 1 and 2 is high, 3 and 4 is medium, 5 and 6 is normal. HHC Center's time window $[e_0, l_0]$ is $[0\text{min}, 540\text{min}]$, and the travel distance between node i and node j is defined as the Euclidean distance. To simplify this problem, let the travel speed and travel cost as units 1, then t_{ij} and c_{ij} both equal to d_{ij} . While the fixed cost cf_k of caregiver with different skill level of normal, medium and high is 80, 100, 120, and the service cost per unit time ϕ_{qk} is 0.8, 1, 1.2, respectively. Other information can be obtained from Table 1.

Table 1. Information on Patients

| patient | coordinate | P | q_{ip} | t_p | e_i | l_i | sc_i | $d_{ik} (k=1,2,\dots,6)$ |
|---------|------------|-----|----------|-------|-------|-------|--------|--------------------------|
| 1 | (52, 62) | d | medium | 32 | 15 | 82 | 0 | [0 0 0 0 1] |
| 2 | (38, 85) | a | normal | 42 | 75 | 190 | 0 | [0 0 0 1 1 0] |
| 3 | (28, 63) | b | normal | 38 | 203 | 300 | 0 | [1 0 1 1 0 0] |
| 4 | (39, 52) | c | medium | 46 | 310 | 430 | 1 | [0 1 0 1 0 0] |
| 5 | (55, 35) | f | high | 55 | 21 | 88 | 0 | [0 0 0 3 0 0] |
| 6 | (72, 60) | b | normal | 38 | 254 | 388 | 0 | [0 0 1 0 0 0] |

| | | | | | | | | |
|----|----------|---|--------|----|-----|-----|---|---------------|
| 7 | (82, 81) | b | normal | 38 | 342 | 422 | 0 | [0 0 0 2 0 0] |
| 8 | (92, 49) | d | medium | 32 | 295 | 404 | 0 | [0 0 0 0 0 0] |
| 9 | (28, 42) | e | high | 62 | 34 | 80 | 0 | [1 2 0 1 2 1] |
| 10 | (25, 12) | b | normal | 38 | 65 | 125 | 0 | [0 0 0 0 0 0] |
| 11 | (9, 54) | c | medium | 46 | 131 | 185 | 1 | [1 0 0 0 0 0] |
| 12 | (48, 30) | d | medium | 32 | 92 | 210 | 0 | [0 0 1 2 0 0] |
| 13 | (60, 22) | a | normal | 42 | 155 | 268 | 0 | [0 0 0 2 0 2] |
| 14 | (74, 39) | f | high | 55 | 320 | 406 | 0 | [0 0 0 1 0 0] |
| 15 | (59, 40) | e | high | 62 | 208 | 292 | 0 | [2 2 0 0 0 0] |

4.2 Model comparison

In this section, We set the parameters as follows: $A=15$, $B=10$, $u_1=A$, $u_2=B$, $\sigma_1=3$, $\sigma_2=3$, $\nu=0.3$, $\hat{t}_{ij}=10$, $\hat{t}_{ik}=10$, $t_{hbc}=10$. Additionally, set the parameters of the NSGA-III algorithm as follows: the initial population size: 100, number of iterations: 400, crossover probability: 0.9, mutation probability: 0.1, number of local search iterations:2. The avg. of the pareto non-dominated solutions and the routing and scheduling plan of the extreme non-dominated solutions under various configurations of the manager's risk preference coefficients Ω and Γ , are presented in Tables 1 and 2, respectively.

Table 2. The Avg. of pareto non-dominated solutions under various configurations of Ω and Γ

| | | $\Omega=0, \Gamma=0$ | $\Omega=5, \Gamma=5$ | $\Omega=10, \Gamma=10$ | $\Omega=15, \Gamma=15$ |
|------|----|----------------------|----------------------|------------------------|------------------------|
| Avg. | CO | 1677.59 | 1691.94 | 1700.61 | 1704.92 |
| | RO | 22.73 | 23.25 | 24.24 | 24.40 |
| | PS | 24.48 | 23.95 | 22.93 | 22.72 |

CO: cost; RO: robustness; PS: patient satisfaction

Table 3. the routing and scheduling plan of the extreme non-dominated solutions under various configurations of Ω and Γ

| | | $\Omega=0; \Gamma=0$ | $\Omega=5; \Gamma=5$ | $\Omega=10; \Gamma=10$ | $\Omega=15; \Gamma=15$ |
|-----------------------------|----|----------------------|----------------------|------------------------|------------------------|
| min CO | CO | <u>1544.08</u> | <u>1551.65</u> | <u>1551.65</u> | <u>1555.21</u> |
| | RO | 19.41 | 20.14 | 20.14 | 20.66 |
| | PS | 22.71 | 23.71 | 23.71 | 24.65 |
| routing and scheduling plan | | 1{5,12,13,15,14,8} | 1{5,12,13,15,14,8} | 1{5,12,13,15,14,8} | 1{1,9,11,4} |
| | | 2{1,9,11,4} | 2{1,9,11,4} | 2{1,9,11,4} | 2{5,12,13,15,8,14} |
| | | 5{10,2,3,7,6} | 5{10,2,3,6,7} | 5{10,2,3,6,7} | 5{10,2,3,6,7} |
| | | | | | |
| min RO | CO | 1881.13 | 1881.13 | 1881.13 | 1881.13 |
| | RO | <u>27.10</u> | <u>27.10</u> | <u>27.10</u> | <u>27.10</u> |
| | PS | 17.26 | 17.26 | 17.26 | 17.26 |
| routing and scheduling plan | | 1{9,11,8} | 1{9,11,8} | 1{9,11,8} | 1{9,11,8} |
| | | 2{5,12,13,15,14} | 2{5,12,13,15,14} | 2{5,12,13,15,14} | 2{5,12,13,15,14} |
| | | 3{1,6,7} | 3{1,6,7} | 3{1,6,7} | 3{1,6,7} |
| | | 4{4} | 4{4} | 4{4} | 4{4} |
| | | 5{2,3} | 5{2,3} | 5{2,3} | 5{2,3} |
| | | 6{10} | 6{10} | 6{10} | 6{10} |
| min PS | CO | 1691.83 | 1684.29 | 1745.77 | 1733.24 |
| | RO | 14.24 | 13.80 | 13.09 | 13.30 |

| PS | 32.15 | 32.15 | 30.86 | 30.85 |
|-----------------------------|--------------------|-------------------|--------------------|--------------------|
| routing and scheduling plan | 1{5,10,13,15,14,7} | 1{5,10,13,15,8,7} | 1{5,10,12,15,6,14} | 1{1,5,11,6,4} |
| | 2{1,9,2,6,8,4} | 2{1,9,2,6,14,4} | 2{1,9,11,7,4} | 2{9,10,12,15,8,14} |
| | 4{12,11,3} | 4{12,11,3} | 4{2,13,3,8} | 4{2,13,3,7} |

From Tables 2 and 3, it is evident that, compared to exact models, the robust optimization model exhibits an increase in the average robustness of Pareto non-dominated solutions as the values of Ω and Γ grow. Simultaneously, the average total costs improve, while the average patient satisfaction decreases. Within the Pareto extreme non-dominated solutions, as the grow of Ω and Γ , the total costs decrease, and patient satisfaction increases. However, due to constraints such as the number of caregivers, time windows and skill levels, the values of the objective functions and scheduling schemes for the Pareto extreme non-dominated solutions optimized for robustness remain unchanged.

The underlying rationale lies in the fact that as Ω and Γ increase, it signifies greater fluctuations in travel and service times, leading to some patients in the original schedule being unable to meet time window constraints. Consequently, it becomes necessary to re-plan routes and schedules. This may involve transferring patients from caregivers with higher skill levels to those with lower skill levels, increasing the number of caregivers, or adjusting the sequence of caregiver visits to patients. These adjustments result in an increase in total costs, an increase in robustness, a decrease in patient satisfaction of skill levels, and an increase in patient satisfaction of service time. When the decrease in patient satisfaction of skill levels surpasses the increase in satisfaction of service time, the overall patient satisfaction declines.

4.3 Objective function relationship analysis

In order to better comprehend and observe the relationships among various objective functions, Figure 2 illustrates a three-dimensional display of the Pareto frontiers of the model. With the set of parameters as follows: $A=15, B=10, u_1=A, u_2=B, \Omega=3, \Gamma=3, \sigma_1=3, \sigma_2=3, v=0.3, \hat{t}_{ij}=10, \hat{t}_{ik}=10, t_{hbc}=10$. Figure 3 is the projection plots of total costs versus patient satisfaction of service time(a), skill level(b), and familiarity(c). Figure 4 is the projection plots of robustness against patient satisfaction of service time(a), skill level(b), and familiarity(c).

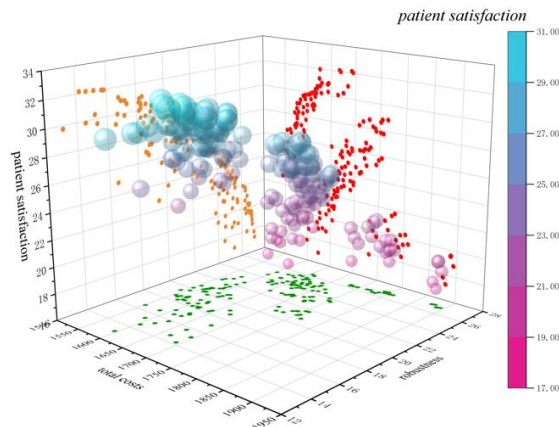
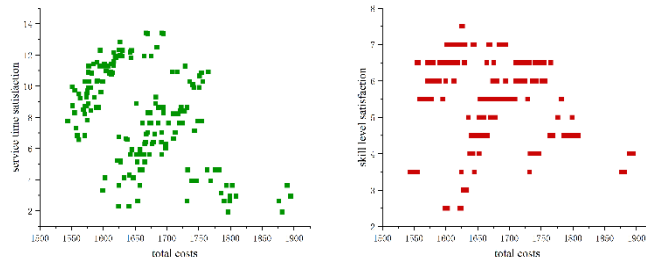
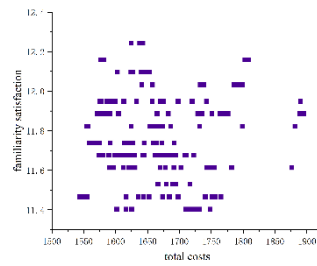


Fig. 2. Three-dimensional Display of the Pareto Front



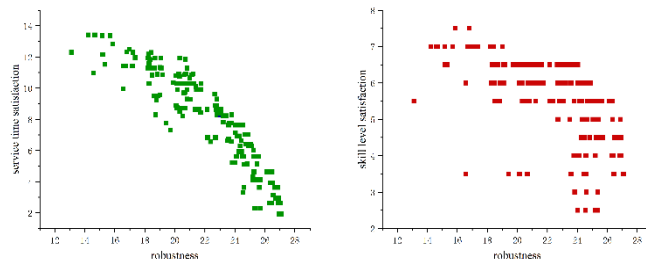
(a)

(b)



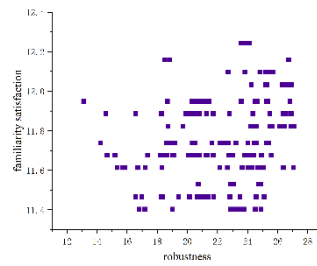
(c)

Fig. 3. Projection plot of total costs with satisfaction of service time, skill level, and familiarity



(a)

(b)



(c)

Fig. 4. Projection of robustness with satisfaction of service time, skill level and familiarity

Figure 2 illustrates that the Pareto non-inferior solutions display four distinct clusters, reflecting the constraints associated with the number of available caregivers in the given scenario. Specifically, these clusters represent instances where 3, 4, 5, and 6 caregivers are dispatched, thus delineating the corresponding dispatching scenarios.

A more detailed analysis of Figures 3 and 4 reveals a consistent trend: as the number of caregivers varies, both costs and robustness exhibit an upward trend. Conversely, the satisfaction levels pertaining to service time and skill level experienced a simultaneous decline. However, under fixed caregiver constraints, the total costs demonstrate a positive correlation with the patient satisfaction of service time and skill level. Notably, a substantial negative correlation becomes apparent between robustness and the patient satisfaction of service time and skill level.

5 Conclusion

In this study, we employ a budget uncertainty set to address the uncertain travel time and service time in HHCRSP, then a HHCRSP multi-objective mixed integer programming model is established, and the NSGA-III algorithm is used to solve the Pareto non-dominated solution set of the model. Subsequently, a comparison is made between the exact model and the robust model under various risk preferences of the managers. In addition, we thoroughly explore the relationship among the three objectives involving total costs, robustness and patient satisfaction from the perspective of the Pareto frontier.

The results demonstrate that robust optimization methods can significantly enhance the robustness of solutions. Moreover, as managers become more conservative, total costs increase, robustness improves, patient satisfaction of service time increases, and patient satisfaction of skill level decreases. Managers can flexibly adjust the values of Ω and Γ to attain suitable routing and scheduling schemes. In addition, with the increase of the number of caregivers, the total costs increase, the robustness of solution increases, however, both service time and skill level of patient satisfaction decreases. When the number of caregivers is fixed, total costs is positively correlated with patient satisfaction of service time and skill level, while robustness is negatively correlated with patient satisfaction of service time and skill level.

For future investigations, we aspire to utilize larger-scale examples to rigorously test the algorithm's performance. Additionally, data in HHCRSP can be collected through big data analysis or internet of Things technology to better identify and monitor its uncertainty. Simultaneously, optimization and improvement of multi-objective algorithms are also worthy of consideration.

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