

# Stochastic Geometry Analysis of Ultra Dense Network and TRSC Green Communication Strategy

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## Abstract

In recent years, with the rapid development of wireless communication, the traditional cellular with isomorphic and regular structure has been unable to meet the increasing number of users and business needs involving data of big volume. The trend is evolving into Ultra Dense Network (UDN) architecture which is covered by cellular of irregular complex structure. In UDN, the spatial distribution of the base station plays an important role in the interference and performance evaluation of the whole cellular network, and the concept of green communication has also been put on agenda. In this paper, stochastic geometry theory is used to model UDN and to analyze the key performance of interference and wireless network. Moreover, a green communication strategy called TRSC is proposed, which is aimed at saving energy and reducing the signal interference among cells to a certain extent.

**Keywords:** UDN, Stochastic geometry, Relay, Modeling analysis, TRSC.

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## 1. Introduction

Mobile communication networks have evolved from the original first generation analog cellular systems to 4G currently. The cell radius is constantly shrinking and the density is increasing. In the future, there will be more and more hotspots of high speed data access getting denser. The existing heterogeneous networks will gradually develop into high speed ultra-dense cells, namely Ultra Dense network (UDN), consisting of regular macro cells and high density irregular small cells.

The statistical characteristics of Inter-cell interference (ICI) are different because UDN is completely different from the traditional cellular network topology, and there are different requirements on interference control. UDN's dynamic irregular network topology needs to be simulated by Poisson's process based on stochastic geometry theory. Under this model, the ICI concentration effect is reduced, and the power distribution curve becomes flat, the trailing effect is remarkable as well. It is necessary to reform the

interference control method technology according to the interference characteristics of UDN.

Earlier, Baccelli *et al* introduced the uniform Poisson point process into base station modeling, but they did not use metrics such as coverage to measure the advantages of modeling [1-2]. In recent years, Andrews *et al* assumed that the base stations were independent of each other and took the uniform Poisson point process as a model. The rate and coverage of the downlink (only related to SINR) were analyzed [3-4]. They derived a universal expression for coverage of interference fading shadows in a wireless communication network that follows any distribution. In particular, the distribution of received power was calculated. The experimental results showed that the coverage rate of the Poisson point process provides the lower bound of the coverage rate of the measured base station distribution, while the hexagonal lattice cell model provides the upper bound of its coverage. Document [5] uses stochastic geometry theory to analyze the performance of multilayer dense homogeneous networks composed of different types of wireless access points. Each layer has different base

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station transmit power, data rate and deployment density. Compared with the actual LTE network performance, the validity of the stochastic geometry theory in multi-layer heterogeneous network performance analysis was verified. Tsinghua University professor Zhisheng Niu team used stochastic geometry theory to analyze the energy efficient base station distribution density [6] in heterogeneous cellular networks.

## 2. Spatial Poisson Point Process Theory

The spatial point process is different from the usual one dimensional point process. It is a higher dimensional point process that does not have the characteristics of temporal ordering of one-dimensional processes. In the spatial point process, although each point has the characteristics of sequential arrival, the spatial point process can be defined by the number of points falling into the unit region rather than by counting process. Human settlements, tourist attractions and macro base stations show the characteristic of entity in space. Spatial point model is based on the spatial location of these points, and researchers are trying to find the potential law of these points.

### 2.1. Spatial Homogeneous Poisson Point Process

If two conditions of a spatial point process  $X$  are satisfied, we called  $X$  as spatial homogeneous Poisson point process.

(1) For any bounded region  $B \subseteq R^2$ ,  $N(B)$  subjects to Poisson distribution whose mean is  $\lambda v_d(B)$  ( $\lambda > 0$ ),

$$P(N(B) = m) = (\lambda v_d(B))^m \cdot \exp \frac{-(\lambda v_d(B))}{m!}.$$

(2) If the bounded regions  $B_1, B_2 \dots B_n$  do not intersect with each other, then  $N(B_1), N(B_2) \dots N(B_n)$  are independent of each other. Thus,  $N(B)$  is the number of points in region  $B$ , and the density  $\lambda$  is a constant, which represents the number of the average points in unit area, and  $v_d(B)$  represents the area of the bounded region.

According to the above definition, we can conclude that spatial homogeneous Poisson point process has following characteristics: (i) The spatial distribution of points is completely random; (ii) The expectation  $\mu_B$  of the number of points on the unit area is a constant, without changing with the change of space.

### 2.2. Applications in Wireless Networks

In spatial stochastic geometry model, it is assumed that the location of the base station is not fixed but random, so usually we suppose that the position of the base station obeys spatial homogeneous Poisson point process. In fact, the distribution of base station is not completely independent, but relevant researches discovered that using spatial stochastic geometry model to calculate the performance of system was close to actual testing, high reliability was provided as well. And after using stochastic

geometry model, it is convenient to obtain system parameters such as outage probability and capacity of multi-cell system model. In that case, the model will help the designers optimize and adjust the parameters of the system easily.

Relevant simulations and comparisons [3] indicated that the performance index calculated by using system model structured through spatial homogeneous Poisson point process have an accuracy extremely close to the data measured in real network, and more precisely than using the hexagonal or square network model. Therefore, the method of stochastic geometry modeling can simulate the cellular network more accurately, and the computational complexity of this method is low, the system performance index can be easily obtained as well.

## 3. Modeling and Analysis of Relays in UDN

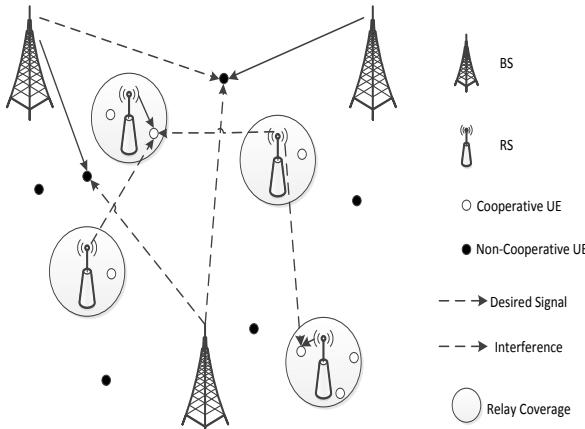
Cooperative relaying is an effective way to expand network coverage and improve network capacity. Compared with macro base station, relay has smaller coverage and lower transmission power. Therefore, the majority of users are able to obtain a higher value of Signal to Interference plus Noise Ratio (SINR). As the relay does not require wired backbone connections, its deployment costs will be greatly reduced. So the cooperative relay technology in UDN can be used with less energy consumption and the cost of deployment to improve network capacity and expand the network coverage, but also do not need to make any changes for existing cellular network architectures. This chapter models for double-layer heterogeneous relays in UDN based on spatial homogeneous Poisson point process, and derives the SINR distribution and the average achievable rate of the cooperative users and non-cooperative users. Meanwhile, the theoretical derivations are verified through system simulation.

### 3.1. Network Model

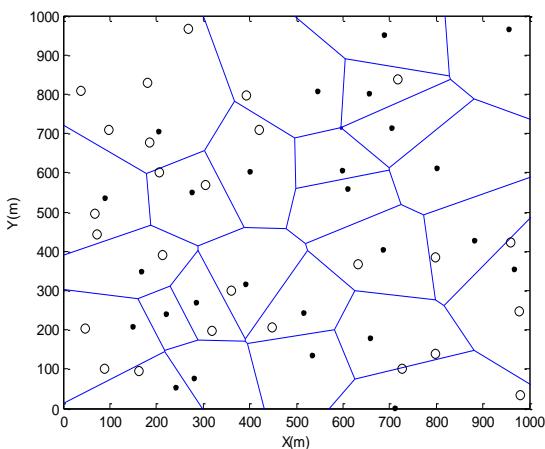
Consider a downlink double layer heterogeneous relay cellular network, as shown in Fig. 1. There are two kinds of access points in the network: macro base station and relay, respectively modeling by the density of  $\lambda_b$  homogeneous Poisson point process with  $\theta_b$  and density of  $\lambda_r$  homogeneous Poisson point process with  $\theta_r$ . The coverage area of each relay is a circle with a radius of  $R_r$ , and the cooperative coding strategy of adaptive source retransmission check bits is adopted in cooperative transmission. Especially, for a provided Poisson point process, the quantity of points in a closed region is a random variable subjected to Poisson distribution. Fig. 2 shows the implementation of a two-layer heterogeneous relay cellular network model consisting of a macro base station and a relay consisting of Poisson distribution, characterized by a grid diagram. The small areas divided into grids are called cells, namely the coverage area of the macro base station.

The area is a random variable, expressed as its probability distribution function which is approximated as follow:

$$F(S) = \lambda_b \sqrt{\frac{1830}{\pi}} (S\lambda_b)^{\frac{5}{2}} \exp(-\frac{7}{2}S\lambda_b) \quad (1)$$



**Figure 1.** Downlink double layered heterogeneous UDN model



**Figure 2.** Voronoi network topology

Considering two types of users, one is homogeneous Poisson point process  $\theta_{nc}$  with the density of  $\lambda_{nc}$  distributed on the whole network plane, communicating directly with the macro base station, and accessed with the nearest macro base station, called non-cooperative users. Its distance distribution from the service station is:  $f(r) = 2\pi\lambda_b r e^{-\lambda_b\pi r^2}$ . The other is homogeneous Poisson point process  $\theta_c$  with the density of  $\lambda_c$  distributed within the round area covered by each relay. It works through the relay and macro base station, called cooperative users, whose distance distribution from the relay in services is:  $f_R(r) = 2r/R_r$ . All these Poisson point processes ( $\theta_b, \theta_r, \theta_{nc}, \theta_c$ ) used for modeling are independent of each other.

Based on the fact that the coverage area of the relay is far less than the coverage area of macro base station, we propose a two level approximation model for the analysis of

subsequent performance. It contains two angles: macroscopic and microscopic. The macroscopic angle refers to the observation in the relay coverage area of the entire network. At this time all the relay coverage area will shrink to a point, and the number of cooperative users is marked. Micro perspective refers to the observed network within the covered area of relay network. At this time the cooperative users in the round coverage of relay are observed. This approximation will avoid the problems of site selection while user accessing with relay and relay accessing with macro base station.

### 3.2. Users Distribution

Take  $V_c$  as the number of cooperative users covered by a relay, then  $V_c \sim \text{Poisson}(\lambda_c \pi R_r)$ . Take  $V_{nc}$  as the number of non-cooperative users covered by a macro base station.

$$\begin{aligned} G_{V_{nc}}(z) &= \int_0^\infty \exp(\lambda_{nc}(y-1)S) F(S) dS \\ &= \frac{341}{8} \sqrt{\frac{7}{2}} \left( \frac{7}{2} - \frac{\lambda_{nc}}{\lambda_b} (y-1) \right)^{\frac{7}{2}} \end{aligned} \quad (2)$$

The relation between the discrete probability distribution and its corresponding probability generating function can be obtained. The distribution of the quantity of non-cooperative users  $V_{nc}$  is as follow:

$$P\{V_{nc} = i\} = \frac{G_{V_{nc}}^{(i)}(0)}{i!}, i = 0, 1, \dots \quad (3)$$

### 3.3. Performance Analysis

Assume a macro base station that the user access distance is  $r$ , occupying a sub channel, whose power is  $P_b = P_B/M_b$ ,  $g$  is the interference channel gain, and  $R$  is the distance from user to the interference source. Then the received SINR of the user is:

$$\gamma = \frac{P_b h r^{-\alpha}}{I_b + \sigma^2} \quad (4)$$

The cumulative probability distribution function of  $\gamma$  is:

$$\Pr(\gamma \leq T) = 1 - \int_0^\infty \exp\left(-\frac{\mu T r^\alpha \sigma^2}{P_b}\right) \mathcal{L}\left(\frac{\mu T r^\alpha}{P_b}\right) f(r) dr \quad (5)$$

$$f(r) = 2\pi\lambda_b r e^{-\lambda_b\pi r^2} \quad (6)$$

Average achievable speed of non-cooperative users is:

$$\bar{\tau}_{nc} = \frac{\sum_{i=1}^{M_{b1}} P\{V_{nc}=i\} + \sum_{i=M_{b1}+1}^{\infty} P\{V_{nc}=i\} \frac{M_{b1}}{i}}{1 - P\{V_{nc}=0\}} \tau_{nc} \quad (7)$$

Likely, average achievable speed of cooperative users is:

$$\bar{\tau}_c = \frac{\sum_{i=1}^{M_r} P\{V_c=i\} + \sum_{i=M_r+1}^{\infty} P\{V_c=i\} \frac{M_r}{i}}{1 - P\{V_c=0\}} \tau_c \quad (8)$$

### 3.4. Simulation Analysis

This section gives the simulation results of the performance of the relay cellular network in terms of SINR and single user rate.

Table 1. This is the simulation configuration of model.

Symbol	Interpretation	Value
$\lambda_b$	Density of BS	10-5BS/m2
$\lambda_r$	Density of RS	9×10-5RS/m2
$P_b/P_r$	Trans power of BS/RS	43dBm/36dBm
M	Quantity of sub channels	320(Mr=16)
R	Coverage radius of RS	25m
$\mu$	Rayleigh fading value	1
$\alpha$	Path loss parameter	4
$\sigma$	Noise power	-90dBm
$\beta$	Cooperative parameter	0.7
$R_{ts}$	Target speed of RS	0.5bits/Hz

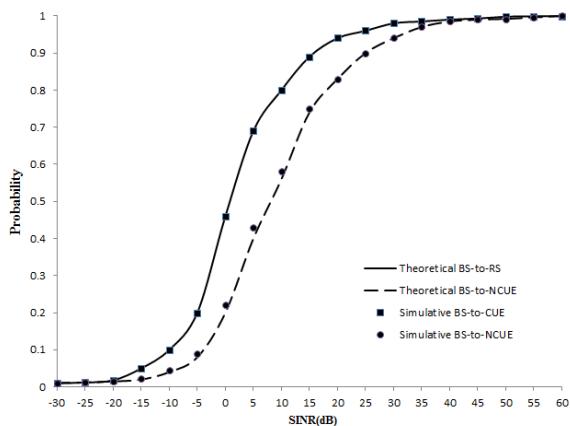


Figure 3. CDF of SINR for CUEs &amp; NCUEs

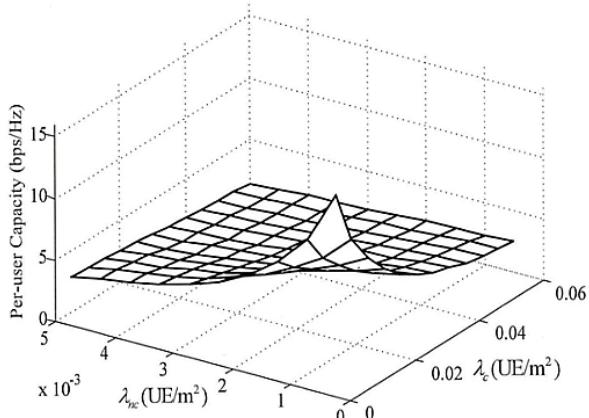


Figure 4. The relationship between single user rate and the density of CUEs &amp; NCUEs

Fig. 3 shows the SINR distribution of both collaborative and non-cooperative users. The outline of non-cooperative users shows that the results are in good agreement with the theoretical results. In the meantime, the correctness of the theoretical analysis is proved. For cooperative users, the theoretical outline shows the SINR distributing of the B-to-R link, indicating that the distribution of SINR of B-to-U link could approximately be B-to-R link, according to the research.

Fig. 4 shows the relation that single user rate varies with the density of CUEs and NCUEs. The result indicates that

the single user rate reached the highest point when the density of two types of users hit the bottom. And also with the growth of their density, single user rate decreased rapidly, reaching the lowest point when the density of two kinds of users reached the peak. This phenomenon exists out of two reasons: One is that the density of users increased, and the number of users was greater than sub channels, on which users got less resources so that the single user rate decreased obviously. The other is that increasing user density augmented the probability of BS or RS occupied a channel, so the effective interference source link is strengthen, which would reduce the user's SINR and decrease the rate of single user.

#### 4. TRSC Green Communication Strategy

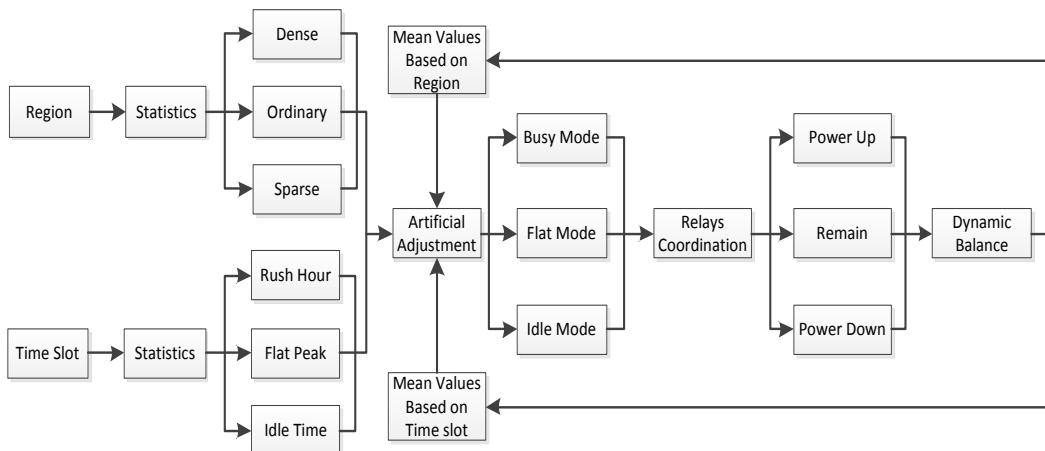
Nowadays, heterogeneity and irregularity of UDN architecture become more and more obvious. Based on the key performance analysis of last chapter, we find that the control of interference and energy efficiency for UDN is one of the most crucial problems that constrain green communication developing. Therefore, this chapter presents a green communication strategy called TRSC (Time slot and Region Stratified Control).

TRSC is the short form of Time slot and Region Stratified Control. This strategy needs some human intervention, but the core idea is using inter communication of cells to build an adaptive control mechanism to optimize the network energy efficiency and interference control. Fig. 5 shows the working mechanism of TRSC, which can be divided into two phases.

The first is constructing phase: firstly, TRSC needs people to investigate relays' working state in intensive, general and sparse scenarios based on region, and record relays' working power in rush hour, flat peak and idle time based on time slot. Then configure different work modes according to the statistical values. Next, the relays start to communicate and coordinate whether the power needs further adjustment (up, down or remain). If the adjustment is necessary, relays take regular increments, step by step, in order to achieve the dynamic balance of the energy efficiency of relays. After then, the data of the mean value of the dynamic balance state will be fed back providing reference for human.

Second is adjusting phase: In fact, according to the feedback data to configure relays' work mode, and then the workload of relays coordination will be greatly reduced comparing to the first stage. Because the space needs to be adjusted is very small, the overhead of relay work will be greatly reduced as well. Once a dynamic balance is reached, a virtuous circle will be formed, and a nice working mechanism will also be formed with little human intervention.

TRSC strategy is based on real time and scenario, so the best working state of relays will be reached under less human intervention: Relays can provide working power values to meet users' communication needs, and won't produce lack of flow or overflow conditions.



**Figure 5.** The working mechanism of TRSC

In other words, TRSC is able to meet the user's demand, meanwhile, energy saving can also reduce the signal interference among relays to some extent. Therefore, TRSC is a green communication strategy.

## 5. Conclusion

In this paper, we modeled and analyzed UDN by using spatial homogeneous Poisson point process of stochastic geometry theory, and derived SINR distribution and average rate of cooperative users and non-cooperative users. Through system simulation, the theoretical derivations are verified. This research indicates that lower user density, weak interference and sufficient resources would lead to the highest rate of single user. On the contrary, a great number of users would be blocked or the single user rate would decrease rapidly. Therefore, this paper proposed a green communication strategy called TRSC, which is able to not only save energy to some extent on the basis of meeting the user's needs, buy also reduce parts of the signal interference among cells.

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