Mean Square Cordial Labeling Of Some Pentagonal Snake Graphs

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Abstract. The A Mean Square Cordial labeling of a Graph G(V,E) with p vertices and q edges is an onto from V to {0, 1} such that each edge uv is assigned the label $(\left\lceil (f(u)^2 + f(v)^2)/2 \right\rceil)$ where $\lceil x \rceil$ (ceil(x)) is the least integer greater than or equal to x with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1 and the number of edges labeled with 0 and the number of edges labeled 0 and the number of edges labeled with 1 differ by at most 1.In this paper we analysed that Pentagonal snake PS_k, Subdivision of a pentagonal snake $S(PS_k)$,Double pentagonal snake $D(PS_k)$ and Alternate pentagonal snake $A(PS_k)$ are mean square cordial graphs.

Keywords: Mean Square Cordial Labeling, Pentagonal snake $S(PS_k)$,Double pentagonal snake $D(PS_k)$, Alternate pentagonal snake $A(PS_k)$.

1 Introduction

Graph theory is one of the most comprehensive and growing areas of mathematics which is a graph analysis concerned with the relationship between vertices and edges. Over 200 graph labeling techniques [1] have been explored in thousands of research papers, leading to the participation of the researchers over the past 60 years. Label graphs serve as useful models with more applications such as coding principles, crystal analysis, radar detection, astro studies, circuit design, communication network addressing, database management, exchange of secret messages, simulating many restricted programming in a finite number of domains. We follow Harary[2] for the basic terms and notations. Cordial labeling was introduced by Cahit[3] and Ponraj et al[4] initiated the mean cordial labeling of a graph. Mean square cordial labeling introduced by A.Nellaimurugan et al and discussed it for some special graphs[5]. Furthermore, the mean square cordial labeling for certain tree and cycle based graphs[6,7] was discussed. Dhanalakshmi et al discussed mean cordial square labelling related to certain cyclic and acyclic graphs and their rough approximations[8,9]. Some interesting findings have been discussed by Dhanalakshmi et al in mean square cordial marking of some star based graphs[10].

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2Preliminaries

Definition 1:Let G = (V,E) be a graph with p vertices and q edges. "A Mean Square Cordial labeling of a Graph G(V,E) with p vertices and q edges is an onto mapping from V to {0, 1} such that each edge uv is assigned the label $(\lceil (f(u)^2 + f(v)^2)/2 \rceil)$ where $\lceil x \rceil$ (ceil(x)) is the least integer greater than or equal to x with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1 and the number of edges labeled with 0 and the number of edges labeled 0 and the number of edges labeled with 1 differ by at most 1".

Definition 2: The pentagonal snake $P(S_k)$ is obtained from a path $u_1,u_2,\ldots\ldots.u_k$ by joining u_i and u_{i+1} for $1 \leq i \leq k-1$,to two new vertices v_i, w_i, x_i and then joining v_i, x_i and x_i ,w_i.That is the path P_n by replacing each edge of the path by a cycle C_5.

Definition 3: Let G be a graph. The subdivision graph S (G) is obtained from G by subdividing each edge of G with a vertex

Definition 4: Double pentagonal snake D (PS_k) is obtained by two pentagonal snakes that have a common path.

Definition 5: An alternate pentagonal snake $A(PS_k)$is obtained from a path $u_1,u_2, \ldots , u_n$ by joining u_i and u_{i+1} to two new vertices v_i ,w_i and by joining v_i and w_i to a new vertex x_i respectively .That is, every alternate edge of a path is replaced by a cycle C_5

3Main Results

Theorem 1: Pentagonal snake PS_kadmits mean square cordial labelling $\forall k \geq 2$.

Proof: Let $P_k$be the path $u_1,u_2,\ldots,u_k$ .Let $V(PS_k) = V(P_k) \cup \{v_i,w_i ,x_i$: i varies from 1 to k-1} and $E(PS_k)=\{[(u_iu_{i+1})$:i varies from 1 to k-1] $\cup [(u_iv_i$: i varies from 1 to k-1 } $\cup [(v_ix_i$:i varies from 1 to k-1] $\cup [(x_iw_i$: i varies from 1 to k-1 } $\cup [(w_iu_i$:i varies from 1 to k-1]

Here $|V| = 4k - 3$ and $|E| = 5k - 5$

Define f maps $V(PS_k)$ to { 0,1}

Case (i) k is odd

$f(u_i) = 0, i$ varies from 1 to $(k+1)/2$

$1, i$ varies from $(k+3)/2$ to k

$f(v_i) = f(w_i) = f(x_i) = 0, i$ varies from 1 to $(k-1)/2$

$1, i$ varies from $(k+1)/2$ to $k-1$

Then the induced edge labelling is as follows

$f(u_i u_{i+1}) = 0, i$ varies from 1 to $(k-1)/2$

$1, i$ varies from $(k+1)/2$ to $k-1$

$$f(u_i v_i) = f(v_i x_i)$$
$$= f(x_i w_i) = f(u_{i+1} w_i) \quad = \quad 0, i \text{ varies from } 1 \text{ to } (k-1)/2,$$
$$1, i \text{ varies from } (k+1)/2 \text{ to } k-1$$

The vertex and edge cardinality of 0 label and 1 label are shown in the following table

T	0	1
$\lvert v_f(t)\rvert$	$2k-1$	$2k-2$
$\lvert e_f(t)\rvert$	$\frac{5k-5}{2}$	$\frac{5k-5}{2}$

Case (ii) k is even

$f(u_i) = 0, i$ varies from 1 to $k/2$

$1, i$ varies from $(k+2)/2$ to k

$f(v_i) = f(x_i) = 0, i$ varies from 1 to $(k-1)/2$

$1, i$ varies from $(k+1)/2$ to $k-1$

$f(w_i) = 0, i$ varies from 1 to $(k-2)/2$

$1, i$ varies from $k/2$ to $k-1$

Then the induced edge labelling is as follows

$f(u_i u_{i+1}) = f(x_i w_i) = f(u_{i+1} w_i) = 0, i$ varies from 1 to $(k-2)/2$

$1, i$ varies from $k/2$ to $k-1$

The vertex and edge cardinality of 0 label and 1 label are shown in the following table

T	0	1
$\lvert v_f(t)\rvert$	$2k-1$	$2k-2$
$\lvert e_f(t)\rvert$	$\frac{5k-6}{2}$	$\frac{5k-4}{2}$

Hence pentagonal snake PS_kadmits mean square cordial labeling$\forall k \geq 2$.

Illustr:

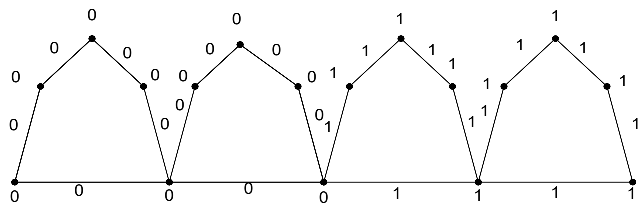


Figure 1: Mean square cordial labeling of pentagonal snake PS_5

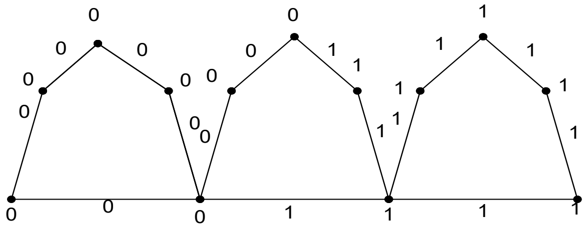


Figure 2: Mean square cordial labeling of pentagonal snake PS_4

Theorem: 2 Subdivision of a pentagonal snake $S(PS_k)$ admits mean square cordial labeling $\forall k \geq 3$ *and k is odd.*

Proof: Let P_k be the path $u_1, u_2, \ldots, u_k$. Let $V(PS_k) = V(P_k) \cup \{v_i, w_i, x_i$: varies from 1 to k-1$\}$ and $V(S(PS_k)) = V(PS_k) \cup \{a_i, b_i, c_i, d_i, e_i$:i varies from 1 to k-1$\}$ Then $E(S(PS_k)) = \{[(u_ib_i)$: i varies from 1 to k-1$] \cup [(b_iv_i)$: i varies from 1 to k-1$] \cup [(v_ic_i)$: i varies from 1 to k-1 $\} \cup [(c_ix_i)$:i varies from 1 to k-1$] \cup [(x_id_i)$:i varies from 1 to k-1$] \cup [(d_iw_i)$: i varies from 1 to k-1 $\cup [(w_ie_i)$: i varies from 1 to k-1 $\cup [(e_iu_{i+1})$: i varies from 1 to k-1 $\} \cup [(a_iu_{i+1})$: i varies from 1 to k-1 $\} \cup [(a_iu_i)$: i varies from 1 to k-1 $\}$

/Here $|V| = 9k - 8$ *and* $|E| = 10k - 10$

Define f maps $V(S(PS_k))$ to $\{0,1\}$

$$f(u_i) = \begin{cases} 0, i \text{ varies from } 1 \text{ to } (k+1)/2 \\ 1, i \text{ varies from } (k+3)/2 \text{ to } k \end{cases}$$

$$f(v_i) = f(w_i) = f(x_i) =$$
$$f(a_i) = f(b_i) = f(c_i) =$$
$$f(d_i) = f(e_i) = \begin{cases} 0, i \text{ varies from } 1 \text{ to } (k-1)/2 \\ 1, i \text{ varies from } (k+1)/2 \text{ to } k-1 \end{cases}$$

Then the induced edge labelling is as follows

$$f(u_ib_i) = f(b_iv_i) = f(v_ic_i) =$$
$$f(c_iw_i) = f(d_iw_i) = f(d_ix_i) =$$
$$f(x_ie_i) = f(e_iu_{i+1}) = f(u_{i+1}a_i) =$$
$$f(a_iu_i) = \begin{cases} 0, i \text{ varies from } 1 \text{ to} (k-1)/2, \\ 1, i \text{ varies from} (k+1)/2 \text{ to } k-1 \end{cases}$$

The vertex and edge cardinality of 0 label and 1 label are shown in the following table

T	0	1
$\lvert v_f(t) \rvert$	$\frac{9k-7}{2}$	$\frac{9k-9}{2}$
$\lvert e_f(t) \rvert$	$5k-5$	$5k-5$

Hence subdivision of a pentagonal snake $S(PS_k)$ admits mean square cordial labeling $\forall k \geq 2$ and k is odd.

Illustration:

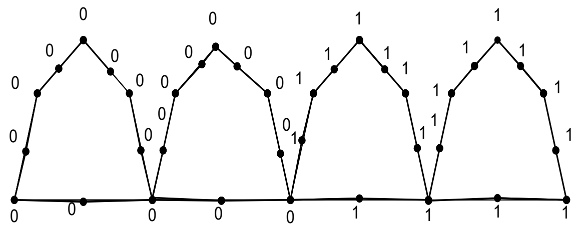


Figure 3: Mean square cordial labeling of pentagonal snake $S(P\,S_5)$

Theorem 3: Double pentagonal snake $D(PS_k)$, k is odd and $k \geq 3$ admits mean square cordial labeling .

Proof: Let P_k be the path $u_1, u_2, u_3, \ldots u_k$. Let $V(D(PS_k)) = V(P_k) \cup \{ v_i, w_i, x_i$: i varies from 1 to k-1} $\cup \{ v_i', w_i', x_i' : i$ varies from 1 to $k-1\}$ and $E(D(PS_k))=\{[u_i u_{i+1} : i$ varies from 1 to $k-1\} \cup [(u_i v_i : i$ varies from 1 to $k-1\} \cup [(v_i w_i : i$ varies from 1 to $k-1] \cup [(w_i x_i : i$ varies from 1 to $k-1\} \cup [(u_{i+1} x_i : i$ varies from 1 to $k-1)\} \cup [(v_i' w_i' : i$ varies from 1 to $k-1] \cup [w_i' x_i' : i$ varies from 1 to $k-1\} \cup [(x_i' u_{i+1} : i$ varies from 1 to $k-1)]$.

Here $|V| = 7k - 6$ and $|E| = 9n - 9$.

Define f maps $V(PS_k)$ to $\{0,1\}$

$f(u_i) = 0, i$ varies from 1 to $(k+1)/2$

$\quad 1, i$ varies from $(k-1)/2$ to k

$f(v_i) = f(w_i) = f(x_i) = 0, i$ varies from 1 to $(k-1)/2$

$\quad 1, i$ varies from $(k+1)/2$ to $k-1$

$f(v_i') = f(w_i') = f(x_i') = 0, i$ varies from 1 to $(k-1)/2$

$\quad 1, i$ varies from $(k+1)/2$ to $k-1$

Then the induced edge labelling is as follows

$f(u_i u_{i+1}) = 0, i$ varies from 1 to $(k-1)/2$

$\quad 1, i$ varies from $(k+1)/2$ to $k-1$

$$f(u_i v_i) = f(v_i w_i) = f(w_i x_i) = f(u_i^{'} v_i^{'}) = f(v_i^{'} w_i^{'}) = f(w_i^{'} x_i^{'}) = \begin{cases} 0, i \text{ varies from } 1 \text{ to } (k-1)/2, \\ 1, i \text{ varies from } (k+1)/2 \text{ to } (k-1)/2, \end{cases}$$

$$f(v_i w_i) = \begin{cases} 0, i \text{ varies from } 1 \text{ to } (k-1)/2 \\ 1, i \text{ varies from } (k+1)/2 \text{ to } k-1, \end{cases}$$

$$f(x_i u_{i+1}) = f(x_i^{'} u_{i+1}) = \begin{cases} 0, i \text{ varies from } 1 \text{ to } (k-1)/2 \\ 1, i \text{ varies from } (k+1)/2 \text{ to } k-1, \end{cases}$$

From the following table , we can conclude that the above vertex labeling, say f, is a mean square cordial labeling

T	0	1
$\lvert v_f(t)\rvert$	$\frac{7k-6}{2}$	$\frac{7k-6}{2}$
$\lvert e_f(t)\rvert$	$\frac{9n-9}{2}$	$\frac{9n-9}{2}$

Illustration:

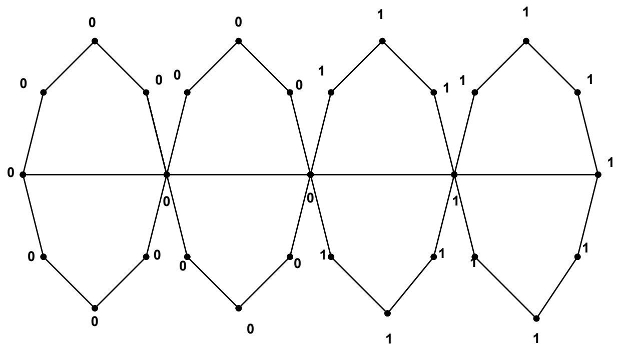


Figure 1: Mean square cordial labeling of pentagonal snake $D(PS_5)$

Theorem: 4 Alternate pentagonal snake $A(PS_k)$, $k \geq 2$ and the pentagon starts from first vertex and ends with last vertex of a path admits mean square cordial labeling.

Proof : Let P_k be the path $u_1, u_1', u_2, u_2', \ldots, u_k, u_k'$. Let $V(A(PS_k)) = V(P_k) \cup \{v_i, w_i, x_i$: i varies from 1 to k} and $E(A(PS_k))$={ [($u_i u_i'$):i varies from 1 to k] $\cup$ [($u_i' u_{i+1}$):i varies from 1 to k] $\cup$ [($u_i v_i$: i varies from 1 to k} $\cup$ [($v_i x_i$:i varies from 1 to k-1] $\cup$ [($v_i x_i$:i varies from 1 to k-1] $\cup$ [($x_i u_i'$: i varies from 1 to k]}.

Here $|V| = 5k$ and $|E| = 6n - 1$.

Define f maps $V(A(PS_k))$ to { 0,1}

Case (i): k is even

$$f(u_i) = f(u_i') = f(v_i) = f(w_i) = f(x_i) = \begin{cases} 0, i \text{ varies from } 1 \text{ to } k/2 \\ 1, i \text{ varies from } (k/2)+1 \text{ to } k \end{cases}$$

Then the induced edge labelling is as follows

$$f(u_i u_{i'}) = \begin{cases} 0, i \text{ varies from } 1 \text{ to } k/2 \\ 1, i \text{ varies from } (k+2)/2 \text{ to } k \end{cases}$$

$$f(u_i u_{i+1}) = \begin{cases} 0, i \text{ varies from } 1 \text{ to } (k/2)-1 \\ 1, i \text{ varies from } k/2 \text{ to } k-1 \end{cases}$$

$$f(u_i v_i) = f(v_i w_i) = f(w_i x_i) = f(x_i u_i') = \begin{cases} 0, i \text{ varies from } 1 \text{ to } (k)/2 \\ 1, i \text{ varies from } (k+2)/2 \text{ to } k \end{cases}$$

From the following table , we can conclude that the above vertex labeling, say f, is a mean square cordial labelling

T	0	1
$\lvert v_f(t)\rvert$	$\frac{5k+1}{2}$	$\frac{5k-1}{2}$
$\lvert e_f(t)\rvert$	$\frac{6n-1}{2}$	$\frac{6n}{2}$

Case (i): k is odd

$$f(u_i) = f(v_i) = f(w_i) = \begin{cases} 0, i \text{ varies from } 1 \text{ to } (k+1)/2 \\ 1, i \text{ varies from } (k+3)/2+1 \text{ to } k \end{cases}$$

$$f(x_i) = f(u_i') = \begin{cases} 0, i \text{ varies from } 1 \text{ to } (k-1)/2 \\ 1, i \text{ varies from } (k+1)/2+1 \text{ to } k \end{cases}$$

Then the induced edge labelling is as follows

$$f(u_i u_{i'}) = \begin{cases} 0, i \text{ varies from } 1 \text{ to } (k-1)/2 \\ 1, i \text{ varies from } (k+1)/2 \text{ to } k \end{cases}$$

$$f(u_i u_{i+1}) = \begin{cases} 0, i \text{ varies from } 1 \text{ to } (k-1)/2 \\ 1, i \text{ varies from } (k+1)/2 \text{ to } k-1 \end{cases}$$

$$f(u_i v_i) = f(v_i w_i) = \begin{cases} 0, i \text{ varies from } 1 \text{ to } (k-1)/2 \\ 1, i \text{ varies from } (k+1)/2 \text{ to } k \end{cases}$$

$$f(w_i x_i) = f(x_i u_i') = \begin{cases} 0, i \text{ varies from } 1 \text{ to } (k+1)/2 \\ 1, i \text{ varies from } (k+3)/2 \text{ to } k \end{cases}$$

From the following table , we can conclude that the above vertex labeling, say f, is a mean square cordial labelling

T	0	1
$\lvert v_f(t)\rvert$	$\frac{5k}{2}$	$\frac{5k}{2}$
$\lvert e_f(t)\rvert$	$\frac{6n-1}{2}$	$\frac{6n}{2}$

Hence alternate pentagonal snake A(PS$_k$), $k \geq 2$ and the pentagon starts from first vertex $u_1$and ends with last vertex u_k'**admits mean square cordial labeling.**

Illustration:

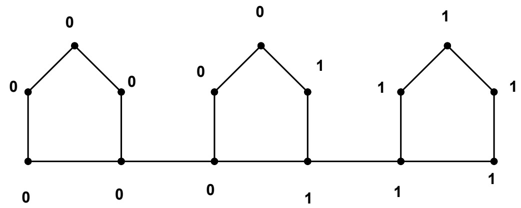


Figure 4: Mean square cordial labeling of pentagonal snake A(PS$_3$)

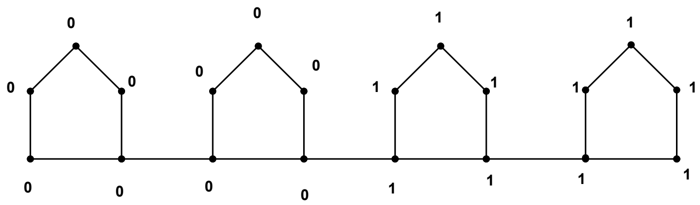
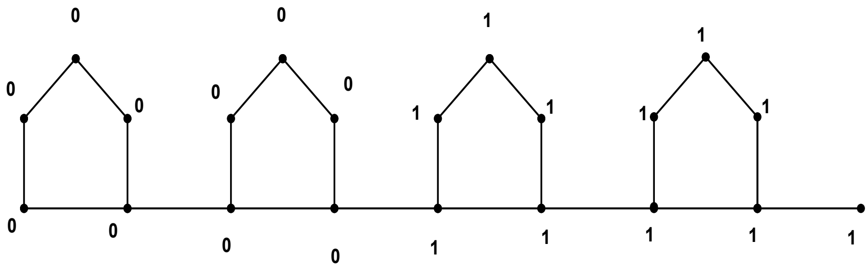


Figure 5: Mean square cordial labeling of pentagonal snake A(PS$_4$)

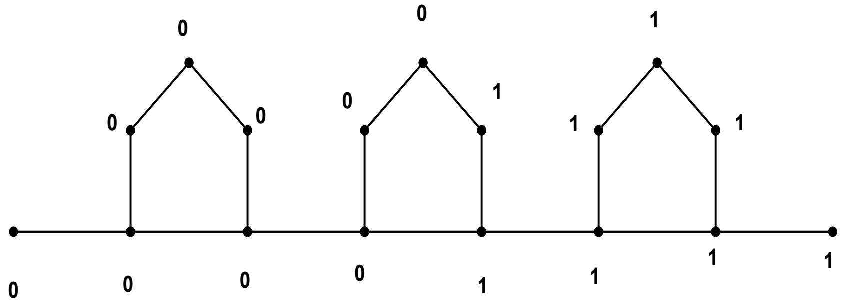
Corollary (i): **Alternate pentagonal snake A(PS$_k$), for all $k \geq 2$ and the pentagon starts from first vertex and ends with second last vertex admits mean square cordial labeling.**

Illustration:



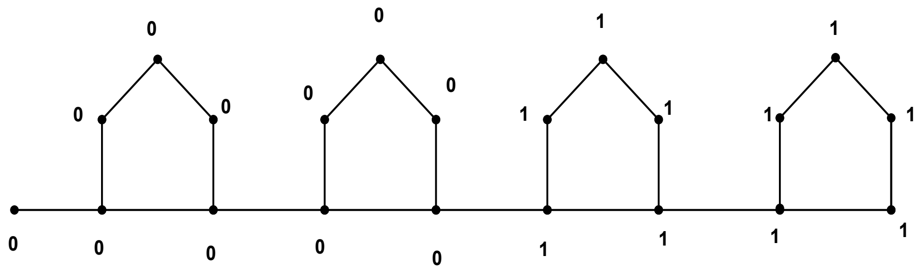
Corollary (ii): **Alternate pentagonal snake A(PS_k), for all $k \geq 2$ and the pentagon starts from second vertex and ends with second last vertex admits mean square cordial labeling.**

Illustration:



Corollary (iii): **Alternate pentagonal snake A(PS_k), for all $k \geq 2$ and the pentagon starts from second vertex and ends with last vertex admits mean square cordial labeling.**

Illustration:



4Conclusion

In this section mean square cordial labeling is investigated for some pentagonal snake graphs. It can be further investigated by the researcher for some more snake related graphs like alternate triangular snake graphs, double triangular snake graphs, alternate quadrilateral snake graphs, double quadrilateral graphs etc.

5Future Scope

The same labelling technique can be verified for some other family of graphs.Also graph operations like union, intersection, corona of two graphs etc., can also be analysed for mean square cordial labeling in future.

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