# Pythagorean Fuzzy Cognitive Maps in Making Optimal Decisions on Feasible Strategies for Inhibiting Electronic Waste

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**Abstract.** Electronic waste is an emerging cataclysm to the environment in recent times and the mounting industrial sectors are the constant contributors to it. Escalation of technology is being hailed but the environment impacts highly intrudes the resilience of the ecosystem. Refraining from technology is quite not possible in this digital epoch but reclamation of the milieu has high probability. Decision making on inhibiting electronic waste demands feasible strategies and it is the most challenging task for the industrial administrators. This paper proposes the new approach of Pythagorean fuzzy cognitive maps to find the viable strategies of electronic waste inhibition. The representation of the elements of the connection matrix as Pythagorean fuzzy sets is a distinctive feature of the research work. The strategies are subjected to the analysis of the experts to investigate their interrelational impacts. The resultant of the decision making model based on Pythagorean fuzzy cognitive maps is the score value of the strategies based on which they are ranked. This paper also presents the comparative analysis of Pythagorean fuzzy cognitive maps over fuzzy cognitive maps.

**Keywords:** Pythagorean Sets, Fuzzy Cognitive Maps, Decision Making, Electronic Waste.

## 1 Introduction

In this world of technology, the Electrical and Electronics Engineering industries are advancing by launching their new and innovative products. Manifold products are designed with the features of enhanced technology that makes the existing products turn obsolete. New product promotion is one of the default annual business targets of such kinds of industries and it is their nature to treat themselves as self-competitors and compete with themselves in terms of profit maximization. The ameliorating tendency of these industries has resulted in the generation of electronic waste and now it has become a great menace to environment sustainability of the nations worldwide. The disposal of electronic waste is nearly around 20 to 30 million tonnes every year universally and in India nearly 5.2 million tonnes is being

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generated annually. Industrial sectors, individual house hold and manufactures are the prime contributors to electronic waste. Nearly 10 states contribute to 70% and 65 cities contribute to 60% of total waste generated in our country [10]. In addition to it, the import of E waste by some companies are also propelling environmental catastrophe. If the electronic waste generation is not monitored and controlled, then India will emerge as one of the leading nation contributing to the electronic waste generation at global level.

Legislative measures are exclusively put forwarded to regulate and minimize the generation of electronic waste, but lack of strict enforcement of these laws is a setback [3]. Researchers are investigating on the optimal methods of mitigating waste generation and treating the waste generated. The electrical components are composed of toxic constituents such as lead, cadmium, mercury and other substances causing adversity to the mankind and environment. The electrical and electronic industrial sectors must devise feasible strategies to decrease waste generation and choose optimal methods of waste treatment. Special focus shall also be laid on product design, product production. If formulating strategies is a challenging task for these industries then selection of optimal strategy is yet another perplexing mission. The industrial sectors shall seek expert's opinion in framing and customizing the strategies that could meet the objectives of inhibiting waste generation. Scientific methods shall be used to make optimal decisions on feasible strategies as it is not possible to evaluate the effectiveness of all the strategies by implementing but rather it shall be stimulated to find the effectiveness of these strategies and their inter impacts. Fuzzy Cognitive Maps (FCM) [6] is a kind of scientific decision making model typically used in optimum decision making. FCM is a directed graph comprising of vertices and edges that represent the factors which are the elements of decision making. The strength of the relationship between the edges takes the values from [-1,1]. If the edge that connects the nodes representing the factors has weight 1, 0 and -1 then the factors has positive influence, no influence and negative influence over one another respectively. The connection matrix of FCM represents the edge weights between the factors. The FCM models are extended to intuitionistic and neutrosophic FCM with the intuitionistic [2,5] and neutrosophic connection matrix [12] respectively. FCM models with linguistic connection matrix was also developed in which the connection matrix consists of linguistic variables representing the inter associations between the factors [8]. These linguistic variables are quantified using fuzzy numbers, intuitionistic and neutrosophic fuzzy numbers. FCM models have wide spread applications in making decisions on pattern recognition, modeling of manufacturing systems, evaluating data interchange phenomenon, data mining, project management, investment analysis [1,4,9]. In this research work as a new initiative is made in developing Pythagorean FCM model in which the linguistic connection matrix is quantified using Pythagorean fuzzy sets. Yager [13] introduced Pythagorean fuzzy sets and it has also varied kind of applications in Multi-criteria decision making. Pythagorean fuzzy sets are also applied in the decision making methods such as DEMATEL, TOPSIS, VIKOR, PROMTHEE to make optimal decisions on various business management aspects such as supply chain management, product production and so on [7]. To the best of our knowledge, Pythagorean fuzzy sets are not used in FCM models and this paper initiates it. As many researchers are using Pythagorean fuzzy sets in different decision making techniques and based on their remarks on the efficiency of Pythagorean fuzzy sets, the authors are motivated to develop FCM model with Pythagorean fuzzy sets representations. In this paper a decision making model on determining the optimal strategies of E-waste generation is developed with the integration of the concepts of Pythagorean fuzzy sets. The scores of each strategy is determined and based on the score values the strategies are ranked and the optimal strategies are found.

The paper is organized as follows: section 2 presents the methodology; section 3 presents the application of the proposed model to the decision making scenario; section 4 discusses the results and the last section concludes the work.

## 2 Methodology

This section comprises of the steps contained in the proposed Pythagorean FCM model.

Step1. The strategies are formulated considering the nature of the company, principles of production, legislative demands and other influencing factors. The strategies are taken as the factors and represented as nodes or vertices of the directed graph.

Step 2. The connection matrix M is formulated using Pythagorean sets. A Pythagorean set P is of the form $\{(x,A(x),C(x)):x\in X\}, A(x):X\to[0,1], C(x):X\to[0,1]$ , where X is the universal set , A(x) & C(x) are the membership & non-membership degrees for each $x\in X$ satisfying the condition of $(A(x))^2 + (C(x))^2 \leq 1$[13]. The Pythagorean fuzzy sets are the special case of intuitionistic fuzzy sets and it is defuzzified [11] by $\frac{A(x)}{A(x)+C(x)}$

Step 3. Consider an initial vector of the form X = (10000..0) and pass on to M, in this initial vector the value 1 represents that the first factor is kept in ON position and other factors in OFF position, threshold the resultant vector by assigning the value 1 to the factor in ON position and the greatest value, assign the value 0 to the remaining. The new vector is X1

Step 4 : Repeat Step 3 to the vector X1 and continue until two threshold vectors are alike. The state of attaining two like vectors indicates the presence of limit cycle.

Step 5: Repeat the Step 3 & 4 for other factors. The final vectors attained after each setting are tabulated (see Table 3.2 ) to determine the final score values of each factors.

Step 6: The optimal factors are determined based on the final score values.

## 3 Decision Making on the Strategies to Minimize Electronic Waste

This section presents the application of the proposed method in determining the optimal strategies. The proposed strategies from the outlook of experts are presented as follows,
ES1: Production of Eco - friendly electronic products
ES2: Designing of products embedded with eco-consciousness.
ES3: Selection of suitable electronic waste management method
ES4: Electronic products must possess features of recycling.
ES5: Product production with strict adherence to environmental regulations
ES6: Periodical review of the electronic waste stock
ES7: Choosing the optimal method of disposal
ES8: Selection of feasible electronic waste treatment methods to mitigate the environmental effects

ES9: Production of multi-faceted products to fulfill multiple needs
ES10: Innovation in creating secondary products from the generated electronic waste
The connection matrix M representing the linguistic inter impacts of the factors

| | ES1 | ES2 | ES3 | ES4 | ES5 | ES6 | ES7 | ES8 | ES9 | ES10 |
|---|---|---|---|---|---|---|---|---|---|---|
| ES1 | VL | VH | H | H | H | H | H | VH | M | H |
| ES2 | VH | VL | VH | VH | VH | H | H | H | H | H |
| ES3 | VH | H | VL | H | H | M | H | H | H | H |
| ES4 | VH | H | H | VL | H | H | H | H | H | H |
| ES5 | VH | VH | VH | VH | VL | VH | VH | VH | VH | VH |
| ES6 | L | L | L | L | VH | M | M | M | L | L |
| ES7 | M | M | M | VH | VH | VH | VL | VH | H | M |
| ES8 | M | M | M | M | VH | VH | VH | VL | H | H |
| ES9 | H | H | H | H | H | VH | H | H | VL | H |
| ES10 | H | H | H | H | H | VH | M | M | M | VL |

**Table 3.1.** Pythagorean Quantification of Linguistic Variable

| **Linguistic Variable** | **Pythagorean Quantification** | **Crisp value** |
|---|---|---|
| Very High | (0.9,0.1) | 0.9 |
| High | (0.7,0.2) | 0.78 |
| Moderate | (0.6,0.5) | 0.55 |
| Low | (0.1,0.9) | 0.1 |
| Very Low | (0,0) | 0 |

The quantified matrix is

| | ES1 | ES2 | ES3 | ES4 | ES5 | ES6 | ES7 | ES8 | ES9 | ES10 |
|---|---|---|---|---|---|---|---|---|---|---|
| ES1 | 0 | 0.9 | 0.78 | 0.78 | 0.78 | 0.78 | 0.78 | 0.9 | 0.55 | 0.78 |
| ES2 | 0.9 | 0 | 0.9 | 0.9 | 0.9 | 0.78 | 0.78 | 0.78 | 0.78 | 0.78 |
| ES3 | 0.9 | 0.78 | 0 | 0.78 | 0.78 | M | 0.78 | 0.78 | 0.78 | 0.78 |
| ES4 | 0.9 | 0.78 | 0.78 | 0 | 0.78 | 0.78 | 0.78 | 0.78 | 0.78 | 0.78 |
| ES5 | 0.9 | 0.9 | 0.9 | 0.9 | 0 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 |
| ES6 | 0.1 | 0.1 | 0.1 | 0.1 | 0.9 | 0 | 0.55 | 0.55 | 0.1 | 0.1 |
| ES7 | 0.55 | 0.55 | 0.55 | 0.9 | 0.9 | 0.9 | 0 | 0.9 | 0.78 | 0.55 |
| ES8 | 0.55 | 0.55 | 0.55 | 0.55 | 0.9 | 0.9 | 0.9 | 0 | 0.78 | 0.78 |
| ES9 | 0.78 | 0.78 | 0.78 | 0.78 | 0.78 | 0.9 | 0.78 | 0.78 | 0 | 0.78 |
| ES10 | 0.78 | 0.78 | 0.78 | 0.78 | 0.78 | 0.9 | 0.55 | 0.55 | 0.55 | 0 |

The first factor ES1 is kept in ON position
Let X1 = (1000000000)
X1*M = (0 0.9 0.78 0.78 0.78 0.78 0.78 0.9 0.55 0.78)
(1100000100) = X2

X2*M = (1.45 1.45 2.23 2.23 2.58 2.46 2.46 1.68 2.11 2.34)
(1000100000) = X3
X3*M = (0.9 1.8 1.68 1.68 0.78 1.68 1.68 1.8 1.45 1.68)
(1100000101) = X4
X4*M = (2.23 2.23 3.01 3.01 3.36 3.36 3.01 2.23 2.66 2.34)
(1000110000) = X5
X5*M = (1 1.9 1.78 1.78 1.68 1.68 2.23 2.35 1.55 1.78)
(1000000100) = X6
X6*M = (0.55 1.45 1.33 1.33 1.68 1.68 1.68 0.9 1.33 1.56)
(1000111000) = X7
X7*M = (1.55 2.45 2.33 2.68 2.58 2.58 2.23 3.25 2.33 2.33)
(1000000100) = X8

X6 = X8 the fixed point is obtained.

By repeating in the same mode, the following resultant vectors are obtained and presented in Table 3.2.

**Table 3.2.** Score values of the Factors

| On position of the Strategies | ES1 | ES2 | ES3 | ES4 | ES5 | ES6 | ES7 | ES8 | ES9 | ES10 |
|---|---|---|---|---|---|---|---|---|---|---|
| (1000000000) | 1.55 | 2.45 | 2.33 | 2.68 | 2.58 | 2.58 | 2.23 | 3.25 | 2.33 | 2.33 |
| (0100000000) | 1.55 | 0.65 | 1.55 | 1.9 | 2.7 | 1.68 | 1.33 | 2.23 | 1.66 | 1.43 |
| (0010000000) | 1.8 | 1.68 | 0.9 | 1.68 | 0.78 | 1.45 | 1.68 | 1.68 | 1.68 | 1.68 |
| (0001000000) | 1.45 | 1.33 | 1.33 | 0.55 | 1.68 | 1.68 | 1.68 | 0.78 | 1.56 | 1.56 |
| (0000100000) | 6.36 | 6.12 | 6.12 | 6.47 | 7.5 | 7.39 | 6.8 | 6.92 | 6 | 6.23 |
| (0000010000) | 1 | 1 | 1 | 1 | 0.9 | 0.9 | 1.45 | 1.45 | 1 | 1 |
| (0000001000) | 1.1 | 1.1 | 1.1 | 1.45 | 1.8 | 1.8 | 0.9 | 0.9 | 1.56 | 1.33 |
| (0000000100) | 1.1 | 1.1 | 1.1 | 1.45 | 1.8 | 1.8 | 0.9 | 0.9 | 1.56 | 1.33 |
| (0000000010) | 1.68 | 1.68 | 1.68 | 1.68 | 0.78 | 1.8 | 1.68 | 1.68 | 0.9 | 1.68 |
| (0000000001) | 0.88 | 0.88 | 0.88 | 0.88 | 1.68 | 0.9 | 1.1 | 1.1 | 0.65 | 0.1 |
| **Total Weight** | 18.47 | 17.99 | 17.99 | 19.74 | 22.2 | 21.98 | 19.75 | 20.89 | 18.9 | 18.67 |
| **Score Values** | 1.847 | 1.799 | 1.799 | 1.974 | 2.22 | 2.198 | 1.975 | 2.089 | 1.89 | 1.867 |
| **Rank** | 8 | 9 | 9 | 5 | 1 | 2 | 4 | 3 | 6 | 7 |


**Fig. 3.1.** Ranking of the Factors

## 4 Discussion

The score values obtained in Table 3.2.are ranked and the factors representing the strategies that has the maximum values are categorized as the optimal strategies. The factors are ranked and it is represented in Fig.3.1. The strategies ES5, ES6, ES8,ES7,ES4 are the most optimal strategies and also they possess the influencing characteristic features. The decision makers shall consider these optimal strategies in making decisions on inhibiting the electronic waste. The electrical and electronics industries shall take up feasible measures based on these strategies to maximize their profit with minimal waste generation rate.

## 5 Conclusion

In this research work a new kind of FCM decision making model integrated with Pythagorean fuzzy sets is developed. The efficiency of Pythagorean fuzzy sets in quantifying the linguistic representation of the expert's opinion is validated by applying to the decision making scenario of optimal strategies on inhibiting electronic waste. The proposed model can be extended with various extended representations of Pythagorean fuzzy sets. This research work has certain limitations, the comparative analysis with other FCM model are not done, substantiation of using Pythagorean fuzzy sets of representation is not highly stated.

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