

Impulsive Control Approach to Stabilization of Delayed Inertial Neural Networks

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Abstract: Stabilization of delayed inertial neural networks based on impulses is investigated in this paper. Delay-dependent sufficient conditions of stabilization results are obtained as linear matrix inequalities via Lyapunov stability theory which involves the construction of Lyapunov-Krasovskii functional. Information of time-delay is taken into account to obtain these results. Here, time-delay is considered to be time-varying and the activation function is assumed to be sector bounded. Derived conditions can be validated via MATLAB. Finally, an example is provided to support the derived results.

Keywords: Inertial neural networks, impulses, Lyapunov-Krasovskii functional, stabilization.

1. Introduction:

Past decades witnessed the dynamical analysis of various types of neural networks (NNs) namely Hopfield NNs, bidirectional associative memory (BAM) NNs, cellular NNs, Cohen-Grossberg NNs and so on. In the year 1981, Babcock and Westervelt published an article on the dynamics of simple electronic NNs which initiated the research on inertial NNs (INNs). In their research work, they mentioned that inertial characteristics added to resistance and capacitance couplings can cause spontaneous oscillation, chaotic response and so on. Dynamics of INNs received much attention among researchers because of its both physical and biological significance, for details one can see [1]-[3], [7]-[12] and therein.

Meanwhile, time-delays which may occur during the process of storage of information and transmission in NNs can cause instability, oscillation in the dynamics of NNs. Hence, there is an increasing interest to investigate the dynamics of NNs with the inclusion of time-delays, for details see [4]-[8]. On the other hand, states of NNs are often condition to abrupt change at certain instants of time due to the switching phenomenon, frequency change called impulse effects. Impulsive NNs belong to the special category which is the combination of continuous and discrete-time systems. Hence, it is necessary to include the effects of impulses in the dynamics of NNs, see [13]-[15]. Sometimes impulsive effects can destabilize stable systems which are considered as the destructive one but on the other hand they can also be used to stabilize the de-stable systems. In this work, impulses are used to stabilize the de-stable systems.

In [13], authors established distributed delay-dependent stability criteria for impulsive INNs in which they considered both discrete and distributed time-delays. Robust stability conditions for inertial BAM NNs with time-delays and uncertainties via impulsive effect are considered in [14]. Stability conditions of stochastic BAM NNs with Markovian jump, impulse

control and mixed time-delays is stated in [15]. Hence from the existing literature, it can be seen that stability analysis of INNs with impulsive effects is taken in to account whereas the problem of stabilization of INNs via impulsive control has not gained attention from the researchers. This motivates us to consider the problem of impulsive stabilization of time-delayed INNs.

Inspired by the above observations, stabilization problem of INNs based on impulsive controls is considered in this work. Here, the activation function of the neural network is assumed to be sector-bounded and the time-delay is taken to be time-varying. Sufficient delay-dependent stabilization results are developed for the considered problem based on the construction of Lyapunov-Krasovskii functional (LKF) involving some quadratic and integral terms. Schur complement lemma is used in the derivation process to convert some nonlinear matrix inequalities into linear matrix inequalities (LMIs). Obtained results can be easily solved through MATLAB software. An example is provided to check the validity of the developed theoretical results.

This paper is structured as follows. In second section, the problem under study is formulated and related preliminaries are given. In the third section, sufficient delay-dependent impulsive stabilization conditions pertaining to impulsive control are presented. In the fourth section, a numerical example is given to illustrate the proposed results and the fifth section concludes the paper.

Notations: In the entire paper, \square^n and $\square^{n \times m}$ denote, the n -dimensional Euclidean space and the set of all $n \times m$ real matrices respectively. The notation $X \geq Y$ ($X > Y$) means that $X - Y$ is positive-semidefinite (positive-definite), where both X and Y represent symmetric matrices; transpose of the matrix M is denoted by M^T ; identity matrix is represented by I which is of appropriate dimension and compatible with the corresponding expressions; $\|\cdot\|$ is the Euclidean norm in \square^n . $PC([- \tau_2, 0], \square^n)$ is the piece-wise continuous function; right-hand and left-hand limits of the function $\psi(s)$ are represented by $\psi(s^+)$ and $\psi(s^-)$ respectively; $\lambda_{\max}(A)$, $\lambda_{\min}(A)$ denote the maximum and minimum eigenvalues of the matrix A respectively, and for the matrices, if its dimension is not stated explicitly, are supposed to have compatible dimensions.

2. Formulation of the Problem and Preliminaries:

$$\ddot{m}_i(t) = -a_i \dot{m}_i(t) - b_i m_i(t) + \sum_{j=1}^n c_{ij} f_j(m_j(t)) + \sum_{j=1}^n d_{ij} f_j(m_j(t - \tau(t))) + I_i \quad (1)$$

for $i = 1, 2, \dots, n$, where $\ddot{m}_i(t)$ denotes the inertial term of the i^{th} neuron at time t , $m_i(t)$ is the state of the i^{th} neuron at time t . $f_j(\cdot)$ denotes the neuron activation function of i^{th} neuron at time t and $f_j(0) = 0$, $j \in \{1, 2, \dots, n\}$, $\tau(t)$ is the time-varying delay, I_i represents the external input on the i^{th} neuron at time t . a_i and b_i are positive constants, c_{ij} and d_{ij} are the connection weights related to the neurons without delays and with delays respectively. Initial condition of (1) is given by $m_i(s) = \varphi_i(s)$ and $\dot{m}_i(s) = \psi_i(s)$, for $-\tau_2 \leq s \leq 0$ where $\varphi_i(s)$ and $\psi_i(s)$ are bounded and continuous.

Now transform the second order differential equation model into a system of first order differential equations using the transformation $p_i(t) = \dot{m}_i(t) + m_i(t)$, $i = 1, 2, \dots, n$ and the resulting system is given as

$$\begin{cases} \dot{m}_i(t) = -m_i(t) + p_i(t) \\ \dot{p}_i(t) = -(a_i - 1)p_i(t) - [b_i + (1 - a_i)]m_i(t) + \sum_{j=1}^n c_{ij} f_j(m_j(t)) + \sum_{j=1}^n d_{ij} f_j(m_j(t - \tau(t))) + I_i \end{cases} \quad (2)$$

with the initial conditions $m_i(s) = \varphi_i(s)$ and $p_i(s) = \varphi_i(s) + \psi_i(s)$, for $-\tau_2 \leq s \leq 0$. Next, we intend to shift the equilibrium of system (2) to the origin by using the transformation $x(t) = m(t) - m^*$ and $y(t) = p(t) - p^*$, where $m(t) = [m_1(t), m_2(t), \dots, m_n(t)]^T$, $p(t) = [p_1(t), p_2(t), \dots, p_n(t)]^T$. Here (m^*, p^*) is the equilibrium point of (2). Therefore, the transformed system in matrix form can be written as

$$\begin{cases} \dot{x}(t) = -x(t) + y(t) \\ \dot{y}(t) = Ay(t) - Bx(t) + Cf(x(t)) + Dg(x(t - \tau(t))) \end{cases} \quad (3)$$

Initial conditions $x(s) = \phi(s) - m^*$ and $y(s) = \psi(s) + \phi(s) - p^*$, where $x(t) = [x_1(t), x_2(t), \dots, x_n(t)]^T$, $y(t) = [y_1(t), y_2(t), \dots, y_n(t)]^T$ are state vectors of (3), $g(x(t)) = [g_1(x_1(t)), g_2(x_2(t)), \dots, g_n(x_n(t))]^T$ with $g(x(t)) = f(x(t) + m^*) - f(m^*)$, $A = \text{diag}\{(a_1 - 1), (a_2 - 1), \dots, (a_n - 1)\}$, $B = \text{diag}\{b_1 + (1 - a_1), b_2 + (1 - a_2), \dots, b_n + (1 - a_n)\}$, $C = (c_{ij})_{n \times n}$, $D = (d_{ij})_{n \times n}$ and $I = \text{diag}\{I_1, I_2, \dots, I_n\}$.

System (3) with continuous and impulsive control becomes

$$\begin{cases} \dot{x}(t) = -x(t) + y(t) + E_1 u_1(t), \quad t \neq t_k, \\ \Delta x(t) = (F_1 - I)x(t^-) + E_2 u_2(t), \quad t = t_k, \\ \dot{y}(t) = Ay(t) - Bx(t) + Cf(x(t)) + Dg(x(t - \tau(t))) + E_3 u_3(t), \quad t \neq t_k, \\ \Delta y(t) = (F_2 - I)y(t^-) + E_4 u_4(t), \quad t = t_k, \end{cases} \quad (4)$$

where $u_1, u_3 \in \mathbb{R}^{n_1}$ are continuous control inputs and $u_2, u_4 \in \mathbb{R}^{n_2}$ are impulse control inputs. E_1, E_2, E_3, E_4, F_1 and F_2 are known constant matrices. $\Delta x(t)|_{t=t_k} = x(t_k^+) - x(t_k^-)$ and $\Delta y(t)|_{t=t_k} = y(t_k^+) - y(t_k^-)$. $x(t_k)$ and $y(t_k)$ denote the impulse at the moment t_k . Here the discrete time sequence t_k satisfies $0 = t_0 < t_1 < t_2 < \dots < t_k < \dots$ and $\lim_{k \rightarrow \infty} t_k = \infty$. Both $x(t_k)$ and $y(t_k)$ are assumed to be right-continuous, i.e., $x(t_k) = x(t_k^+)$ and $y(t_k) = y(t_k^+)$. Initial conditions $\phi(t), \psi(t) \in PC([-\tau_2, 0], \mathbb{R}^n)$ are piece-wise continuous functions at finite number of points.

Next design the controllers u_1, u_2, u_3 and u_4 as follows

$$u_1(t) = K_1 x(t), u_2(t^-) = K_2 x(t^-), u_3(t) = K_3 y(t), u_4(t) = K_4 y(t^-), \quad (5)$$

where K_1, K_2, K_3 and K_4 are control gain matrices to be designed. Also use the fact that $\Delta x(t) = x(t_k) - x(t_k^-)$ and $\Delta y(t) = y(t_k) - y(t_k^-)$ then system (4) becomes

$$\begin{cases} \dot{x}(t) = -x(t) + y(t) + E_1 K_1 x(t), \quad t \neq t_k, \\ \Delta x(t) = (F_1 - I)x(t^-) + E_2 K_2 x(t^-), \quad t = t_k, \\ \dot{y}(t) = Ay(t) - Bx(t) + Cf(x(t)) + Dg(x(t - \tau(t))) + E_3 K_3 y(t), \quad t \neq t_k, \\ \Delta y(t) = (F_2 - I)y(t^-) + E_4 K_4 y(t^-), \quad t = t_k. \end{cases} \quad (6)$$

Next, we give assumptions, definition and lemma which are essential to obtain the results.

Assumption 2.1: The neuron activation function $g_i(\cdot)$ in (6) is Lipschitz globally in $x(t)$, $g(0) = 0$ and it follows that

$$\left(g(x(t)) - G_1 x(t) \right)^T \left(g(t, x(t)) - G_2 x(t) \right) \leq 0, \quad \forall x(t) \in \mathbb{R}^n,$$

where G_1 and G_2 are known real constant matrices of appropriate dimensions.

Assumption 2.2: Time-varying delay $\tau(t)$ satisfies

$$0 \leq \tau(t) \leq \tau_2, \quad \dot{\tau}(t) \leq \mu < 1,$$

where τ_2 and μ are constants.

Definition 2.3: The equilibrium point of INNs with impulsive control and time-delay (6) is said to be exponentially stabilizable under impulsive control with positive convergence rate α if there exists $\lambda > 0$ such that

$$\|x(t)\|^2 + \|y(t)\|^2 \leq \lambda e^{-2\alpha(t-t_0)} \text{Sup}_{-\tau_2 \leq s \leq 0} [\|\phi(s)\|^2 + \|\psi(s)\|^2], \quad \forall t \geq t_0.$$

Definition 2.4: The function $V: [0, \infty) \times \mathbb{R}^n \times \mathbb{R}^n \rightarrow [0, \infty)$ belong to class Ω_0 if

(1) the function V defined above is continuous on all of the sets $[t_{k-1}, t_k) \times \mathbb{R}^n \times \mathbb{R}^n$, $\forall t \geq 0$ and it vanishes at $V(t, 0, 0)$,

(2) $V(t, x, y)$ is locally Lipschitzian,

(3) for every $k \in \mathbb{N}$, $\lim_{(t, \bar{x}, \bar{y}) \rightarrow (t_k^-, x, y)} V(t_k^-, x, y)$ exists and it is finite and further

$$\lim_{(t, \bar{x}, \bar{y}) \rightarrow (t_k^+, x, y)} V(t_k^+, x, y) \text{ with } V(t_k^+, x, y) = V(t_k^-, x, y).$$

3. Stabilization results for impulsive INNs:

In this section, we derive the stabilization conditions for the time-delay INNs based on impulsive control. Here, control gain matrices introduced in (5) is taken as

$$K_1 = L_1 X^{-1}, \quad K_2 = L_2 X^{-1}, \quad K_3 = L_3 Y^{-1}, \quad K_4 = L_4 Y^{-1}, \quad (7)$$

where L_1, L_2, L_3 and L_4 are unknown matrices to be determined.

Theorem: System (6) is exponentially stable under the Assumptions 2.1 and 2.2 if there exist scalars α, λ , positive-definite symmetric matrices P_1, P_2, \hat{Q} and R , matrices L_1, L_2, L_3 and L_4 such that the symmetric LMIs given below hold

$$\Omega_1 < 0, \Omega_2 < 0, \Omega_3 < 0, \quad (8)$$

$$\text{where } \Omega_1 = \begin{bmatrix} \hat{\Omega} & S_1 & S_2 \\ * & -I & 0 \\ * & * & -I \end{bmatrix}, \quad \Omega_2 = \begin{bmatrix} -X & XF_1^T + L_2^T E_2^T \\ * & -X \end{bmatrix}, \quad \Omega_3 = \begin{bmatrix} -Y & YF_2^T + L_4^T E_4^T \\ * & -Y \end{bmatrix}$$

with the stabilizing gains given by (7). Here $\hat{\Omega} = (\hat{\Omega}_{p,q})_{5 \times 5}$ with

$$\hat{\Omega}_{1,1} = -2X + \hat{Q} + 2\alpha X + E_1 L_1, \quad \hat{\Omega}_{1,3} = Y - XB^T, \quad \hat{\Omega}_{1,4} = X(G_1^T + G_2^T), \quad \hat{\Omega}_{\{2,2\}} = -e^{-2\alpha\tau_2} \hat{Q}, \quad \hat{\Omega}_{3,3} = -2AY + 2\alpha Y + E_3 L_3, \quad \hat{\Omega}_{3,4} = C, \quad \hat{\Omega}_{3,5} = D, \quad \hat{\Omega}_{4,4} = R - 2I, \quad \hat{\Omega}_{5,5} = -(1 - \mu)R, \quad S_1 = [G_1 X \ 0 \ 0 \ 0 \ 0]^T, \quad S_2 = [G_2 X \ 0 \ 0 \ 0 \ 0]^T$$

and the rest of the entries of symmetric block are zero.

Proof: Stabilization results for system (6) can be obtained from the construction of LKF as follows

$$V(t, x(t), y(t)) = x^T(t) P_1 x(t) + y^T(t) P_2 y(t) + \int_{t-\tau_2}^t e^{2\alpha(s-t)} x^T(s) Q x(s) ds + \int_{t-\tau(t)}^t e^{2\alpha(s-t)} g^T(x(s)) R g(x(s)) ds, \quad (9)$$

where P_1, P_2, Q and R are unknown positive-definite symmetric matrices of appropriate dimensions. From Definition 2.4, one can see that (9) is locally Lipschitz for each $t \in [t_{k-1}, t_k)$ and $V(t, 0, 0) = 0$. Moreover, all the conditions of Definition 2.4 hold and hence $V(t, x(t), y(t)) \in \Omega_0$ for each $t \in [t_{k-1}, t_k)$. Dini's upper right hand derivative of (9) along trajectory of (6) can be determined as follows:

$$D^+ V(t, x(t), y(t)) \leq 2x^T(t) P_1 \dot{x}(t) + 2y^T(t) P_2 \dot{y}(t) + x^T(t) Q x(t) - e^{-2\alpha\tau_2} x^T(t - \tau_2) Q x(t - \tau_2)$$

$$+g^T(x(t))Rg(x(t)) - (1 - \mu)g^T x(t - \tau(t))Rg(x(t - \tau(t))) \\ -2\alpha \left(\int_{t-\tau_2}^t e^{2\alpha(s-t)} x^T(s)Qx(s)ds + \int_{t-\tau(t)}^t e^{2\alpha(s-t)} g^T(x(s))Rg(x(s))ds. \right)$$

(10)

According to Assumption 2.1, we have

$$\begin{bmatrix} x(t) \\ g(x(t)) \end{bmatrix}^T \begin{bmatrix} -(G_1^T G_2 + G_2^T G_1) & G_1^T + G_2^T \\ * & -2I \end{bmatrix} \begin{bmatrix} x(t) \\ g(x(t)) \end{bmatrix} \geq 0. \quad (11)$$

Combining inequalities (10) and (11), we get

$$D^+V(t, x(t), y(t)) + 2\alpha V(t, x(t), y(t)) \leq \xi^T(t)\Omega\xi(t) < 0 \quad (12)$$

which implies that

$$\Omega < 0, \quad (13)$$

$\forall t \in [t_{k-1}, t_k]$ where $\xi^T(t) = [x^T(t) \quad x^T(t - \tau_2) \quad y^T(t) \quad g^T(x(t)) \quad g^T(x(t - \tau(t)))]$
and $\Omega = (\Omega_{i,m})_{5 \times 5}$ with

$$\begin{aligned} \Omega_{1,1} &= -2P_1 + Q + 2\alpha P_1 + P_1 E_1 K_1 - (G_1^T G_2 + G_2^T G_1), & \Omega_{1,3} &= P_1 - B^T P_2, & \Omega_{1,4} &= G_1^T + G_2^T, \\ \Omega_{2,2} &= -e^{-2\alpha\tau_2} Q, & \Omega_{3,3} &= -2P_2 A + 2\alpha P_2 + P_2 E_3 K_3, & \Omega_{3,4} &= P_2 C, \\ & & \Omega_{3,5} &= P_2 D, & \Omega_{4,4} &= R - 2I, \\ & & & & \Omega_{5,5} &= -(1 - \mu)R. \end{aligned}$$

From (9), at $t = t_k$, we have

$$\begin{aligned} V(t_k, x(t_k), y(t_k)) - V(t_k^-, x(t_k^-), y(t_k^-)) &= x^T(t_k)P_1 x(t_k) + y^T(t_k)P_2 y(t_k) - x^T(t_k^-)P_1 x(t_k^-) \\ &\quad - y^T(t_k^-)P_2 y(t_k^-) \\ &= x^T(t_k^-) \{-P_1 + (K_2^T E_2^T + F_1^T)P_1(F_1 + E_2 K_2)\} x(t_k^-) \\ &\quad + y^T(t_k^-) \{-P_2 + (K_4^T E_4^T + F_2^T)P_2(F_2 + E_4 K_4)\} y(t_k^-) \end{aligned}$$

which implies that

$$\begin{aligned} -P_1 + (K_2^T E_2^T + F_1^T)P_1(F_1 + E_2 K_2) &< 0 & (14) \\ -P_2 + (K_4^T E_4^T + F_2^T)P_2(F_2 + E_4 K_4) &< 0. & (15) \end{aligned}$$

One can notice that inequalities (13)-(15) are not LMIs and hence cannot be directly solved by using MATLAB LMI toolbox. So, we make use of Schur complement lemma to convert the inequalities (14) and (15) as

$$\begin{bmatrix} -P_1 & (F_1^T + K_2^T E_2^T)P_1 \\ * & -P_1 \end{bmatrix} < 0 \quad \text{--- (a)}, \quad \begin{bmatrix} -P_2 & (F_2^T + K_4^T E_4^T)P_2 \\ * & -P_2 \end{bmatrix} < 0 \quad \text{--- (b)}. \quad (16)$$

Now pre- and post-multiply inequalities (16a&b) respectively by $\text{diag}\{X, X\}$ and $\text{diag}\{Y, Y\}$ on both sides and using the relation $X = P_1^{-1}$, we get Ω_2 and Ω_3 . In order to get Ω_1 , pre- and post-multiply both sides of (13) by $\text{diag}\{X, I, Y, I, I\}$, where $Y = P_2^{-1}$ and also use the relation $-2X(G_1^T G_2)X \leq XG_1^T G_1 X + XG_2^T G_2 X$. After some algebraic manipulations, one can get Ω_1 .

From inequality (12), one can get

$$\begin{aligned} V(t, x(t), y(t)) &\leq e^{-2\alpha(t-t_{k-1})} V(t_{k-1}, x(t_{k-1}), y(t_{k-1})) \text{ for each } t \in [t_{k-1}, t_k] \text{ and} \\ V(t, x(t), y(t)) &\leq e^{-2\alpha(t-t_0)} V(t_0, x(t_0), y(t_0)). \end{aligned}$$

From LKF, we get

$$\begin{aligned} V(t, x(t), y(t)) &\geq \lambda_1 [\|x(t)\|^2 + \|y(t)\|^2], \quad \text{where } \lambda_1 = \lambda_{\min}(P_1) + \lambda_{\min}(P_2) \text{ and} \\ V(t_0, x(t_0), y(t_0)) &\leq \lambda_2 \text{Sup}_{-\tau_2 \leq s \leq 0} [\|\phi(s)\|^2 + \|\psi(s)\|^2], \quad \text{where } \lambda_2 = \lambda_{\max}(P_1) + \\ &\lambda_{\max}(P_2) + \tau_2 \lambda_{\max}(Q) + \tau_2 \lambda_{\max}(R) \text{ and hence} \end{aligned}$$

$\|x(t)\|^2 + \|y(t)\|^2 \leq \lambda e^{-2\alpha(t-t_0)} \text{Sup}_{-\tau_2 \leq s \leq 0} [\|\phi(s)\|^2 + \|\psi(s)\|^2], \lambda = \frac{\lambda_2}{\lambda_1}$. Hence by definition of exponential stability, system (6) is stable exponentially with stabilizing gains (7). This completes the proof of the theorem.

Remark: This research work investigates the stabilization analysis of INNs with time-delays under the influence of impulsive controls. Here information of both time-varying delay and its derivative is considered. Even though the stabilization problem of INNs with both discrete and distributed delay under impulsive control is investigated in [13], information on the bound of discrete delay is not considered in [13]. This work is focused to derive stabilization results of INNs which include the data on both time-delay and its derivative.

4. Numerical Example:

This section presents a numerical example through which we can see the validity of the derived results.

Example: Consider system (6) with the following parameters

$$A = \text{diag}\{4, 4\}, B = \text{diag}\{12, 11\}, C = \begin{bmatrix} 0.3 & 0 \\ -0.6 & -0.4 \end{bmatrix}, D = \begin{bmatrix} 0.2 & 0.3 \\ 0.4 & 0.7 \end{bmatrix}, E_1 = 0.3I, E_2 = 0.2I, \\ E_3 = E_4 = 0.2I, F_1 = F_2 = 0.1I, G_1 = 0.2I, G_2 = 0.1I, \tau_2 = 0.6, \mu = 0.2, \alpha = 0.1,$$

Now, the delay-dependent stabilization conditions obtained in Theorem 3.1 is solved through MATLAB LMI solvers for the above parameters and it can be seen that the considered system (6) is exponentially stabilizable. Feasible matrices that occur in the corresponding conditions of Theorem 3.1 are given below

$$X = \begin{bmatrix} 10.39 & -0.31 \\ -0.31 & 11.26 \end{bmatrix}, Y = \begin{bmatrix} 68.92 & -1.68 \\ -1.68 & 68.16 \end{bmatrix}, Q = \begin{bmatrix} 37.76 & 0 \\ 0 & 37.76 \end{bmatrix}, R = \begin{bmatrix} 4.61 & 2.17 \\ 2.17 & 7.97 \end{bmatrix}.$$

Stabilizing gains are given by

$$K_1 = \begin{bmatrix} -15.32 & -0.38 \\ 0.15 & -12.72 \end{bmatrix}, K_2 = K_4 = \begin{bmatrix} -0.5 & 0 \\ 0 & -0.5 \end{bmatrix}, K_3 = \begin{bmatrix} 16.65 & 0.02 \\ -0.04 & 16.12 \end{bmatrix},$$

$\lambda_1 = 0.10, \lambda_2 = 27.23, \lambda = 266.36$. Hence we get

$$\|x(t)\|^2 + \|y(t)\|^2 \leq 266.36 e^{-0.2t} \text{Sup}_{-0.6 \leq s \leq 0} [\|\phi(s)\|^2 + \|\psi(s)\|^2]$$

which shows that system (6) is exponentially stabilizable.

5. Conclusion:

In this work, stabilizability problem of INNs with time-delay under impulsive control is investigated. Both time-varying and constant type of time-delay is taken into account and the corresponding results are presented. LKF involving exponential terms are utilized in the process of obtaining stabilization results. Those conditions are checked using MATLAB through LMI solvers. The problem considered in this research work can be further extended with the incorporation of multiple time-delays in the place of single delay and also with parameter uncertainties.

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