Controllability and Asymptotically Almost Periodic Result of Nonlinear Neutral Integrodifferential Evolution System

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Abstract. This paper deals the existence of periodicity solutions for neutral integrodifferential evolution equation in Banach space. The results are obtained by using resolvent operators and a fixed point technique. The analysis begins with the almost periodic solution for the evolution equation. Further, the almost periodic solution of the integrodifferential evolution equation has been shown to be asymptotically almost periodic. Then controllability result has been shown to the same system.

Keywords: Controllability, neutral integrodifferential equation, delay differential equation, asymptotically almost automorphic ,2010 AMS Subject Classification: 93B05, 47D06, 34K40.

1 Introduction

Recently, the study of periodicity is one of the most attractive topics in mathematical analysis. Almost periodicity is the generalization of classical periodicity. It performs a significant role in several fields including harmonic analysis, physics, and dynamical system. Bochner [4] was first introduced these functions, which begin with abstract differential equations with periodic functions. Zaidman [22] explained that the existence of almost periodic solution for abstract differential equation with C_0-Semi group. Later, Guerekata [13] a studied the existence and uniqueness of almost automorphic solutions of semilinear evolution equations.

Almost periodic solutions of differential equations and their natures have been studied so many authors [2, 3, 8-10, 15, 16] in the very beginning of this century. Almost periodic solutions of evolution problems and time dependent evolution equations has been discussed in Francois et al,[12]. The existence and uniqueness of a compact almost automorphic solution for dissipative differential equations in Banach spaces were discussed by Drisi et al.,[11]and Ding [7] proved asymptotically almost automorphic solution for the differential equations. The new existence theorem for semilinear evolution equations with the asymptotically almost automorphic mild solutions of the following form.

$$x'(t) = A(t)x(t) + F(t, x(t))$$

has been proved by Cao [6]. By using measure of non compactness and generalization of Darbo's fixed point theorem Benchohra et al. investigated the asymptotically almost

ICCAP 2021, December 07-08, Chennai, India

DOI 10.4108/eai.7-12-2021.2314626

automorphic mild solutions for non autonomous semilinear evolution equations by Benchohra [5].

Controllability and almost periodic results for neutral impulsive evolution system, using analytic semigroup and fixed point principle has been studied by Radhakrishnan [20]. This fact that the present work is much more interested in authors. From these, our main contributions are highlighted as follows:

- A new set of sufficient conditions are established for the periodicity results of the nonlinear neutral integrodifferential evolution system.
- Most of the available literature, for the first time controllability with periodicity results involving resolvent operator, are have been investigated.
- The mild solution of the system has been shown to be Asymptotically Almost Periodic Mild (AAPM) solution.
- Subsequently, Controllability of the AAPM solution of the neutral integrodifferential evolution system has been proved.
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The structure of the work is planned as follows, in the second section basic definitions, Lemmas are given. Section 3 deals with the periodicity result. In section 4 controllability result of the nonlinear neutral integrodifferential system has been discussed.

2 Preliminaries

In this paper, consider the neutral integrodifferential evolution equation with delay

$$\left.\begin{aligned} \mathbb{D}\left[w(\eta) - \int_{-r}^{\eta} \mathbb{Y}(\eta,z)w(z)dz\right] &= A(\eta)w(\eta) + \int_{-r}^{\eta} \mathbb{Y}_2(\eta,z)w(z)dz \\ &\quad + d(\eta, w_\eta), \qquad \eta \in I, \\ w(0) + g(w) &= w_0, \end{aligned}\right\} \qquad (2.1)$$

where $\mathbb{D} = \frac{d}{d\eta}$, the state $w(\cdot)$ takes values in a Banach space $\mathcal{S}$. Here $A(\eta)$,$\mathbb{Y}(\eta,s)$ and $\mathbb{Y}_2(\eta,s)$, $0 \le s \le \eta \le b$ are closed and bounded linear operators defined on the common domain $\mathcal{D}(A)$ which is dense in $\mathcal{S}$. $d: I \times \mathcal{B} \to \mathcal{S}$, g are appropriated functions. The history $w_\eta: (-r,0] \to \mathcal{S}$, $w_\eta(\theta) = w(\eta+\theta)$, $\theta \in (0,\infty)$,belongs to an abstract phase space $\mathcal{B}$ defined axiomatically.

Initially consider the linear integrodifferential equation,

$$\frac{dw(\eta)}{d\eta} = A(\eta)w(\eta) + \int_{-r}^{\eta} \mathbb{Y}_2(\eta,s)w(z)dz, \qquad (2.2)$$

$$w(0) = \delta, \qquad \delta \in \mathcal{S}. \qquad (2.3)$$

Definition 2.1 *A linear operator* $\Re(\eta,z)$, $0 \le z \le \eta \le b$ *on* $\mathcal{S}$ *which is bounded and it is said to be a resolvent operator of* (2.2) - (2.3). *The properties of resolvent operator can be found in [19].*

Definition 2.2 *[13] A function* $\mathcal{G} \in \mathcal{C}(\mathcal{R}, Z)$ *is almost periodic if for every* $\tau > 0$ *there exists a relatively dense subset of* $\mathcal{R}$, *denoted by* $\mathcal{I}(\tau,\mathcal{G},Z)$, *such that*

$$\| \mathcal{G}(\eta+\mu) - \mathcal{G}(a) \|_Z < \tau, a \in \mathcal{R},\ \mu \in \mathcal{I}(\tau,\mathcal{G},Z).$$

Definition 2.3 *[13] A function* $\mathcal{G} \in \mathcal{C}([0,\infty), Z)$ *is asymptotically almost periodic if there exists an almost periodic function* $v_1(\cdot)$ *and* $v_2 \in \mathcal{C}_0([0,\infty), Z)$ *such that* $\mathcal{G}(\cdot) = v_1(\cdot) + v_2(\cdot)$. The following developed integral solution is based on the works [1, 18].

Definition 2.4 *A function* $\mathcal{Q}_i: [0,b] \to \mathcal{S}$, $i = 1,2$ *and it is addressed by*

$$\mathcal{Q}_1(\eta) = \int_{-r}^{0} \mathbb{Y}(\eta, z) w(z) dz$$

and

$$\mathcal{Q}_2(\eta) = \int_{-r}^{0} \mathbb{Y}_2(\eta, z) w(z) dz$$

in (2.1). Then the function $w: [0, b] \to \mathcal{S}$ will be a mild solution of equation (2.1) on $[0, b]$ if and only if $w(0) = w_0 - g(w)$, $\mathcal{Q}_1$ is differentiable on $[0, b]$, $\mathcal{Q}_1', \mathcal{Q}_2 \in \mathcal{L}^1([0, b], \mathcal{S})$ and the following integral equation

$$\left.\begin{aligned} w(t) \quad = \quad & \Re(\eta, 0)[w_0 - g(w)] + \int_0^{\eta} \Re(\eta, z)[\mathcal{Q}_1'(z) + \mathcal{Q}_2(z)]dz \\ & + \int_0^{\eta} \Re(\eta, z) d(z, w_z) dz, \quad \eta \in I, \end{aligned}\right\} \tag{2.4}$$

is satisfied.

3 Periodicity Results

The following assumptions are needed to establish our results.

(H1) The function $g: \mathcal{S} \to \mathcal{S}$ is a continuous and there exist a constants $K_g, M_2 > 0$ such that

$\parallel g(v_1) - g(v_2) \parallel \leq K_g \parallel v_1 - v_2 \parallel, \ for \ v_1, v_2 \in \mathcal{S},$

and $\parallel g(0) \parallel \leq M_2$.

(H2) (a) For each $\eta \in I$, the function $d(t, \cdot): \mathcal{B} \to \mathcal{S}$ is continuous.

(b) The function d is continuous and there exists a constant $K_d > 0$ such that

$\parallel d(\eta, \eta_1) - s(\eta, \eta_2) \parallel \leq K_d \parallel \eta_1 - \eta_2 \parallel, \ \eta \in [0, b].$

(c) There exists a continuous function $d_g \in \mathcal{C}([0, b], [0, \infty])$ and a non-decreasing continuous function $L_g: \mathcal{R}^+ \to \mathcal{R}^+$ such that

$\parallel d(a, y) \parallel \leq d_g(a) L_g(\parallel y \parallel_{\mathcal{B}}).$

(H3) The functions $\mathcal{Q}_1(z), \mathcal{Q}_2(z)$ are continuous and there exists a constant $\widetilde{K}_2 > 0$, such that $\parallel \mathcal{Q}_1'(y) + \mathcal{Q}_2(y) \parallel \leq \widetilde{K}_2(y), y \in I$.

The following frames present the existence result of periodicity,

Theorem 3.1 *If $\Gamma(\cdot): [0, \infty) \to \mathcal{S}$ is defined by*

$$\Gamma(\eta) = \int_0^{\eta} \Re(\eta, z) \int_0^{z} \mathbb{Y}_2(\eta, z) w(\tau) \ d\tau dz = \int_0^{\eta} \Re(\eta, z) \mathcal{Q}_2(z) dz$$

$\eta \geq 0$, then $\Gamma(\eta)$ is almost periodic function.

Proof. Using the assumptions,

$$\Gamma(\eta) = \int_0^{\eta} \Re(\eta, z)[\mathcal{Q}_2(z)]_{\mathcal{L}_1} dz$$

$$\begin{aligned} \parallel \Gamma(\eta + \xi) - \Gamma(\eta) \parallel & \leq \int_0^{\eta+\xi} \Re(\eta + \xi, z)[\mathcal{Q}_2(z)]_{\mathcal{L}_1} dz - \int_0^{\eta} \Re(\eta, z)[\mathcal{Q}_2(z)]_{\mathcal{L}_1} ds \parallel \\ & \leq \int_0^{\eta+\xi} \parallel \Re\eta + \xi, s) \parallel\parallel \mathcal{Q}_2(z) \parallel_{\mathcal{L}_1} ds - \int_0^{\eta} \parallel \Re(\eta, z) \parallel\parallel \mathcal{Q}_2(z) \parallel_{\mathcal{L}^1} dz \\ & \leq M_1 \sup \parallel \mathcal{Q}_2(z) \parallel_{\mathcal{L}_1} (\eta + \xi) - M_1 \sup \parallel \mathcal{Q}_2(z) \parallel_{\mathcal{L}_1} (\eta) \\ & \leq M_1 \xi \ k_0 \\ & \leq \epsilon, \end{aligned}$$

where $\epsilon = M_1 \xi \ k_0$, $k_0 = \sup \parallel \mathcal{Q}_2(z) \parallel_{\mathcal{L}_1}$. Hence the function $\Gamma(\eta) = \int_0^{\eta} \Re(\eta, z) \mathcal{Q}_2(z) dz$ is almost periodic. This completes the proof.

Theorem 3.2 *Suppose that the assumptions* $(H1)$ *and* $(H3)$ *are satisfied, then mild solution* $w(\cdot)$ *of the neutral integrodifferential evolution system* (2.1) *is asymptotically almost periodic function.*

Proof. Assume that the mild solution $w(\eta) = \mathcal{Z}(\eta) + \mathcal{Y}(\eta)$, where

$$\mathcal{Z}(\eta) = \int_0^\eta \Re(\eta, z) \int_0^z \mathbb{Y}_2(\eta, \tau) w(\tau)\ d\tau dz = \int_0^\eta \Re(\eta, z) \mathcal{Q}_2(z) dz$$

$$\mathcal{Y}(\eta) = \Re(\eta, 0)[w_0 - g(w)] + \int_0^\eta \Re(\eta, z) \mathcal{Q}_1'(z) dz$$

$$+ \int_0^\eta \Re(\eta, z) d(z, w_z) dz.$$

By the Theorem 3.1 the defined function $\mathcal{Z}(\eta)$ is almost periodic function and $\mathcal{Y}(\eta) \in \mathcal{C}_0([0, \infty), \mathcal{S})$. Hence the mild solution $w(\eta)$ is asymptotically almost periodic function.

4 Controllability

In this section, controllability of neutral integrodifferential evolution system has been discussed. Control theory arises from the most modern applications. It is one of the most interdisciplinary fields of research. Further, it is a mixture of several mathematical concepts and techniques. In control theory, the problem of controllability is to find an objective can be reached by some suitable control function. It occurs when a system described by a state $w(\eta)$ is controlled by a given differential equation. Balachandran [1] appeared on the problem of controllability nonlinear and integrodifferential systems in Banach spaces using fixed point principles. Controllability of an integrodifferential evolution and neutral evolution system with nonlocal initial condition and infinite delay presented in[21]. Theory of partial neutral integrodifferential with delay equations has been studied by several authors [14, 17] in Banach spaces. The notion of controllability is of royal significance in mathematical control theory. Control theory is a part of applied mathematics that considers the fundamental laws carrying the analysis and design of control systems.

Consider an integrodifferential neutral evolution system with bounded delay

$$\left.\begin{aligned} \mathbb{D}[w(\eta) - \int_{-r}^\eta \mathbb{Y}(\eta, z) w(z) dz] &= A(\eta) w(\eta) + Bu(\eta) + \int_{-r}^\eta \mathbb{Y}_2(\eta, z) w(z) dz \\ &\quad + d(\eta, w_\eta), \qquad \eta \in I, \\ w(0) + g(w) &= w_0, \end{aligned}\right\} \tag{4.1}$$

Here B is a bounded linear operator from U into $\mathcal{S}$ and the control function $u(\cdot)$ is given in $\mathcal{L}_2(I, U)$, a Banach space of admissible control functions with U as a Banach space the interval $I = [0, b]$. Based on the definition (2.4) the following integral solution is formed.

Definition 4.1 *If* B *is a bounded linear operator and* u *is the control function of the system* (4.1) *then the following integral equation*

$$\left.\begin{aligned} w(\eta) &= \Re(\eta, 0)[w_0 - g(w)] + \int_0^\eta \Re(\eta, z)[\mathcal{Q}_1'(z) + \mathcal{Q}_2(z)] dz \\ &\quad + \int_0^\eta \Re(\eta, z)[Bu(z) + d(s, w_z)] dz, \qquad \eta \in I, \end{aligned}\right\} \tag{4.2}$$

is the mild solution for the system (4.1).

Next, to prove the control formula the following assumption is needed.

(H4) The linear operator $W: \mathcal{L}^2(I, U) \to \mathcal{S}$ is defined by

$$Wu = \int_0^b \Re(b, z) Bu(z) dz,$$

has an induced inverse operator W^{-1} which takes values in $\mathcal{L}^2(I, U)/kerW$ and there exists a positive constant N such that $\| BW^{-1} \| \leq$ N.

Definition 4.2 *[1] The system* (4.1) *is said to be controllable on the interval I iff, for every* $w_0, w_b \in \mathcal{S}$, *there exists a control* $u \in \mathcal{L}^2(I, U)$ *such that the solution* $w(\cdot)$ *of the system* (4.1) *satisfies* $w(0) = w_0, \quad w(b) = w_b$.

Theorem 4.3 *For* $w_b \in \mathcal{S}$, *define the control*

$$\left.\begin{aligned} u(\eta) \quad = \quad & W^{-1}\{w_b - \Re(b,0)[w_0 - g(w)] - \int_0^b \Re(b,z)[\mathcal{Q}_1'(z) + \mathcal{Q}_2(z)]dz \\ & - \int_0^b \Re(b,z)d(z,w_z)dz\}(\eta) \end{aligned}\right\} \quad (4.3)$$

transfers initial state w_0 to final state

$$\left.\begin{aligned} w(b) \quad = \quad & \Re(b,0)[w_0 - g(w)] + \int_0^b \Re(b,z)[\mathcal{Q}_1'(z) + \mathcal{Q}_2(z)]dz \\ & + \int_0^b \Re(b,z)[Bu(z) + d(z,w_z)]dz, \quad \eta \in I, \end{aligned}\right\} \quad (4.4)$$

at time a=b.

Proof *:* By substituting this control (4.3) in equation (4.4), then the folowing equation is obtained at $\eta = b$.

$$\begin{aligned} & w(b) = \Re(b,0)[w_0 - g(w)] + \int_0^b \Re(b,z))[\mathcal{Q}_1'(z) + \mathcal{Q}_2(z)]dz \\ & + \int_0^b \Re(b,z)BW^{-1}\{w_b - \Re(b,0)[w_0 + g(w)] - \int_0^b \Re(b,z)[\mathcal{Q}_1'(z) + \mathcal{Q}_2(z)]dz \\ & - \int_0^b \Re(b,z)d(z,w_z)\}(z)dz + \int_0^b \Re(b,z)d(z,w_z)dz \\ & = \Re(b,0)[w_0 - g(w)] + \int_0^b \Re(b,z)[\mathcal{Q}_1'(z) + \mathcal{Q}_2(z)]dz \\ & + WW^{-1}\{w_b - \Re(b,0)[w_0 + g(w)] - \int_0^b \Re(b,z)[\mathcal{Q}_1'(z) + \mathcal{Q}_2(z)]dz \\ & - \int_0^b \Re(b,z)d(z,w_z)\} + \int_0^b \Re(b,z)d(z,w_z)dz = w_b. \end{aligned}$$

Hence the proof.

Theorem 4.4 *If the assumptions* $(H1) - (H4)$ *are satisfied. Then the existence of an asymptotically almost periodic mild solution to the evolution system,* (4.1) *is controllable.*

Proof : Consider the Banach space $\mathcal{Z} = C(I, \mathcal{S})$ with the norm $\| w \| = \sup\{|w(t)|: t \in I\}$. and Set $\mathcal{B}_r = \{w \in I: \| w \| < r\}$. Using $(H3)$ for an arbitrary function $w(\cdot) \in C(I, \mathcal{S})$, and define an operator Ω as,

$$\begin{aligned} & (\Omega w)(\eta) \le \Re(\eta,0)w_0 + g(w) + \int_0^\eta \Re(\eta,z)\mathcal{Q}_1'(z) + \mathcal{Q}_2(z)dz \\ & + \int_0^\eta \Re(\eta,z)BW^{-1}\{w_b - \Re(b,0)[w_0 + g(w)] \\ & - \int_0^b \Re(b,z)[\mathcal{Q}_1'(z) + \mathcal{Q}_2(z)]dz + \int_0^b \Re(b,z)d(s,w_z)ds\}(z)dz \\ & + \int_0^\eta \Re(\eta,z)d(z,w_z)dz. \end{aligned}$$

Now the operator Ω is subdivided into two operators Ω_1 and Ω_2 on $\mathcal{B}_n$, we have

$$\begin{aligned} & (\Omega_1 w)(\eta) = \Re(\eta,0)w_0 + g(w) \\ & + \int_0^\eta \Re(\eta,z)BW^{-1}\{w_b - \Re(b,0)[w_0 + g(w)] \\ & - \int_0^b \Re(b,z)[\mathcal{Q}_1'(z) + \mathcal{Q}_2(z)]dz + \int_0^b \Re(b,z)d(s,w_z)ds\}(z)dz \\ & (\Omega_2 w)(\eta) = \int_0^\eta \Re(\eta,z)[\mathcal{Q}_1'(z) + \mathcal{Q}_2(z)]dz \\ & + \int_0^\eta \Re(\eta,z)d(z,w_z)dz. \end{aligned}$$

Next, to show that when using the control $u(t)$, the operator $\Omega = \Omega_1 + \Omega_2$ has a fixed point $w(\cdot)$. This fixed point is the solution to system (2.1), implying that the system is controllable. For any $w, x \in \mathcal{B}_r$, we get

$$\begin{aligned} & \| \Omega_1 w(\eta) + \Omega_2 x(\eta) \| \le \| \Re(\eta,0) \| \| w_0 + g(w) \| + \int_0^\eta \| \Re(\eta,z) \| \| \mathcal{Q}_1'(z) + \\ & \qquad \mathcal{Q}_2(z) \| dz + \int_0^\eta \| \Re(\eta,z) \| \| BW^{-1} \| \{\| w_b \| - \| \Re(b,0)[w_0 + g(w)] \| \end{aligned}$$

$- \int_0^b \parallel \Re(b,z) \parallel\parallel Q_1'(z) + Q_2(z) \parallel dz$
$+ \int_0^b \parallel \Re(b,z) \parallel\parallel d(s,w_z) \parallel ds\}(z)dz$
$+ \int_0^\eta \parallel \Re(\eta,z) \parallel\parallel d(z,x_z) \parallel dz.$
$\leq M_1 \parallel w_0 \parallel + K_g(w) + M_2 + M_1\widetilde{K}_2\eta + M_1N\eta\{\parallel w_b \parallel - M_1 \parallel w_0 \parallel - K_g(w)$
$+ M_2 + M_1\widetilde{K}_2 b - + M_1 d_g(s) w_g(b)\} + M_1 d_g(a) x_g(\eta)$
$< r.$

Thus Ω deduce to r. Next to prove that Ω_2 is continuous and compact. Since Ω_2 is uniformly bounded on $\mathcal{B}_r$. This follows from the inequality

$(\Omega_2 w)(\eta) \leq \int_0^\eta \parallel \Re(\eta,z) \parallel\parallel Q_1'(z) + Q_2(z) \parallel dz$
$+ \int_0^\eta \parallel \Re(\eta,z) \parallel\parallel d(z,w_z) \parallel dz$
$\leq \rho^*,$

where $\rho^* = M_1 K_2(\eta) + M_1 d_g w_g(\eta)$. Let $\{w_n\}$ be a sequence in $\mathcal{B}_r$, such that $w_n \to w$ in $\mathcal{B}_r$ then $d(z,w_{nz}) \to d(z,w_z)$ and $Q_1'(z_n) + Q_2(z_n) \to Q_1'(z) + Q_2(z)$ as $n \to \infty$ because the function d and Q_1, Q_2 are continuous on $I \times \mathcal{S}$. Now for each $\eta \in I$, we have

$\parallel \Omega_2 w_n(t) - \Omega_2 w(t) \parallel \leq \int_0^\eta \parallel \Re(\eta,z) \parallel\parallel Q_1'(z_n) + Q_2(z_n) \parallel dz$
$- \int_0^\eta \parallel \Re(\eta,z) \parallel\parallel Q_1'(z) + Q_2(z) \parallel dz -$
$+ + \int_0^\eta \parallel \Re(\eta,z) \parallel\parallel d(z,w_{nz}) \parallel dz - \int_0^\eta \parallel \Re(\eta,z) \parallel\parallel d(z,w_z) \parallel dz$
$\to 0 \quad as \quad n \to \infty,$

which shows that Ω_2 is continuous. Next we prove that $\{\Omega_2 x(t): x \in \mathcal{B}_r\}$ is relatively compact in $\mathcal{S}$ for all $t \in \mathcal{S}$. Obviously $\{\Omega w(0): w \in \mathcal{B}_r\}$ is compact.

Let $\eta > 0$ be fixed for each $q \in (0,t)$ and $w \in \mathcal{B}_r$ define the operator Ω_2^q by

$$\Omega_2^q w(\eta) \leq \int_0^{\eta-q} \Re(\eta,z)[Q_1'(z) + Q_2(z)]dz + \int_0^{\eta-q} \Re(\eta,z) d(z,w_z)dz.$$

As Q_1 and Q_2 are completely continuous obviously the set $\{\Omega_2^q w(\eta): w \in \mathcal{B}_r\}$ is pre compact in $\mathcal{S}$ for every $q, 0 < q < t$ and as it is compact at $t = 0$ we have relatively compactness in $\mathcal{S}$ for all $\eta \in I$. Moreover, for every $w \in \mathcal{B}_r$ we have

$\parallel \Omega_2 w(\eta) - \Omega_2^q w(\eta) \parallel \leq \int_0^{\eta-q} \parallel \Re(\eta,z) \parallel\parallel Q_1'(z) + Q_2(z) \parallel dz$
$+ \int_0^{\eta-q} \parallel \Re(\eta,z) \parallel\parallel d(z,w_z) \parallel dz - \int_0^\eta \parallel \Re(\eta,z) \parallel\parallel Q_1'(z) + Q_2(z) \parallel dz$
$+ \int_0^\eta \parallel \Re(\eta,z) \parallel\parallel d(z,w_z) \parallel dz$

which tends to zero as $q \to 0$. Hence the set $\{\Omega_2^q w(t): w \in \mathcal{B}_r\}$ is precompact in $\mathcal{S}$. Now $\eta \in I,\ x,y \in \mathcal{B}_r$, then

$\parallel \Omega_1 x(\eta) - \Omega_1 y(\eta) \parallel = \parallel \Re(\eta,0) \parallel\parallel x_0 - y_0 \parallel + \parallel g(x) - g(y) \parallel$
$+ \int_0^\eta \parallel \Re(\eta,z) \parallel\parallel BW^{-1} \parallel \{\parallel x_b - y_b \parallel - \parallel \Re(b,0)[x_0 - y_0 + g(x) - g(y)] \parallel$
$+ \int_0^b \parallel \Re(b,z) \parallel\parallel d(s,x_z) - d(s,y_z) \parallel ds\}(z)dz$
$\leq [M_1 + K_g + M_1\eta N[1 + M_1 + K_g + M_1 b K_d] + \eta M_1 K_d] \parallel w - y \parallel$
$\leq \Delta \parallel x - y \parallel.$

Since $\Delta < 1,$ which shows that Ω_1 is a contraction mapping.

Now let us prove that $\Omega_2 w,\ w \in \mathcal{B}_r$ is equicontinuous. Let $\eta_1, \eta_2 \in \mathcal{S},\ \eta_1 < \eta_2$ and let $w \in \mathcal{B}_r$ then

$\parallel \Omega_2 w(\eta_1) - \Omega_2 w(\eta_2) \parallel \leq \int_0^{\eta_1} \parallel \Re(\eta,z) \parallel\parallel Q_1'(z) + Q_2(z) \parallel dz + \int_0^{\eta_1} \parallel \Re(\eta_1,z) \parallel \parallel d(z,w_z) \parallel dz$
$- \int_0^{\eta_2} \parallel \Re(\eta_2,z) \parallel\parallel Q_1'(z) + Q_2(z) \parallel dz + \int_0^{\eta_2} \parallel \Re(\eta_2,z) \parallel\parallel d(z,w_z) \parallel dz$

$$\leq \int_0^{\eta_1} \parallel \Re(\eta_2, z) - \Re(\eta_1, z) \parallel\parallel \mathcal{Q}_1'(z) + \mathcal{Q}_2(z) \parallel dz$$
$$+ \int_0^{\eta_1} \parallel \Re(\eta_2, z) - \Re(\eta_1, z) \parallel\parallel d(z, w_z) \parallel dz$$

as $t_1 \to t_2$ the right hand side of above inequality tends to zero. The set $\Omega_2(\mathcal{B}_r)$ is equicontinuous. Thus we have prove that $\Omega_2(\mathcal{B}_r)$ is relatively compact for $\eta \in \mathcal{S}$. By Arzela-Ascolis theorem, Ω_2 is compact. Hence by the Kranoselskii's theorem, there exists a fixed point $w \in \mathcal{B}_r$ such that $(\Omega_1 + \Omega_2)w = w$ which is a solution of equation (4.1). Hence $(\Omega_1 + \Omega_2)w(b) = w_b$, which implies that the given system is controllable.

5 Conclusion

In this paper, the author achieved the existence of the asymptotically almost periodic solution to the nonlinear neutral integrodifferential evolution system. Moreover, controllability result is also proved for the same system by using the Kranoselskii's fixed point theorem.

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