

The Influence of Inclusion and Porosity on Thermal Conductivity of Metal Matrix Composite using Mathematical Model

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Abstract. The mathematical modelling programme MATLAB was used to examine the effective thermal conductivity of heterogeneous materials, and the results are given in this research. The prediction of metal matrix composite materials' thermal conductivity in relation to different relevant variables such as inclusion size and volume %. Ceramic compounds and non-thermal conductive inclusions are common. It appears that determining the effect of inclusion on the thermal conductivity of heterogeneous materials using standardised tests is challenging. As a result, the created mathematical tool may be used to demonstrate the effect of inclusions on thermal conductivity. For the evaluation of heterogeneous metals, the volume proportion of inclusion has been estimated to be up to 50%. For the thermal conductivity test, metals such as aluminium, iron, magnesium, and zinc were chosen.

Keywords: Ceramic, thermal conductivity, metals.

1 Introduction

The rate of heat transmission through a unit thickness of materials per unit area and unit temperature gradient is defined as thermal conductivity of the materials. Greater thermal conducting materials, such as iron, copper, and aluminium, have a higher thermal conductivity [1]. Non-thermal conducting particles (oxides, carbides, and nitrides), as well as other types of inclusions including holes, fractures, and voids, have an impact on the thermal characteristics of materials. The thermal conductivity of materials is determined by the degree of inclusions present [2]. There has been a tremendous amount of research done and continues to be done in order to fully comprehend the phenomena of steady-state heat transfer in general and via composite materials in particular.

Guocheng Zhu [1] has devised a mathematical model for heterogeneous materials with a basic structure that determines the effective thermal conductivity (ETC) of materials with more than two types. R. H. Davis [2] has developed a novel model for determining the effective thermal conductivity of a composite material with spherical inclusions. For materials with continuous and scattered phases, the Maxwell–Eucken model [3,4] was employed. It presupposes a dispersion of tiny spheres inside a continuous matrix of a different component, with the spheres spaced far enough apart that local temperature distribution distortions around each of the spheres do not influence with the temperature distributions of their neighbours. Guarded hot plate, Laser flash diffusivity, Absolute technique, Comparative technique, Radial heat flow method, Parallel conductance method, Electrical heating methods,

Pulsed power technique, 3 method, FDTR technique, Hot-wire method (needle-probe method), Laser flash method, and Transient plane source (TPS) method are some of the methods used to predict the effective thermal conductivity of heterogeneous metallic materials. Apart from the different experimental approaches, computer-based numerical analysis and mathematical modelling are emerging as useful tools for predicting thermal conductivity.

In his famous work on electricity and magnetism, Maxwell was the first to offer analytical formulas for effective conductivity of heterogenic media. He looked at the problem of dilute dispersion of spherical conductivity k_1 particles implanted in a continuous matrix of conductivity k_m , with thermal interactions between filler particles disregarded [3]. The thermal model takes into account a variety of variables depending on the function to be performed. This approach entails creating a basic analytical model that may be used to estimate thermal conductivity. The mathematical modelling is a simplification of the real-time problem and does not take into account all of the real-time conditions and scenarios. This mathematical model can accurately estimate the effect of inclusions on metallic materials' thermal conductivity. [5] Jhon Constantine (2016) used a mathematical model to determine the heat conductivity of a number of different polymer homogenous composite materials. Ideal spherical particles are encompassed by a homogenous interface area in this form of composite, and the matrix is considered isotropic. To evaluate the property, particle contiguity was considered in conjunction with the idea of interphase.

As a result, when the partials are stiffer than the matrix, the interphase magnitude increases as the thickness increases. Meanwhile, the partial interactions contribute to thermal properties for composites with cubic arrangements, as evidenced by the transformation of the proposed micro structural model. Based on the law of least thermal resistance and the equal law of specific equivalent thermal conductivity, Alok Agrawal (2015) developed a theoretical model of heat transmission in polymer composites with hybrid fillers. The suggested model is used to evaluate the effective thermal conductivities of polymers with various conductive and non-conductive particle fillers. The inclusion of two different particle fillers contributes to the change of the thermal conductivity value of such hybrid filler composites, according to the findings. The experimental effort confirms the usage of the proposed model for prediction purposes by fabricating several sets of composites and measuring their conductivity. The effective thermal conductivities of a number of composites have a very high agreement between theoretical and empirically observed values. R. H. Davis (1986) demonstrated how to calculate the effective thermal conductivity of a composite material with spherical inclusions. A big body's surface is expected to be kept at a constant temperature. This body is in touch with an infinitely long composite material with a lower temperature than the heated body. The rate of heat transfer from the heated body to the composite material is then calculated using Green's theorem. Y.P. Mamunya [7] In policy assessments involving the transportation system, mathematical models are often used. There are many different types of mathematical models, with econometric models being the most used in the field of automobile transportation policy. Models may offer a lot of insight to the policymaking process when used correctly, however model output should only be treated as approximations. Mathematical modelling can only improve the policy-making process if policymakers are aware of the limits of models. Rao V Dukkipati (2008) [8] addressed the fundamentals of MATLAB and how to use it to tackle engineering issues, such as thermal problems and numerical techniques.

2 Methods and simulation

There are certain considerations and parameters that are necessary while performing a thermal analysis some of the main considerations and parameters are,

- Temperature [K]
- Temperature Gradient [K/m]
- Heat flux [W/m²]
- Heat Source for point, line, surface, volume [W], [W/m], [W/m²], [W/m³]
- Temperature [K]
- Heat flow resultant [W]
- Thermal conductivity [W/m-K]

Thermal analysis input properties

The following is a list of the thermal analysis material attributes that are taken into account in this simulation approach. Only the conductivities are theoretically required for a steady state analysis, thus the thermal conductivity is requested in this mathematical simulation method. The product of mass density and specific heat is used in any transient (time-dependent) thermal study. In the same way, the solid model utilised in this simulation contains information on the part's borders, faces, and interior. The mathematical description includes information that defines whether a given position is on the boundary surface, within it, or outside it. Primitive construction and Boolean operations, surface operations, protrusion operations, pick-and-place, feature-based modelling, and parametric modelling are all examples of solid model modelling.

This system was analysed by the model under the following circumstances.

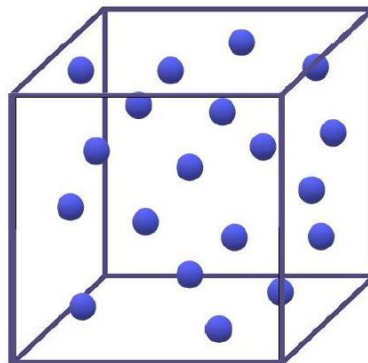
(I) In a composite mixture, none of the constitutive parameters are affected by the electromagnetic field intensity.

(II) Distances between inclusions are larger than their typical sizes.

(III) Inclusions' characteristic size changes depending on their volumetric proportion of the system's total volume.

(IV) Inclusions are scattered haphazardly.

The schematic of such material is shown in Fig 1. Maxwell's expression is as follows:



$$K_e = K_1 \frac{2K_1 + K_2 - 2(K_1 - K_2)v_2}{2K_1 + K_2 + (K_1 - K_2)v_2} \quad \text{--- eq (1)}$$

K_e is the effective thermal conductivity of heterogeneous metallic materials
 K_1 is the thermal conductivity of the major material
 K_2 is the thermal conductivity of the inclusion material
 v_2 is the volumetric fraction of the inclusion material to the main material

The thermal conductivity of different metals Al, Fe, Zn, and Mg is first indicated using variables K_1 , K_2 , K_3 , and K_4 in this MATLAB application. To begin, a primary function called "ETCmaxeuc" is constructed and named, inside which the entire procedure takes place. The four variables are then saved as integers and utilised or called as needed. The thermal conductivity of the materials may be adjusted according to the requirements. There are a lot of sub-functions in the main function "ETCmaxeuc."

The first sub-function calculates the effective thermal conductivity of any two-material combination. K_1 , K_2 , and v_2 are the three variables that make up this complete sub-system. The volumetric proportion of the inclusions introduced to the material is referred to as v_2 . "Keff=maxeuc (K_1, K_2, v_2)" is the name of the function. The function then uses the Maxwell-Eucken formula to calculate the effective thermal conductivity.

$$K_{eff} = k_1 * (((2*k_1) + k_2 - (2*(k_1 - k_2)*v_2)) / ((2*k_1) + k_2 + (k_1 - k_2)*v_2)) \quad \text{-- eq (2)}$$

Except for the variable v_2 , which is obtained using another sub-function "vf," which is used to determine the volumetric percent of the inclusions added to the material. "vf=vfrac (a)" is used to represent it. The function makes use of variables such as "A, Vol, X1,a." The variable "A" represents the cube's length, while the variable "vol" represents the cube's volume, which is three times "A" because the cube's sides have the same length in all directions. The volumetric fraction of the inclusions added is then projected using the proportion of the inclusion added to the substance "a."

$$\begin{aligned} \text{vol} &= A * A * A \\ X1 &= (\text{vol} / 100) * a \\ \text{vf} &= X1 / (\text{vol} + X1) \end{aligned}$$

After defining the functions, an analytical model is used to forecast various situations for various materials. For all situations in this model, the thermal conductivity of material 1, which is the thermal conductivity of aluminium, remains constant. Each kind is made up of five instances that differ in the volumetric proportion of the various inclusions added to the material, which ranges from 10 to 50%.

$$\begin{aligned} V22 &= \text{vfrac}(10) \\ Ke11 &= \text{maxeuc}(K1, K2, V22) \end{aligned}$$

After entering cases for the first combination the graph that determines the relation between the effective thermal conductivity to the volume of the inclusions added is drawn using commands.

```

plot(X1,Y1,'-ro','LineWidth',1,'MarkerFaceColor',[1 0 0])
title('Aluminium - Ferrous')
xlabel('Volume content of inclusion(%)')
ylabel('Effective thermal conductivity (W/m.K)')

```

This approach is used for the last two situations, and after finishing the complete set of programmes, the entire function is terminated with the command "end." The programme is then saved with the main function name "ETCmaxeuc." When the function "ETCmaxeuc" is invoked or the complete system runs, a graph showing the relationship between the effective thermal conductivity and the inclusion materials is added to the command window.

This model is only used to calculate the effective thermal conductivity of two or more materials using the Maxwell-Eucken formula; depending on the circumstances and parameters, alternative analytical models can be employed.

```

functionETCmaxeuc()
K1=238; %TC of Al
K2=146; %TC of Mg
K3=105; %TC of Zn
K4=43.3; %TC of Fe
functionKeff=maxeuc(k1,k2,v2)
Keff=k1*(((2*k1)+k2-(2*v2*(k1-k2)))/((2*k1)+k2+(v2*(k1-k2)))); % EffectiveTC using
Maxwell-Eucken
%Keff=k1*(((2*k1)+k2-(2*(k1-k2)*v2))/((2*k1)+k2+(k1-k2)*v2));
%disp(Keff);
end
functionvf=volfrac(a)
A=50; %length 50mm in cube all sides are same
vol= A*A*A; %vol--> volume of cube
x1=(vol/100)*a; %a--> %of inclusion
vf=x1/(vol+x1); %vf--> volume fraction of material 2
%disp(vf);
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%ETC for K1 and K2
V22=volfrac(10); % volume
Ke11=maxeuc(K1,K2,V22); %
V22=volfrac(20);
Ke12=maxeuc(K1,K2,V22);
V22=volfrac(30);
Ke13=maxeuc(K1,K2,V22);
V22=volfrac(40);
Ke14=maxeuc(K1,K2,V22);
V22=volfrac(50);
Ke15=maxeuc(K1,K2,V22);
X1=[10 20 30 40 50];
Y1=[Ke11 Ke12 Ke13 Ke14 Ke15];
figure(1);

```

```

plot(X1,Y1,'-ro','LineWidth',1,'MarkerFaceColor',[1 0 0]);
title('Aluminum - Ferrous');
xlabel('Volume content of inclusion(%)' );
ylabel('Effective thermal conductivity (W/m.K)');
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% ETC for K1 and K3
V22=volfrac(10);
Ke21=maxeuc(K1,K3,V22);
V22=volfrac(20);
Ke22=maxeuc(K1,K3,V22);
V22=volfrac(30);
Ke23=maxeuc(K1,K3,V22);
V22=volfrac(40);
Ke24=maxeuc(K1,K3,V22);
V22=volfrac(50);
Ke25=maxeuc(K1,K3,V22);
X2=[10 20 30 40 50];
Y2=[Ke21 Ke22 Ke23 Ke24 Ke25];
figure(2);
plot(X2,Y2,'-bs','LineWidth',1,'MarkerFaceColor',[0 0 1]);
title('Aluminum - Zinc');
xlabel('Volume content of inclusion(%)' );
ylabel('Effective thermal conductivity (W/m.K)');
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% ETC for K1 and K4
V32=volfrac(10);
Ke31=maxeuc(K1,K4,V32);
V32=volfrac(20);
Ke32=maxeuc(K1,K4,V32);
V32=volfrac(30);
Ke33=maxeuc(K1,K4,V32);
V32=volfrac(40);
Ke34=maxeuc(K1,K4,V32);
V32=volfrac(50);
Ke35=maxeuc(K1,K4,V32);
X3=[10 20 30 40 50];
Y3=[Ke31 Ke32 Ke33 Ke34 Ke35];
figure(3);
plot(X3,Y3,'-g^','LineWidth',1,'MarkerFaceColor',[0 1 0]);
title('Aluminum - Magnesium');
xlabel('Volume content of inclusion(%)' );
ylabel('Effective thermal conductivity (W/m.K)');
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
figure(4);
plot(X1,Y1,'-ro',X2,Y2,'-bs',X3,Y3,'-g^');

```

```
title('Overall');
xlabel('Volume content of inclusion(%));
ylabel('Effective thermal conductivity (W/m.K)');
legend('Al-Fe','Al-Zn','Al-Mg');
end
```

3 Results and Discussion

Simulation 1. Ferrous inclusions of spherical type – were investigated and the volume fractions of inclusions were from 10% to 50%. And the inclusion sizes are kept constant.

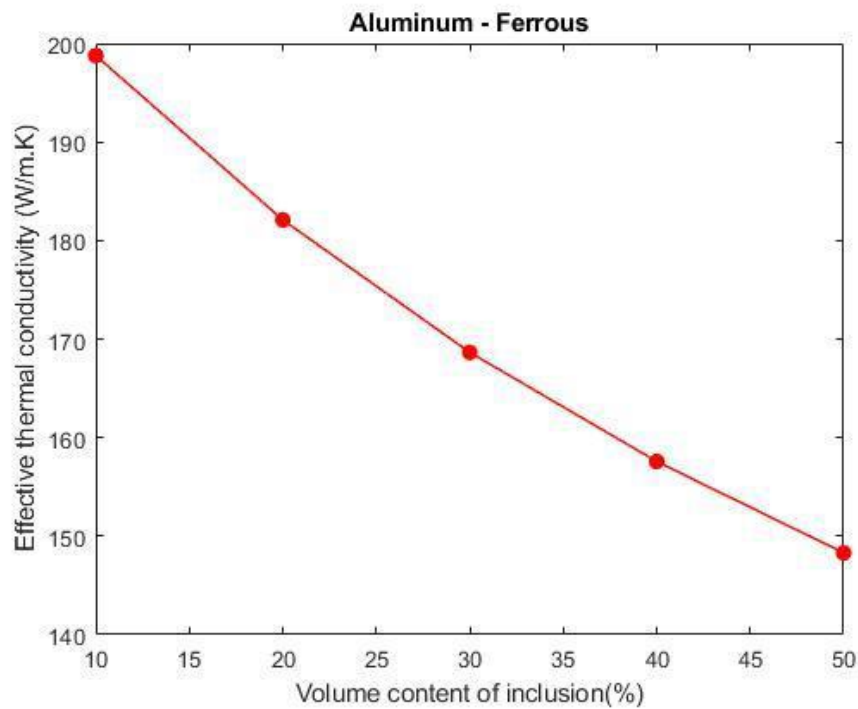


Figure.2. Thermal Conductivity of Al-Fe composite

The thermal conductivity of the composite material decreases as the percentage of the ferrous inclusions added to the aluminium metal. The curves shows the drop of the thermal conductivity of the composite material.

Simulation 2. Zinc inclusions of spherical type – were investigated and the volume fractions of inclusions were from 10% to 50%. And the inclusion sizes are kept constant.

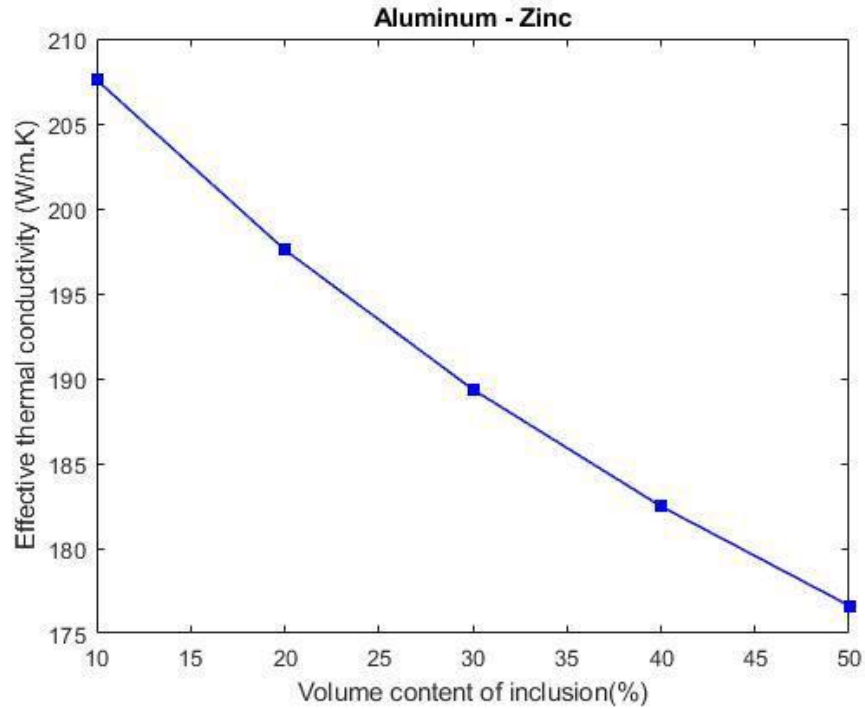


Figure.3. Thermal Conductivity of Al-Zn composite

Comparing to the previous results the aluminium-zinc composite material the thermal conductivity drops higher and faster when compared to the aluminium-ferrous combination this is because the thermal conductivity of the zinc material is lower than the ferrous material.

Simulation 3. Magnesium inclusions of spherical type – were investigated and the volume fractions of inclusions were from 10% to 50%. And the inclusion sizes are kept constant.

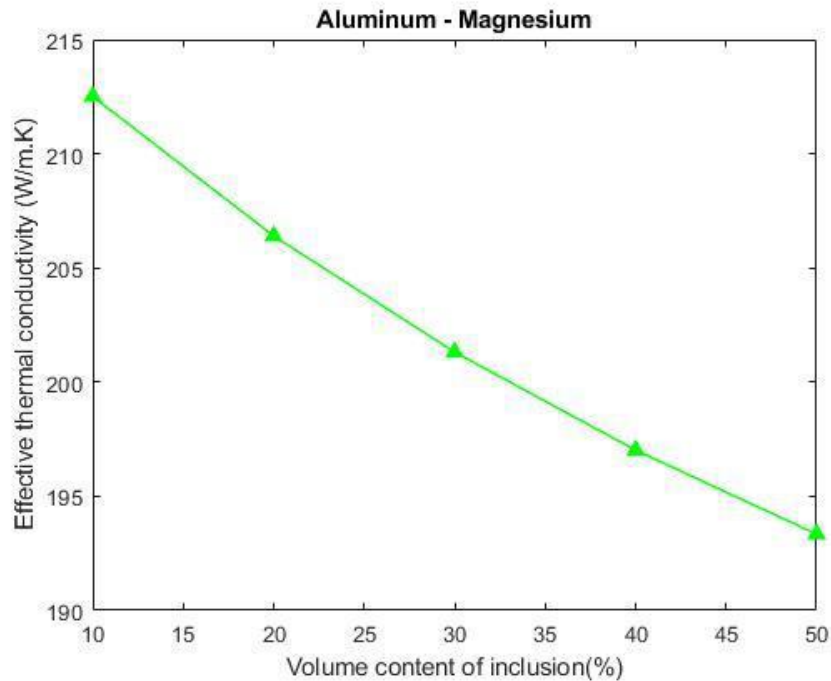


Figure.4. Thermal Conductivity of Al-Mg composite

From all the samples the magnesium has the highest thermal conductivity among the inclusion materials. These predicted values are similar to the reference values when compared to the references from the previous papers. Thus, the effective thermal conductivity of the Al based composite materials is evaluated using mathematical modelling by using MATLAB and study is made for different cases. The model explains that the mathematical simulation can provide accurate prediction based on the analytical methods used.

4 Conclusion

Analytical models and simulations were studied in order to investigate the differences among the different combinations of composite materials. This model gives the mathematical modelling method for predicting the effective thermal conductivity of aluminium based composite materials under various compositions.

Future work

- Mathematical model and thermal analysis on various complicated shapes and sizes of the inclusions added which is more relevant and accurate as the normal methods.
- Similar other kind of analysis operations using mathematical model can be used which gives almost the accurate prediction to the tested values.
- Generate similar mathematical models for different kind of test process used in the automobile industry.

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