(γ, α_β)AND (α_γ, α_β)-Generalized Closed Mappings Intopological Spaces

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Abstract. The Inthis paper the conceptsof(γ, α_β)and (α_γ, α_β)-generalized closed mappings are introduced and some of their properties are studied. Further(γ, α_β)and (α_γ, α_β)-homeomorphism are initiated and Investigated some of their properties.

Keywords: open set,α_γ-open set,τ_{α_γ}-int, τ_{α_γ}-cl ,$\alpha_\gamma T_i$ spaces (i = 0,$\frac{1}{2}$,1,2), (γ, α_β) -generalized closed Mappings, (α_γ, α_β) -generalized closed mappings. Mathematics Subject Classification: AMS(2000)54A05,54A10.

1 Introduction

The α -open sets, operation on topological spaces, $\tau_{\alpha-\gamma}$, $\tau_{\alpha-\gamma-I}$, $\tau_{\alpha-\gamma-I}$- interior and $\tau_{\alpha-\gamma-I}$ -closure operators and α -(γ, γ') -open sets were introduced respectively by Njastad [7], Kasahara [4,5], Ogata [8,9]and Kalaivani, SaiSundara Krishnan[1,2,3].In this paper, the concept of ($\boldsymbol{\gamma}, \boldsymbol{\alpha_\beta}$)-generalized closed mappings and are ($\boldsymbol{\alpha_\gamma}$, $\boldsymbol{\alpha_\beta}$) -Generalized Closed Mappingsare introduced some of their properties are studied. Further their the corresponding homeomorphisms are introduced and characterize them using ($\boldsymbol{\gamma}, \boldsymbol{\alpha_\beta}$) -generalized closed mappings and are ($\boldsymbol{\alpha_\gamma}$, $\boldsymbol{\alpha_\beta}$) -Generalized Closed Mappings.

2.(γ, α_β)-GENERALIZED CLOSED MAPPINGS

Definition 2.1. A mapping$f: X_{tTS} \rightarrow Y_{tTS}$is said to be a(γ, α_β)-generalized closed mapping[denoted as (γ, α_β)-ge cl ma]if and only if for eachγ -*closed set* $H \in X_{tTS}$, the image f (H) is an α_β- generalized *closed set*(ge cl se) inY_{tTS}.

Definition 2.2. A mapping$f: X_{tTS} \rightarrow Y_{tTS}$is said to be a (γ, α_β)-generalized open mapping [denoted as (γ, α_β)-ge op ma] if and only if for eachγ -*open set* $H \in X_{TS}$, the image f (H) is an α_β-generalized *open set* (ge op se) in Y_{tTS}.

ICCAP 2021, December 07-08, Chennai, India

DOI 10.4108/eai.7-12-2021.2314527

Definition 2.3. A mapping $f: X_{tTS} \to Y_{tTS}$ is said to be a (γ, α_β) - closed mapping [(γ, α_β) -cl ma] if and only if for each γ-*closed set* $H \in X_{tTS}$, the image $f(H)$ is an α_β-*closed set* in Y_{tTS} .

Definition 2.4. A ma $f: X_{tTS} \to Y_{tTS}$ is said to be a(γ, α_β)- open mapping if and only if for each γ-*open set* $H \in X_{tTS}$, the image $f(H)$ is an α_β-*open set* in Y_{tTS} .

Remark 2.1. From the Definitions 3.1, 3.2, 3.3and 3.4, we can conclude that every (γ, α_β)- closed (open)ma is a(γ, α_β)- generalized closed (open) ma. But the converse need not be true.

The above Remark 3.1. follows from the example 3.1.

Example 2.1. Let $X_{tTS} = \{a, b, c\}, \tau = \{\varphi, X_{tTS}, \{a\}, \{c\}, \{a, c\}, \{a, b\}\}, Y_{tTS} = \{a, b, c\}$ and $\sigma = \{\varphi, Y_{tTS}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$.

Theoperations γ, β on τ and σ aredefined as $A^\gamma = \begin{cases} cl(A) \text{ if } b \in A \\ A \cup \{c\} \text{ if } b \notin A \end{cases}$ for every $A \in \tau$, then $\tau_{\alpha_\gamma} =$ $\{\varphi, X_{tTS}, \{a\}, \{c\}, \{a, c\}, \{a, b\}\}$ and $B^\beta = \begin{cases} B \cup \{a\} \text{ if } B = \{a\}, \{b\} \text{ or } \{c\} \\ B \text{ otherwise} \end{cases}$ forevery $B \in \sigma$, then $\sigma_{\alpha_\beta} = \{\varphi, Y_{tTS}, \{a, c\}, \{b, c\}\}$.

The mapping f is defined as f (a) = b, f (b) = c and f (c) = a. Then the image of every γ-closed(open set) is an α_β-generalized closed (open) set under the mapping f . Hence f is an (γ, α_β)-generalized closed (open) mapping. But f is not a(γ, α_β)-open mapping, since {a} is aγ-open set in X_{TS} , but f({a})= {b} is not an α_β -open set in Y_{TS} .Similarly, f is not a (γ, α_β) - closed mapping, since {b} is aγ -closed set in X_{TS} , but f({b}) = {c} is not an α_β - closed set in Y_{TS} .

Theorem 2.1.A surjective mapping $f: X_{tTS} \to Y_{tTS}$ is a(γ, α_β)-generalizedclosed mapping if and only if for each subset B of Y_{tTS} and each γ-*open set* U of X_{tTS} containing f^{-1} (B), there exists an α_β- generalized*open set* V of Y_{tTS} such that B ⊆V and f^{-1} (V) ⊆ U.

Proof: Necessary Part: Suppose that f is a (γ, α_β)-generalized closed mapping. Let B be a subset of Y_{tTS} and U is a γ-*open set* of X_{tTS} containing f^{-1}(B). Put V = $Y_{tTS} - f(X_{tTS} - $U).Then V is an α_β- generalized *open set* in Y_{TS} , B ⊆V and f^{-1} (V) ⊆U.

Sufficient Part: Let F be a γ -*closed set* of X_{tTS}. Put B = $Y_{tTS} - f$(F), then $f^{-1}(B) \subseteq X_{tTS} -$ F and X_{tTS} – F is a γ -*open set* in X_{tTS} . There exists an α_β- generalized *open set* V of Y_{tTS} such that $B = Y_{tTS} - f$(F) ⊆ V and f^{-1}(V) ⊆ $X_{tTS} -$ F. Therefore, f(F) = $Y_{tTS} -$ V and hence f(F) is an α_β- generalized *closed set* in Y_{tTS}.This proves that f is a (γ, α_β)-generalized closed mapping.

Remark 2.2. Necessity of Theorem 2.1. is proved without assuming that $f$is surjective. Therefore, we can obtain the following Corollary.

Corollary 2.1. If $f: X_{tTS} \rightarrow Y_{tTS}$is a (γ, α_β)-generalized closed mapping, then for any β-$closed\ set F$ of $Y_{tTS}$and for anyγ-open set U of X_{tTS}containing f^{-1} (F), there exists an α_β-$open\ set V$ of Y_{tTS} such that $F \subseteq V$ and $f^{-1}(V) \subseteq U$.

Proof: By Necessity of Theorem 3.1, there exists anα_β-generalized $open\ set W$ of Y_{tTS} such that$F \subseteq W$ and $f^{-1}(W) \subseteq U$. Since F is aβ-$closed\ set$, By Definition of the α_β-generalized $closed\ set$, we have $F \subseteq \sigma_{\alpha_\beta}$-int($W$).Put V= σ_{α_β}-int(W), then $V \in \sigma_{\alpha_\beta}$, $F \subseteq V$ and $f^{-1}(V) \subseteq U$.

Definition 2.4. A mapping$f: X_{tTS} \rightarrow Y_{tTS}$is said to be an ($\alpha_\gamma$,$\beta$) -generalized continuous mapping [denoted as (α_γ,β) -ge con ma] if and only if for eachβ - $closed\ set B \in Y_{tTS}$, the inverse image $f^{-1}(B)$is an α_γ- generalized $closed\ set$ inX_{tTS} .

Theorem 2.2. Let $f: X_{tTS} \rightarrow Y_{tTS}$be a bijective mapping. Then the following statements are equivalent:

(i)f is a(γ, α_β)-generalized open mapping;

(ii)f is a (γ, α_β)-generalized closed mapping;

(iii)f^{-1} is an (α_β, γ)-generalized continuous mapping.

Proof: (i) ⇒ (ii)The proof follows from the Definitions 2.1, 2.2.

(ii)⇒ (iii) Let F be aγ -$open\ set$ in X_{tTS} and let Bbe anα_β-$closed\ set$ in Y_{tTS} such that $B \subseteq f$(F). This implies that f^{-1}(B) $\subseteq F$.Then by(ii) and corollary 2.1, there exists an α_β-$open\ set V$ of Y_{tTS} such that $B \subseteq V$ and $f^{-1}(V) \subseteq F$.Therefore $B \subseteq \tau_{\alpha_\beta}$-int($V$) and $V \subseteq f(F)$ and hence $B \subseteq \tau_{\alpha_\beta}$-int($f(F)$). Thus$(f^{-1})^{-1}(F)$ =$f(F)$ is anα_β- generalized $open\ set$ inY_{tTS}. Then by definition 2.4, $f^{-1}$is an(α_β, γ) -generalized continuous mapping.

(iii)⇒ (i) Let D be aγ-$open\ set$ in X_{tTS}. Then $X_{tTS} - D$ is a γ-$closed\ set$ in X_{TS} . Since $f^{-1}$is an (α_β, γ)-generalized continuous mapping, $(f^{-1})^{-1}(X_{tTS} - D)$ is an α_β-generalized $closed\ set$in Y_{TS} . But$(f^{-1})^{-1}(X_{tTS} - D) = f\ (X_{tTS} - D) = Y_{tTS} - f(D)$.Thus $f(D)$ is an α_β-generalized $open\ set$ inY_{tTS}. This proves that f is a (γ, α_β)-open mapping. Hence the proof.

Definition 2.6.A mapping$f: X_{tTS} \rightarrow Y_{tTS}$ is said to be an (α_γ,β) - homeomorphism, if f is bijective, (α_γ,β)- continuous mapping andf^{-1} is an (α_β, γ)- continuous mapping.

Remark 2.2. From Theorem 4.3[2], every bijective, (α_γ,β) - continuous mapping and (γ, α_β)-closed mappingis an (α_γ,β) - homeomorphism.

Theorem 2.3. Let $f: X_{tTS} \rightarrow Y_{tTS}$be an ($\alpha_\gamma$,$\beta$) -homeomorphism. If X_{tTS} is a γ- $T_{\frac{1}{2}}$space, then Y_{tTS} is an α_β -$T_{\frac{1}{2}}$ space.

Proof: Let {y}be a singleton set of Y_{tTS}. Then there exists a point x of X_{tTS} such that y=f(x).It follows from the assumption and Proposition 4.10(i)[8] that{x}is γ -open or γ-closed. By Theorem 2.1, we have {y}is an α_β-open set or anα_β-closed set. Then by Theorem 2.2,Y_{tTS} is an α_β -$T_{\frac{1}{2}}$ space.

Definition 3.7.A mapping$f: X_{tTS} \rightarrow Y_{tTS}$ is said to be an(α_γ,β) -generalized-homeomorphism if f is bijective, (α_γ,β)- generalized continuous ma and f^{-1} is an (α_β, γ)-continuous ma.

Remark 3.3. From the Theorem 2.2,Every bijective,(α_γ,β) -generalized continuous and (γ, α_β)-generalized closed mapping is an (α_γ,β) -generalized homeomorphism.

Remark3.4. It is evident that (α_γ,β) - homemorphismmapping imply (α_γ,β) -generalized homemorphism mapping. But the converse is not true. It is evident from the following example.

Let $X_{tTS} = \{a, b, c\}$, $\tau = \{\varphi, X_{tTS}, \{a\}, \{c\}, \{a, c\}, \{a, b\}\}$, $Y_{tTS} = \{a, b, c\}$ and $\sigma = \{\varphi, Y_{tTS}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$.

The operations γ, βon τ and σ are defined as $A^\gamma = \begin{cases} cl(A)\ if\ b \in A \\ A \cup \{c\}\ if b \notin A \end{cases}$for every A $\in \tau$, then τ_{α_γ}=

$\{\varphi, X_{tTS}, \{a\}, \{c\}, \{a, c\}, \{a, b\}\}$ and$B^\beta = \begin{cases} B \cup \{a\}\ if B = \{a\}, \{b\} or\ \{c\} \\ B\ \ \ otherwise \end{cases}$for every $B \in \sigma$, then $\sigma_{\alpha_\beta} = \{\varphi, Y_{tTS}, \{a, c\}, \{b, c\}\}$.

The mapping f is defined as f (a) = b, f (b) = c and f (c) = a. Here f is an (γ, α_β)-generalized homeomorphism. But f^{-1}({a, b})= {c, a} is not an γ -closed set in X_{tTS} for the α_β-closed set {a, b}of Y_{tTS}.

Theorem 3.4. A mapping$f: X_{tTS} \rightarrow Y_{tTS}$be a bijective and ($\alpha_\gamma$,$\beta$) -generalized continuous mapping.

Then the following statements are equivalent:

(i)f is an (γ, α_β)-generalized open mapping.

(ii)f is an (γ, α_β) -generalized closed mapping.

(iii)$f$is an (γ, α_β)-generalized homeomorphism

Proof. Followsfrom the Theorem3.3, Definition2.5 and Remark2.4.

3. (α_γ, α_β)-GENERALIZED CLOSED MAPPINGS

Definition 3.1. A mapping $f: X_{tTS} \rightarrow Y_{tTS}$is said to be a (α_γ, α_β)-generalized closed mapping [denoted as (α_γ, α_β)-ge cl ma] if and only if for each α_γ -generalized closed set $H \in X_{tTS}$, the image f (H) is an α_β- generalized *closed set* (ge cl se) in Y_{tTS} .

Definition 3.2. A mapping $f: X_{tTS} \rightarrow Y_{tTS}$is said to be a (α_γ, α_β)-generalized open mapping [denoted as ($\alpha_\gamma \alpha_\beta$)-ge op ma] if and only if for each α_γ-generalized open set $H \in X_{TS}$, the image f (H) is an α_β-generalized *open set* (ge op se) in Y_{tTS}.

Definition 3.3. A mapping $f: X_{tTS} \rightarrow Y_{tTS}$ is said to be a (α_γ, α_β) - closed mapping [(α_γ, α_β) -cl ma] if and only if for each α_γ-*closed set* $H \in X_{tTS}$, the image $f(H)$ is an α_β - *closed set* in Y_{tTS} .

Definition 3.4. A mapping $f: X_{tTS} \rightarrow Y_{tTS}$ is said to be a (α_γ, α_β) - open mapping if and only if for each α_γ -*open set* $H \in X_{tTS}$, the image $f(H)$ is an α_β-*open set* in Y_{tTS} .

Remark 3.1. From the Definitions 3.1, 3.2, 3.3 and 3.4, we can conclude that every (α_γ, α_β)- closed (open) mapping and (α_γ, α_β) - generalized closed (open) mapping are independent.

Remark 3.2. From the Definitions 3.1, 3.2, 3.3 and 3.4, we can conclude that every (α_γ, α_β) -generalized closed (open) mapping is a (γ, α_β) - generalized closed (open) mapping. But the converse need not be true.

The above Remark 3.2. follows from the example 3.1.

Example 3.1. Let $X_{tTS} = \{a, b, c\}$, $\tau = \{\varphi, X_{tTS}, \{a\}, \{c\}, \{a, c\}, \{a, b\}\}, Y_{tTS} = \{a, b, c\}$ and $\sigma = \{\varphi, Y_{tTS}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$.

The operations γ, β on τ and σ are defined as $A^\gamma = \begin{cases} cl(A)\ if\ b \in A \\ A \cup \{c\}\ if b \notin A \end{cases}$ for every A $\in \tau$, then $\tau_{\alpha_\gamma} =$

$\{\varphi, X_{tTS}, \{a\}, \{c\}, \{a, c\}, \{a, b\}\}$ and $B^\beta = \begin{cases} B \cup \{a\} if B = \{a\}, \{b\} or\ \{c\} \\ B\ otherwise \end{cases}$ for every $B \in \sigma$, then $\sigma_{\alpha_\beta} = \{\varphi, Y_{tTS}, \{a, c\}, \{b, c\}\}$.

The mapping f is defined as f (a) = b, f (b) = c and f (c) = a. Then the image of every γ-generalized closed is an α_β- generalized closed set under the mapping f . Hence f is a (γ, α_β) - generalized closed mapping. But f is not a ((α_γ, α_β) -generalized open mapping, since {a} is aγ -open set in X_{TS} , but f({a}) = {b} is not an α_β -open set in Y_{TS} . Similarly, f is not a (γ, α_β) - closed mapping, since {b} is aγ -closed set in X_{TS} , but f({b}) = {c} is not an α_β -closed set in Y_{TS} .

Theorem 3.1. A surjective mapping $f: X_{tTS} \rightarrow Y_{tTS}$ is a (α_γ, α_β)-generalized closed mapping if and only if for each subset B of Y_{tTS} and each α_γ- *open set* U of X_{tTS} containing f^{-1} (B), there exists an α_β- generalized *open set* V of Y_{tTS} such that B ⊆V and f^{-1} (V) ⊆ U.

Proof: Necessary Part: Suppose that f is an (α_γ, α_β)-generalized closed mapping. Let B be a subset of Y_{tTS} and U is an α_γ- *open set* of X_{tTS} containing f^{-1} (B). Put V = $Y_{tTS} - f(X_{tTS} - U)$. Then V is an α_β- generalized *open set* in Y_{TS} , B ⊆V and f^{-1} (V) ⊆U.

Sufficient Part: Let F be a α_γ - *generalized closed set* of X_{tTS}. Put B = $Y_{tTS} - f$(F), then $f^{-1}(B) \subseteq X_{tTS}$ − F and X_{tTS} – F is a α_γ - *generalized open set* in X_{tTS} . There exists an α_β- generalized *open set* V of Y_{tTS} such that B = $Y_{tTS} - f$(F) ⊆ V and f^{-1}(V) ⊆ X_{tTS} − F.

Therefore, f(F) = Y_{tTS}– V and hence f (F) is an α_β - generalized $closed\ set$ inY_{tTS}.This proves that f is a (γ, α_β)-generalized closed mapping.

Remark 3.2. Necessity of Theorem 3.1. is proved without assuming that f is surjective. Therefore, we can obtain the following Corollary.

Corollary 3.1. If $f: X_{tTS} \rightarrow Y_{tTS}$is a (γ, α_β)-generalized closed mapping, then for any β-$closed\ set F$ of $Y_{tTS}$and for any α_γ -open set U of X_{tTS}containing f^{-1} (F), there exists an α_β-$open\ set V$ of Y_{tTS} such that $F \subseteq V$ and f^{-1} $(V) \subseteq U$.

Proof: By Necessity of Theorem 3.1, there exists anα_β -generalized $open\ set W$ of Y_{tTS} such that $F \subseteq W$ and $f^{-1}(W) \subseteq U$. Since F is a β - $closed\ set$, By Definition of the α_β - generalized $closed\ set$, we have $F \subseteq \sigma_{\alpha_\beta}$-int (W). Put $V = \sigma_{\alpha_\beta}$-int (W), then $V \in \sigma_{\alpha_\beta}$, $F \subseteq V$ and f^{-1} $(V) \subseteq U$.

Theorem 3.2. If$f: X_{tTS} \rightarrow Y_{tTS}$is a (γ,$\beta$)-continuous mapping, ($\alpha_\gamma$, α_β)- closed mapping and H is an α_γ-generalized closed set in X_{tTS}, then $f(H)$ is an α_β - generalized closed set in Y_{tTS}.

Proof: Let V be any β-open set of Y_{tTS} containing $f(H)$. Then $H \subseteq f^{-1}(V)$ and $f^{-1}(V)$is a γ-open set inX_{tTS} . Since H is α_γ-generalized closed set in X_{tTS},τ_{α_γ}-$cl(V) \subseteq f^{-1}(V)$and hence $f(H) \subseteq f\left(\tau_{\alpha_\gamma} cl(H)\right) \subseteq V$. Since f is (α_γ, α_β)-closed mapping and τ_{α_γ}- $cl(H)$ is an α_γ-closed set inX_{tTS}, this implies that $f\left(\tau_{\alpha_\gamma} cl(H)\right)$ is an α_β-closedset inY_{tTS} and hence $\sigma_{\alpha_\beta} cl(f(H)) \subseteq f\left(\tau_{\alpha_\gamma} cl(H)\right) \subseteq V$.Therefore $f(H)$ is an α_β - generalized closed set in Y_{tTS}.

Definition 3.4. A mapping $f: X_{tTS} \rightarrow Y_{tTS}$is said to be an ($\alpha_\gamma$,$\alpha_\beta$) -generalized continuous mapping [denoted as (α_γ,α_β) -ge con ma] if and only if for each ,α_β - generalized$closed\ set B$ $\in$ Y_{tTS}, the inverse image $f^{-1}(B)$ is an α_γ- generalized $closed\ set$ in X_{tTS} .

Theorem 3.3. Let $f: X_{tTS} \rightarrow Y_{tTS}$be a bijectivemapping. Then the following statements are equivalent:

(i) f is an (α_γ, α_β) -generalized open mapping;
(ii) f is an (α_γ, α_β) -generalized closed mapping;
(iii) f^{-1} is an (α_β,α_γ) -generalized continuous mapping.

Proof: (i) $\Rightarrow$ (ii) The proof follows from the Definitions 3.1, 3.2.

(ii) $\Rightarrow$ (iii) Let F be a α_γ - $generalized\ open\ set$ in X_{tTS} and let B be an α_β-$closed\ set$ in Y_{tTS} such that $B \subseteq f$ (F). This implies that f^{-1}(B) $\subseteq F$ .Then by (ii) and corollary 4.1, there exists an α_β-$open\ set V$ of Y_{tTS} such that $B \subseteq V$ and $f^{-1}(V) \subseteq F$. Therefore $B \subseteq \tau_{\alpha_\beta}$-int$(V)$ and $V \subseteq f(F)$ and hence $B \subseteq \tau_{\alpha_\beta}$-int $(f(F))$. Thus$(f^{-1})^{-1}(F) =$

$f(F)$ is an α_β - generalized $open\ set$ in Y_{tTS}. Then by definition 3.4, f^{-1} is an (α_β,α_γ) - generalized continuous mapping.

(iii) $\Rightarrow$ (i) Let D be a α_γ- generalized $open\ set$ in X_{tTS}. Then $X_{tTS} - D$ is an α_γ - $generalized\ closed\ set$ in X_{TS}. Since f^{-1} is an (α_β,α_γ)-generalized continuous mapping, $(f^{-1})^{-1}$ $(X_{tTS} - D)$ is an α_β-generalized $closed\ set$ in Y_{TS}. But $(f^{-1})^{-1}$ $(X_{tTS} - D)$ = f $(X_{tTS} - D)$ = $Y_{tTS} - f(D)$.Thus $f(D)$ is an α_β -generalized $open\ set$ in Y_{tTS}. This proves that f is a (α_γ, α_β) -generalized open mapping. Hence the proof.

Definition 3.6. A mapping $f: X_{tTS} \to Y_{tTS}$ is said to be an (α_γ,α_β) - generalized homeomorphism, if f is bijective, (α_γ,α_β)- generalized continuous mapping and f^{-1} is an (α_β,α_γ)- generalized continuous mapping.

Remark 3.4. From Theorem 3.4, every bijective, (α_γ,α_β)) - generalized continuous mapping and (α_γ, α_β)- generalized closed mapping is an (α_γ,α_β) - generalized homeomorphism.

Definition 3.7.A mapping $f: X_{tTS} \to Y_{tTS}$ is said to be an (α_γ,α_β)-homeomorphism if f is bijective, (α_γ,α_β) -continuous mapping and f^{-1} is an (α_β,α_γ)-continuous mapping.

Remark 3.5. Every(α_γ,α_β) - homeomorphismis (α_γ,β)-homeomorphism. But the converse need not be true.

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Remark 3.7. It is evident that (α_γ,α_β) - generalized homemorphism mapping imply (α_γ,β) -generalized homemorphism mapping but the converse need not be true.

Let $X_{tTS} = \{a, b, c\}$, $\tau = \{\varphi, X_{tTS}, \{a\}, \{c\}, \{a, c\}, \{a, b\}\}$, $Y_{tTS} = \{a, b, c\}$ and $\sigma = \{\varphi, Y_{tTS}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$.

The operations γ, β on τ and σ are defined as $A^\gamma = \begin{cases} cl(A)\ if\ b\ \in A \\ A \cup \{c\}\ if b\ \notin\ A \end{cases}$ for every A $\in \tau$, then τ_{α_γ}=

$\{\varphi, X_{tTS}, \{a\}, \{c\}, \{a, c\}, \{a, b\}\}$ and $B^\beta = \begin{cases} B\ \cup \{a\}\ \ if B = \{a\}, \{b\} or\ \{c\} \\ B \qquad otherwise \end{cases}$ for every $B \in \sigma$, then $\sigma_{\alpha_\beta} = \{\varphi, Y_{tTS}, \{a, c\}, \{b, c\}\}$.

The mapping f is defined as f (a) = b, f (b) = c and f (c) = a. Here f is an (γ, α_β)-generalized homeomorphism. But f^{-1}({a, b}) = {c, a} is not an γ -closed set in X_{tTS} for the α_β-closed set {a, b} of Y_{tTS}.

Theorem 3.4. A mapping $f: X_{tTS} \to Y_{tTS}$ be a bijective and (α_γ,α_β) -generalized continuous mapping.

Then the following statements are equivalent:
(i) f is an (α_γ, α_β) -generalized open mapping.
(ii) f is an (α_γ, α_β) -generalized closed mapping.
(iii) f is an (α_γ, α_β) -generalized homeomorphism
Proof. Follows from the Theorem 3.3, Definition 3.6 and Remark 3.4.

Conclusion 4: In this paper the (γ, α_β) -generalized homeomorphism ((α_γ,α_β) – homeomorphism) introduced and characterize it using (γ, α_β) - generalized closed (open) mappings ((α_γ,α_β) -generalized closed (open) mappings). Final some of its properties are studied and the proposed mappings is compared with already existing mappings.

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