Solving lorenz system of equation by Laplace homotopy analysis method

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Abstract. The Laplace Homotopy Analysis Method Via Modified Riemann-Liouville Integral has been explored from a new perspective. The important items are assimilated, and the result is proven using well-defined evidence. A combination of the homotopy analysis approach and the suggested integral transform is used to find fractional differential equations

Keywords: Homotopy, Homotopy Analysis Method, Laplace Transform, Fractional Differential Equations.

1 Introduction

Many researchers have recently become interested in the theoretical studies of various integral transforms as a methodical mathematical strategy for converting fractional differential equations into simple algebraic formulas. Chowdhury et al (2007) introduced MHPM to handle the Lorenz system in the literature. The DTM was used by Mossa et al (2008) to solve a non-linear differential equation.

Jumarie (2009) presented a fractional-order Laplace's transform definition for functions that are fractionally differentiable but not differentiable. Alomari et al (2011) modified the DTM to achieve continuous and analytic solutions for each interval when solving non-linear fractional differential equations. The Laplace transform was first introduced to the field by Liang et al (2015).Medina et al (2017) an investigated the effect of the fractional Laplace Transform incorporated in the RiemannLiouville Fractional Derivative. For various fractional linear differential equations with constant coefficients, Silva et al (2018) studied the pleasant fractional derivative. The integral trans-form is used in conjunction with the Homotopy Analysis Method to solve nonlinear differential equations (HAM). Hariharan (2017) proposed the homotopy analysis technique (HAM) for solving a few partial differential equations in the literature. Mohammed et al (2017) used modified Laplace Homotopy Analysis to solve a nonlinear system of fractional partial differential equations.

The Fractional Laplace Transform through Modified Riemann-Liouville derivative [Jumarie (2009)] is the subject of this work. It is coupled with HAM, which was proposed by Liao (1992), to create a novel hybrid approach called Laplace Homotopy Analysis

through Modified RiemannnLiouville Integral.Nonlinear fractional differential equations can be solved using this hybrid approach.

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Edward Lorenz was the first to study the Lorenz system, which is a set of ordinary differential equations. For specific parameter values and starting circumstances, it is renowned for having chaotic solutions. The Lorenz attractor, in particular, is a collection of chaotic Lorenz system solutions.

This research motivates to study the Fractional Laplace transform via Modified Riemann-Liouville [Jumarie (2009)] and it is combined with HAM introduced by Liao (1992), which provides the new hybrid technique Laplace Homotopy Analysis method via Modified Riemannn-Liouville Integral.

Non-chaotic behaviour is obtained in this work by employing modal series solutions with Rayleigh number R parameter values below the critical value. Throughout this study, we assume σ = 10, b=-8/3, and change the Rayleigh number R to get different dynamical behaviours and assess the recommended approach. Pandemonium is well known to occur around the critical parameter value R = 24.74. [10]

2 Preliminaries

This part contains the essential definitions for the study, as well as other fundamental results that can be found in Jumarie (2009).

Definition 2.1: The Mittag–Leffler funεεction which is a generalization of exponential function is defined as

$$E_{\zeta}(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\zeta n+1)} \qquad (1)$$

Where ζ ϵ C, R (ζ) >0.

Definition 2.2: The continuous function $g{:}R \rightarrow R$, $t \rightarrow g(t)$ has a fractional derivative of order $k\zeta$. For any positive integer k and for any ζ, $0 < \zeta \leq 1$, the Taylor's series of

Fractional order can be expressed as

$$g(t+h) = \sum_{k=0}^{\infty} \frac{h^{\zeta k}}{(\zeta k)!} g^{(\zeta k)}(t), 0 < \zeta \leq 1$$

Where Γ(1+ζ k) = (ζ k)!.

Definition 2.3: (Modified Riemann Liouville derivative) Let $q{:}R \rightarrow R$, denote a continuous (but not necessarily differentiable) function.

i. Assume that $q(y)$ is a constant K. Then its fractional derivative of order ζ is

$$D_y^{\zeta} = K\,\Gamma^{-1}(1-\zeta)y^{-\zeta}, \zeta \leq 0, \qquad (2)$$
$$= 0, \zeta > 0$$

ii. When $q(y)$ is not a constant, then we will set

$q(y) = q(0) + (q(y) - q(0))$,(3)

in which, for negative ζ, one has

$$D_y^{\zeta}(q(y) - q(0)) = D_y^{\zeta} q(y) = D_y(q^{(\zeta-1)}(y)).$$

(4) $\quad When\, n \leqslant \zeta < n+1\,,\; we\;\; will\; set$

$$q^{(\zeta)}(y) = (q^{(\zeta-n)}(y))^{(n)}, n \geqslant 1. \qquad (5)$$

In order to find the fractional derivative of compound functions, equation (6) is used.

$d^{\zeta}q \cong \Gamma(1+\zeta)dq, 0 < \zeta < 1.$ (6)

Definition 2.4: $If 0 < \zeta < 1, then$

$D_y^{\zeta} y^{\eta} = \Gamma(\eta+1)\Gamma^{-1}(\eta+1-\zeta)y^{\eta-\zeta}, \eta > 0,$ (7)

Or, if $\zeta = n + \theta, n \in \mathbb{N}$, $then$

$D_y^{n+\theta} y^{\eta} = \Gamma(\eta+1)\Gamma^{-1}(\eta+1-n-\theta)y^{\eta-n-\theta}, 0 < \theta < 1,$ (8)

3 Fractional Laplace Transform Via Modified Riemann-Liouville Integral

Definition 3.1: Let $k(y)$ denote a function that vanishes for the negative value of y. Its Laplace's transformation$L_{\zeta}k(y)$ of orderζdefined by the following expression, where it is finite:

$$L_{\zeta}[k(y)] = K_{\zeta}(s) = \int_0^{\infty} E_{\zeta}[-s^{\zeta}y^{\zeta}]k(y)(dy)^{\zeta}, \qquad (9)$$

$$= \lim_{M\to\infty} \int_0^M \quad E\zeta[-s^{\zeta}y^{\zeta}]k(y)(dy)^{\zeta} \qquad (10)$$

$where\ s \in C, and\ E_{\zeta}$ is the Mittag-Leffler function$\sum u^k/(k\zeta)$!.

Theorem 3.1: if $L_{\zeta}[k(y)] = K_{\zeta}(s)\ \ then$

Scaling property

$(i) L_{\zeta}[k(ay)]_s = \frac{1}{a}^{\zeta} L_{\zeta}[k(y)]_{\frac{s}{a}},$ (11)

Shifting Property

$(ii)\ L_{\zeta}[k(y-b)] = E_{\zeta}(-s^{\zeta}b^{\zeta})L_{\zeta}[k(y)],$ (12)

Frequency Shifting Property

$(iii) L_{\zeta}[E_{\zeta}(-c^{\zeta}y^{\zeta})k(y)]_s = L_{\zeta}[k(y)]_{s+c}$,(13)

Derivative Property

$(iv) L_{\zeta}[-y^{\zeta}k(y)]_s = D_s^{\zeta} L_{\zeta}[k(y)]$,(14)

Laplace transform of fractional derivative

$(v) L_{\zeta}[k^{\zeta}(y)] = s_{\zeta} L_{\zeta}[k(y)] - \Gamma(1+\zeta)g(0).$ (15)

Theorem 3.2: Let the convolution of the two functions k(y) and l(y) of order $\zeta$is given by

$(k(y) * l(y))_{\zeta} = \int_0^y k(y-u)l(y)(du)^{\zeta},$ (16)

then one has the equality

$L_{\zeta}[(k(y) * l(y))_{\zeta}] = L_{\zeta}[k(y)]L_{\zeta}[l(y)].$ (17)

Coming up next are the main ends for the Laplace fractional change of standard functions, as displayed in Table 1:

Laplace Fractional transform of standard functions		
S.N	g(y)	$L_{\zeta}[g(y)] = G_{\zeta}(s)$
1	1	$\frac{1}{s^{\zeta}}\Gamma(\zeta+1)$

2	t^{ζ}	$\frac{1}{s^{2\zeta}}\Gamma^2(\zeta+1)$
3	$t^{2\zeta}$	$\frac{1}{s^{3\zeta}}\Gamma^3(\zeta+1)\Gamma(3)$
4	$t^{n\zeta}$	$\frac{1}{s^{(n+1)\zeta}}\Gamma^{n+1}(\zeta+1)\Gamma(n+1)$
5	$E_{\zeta}[a^{\zeta}y^{\zeta}]$	$\frac{1}{(s-a)^{\zeta}}\Gamma(\zeta+1)$
6	$E_{\zeta}[-a^{\zeta}y^{\zeta}]$	$\frac{1}{(s+a)^{\zeta}}\Gamma(\zeta+1)$
7	$E_{\zeta}[i^{\zeta}a^{\zeta}y^{\zeta}]$	$\frac{1}{(s-ia)^{\zeta}}\Gamma(\zeta+1)$
8	$E_{\zeta}[-i^{\zeta}a^{\zeta}y^{\zeta}]$	$\frac{1}{(s+ia)^{\zeta}}\Gamma(\zeta+1)$
9	$sin(a^{\zeta}y^{\zeta})$	$\frac{a^{\zeta}}{(s^2+a^2)^{\zeta}}\Gamma(\zeta+1)$
10	$cos(a^{\zeta}y^{\zeta})$	$\frac{s^{\zeta}}{(s^2+a^2)^{\zeta}}\Gamma(\zeta+1)$
11	$sinh(a^{\zeta}y^{\zeta})$	$\frac{a^{\zeta}}{(s^2-a^2)^{\zeta}}\Gamma(\zeta+1)$
12	$cosh(a^{\zeta}y^{\zeta})$	$\frac{s^{\zeta}}{(s^2-a^2)^{\zeta}}\Gamma(\zeta+1)$
13	$E_{\zeta}[a^{\zeta}y^{\zeta}]sin[b^{\zeta}y^{\zeta}]$	$\frac{b^{\zeta}}{(s^{\zeta}-a^{\zeta})^{2\zeta}+b^{2\zeta}}\Gamma(\zeta+1)$
14	$E_{\zeta}[a^{\zeta}y^{\zeta}]cos[b^{\zeta}y^{\zeta}]$	$\frac{(s-a)^{\zeta}}{(s^{\zeta}-a^{\zeta})^{2\zeta}+b^{2\zeta}}\Gamma(\zeta+1)$
15	$E_{\zeta}[a^{\zeta}y^{\zeta}]sinh[b^{\zeta}y^{\zeta}]$	$\frac{b^{\zeta}}{(s^{\zeta}-a^{\zeta})^{2\zeta}-b^{2\zeta}}\Gamma(\zeta+1)$
16	$E_{\zeta}[a^{\zeta}y^{\zeta}]cos[b^{\zeta}y^{\zeta}]$	$\frac{(s-a)^{\zeta}}{(s^{\zeta}-a^{\zeta})^{2\zeta}-b^{2\zeta}}\Gamma(\zeta+1)$

The Laplace transform will be utilized related to the fractional homotopy analysis way to deal with tackle both linear and nonlinear differential conditions (Fractional Homotopy Analysis method).

4 Fractional laplace homotopy analysis

METHOD (FLHAM)

Consider the fractional time nonlinear differential condition with the accompanying beginning condition:

$D^{\zeta}v(y,t) + R(y,t) + Nv(y,t) = q(y,t), v(y,0) = k(y),$ (18)

where D^{ζ} is the fractional differential operator $D^{\zeta} = \frac{\partial^{\zeta}}{\partial t \quad \zeta}$ R is the differential linear operator, N is the differential non-linear operator and q(y; t) is source term.

To solve the non-linear partial differential condition,

Embrace the accompanying organized method:

Step 1: Apply fractional laplace transform, the equation (18),

$[D^{\zeta}v(y,t)] + L_{\zeta}[Rv(y,t)] + L_{\zeta}[Nv(y,t)] = L_{\zeta}[q(y,t)].$ (19)

Step 2: Applying the derivative of fractional Laplace transform,

The condition (19), can communicate as

$L_{\zeta}[v(y,t)] - \frac{1}{s^{\zeta}}\Gamma(\zeta+1)v(y,0) + \frac{1}{s^{\zeta}}L_{\zeta}[Rv(y,t)] + \frac{1}{s^{\zeta}}L_{\zeta}[Nv(y,t)] - \frac{1}{s^{\zeta}}L_{\zeta}[q(y,t)] = 0.$ (20)

Step 3: The nth order deformation equation

$v_n(y,t) = \chi_n v_{n-1}(y,t) + hL^{-1}[R_n(v_{n-1}(y,t))],$ (21)

Where,

$$R_{n-1}[v_{n-1}(y,t)] = [L_{\zeta}[v_{n-1}(y,t)] - \frac{1}{s^{\zeta}}\Gamma(\zeta+1)v(y,0) + \frac{1}{s^{\zeta}}L_{\zeta}[Rv_{n-1}(y,t)] + \frac{1}{s^{\zeta}}L_{\zeta}[Nv_{n-1}(y,t)] - \frac{1}{s^{\zeta}}L_{\zeta}[q(y,t)],$$

Where,

$$\chi_n = \begin{cases} 0 & n \le 1 \\ 1 & n > 1. \end{cases}$$

5. Application

Consider the well-known Lorenz system in this study.

$$D^{\zeta}x = \sigma(y(t) - x(t)) \quad (22)$$
$$D^{\zeta}y = Rx(t) - y(t) - x(t)z(t) \quad (23)$$
$$D^{\zeta}z = x(t)y(t) + bz(t) \quad (24)$$

where convective velocity, temperature differential between descending and ascending flows, and mean convective heat flow are proportional to x, y, and z, and σ, b, and the so called bifurcation parameter R are real constants. With the initial condition x(0) = c1; y(0) = c2; z(0) = c3.

$$\frac{d^{\zeta}x}{dt^{\zeta}} = \sigma(y(t) - x(t)) \quad (25)$$

$$\frac{d^{\zeta}y}{dt^{\zeta}} = Rx(t) - y(t) - x(t)z(t) \quad (26)$$

$$\frac{d^{\zeta}z}{dt^{\zeta}} = x(t)y(t) + bz(t) \quad (27)$$

with the initial condition $x(0) = c_1, y(0) = c_2, z(0) = c_3$. Apply L_{ζ} on both sides of the equation (25)

$$L_\zeta\left[\frac{d^\zeta x}{dt^\zeta}\right] = L_\zeta\left[\sigma\big(y(t) - x(t)\big)\right] \qquad (28)$$

$$s^\zeta L_\zeta[x(t)] - \Gamma(\zeta+1)x(0) = \sigma\left[L_\zeta\big(y(t) - x(t)\big)\right] \qquad (29)$$

$$s^\zeta L_\zeta[x(t)] - \Gamma(\zeta+1)x(0) = \sigma\left[L_\zeta\big(y_{m-1}(t)\big) - L_\zeta\big(x_{m-1}(t)\big)\right] \quad (30)$$

The nth order deformation equation for x(t) is defined as

$$R_n[x_{n-1}(t)] = [L_\zeta[x_{n-1}(t)] - \frac{1}{s^\zeta}\Gamma(\zeta+1)x(0)(1-\chi_n) - \frac{1}{s^\zeta}\sigma L_\zeta[y_{n-1}(t)] + \frac{1}{s^\zeta}\sigma L_\zeta[x_{n-1}(t)] \quad (31)$$

$$R_n[y_{n-1}(t)] = [L_\zeta[y_{n-1}(t)] - \frac{1}{s^\zeta}\Gamma(\zeta+1)y(0)(1-\chi_n) - \frac{1}{s^\zeta}RL_\zeta\left[(x_{n-1}(t)] + \frac{1}{s^\zeta}L_\zeta[y_{n-1}(t)] + \frac{1}{s^\zeta}L_\zeta\left[\sum_{i=0}^{m-1} x_i(t)z_{m-1-i}(t)\right]\right] \quad (32)$$

$$R_n[z_{n-1}(t)] = [L_\zeta[z_{n-1}(t)] - \frac{1}{s^\zeta}\Gamma(\zeta+1)z(0)(1-\chi_n) - \frac{1}{s^\zeta}L_\zeta[\sum_{i=0}^{m-1} x_i(t)y_{m-1-i}(t)] - \frac{1}{s^\zeta}bL_\zeta[z_{n-1}(t)] \qquad (33)$$

On both sides of the equation (31),(32) , and (33), use the inverse fractional Laplace transform

$$x_n(t) = \chi_n x_{n-1}(t) + hL^{-1}[R_n(x_{n-1}(t)] \qquad (34)$$

$$y_n(t) = \chi_n y_{n-1}(t) + hL^{-1}[R_n(y_{n-1}(t)] \qquad (35)$$

$$z_n(t) = \chi_n z_{n-1}(t) + hL^{-1}[R_n(z_{n-1}(t)] \qquad (36)$$

On solving the (34) (35) (36) equations for n=1, 2, 3, 4,.......

$$x_1(t) = h\sigma(-c_2 + c_1)\frac{t^\zeta}{\Gamma(\zeta+1)}$$

$$y_1(t) = -h(Rc_1 - c_2 - c_1c_3)\frac{t^\zeta}{\Gamma(\zeta+1)}$$

$$z_1(t) = -h(c_1c_2 + bc_3)\frac{t^\zeta}{\Gamma(\zeta+1)}$$

$$x_2(t) = [h\sigma(-c_2 + c_1)\frac{t^\zeta}{\Gamma(\zeta+1)} + h^2\sigma(Rc_1 - c_2 - c_1c_3)\frac{t^{2\zeta}}{2\Gamma^2(\zeta+1)} + h^2\sigma^2(-c_2 + c_1)\frac{t^{2\zeta}}{2\Gamma^2(\zeta+1)}]$$

$$y_2(t) = \left[h(Rc_1 - c_2 - c_1c_3)\frac{t^\zeta}{\Gamma(\zeta+1)} + h^2(Rc_1 - c_2C_1c_3)\frac{t^\zeta}{\Gamma(\zeta+1)}in - Rh^2\sigma(-c_2 + c_1)\frac{t^{2\zeta}}{2\Gamma^2(\zeta+1)} - h^2(Rc_1 - c_2 - c_1c_3)\frac{t^{2\zeta}}{2\Gamma^2(\zeta+1)} - h^2c_1(c_1c_2 + bc_3)\frac{t^{2\zeta}}{2\Gamma^2(\zeta+1)} + h^2\sigma(-c_2 + c_1)c_3\frac{t^{2\zeta}}{2\Gamma^2(\zeta+1)}\right]$$

$$z_2(t) = [h(c_1c_2 + bc_3)\frac{t^{\zeta}}{\Gamma(\zeta+1)} + h^2(c_1c_2 + bc_3)\frac{t^{\zeta}}{\Gamma(\zeta+1)} + c_1h^2(Rc_1 - c_2 - c_1c_3)\frac{t^{2\zeta}}{2\Gamma^2(\zeta+1)} - h^2\sigma(-c_2 + c_1)c_2\frac{t^{2\zeta}}{2\Gamma^2(\zeta+1)} + bh^2(c_1c_2 + bc_3)\frac{t^{2\zeta}}{2\Gamma^2(\zeta+1)}]$$

Similarly x3, x4,....... y3,y4,.......and z3,z4,.......can be assessed, and a sequence of solutions can be found as follows:

$$x(t) = \sum_{n=0}^{N} x_n(t) \qquad (37)$$

$$y(t) = \sum_{n=0}^{N} y_n(t) \qquad (38)$$

$$z(t) = \sum_{n=0}^{N} z_n(t) \qquad (39)$$

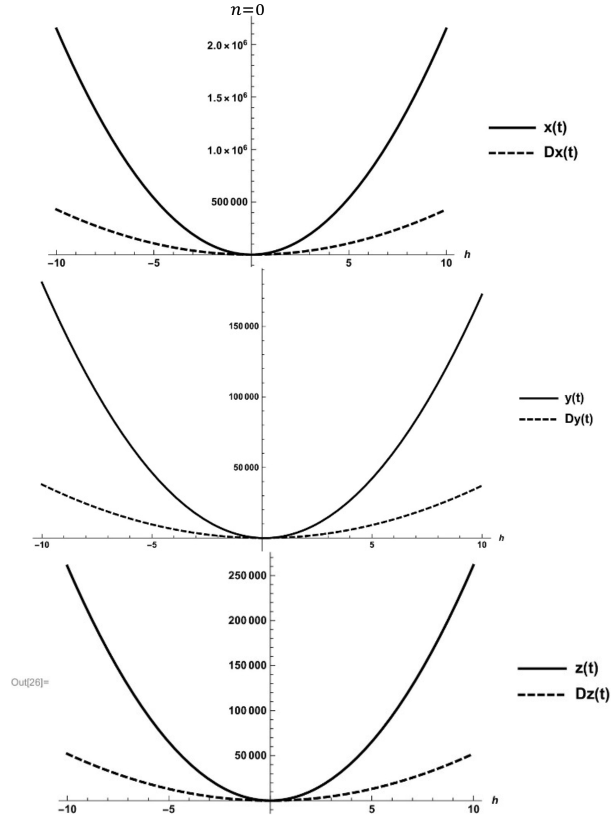

Fig. 1: The h-curve for the Lorenz system of equation x(t) (1a), y(t) (1b) and z(t) (1c) with the convergence region for the auxiliary parameter (h ϵ [-1; 0]).

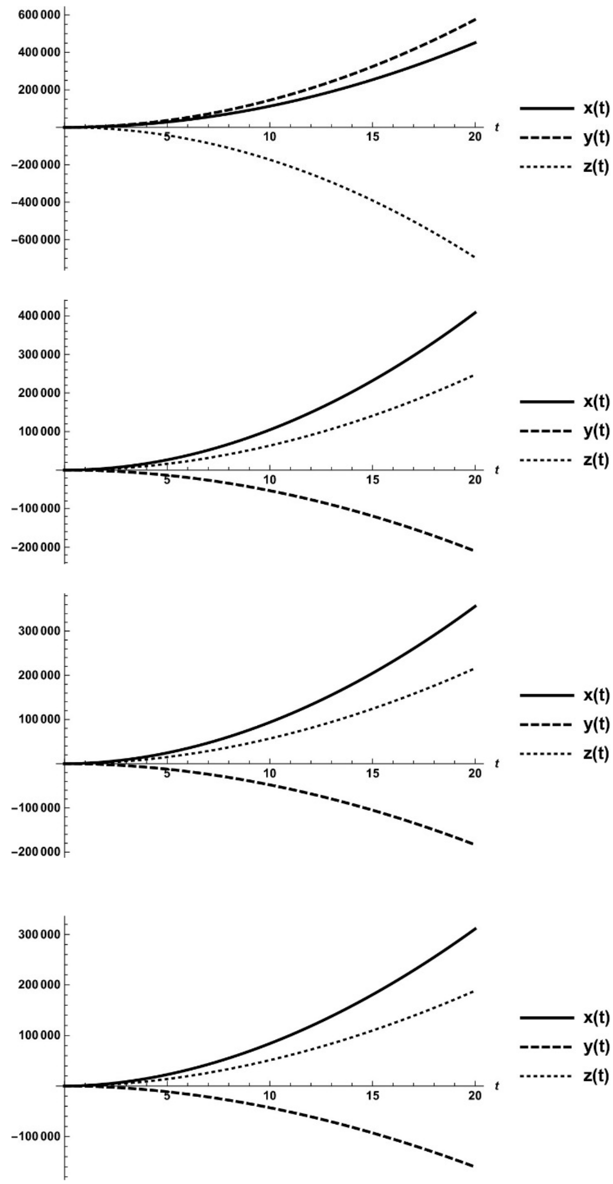

Fig. 2: The Non chaotic Solution curves for Lorenz system of equation when ζ =1,0.98,0.96,0.94 respectively (2a), (2b), (2c), (2d).

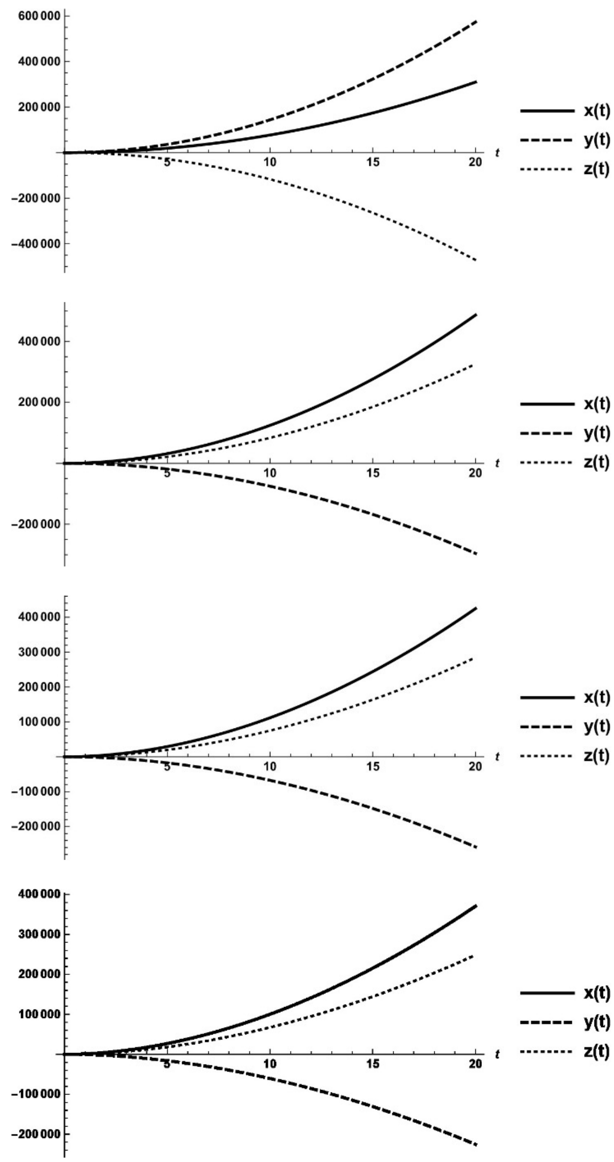

Fig. 3: The Chaotic Solution curves for Lorenz system of equation when ζ =1,0.98,0.96,0.94 respectively (3a), (3b), (3c),(3d)

To decide the values of h we plot the h−curve for the equation (37),(38) and (39) in various figures from $11-13$.From these figures, it is noted that the convergence region of h lies between the range $h \in [-1,\ 1]$.

The solution curve of the Lorenz system are displayed in figure 7 − 8 for the different values of ζ. ($i.e, \zeta = 0.94, 0.96, 0.98, 1$) and for a comparison with A.K.Alomari et al [1].we set $\sigma = 10$, $b = -8/3$ we take initial conditions $x(0) = -15.8$, $y(0) = -17.48$, $z(0) = 35.64$ as in [1]. Which demonstrates the excellence of the proposed Lorenz system.

TABLE I: Numerical results of Non Chaotic Solution for Lorenz system of equation when $x(t), y(t), z(t)$ for the fractional parameter $\zeta = 1$ that are compared among the two methods LHAM, HAM.

	x(t)		y(t)		z(t)	
t	LHAM	HAM	LHAM	HAM	LHAM	HAM
0	10	10	0	0	10	10
0.1	12.112	12.805	7.67927	7.3261	14.8467	15.2658
0.2	38.4705	41.1666	2.53492	1.16086	34.2577	35.8881
0.3	88.1468	94.1153	-14.7983	-17.8402	67.7334	71.3427
0.4	160.717	171.207	-44.0691	-49.415	115.031	121.374
0.5	255.899	272.143	-85.1156	-93.3944	175.984	185.808
0.6	373.479	396.7	-137.819	-149.653	250.469	264.512
0.7	513.288	544.7	-202.084	-218.094	338.385	357.381
0.8	675.182	715.992	-277.834	-298.633	439.647	464.326
0.9	859.042	910.449	-365.003	-391.203	554.184	585.271
1	1064.76	1127.96	-463.533	-495.743	681.932	-1531.32

TABLE II: Numerical results of Non Chaotic Solution for Lorenz system of equation when $x(t), y(t), z(t)$ for the fractional parameter $\zeta = 0.98$ that are compared among the two methods LHAM, HAM.

	x(t)		y(t)		z(t)	
t	LHAM	HAM	LHAM	HAM	LHAM	HAM
0	-15.8	-15.8	-17.48	-17.48	35.64	35.64
0.1	-6.1754	-6.1754	17.7329	17.7329	36.2734	36.2734
0.2	26.0584	26.0584	81.5131	81.5131	1.94479	1.94479
0.3	80.9014	80.9014	173.861	173.861	-67.3458	-67.3458
0.4	158.354	158.354	294.776	294.776	-171.598	-171.598
0.5	258.415	258.415	444.258	444.258	-310.813	-310.813
0.6	381.086	381.086	622.308	622.308	-484.99	-484.99
0.7	526.365	526.365	828.925	828.925	-694.128	-694.128
0.8	694.254	694.254	1064.11	1064.11	-938.229	-938.229
0.9	884.753	884.753	1327.86	1327.86	-1217.29	-1217.29
1	1097.86	1097.86	1620.18	1620.18	-1531.32	-1531.32

TABLE III: Numerical results of Non Chaotic Solution for Lorenz system of equation when $x(t)$,$y(t)$,$z(t)$ for the fractional parameter ζ = 0.96 that are compared among the two methods LHAM, HAM.

t	x(t)		y(t)		z(t)	
	LHAM	HAM	LHAM	HAM	LHAM	HAM
0	10	10	0	0	10	10
0.1	15.295	14.0435	7.94651	7.17223	15.295	16.2133
0.2	35.5032	45.7982	2.40102	-0.529008	35.5032	38.9782
0.3	69.6115	103.203	-15.3279	-21.71	69.6115	77.1807
0.4	117.143	185.341	-44.7415	-55.8294	117.143	130.293
0.5	177.777	291.606	-85.5242	-102.543	177.777	197.961
0.6	251.276	421.549	-137.446	-161.598	251.276	279.92
0.7	337.448	574.81	-200.329	-232.799	337.448	375.958
0.8	436.136	751.095	-274.023	-315.982	436.136	485.899
0.9	547.203	950.15	-358.406	-411.012	547.203	609.594
1	670.532	1171.76	-453.369	-517.77	670.532	-1531.32

TABLE IV: Numerical results of Non Chaotic Solution for Lorenz system of equation when $x(t)$,$y(t)$,$z(t)$ for the fractional parameter ζ = 0.94 that are compared among the two methods LHAM, HAM.

t	x(t)		y(t)		z(t)	
	LHAM	HAM	LHAM	HAM	LHAM	HAM
0	10	10	0	0	10	10
0.1	12.9953	15.4907	8.20776	6.93477	15.7878	17.2969
0.2	41.7532	50.9384	2.22729	-2.45825	36.8311	42.3856
0.3	93.3793	113.064	-15.9186	-25.9604	71.5903	83.4944
0.4	166.644	200.452	-45.4889	-62.7352	119.363	139.808
0.5	260.751	312.181	-86.0223	-112.258	179.689	210.79
0.6	375.117	447.573	-137.188	-174.149	252.223	296.04
0.7	509.28	606.093	-198.727	-248.114	336.697	395.242
0.8	662.863	787.303	-270.433	-333.912	432.885	508.138
0.9	835.547	990.831	-352.129	-431.342	540.6	634.504
1	1027.06	1216.35	-443.664	-540.23	659.676	774.151

TABLE V: Numerical results of Chaotic Solution for Lorenz system of equation when $x(t)$,$y(t)$,$z(t)$ for the fractional parameter ζ = 1 that are compared among the two methods LHAM, HAM.

t	x(t)		y(t)		z(t)	
	LHAM	HAM	LHAM	HAM	LHAM	HAM
0	-15.8	-15.8	-17.48	-17.48	35.64	35.64
0.1	-9.7304	-9.7304	10.6004	10.6004	41.8903	41.8903
0.2	11.8384	11.8384	67.2031	67.2031	24.4124	24.4124
0.3	48.9064	48.9064	152.328	152.328	-16.7937	-16.7937

0.4	101.474	101.474	265.976	265.976	-81.728	-81.728
0.5	169.54	169.54	408.145	408.145	-170.391	-170.391
0.6	253.106	253.106	578.838	578.838	-282.781	-282.781
0.7	352.17	352.17	778.052	778.052	-418.9	-418.9
0.8	466.734	466.734	1005.79	1005.79	-578.747	-578.747
0.9	596.798	596.798	1262.05	1262.05	-762.323	-762.323
1	742.36	742.36	1546.83	1546.83	-969.626	774.151

TABLE VI: Numerical results of Chaotic Solution for Lorenz system of equation when $x(t)$,$y(t)$,$z(t)$ for the fractional parameter $\zeta = 0.98$ that are compared among the two methods LHAM, HAM.

	x(t)		y(t)		z(t)	
t	LHAM	HAM	LHAM	HAM	LHAM	HAM
0	10	10	0	0	10	10
0.1	14.5384	15.364	9.75975	9.26264	17.2731	17.8248
0.2	47.9105	51.1228	1.51521	-0.418838	43.6977	45.8443
0.3	109.045	116.156	-23.8592	-28.1408	88.6316	93.3838
0.4	197.445	209.942	-66.0136	-73.5382	151.758	160.11
0.5	312.775	332.13	-124.722	-136.375	232.861	245.794
0.6	454.786	482.454	-199.819	-216.477	331.776	350.265
0.7	623.275	660.702	-291.171	-313.706	448.373	473.384
0.8	818.074	866.698	-398.671	-427.947	582.539	615.033
0.9	1039.04	1100.29	-522.226	-559.103	734.181	775.112
1	1286.04	1361.34	-661.754	-707.091	903.216	615.033

TABLE VII: Numerical results of Chaotic Solution for Lorenz system of equation when $x(t)$,$y(t)$,$z(t)$ for the fractional parameter $\zeta = 0.96$ that are compared among the two methods LHAM, HAM.

	x(t)		y(t)		z(t)	
t	LHAM	HAM	LHAM	HAM	LHAM	HAM
0	10	10	0	0	10	10
0.1	15.1429	16.9522	10.0782	8.98843	17.9128	19.1219
0.2	49.9584	56.8052	1.24451	-2.87955	45.4098	49.9851
0.3	112.265	127.178	-24.6996	-33.6825	91.1899	101.156
0.4	201.084	226.994	-67.0605	-82.6669	154.631	171.946
0.5	315.77	355.537	-125.398	-149.352	235.317	261.892
0.6	455.84	512.276	-199.392	-233.386	332.934	370.647
0.7	620.913	696.787	-288.79	-334.492	447.231	497.934
0.8	810.67	908.718	-393.387	-452.445	578.002	643.523
0.9	1024.84	1147.77	-513.008	-587.053,	725.068	807.216
1	1263.2	1413.69	-647.503	-738.149	888.276	615.033

TABLE VIII: Numerical results of Chaotic Solution for Lorenz system of equation when $x(t)$,$y(t)$,$z(t)$ for the fractional parameter $\zeta = 0.94$ that are compared among the two methods LHAM, HAM.

	x(t)		y(t)		z(t)	
t	LHAM	HAM	LHAM	HAM	LHAM	HAM
0	10	10	0	0	10	10
0.1	15.8212	18.7946	10.3855	8.59375	18.6138	20.6007
0.2	52.155	63.099	0.915056	-5.67976	47.2329	54.5463
0.3	115.672	139.126	-25.6283	-39.762	93.8828	109.556
0.4	204.93	245.212	-68.2145	-92.4884	157.65	184.568
0.5	318.993	380.271	-126.201	-163.127	237.93	278.88
0.6	457.17	543.5	-199.125	-251.147	334.276	391.967
0.7	618.916	952.054	-286.626	-356.137	446.333	523.417
0.8	803.786	908.718	-388.414	-477.76	573.808	672.889
0.9	1011.4	1196.42	-504.241	-615.733	716.451	840.091
1	1241.43	1466.98	-633.898	-769.812	874.049	672.889

The observation from Tables I, II, III, IV, V, VI, VII, VIII show that the suggested approach has a high level of agreement with HAM this study shows that LHAM is a good mathematical tool for tackling fractional Laplace Homotopy Analysis Method problems. The above tables clearly shows the changes in the Non Chaotic and chaotic situations in Lorenz system of equation with the fractional parameter $\zeta = 1$, 0.94, 0.96, 0.98.

6. Conclusion

In this present work continuous solution for fractional

Lorenz system of equation is obtained by Fractional Laplace Homotopy Analysis Method Via Modified Riemann-Liouville Integral. This solutions are exactly coincide with the solution of A.K.Alomari et al [1].

References

[1] A.K. Alomari A new analytic solution for fractional chaotic dynamical systems using the differential transform method (2011).

[2] A.K.Alomari,M.S.M.Noorani,R.Nazar,C.P.Li,Homotopy analysis method for solving fractional Lorenz system,commun Nonlinear Sci Number Simulat(2009).

[3] Bagyalakshmi.M,Sai Sundara Krishnan,G.Tarig Projected Differential Transform Method to solve Fractional Non linear Partial Differential Equations,Boletim da Sociedad Paranaense de Mathematica,(2018).

[4] B.K.Dutta and L.K.Arora,Fractional Elzki's Transform,PhD thesis,North Regional Institute of Science and Technology,Nirijuli-791109,Arunachal Pradesh, (2014).

[5] Fernando S.Silva,Davidson M.Moreira and Marcclo A.Moret,Conformable Laplace Transform of Fractional Differential Equation,(2018).

[6] Guy Jumarie Laplace's transform of fractional order via the Mittag Leffler function and modified RiemannLiouville derivative,Applied Mathematics Letters(2009).

[7] Guy Jumarie,Table of some basic fractional calculus formulae derived from a modified Riemann-Liouville derivative for non-differentiable functions,Applied Mathematics Letters,(2008).

[8] G.Hariharan,A.Homotopy analysis method for the nonlinear partial differential equation arising in engineering,International journal for Computational Methods in Engineering Science and mechanics,(2017).
[9] Gustavo D.Medina,Nelson R. Ojeda,Jose H.Pereira and Lui G. Romero,Fractional Laplace Transform and Fractional Calculus,International Mathematical Forum,(2008).
[10] Lorenz, E.: Deterministic nonperiodic flow1. J. Atmosph. Sci. 20, 130 (1963)
[11] M.Mossa Al-sawlha,M.S.M.Noorani On Solving the Lorenz System by Differential Transformation Method, (2008).
[12] M.S.H.Chowdhury, I.Hahim, S.Momani The multistage homotopy-perturbation method- A powerful scheme for handling the Lorenz system,(2007).
[13] Song Liang,Ranchao Wu,Liping Chen,Laplace Transform of Fractional Order Differential Equation,Electronic Of Differential Equation,(2015).
[14] Zaid M.Odibat, Cyrille Bertelle, M.A.Aziz-Alaoui, Gerard H.E.Duchamp A multi-step differential transform method and application to non-chaotic or chaotic systems (2009).