A Comprehensive Analysis of Snowball Structured Products: Characteristics, Return Distribution, and Risk Metrics

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Abstract. This study discusses in detail the characteristics, analytical methodology, return distribution and risk metrics of “snowball structure” products. As a complex OTC option product that has gained popularity in the Chinese market in recent years, the core of the Snowball Structured product is that investors sell put options with triggering conditions to brokers in order to realize a “mild bullishness” within a certain range of asset price fluctuations and at the same time obtain downside protection. The paper analyzes the final return distribution of the snowball structure through Geometric Brownian Motion (GBM) and risk-neutral assumptions, pointing out that it essentially provides customers with a short-term limited gain, unlimited loss put option short. In terms of research methodology, this paper then uses the Finite Difference Method (FDM) to calculate the Greek letters of options (e.g., Delta, Gamma, Theta, Vega, Rho) and simulates the price movements of the Shanghai and Shenzhen 300 indices through Monte Carlo simulation methods in order to assess the potential returns and risks of the Snowball product. The results show that the average survival time of Snowball products[1] is about six months, the average probability of elimination is about 73.7%, and the average return is about 5.7%. Also, the paper assessed the potential loss of the products at different confidence levels through VaR analysis.

Keywords: Snowball structural products; Finite difference method (FDM); Monte Carlo simulation; VaR analysis

1 Introduction

The Snowball Structures product[1] is essentially an exotic option[2], and it has been one of the hottest products among OTC options[3] sold in the country in recent years. At their core, investors sell put options[4] with trigger conditions to brokerage firms. This type of option structure[5] is to provide a certain degree of downside protection while expressing a “mildly bullish”[6] view of the market, as long as the price of the underlying asset is always fluctuating within a certain range, the longer the customer's holding

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period, the higher the profit. This process is like rolling a snowball, as long as there are no major potholes in the road, the snowball will roll bigger and bigger, hence the name “snowball”. Nowadays, snowball structure products have been widely adopted by brokerage management and bank wealth management. In the brokerage management products, most of the standard snowball structure. For example, “Guotai Junan Xinying Snowball Profit Enhancement No. 3”.

Under the assumption of GBM+[7] risk-neutrality, we would like to look at the final return[8] distribution of the snowball itself. We find that from an option structure perspective, the snowball is equivalent to a put option short for the client, with limited gains and unlimited losses. A look at the Snowball return distribution shows that Snowball is indeed a finite gain, infinite loss distribution that carries with it a great deal of tail risk. Under the assumption of a risk-neutral + GBM track[9], the maximum loss can reach over 40%.

2 Research Methods

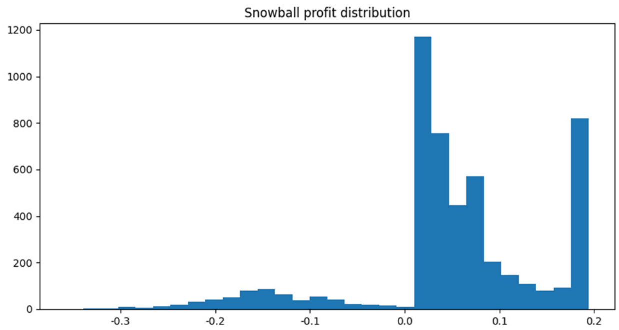

Fig. 1. Snowball profit distribution.

In the Fig. 1 Subsequently, we analyze the value of snowballs in different contexts. It can be seen that:

- Under GBM, the overall average survival time of snowballs is around six months.
- The average probability of a snowball knockout is about 73.7%. On average, it is able to earn 5.7%. If the snowball is certain to be knocked out in the future, its survival time is about three months.
- The probability that Snowball will get the full coupon is 13.7%.
- Snowball's probability of making a profit is more than 85%. However, it also has a 12.6% probability of loss, which is not small overall, and its maximum loss can be -41.4%. In this case, Snowball is worth -14.4%.

Table 1. Snowball Value Chart.

	Snowball structure as a whole	knocked out	Knocked in but not knocked out	neither knocked in nor knocked out
Value	0.050996	0.057661	0.14375	0.194089
Maximum loss	-0.41413	0.016625	0.41413	0.194089
Existing trading days	120.6997	73.72805	252	252
Probability		0.736517	0.12613	0.137353

Form the Table 1 Subsequently, we utilize the finite difference method[10] to compute the Greek letters of the snowball: delta, gamma, theta, vega, rho. we do not use the Monte Carlo method, mainly because Monte Carlo[11] is the result of an orbital simulation, which has inherently higher variance and less stability in pricing. For example, delta, the Monte Carlo calculation has a lot of burrs:

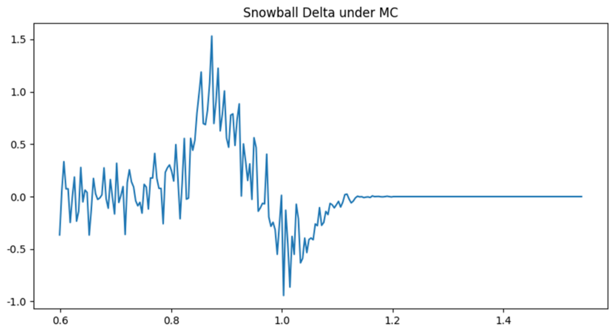

Fig. 2. Snowball Dela under MC.

In the Fig. 2. Therefore all subsequent Greek letters are computed using finite differences. The specific analysis is as follows.

The delta is used to measure the effect of a change in the underlying price on the option price, i. e. There are:

$$delta = \frac{\partial f}{\partial s} \tag{1}$$

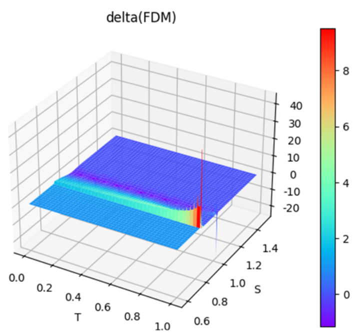

Fig. 3. delta(FDM).

In the Fig. 3. We find that the snowball will knock out when the underlying rises rapidly, when the delta is close to 0. And the snowball delta will spike when it approaches near the knock-in price. This is because a knock-in of the underlying (if it doesn't rally to the knock-out line) will turn the snowball structure into a short put option.

The gamma is a measure of the impact of the underlying market volatility on the option price, and also measures how difficult it is for the snowball seller to hedge the delta, i.e., there is:

$$gamma = \frac{\partial^2 f}{\partial S^2} \tag{2}$$

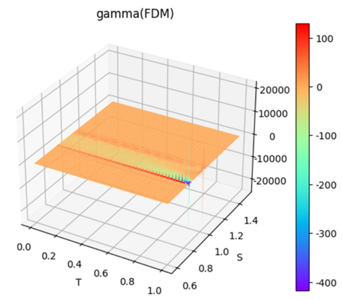

Fig. 4. gamma(FDM).

In the Fig. 4. We find that, again, near the knock-in price, gamma changes more abruptly, and that the extent of this abrupt change increases more and more dramatically with time. This implies that the second-order risk factor for a product like Snowball is difficult to estimate near the knock-in price, and in general, sellers of Snowball tend to hedge only the first-order risk (i.e., delta).

Theta is used to measure the effect of a change in expiration time on the option price, i.e., there is:

$$theta = \frac{\partial f}{\partial t} \tag{3}$$

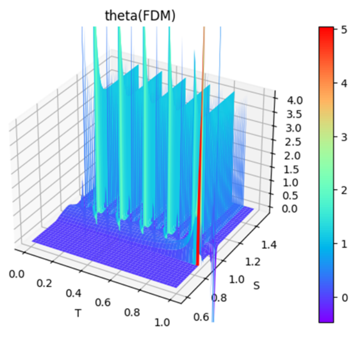

Fig. 5. theta(FDM).

Form the Fig. 5. Theta values are relatively complex: if the price is high, Snowball = a series of zero-coupon bonds with conditions, and zero-coupon bonds have a negative theta. If the price is low, snowball = short European put option, at which point theta should increase and then decrease as the underlying price changes.

Principal and financial income

The floating returns of this financial product are linked to the level of the Shanghai and Shenzhen 300 Index. The Shanghai and Shenzhen 300 Index referred to in the article refers to its market trading prices. The financial return rate is the net return rate after deducting relevant expenses.

Determination of yield

The yield is determined based on the following agreement (excluding taxes and fees related to derivative transactions):

During the observation period, if the fixed price of the Shanghai and Shenzhen 300 Index remains within the knock in and knock out boundaries, the return rate is 20% (annualized). In this case, the formula for calculating the financial return per unit of income is: Financial return per unit of income=Unit share of calculated income x Financial rate of return x Financial product term ÷ 365 (the calculation result is accurate to two decimal places).

During the observation period, if the fixed price of the Shanghai and Shenzhen 300 Index is higher than or equal to the knock out boundary, the financial rate of return is: Financial rate of return (annualized)=20% × (t/T). If at the beginning of the observation period, the time period when the index exceeds the knock out boundary is observed, and during the observation period, the fixed price of the CSI 300 index first falls below the knock in boundary and then rises above the knock out boundary, the same situation applies. The financial rate of return will be rounded to two decimal places. In this case, the formula for calculating the financial return per unit of income is: Financial return per unit of income=Unit share of calculated income x Financial rate of return x Financial product term ÷ 365 (rounded to two decimal places).

During the observation period, if the fixed price of the Shanghai and Shenzhen 300 Index is lower than or equal to the knock in boundary and the final price is still lower than the knock in boundary, the investment return rate is 0 (annualized): the calculation of financial return per unit of income=the unit share of calculated income x financial return rate x wealth management product term ÷ 365 (the result is accurate to two decimal places and rounded to the nearest two decimal places).

Based on the factors mentioned in the previous text, this article will conduct a simulation analysis:

The Monte Carlo method is a numerical method for obtaining derivative prices by simulating the random movement path of underlying asset prices. The advantage of Monte Carlo method lies in its ability to price various complex derivative products and its straightforward approach. However, when using this method to simulate the underlying asset price path in this article, for any path, we need to determine whether the index price exceeds the knock in or knock out limit. Therefore, we need to compare the daily underlying asset prices with the knock in limit, and compare the underlying asset prices on the last trading day of each month with the knock out limit. When conducting Monte Carlo simulations, it is necessary to save daily underlying asset prices, compare their differences with knock in and knock out limits, and calculate returns. This article simulates the price movement trend of the Shanghai and Shenzhen 300 Index, obtains the closing price of the product, and compares it with the barrier price. This article assumes that the Shanghai and Shenzhen 300 prices follow geometric Brownian motion, namely:

$$dS = \hat{\mu} S dt + \sigma S dZ \tag{4}$$

Discretize the above equation to obtain:

$$\Delta S = \hat{\mu} S \Delta t + \sigma S \varepsilon \sqrt{\Delta t} \tag{5}$$

VaR Risk Analysis[12]

VaR refers to the value at risk, which indicates the maximum value of the possible loss of an investment or asset portfolio at a given time and confidence interval, and is firstly defined as follows: W_0 indicates the initial value of the portfolio at the beginning of the period; W indicates the expected return of the portfolio at the end of the period; $\hat{r}$ denotes the minimum rate of return; μ denotes the expected rate of return; $1-\alpha$ is a given confidence interval; $VaR_{1-\alpha}$ denotes the maximum value of the loss within the confidence interval, and the maximum loss is the difference between the expected value of the portfolio at the moment T and the value at the end of the period (relative VaR):

$$VaR = E(\widetilde{W}) - \widehat{W} = -W_0(\hat{r} - \mu) = \alpha \sigma W_0 \sqrt{\Delta t} \tag{6}$$

The probability that the portfolio loss exceeds $VaR_{1-\alpha}$ is only α, which can be obtained as

$$P(E(\widetilde{W}) < \widehat{W}) = \alpha \tag{7}$$

The calculation of VaR relies on the form of distribution of asset returns assuming that they follow a normal distribution, with:

$$VaR_{1-\alpha} = \alpha \sigma W_0 \sqrt{\Delta T} \tag{8}$$

where σ is the standard deviation of the asset portfolio distribution. Based on the return distribution graph derived from Monte Carlo simulation, the VAR is calculated for the product under different confidence intervals.

3 Result & Discussion

- The image shows multiple simulated stock price paths that are based on the geometric Brownian motion assumption[13]. It can be seen that most of the price paths fluctuate around the initial price during the simulation.
- The Knock-in Level is shown as a red dashed line. Most of the paths do not reach the Knock-in Level during the entire period of the simulation, which means that the price is above the Knock-in Line most of the time.

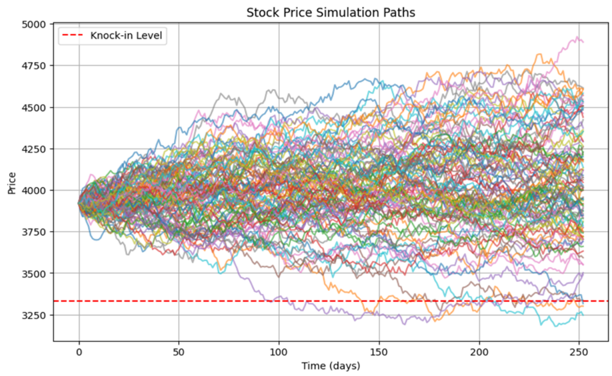

Fig. 6. Stock Price Simulation Paths.

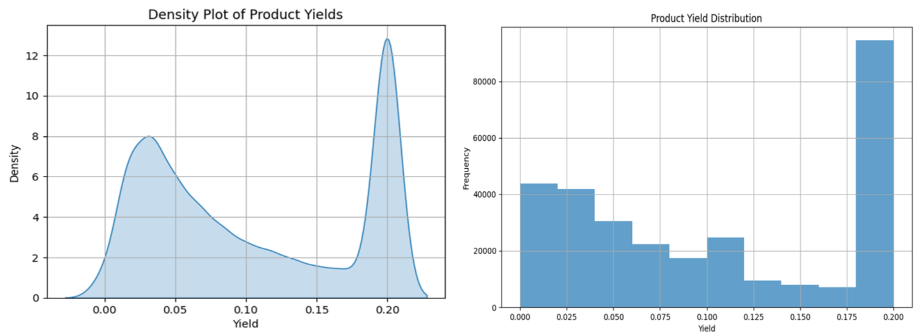

Fig. 7. Density Plot of Product Yields and Product Yield Distribution.

- In the Fig.6, 7. The density plot shows the probability distribution of product returns. Most of the returns are concentrated in the positive return region, which implies that the product returns are generally positive.[14]
- The peak in the plot is at around 0.2, which suggests that the returns are close to the coupon in most scenarios of the simulation.
- The Product Yield Distribution The chart shows the distribution of product returns calculated based on different simulation paths. The returns are calculated based on the stock price path in relation to the knock-out and knock-in points.
- According to the chart, most of the simulated returns are concentrated in a small range and some of the paths have lower returns due to non-triggered knock-out and knock-in conditions.

Based on the given simulations, the following trends and interpretations can be observed:

1. Probability of the index going up:

- The probability of the index going up is 62.63%. This means that more than half of the simulated paths ended the period with the index price higher than the initial price. This indicates an overall upward trend in the market.

2. Product VaR:

- The VaR (Value at Risk)[15] of the product at 80% confidence level is 0.2. This means that in the most unfavorable 20% of the scenarios, the product's return could be less than 20%.

3. stock index return VaR:

- With a confidence level of 80%, the VaR of stock index returns is 0.1007. This means that in the most unfavorable 20% of cases, the return on the stock index could be less than 10.07%. The lower VaR of the stock index compared to the product VaR indicates that the stock index is relatively less volatile.

4. Snowball Product Price:

- The calculated Snowball Product Price is 0.1053. This price reflects the average return of the Snowball Product in a simulated environment.

5. Knock-out probability:

- The Knock Out Probability of 70.12% indicates that most of the paths in the simulation satisfy the Knock Out condition, i.e., the index price exceeds the Knock Out line at the end of the period. This implies that the product is more likely to terminate early and earn higher returns when the market performance is strong.

6. Knock-in probability:

- The knock-in probability is 2.16%, which indicates that very few paths triggered the knock-in condition during the simulation period, suggesting that the market did not experience significant declines for most of the period.

7. Survival Probability:

- The Survival Probability of 27.72% indicates that a significant portion of paths did not meet the knock-out or knock-in conditions and the product persisted throughout the simulation period. The returns for these paths are typically determined by the final index price.

These results show that the market is generally in an uptrend and that the Snowball product performs well in most cases with a high probability of knock-out and a low probability of knock-in, indicating a more robust product design that provides stable returns in most simulated scenarios.

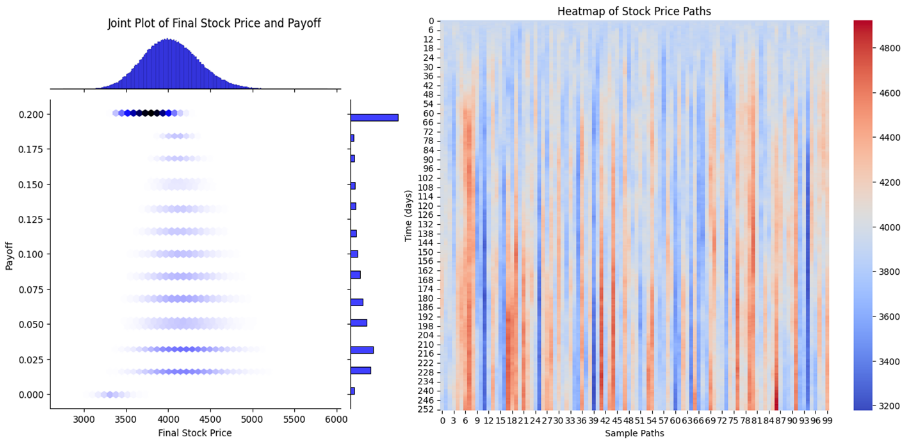

Fig. 8. Joint Plot of Final Stock Price and Payoff and Heatmap of Stock Price Paths.

In the Fig.8 we can know: The joint distribution graph shows the relationship between final stock prices and product returns. The figure shows that as the final stock price increases, the distribution of product returns shifts upward, exhibiting a positive correlation. The areas of higher density show that the product returns are concentrated when the stock price is in a specific range (close to the initial price).

- In the Heatmap of Stock Price Paths. The heat map shows the price changes of the first 100 simulated paths. The colors transition from blue (low prices) to red (high prices), clearly showing the volatility of the stock index prices.
- Most of the paths show a larger orange area, indicating that in most cases the stock index price tends to go up.

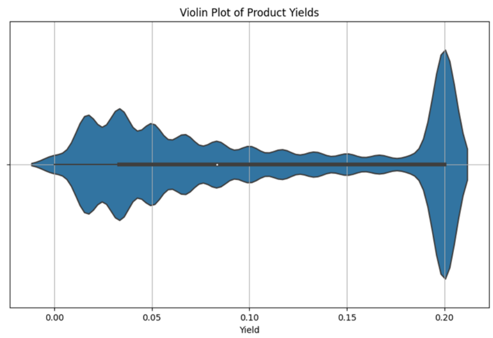

Fig. 9. Violin Plot of Product Yields

- In the Fig. 9. TREND: The violin plot further illustrates the probability density distribution of product returns. The plot shows that most of the returns are concentrated in the upper-middle region, which is consistent with the observation of the box-and-line plot. However, the violin plot also reveals the symmetry and multi-peakedness of the return distribution, showing that there is some distributional probability in the higher and lower return ranges. Overall, the distribution of returns is more compact, suggesting that most of the returns are concentrated in a smaller range.

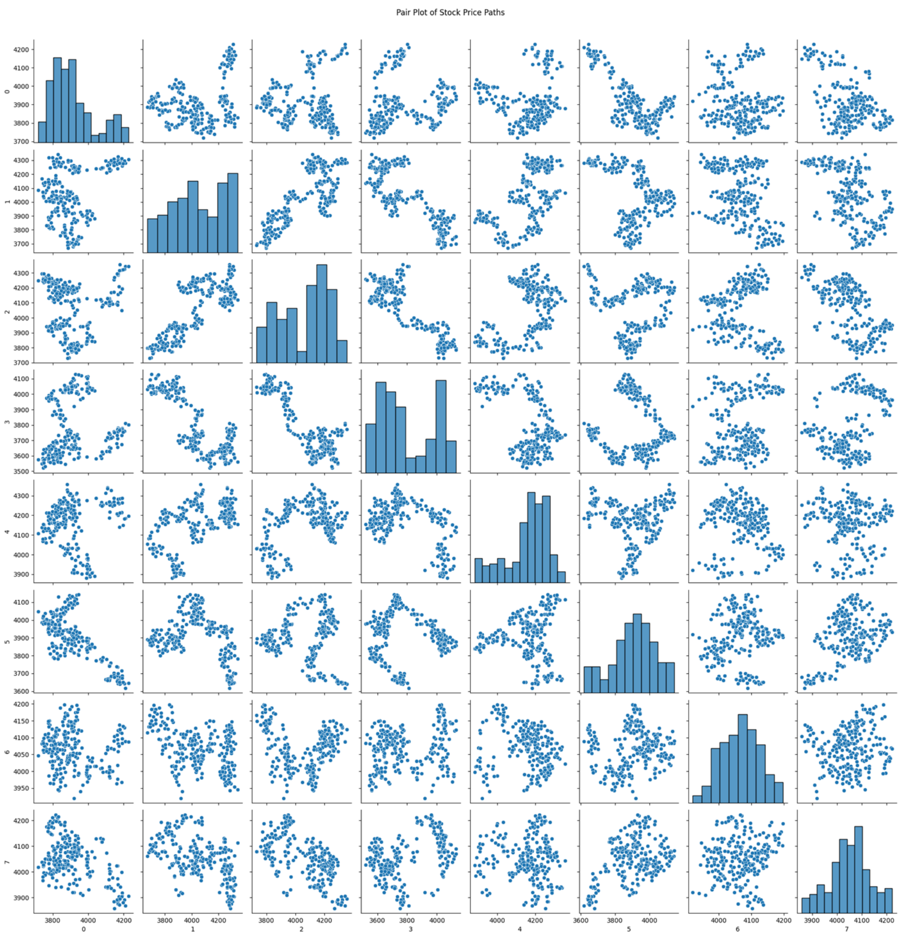

Fig. 10. Pair Plot of Stock Price Paths.

In the Fig. 10. Trend: paired relationship plots show the two-by-two relationship between multiple variables in the stock price path. The scatterplot shows that there is a strong positive correlation between stock prices at different time nodes, which implies that the trend of stock prices is consistent throughout the simulation. The price distribution plots at each time node also demonstrate the trend of price changes over time, showing a gradual increase in stock prices over time, while the range of price fluctuations at each time node gradually widens, reflecting a gradual increase in uncertainty over time.

Table 2. VaR analysis indicates.

confidence interval	95%	75%	50%	25%	20%	15%	10%	5%	1%
Product yield VaR	1.67%	3.33%	8.33%	20%	20%	20%	20%	20%	20%

Form the Table 2. VaR analysis indicates that the overall return of the product is positive. Within a 25% confidence interval, the highest return rate can be achieved at 20%, while within a 99% confidence interval, the return remains positive. The probability of zero returns is low, indicating that the risk of the product is relatively low.

4 Conclusions

This article designs a snowball option product linked to the Shanghai and Shenzhen 300 Index. The revenue of this product consists of the lowest, highest, and intermediate rates of return. During the holding period of the product, if the closing price of the underlying asset is higher than the strike price, the product will end early and investors can receive a return of 20% * (t/T); If the closing price of the underlying asset does not exceed or fall below the strike price throughout the entire term, investors will receive a maximum return of 20%; If the price of the underlying asset is lower than the strike in price at maturity, investors will have no returns. VaR analysis shows that the product has low risk and high principal safety. Investors have a 95% chance of earning a return of over 1.67% and a 25% chance of earning a high return of 20%. For investors who believe that the price of the underlying asset will be relatively stable and less volatile in the future, this product has great appeal.

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