

A Study on Dual-Channel Green Supply Chain Decision Making Considering Free-riding Behavior under Government Subsidies

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Abstract. Amidst the context of government subsidies provided to manufacturers, this paper explores the influence of consumer free-riding behavior in the market on the pricing strategies of dual-channel green supply chains. By conducting comparative analyses, the study discloses the effects of consumer free-riding presence or absence on pricing strategies and profit margins within dual-channel green supply chains. This analysis demonstrates how consumer free-riding behavior, whether present or absent, influences supply chain decision-making processes and profitability. The study's findings indicate that, firstly, the profits of manufacturers tend to follow an inverted U-shaped curve rather than showing continuous growth with increasing consumer free-riding coefficients. This suggests that beyond a certain threshold of consumer free-riding, there is a potential decline in manufacturers' profits. Secondly, consumer free-riding behavior does not always favor manufacturers; in some instances, it may actually be detrimental to their interests. Lastly, the phenomenon of consumer "free-riding" could negatively impact environmental conservation by diminishing manufacturers' incentives for green production and lowering retailers' commitment to promotional activities. These insights are critical for comprehending the influence of consumer behavior on supply chain decisions and for formulating effective supply chain management tactics.

Keywords: Government subsidies, Two-channel Supply chain, Free-riding Behavior, Stackelberg game

1 Introduction

With global environmental issues escalating, the importance of protecting the environment and promoting green development cannot be overlooked. Governments worldwide have enacted policies to encourage green development and assist consumers in transitioning to more environmentally friendly practices. For instance, the Chinese government's Implementation Plan for the Promotion of Green Consumption explicitly aims for widespread adoption of green consumption patterns by 2025, necessitating that enterprises fully embrace green manufacturing. Additionally, the "2022 China E-tailing Market Development Report" from the Department of E-commerce and Informatization

under the Ministry of Commerce of China highlighted that national online retail sales will reach 13.79 trillion yuan in 2022, marking a 4% increase from 2021, while B2C online retail sales will see a 5.6% year-on-year rise. The "new crown epidemic" has thus not hindered the growth of China's Internet economy; on the contrary, it has contributed to the further expansion of "rural e-commerce," "live e-commerce," and other digital sectors. Consequently, in an effort to broaden market reach, many traditional businesses are leveraging online platforms to establish dual-channel sales models that integrate both online and offline retail. However, the emergence of online retail channels inherently impacts traditional retail channels, leading to competition between them and incidents of consumer free-riding behavior. This phenomenon of "free-riding behavior" results in reduced overall profits for the supply chain and negatively impacts carbon emissions^[1,2]. The research focuses on both government subsidies and consumer free-riding behavior. Additionally, it examines how consumer free-riding behavior affects the decision-making of supply chain firms when manufacturers receive government subsidies.

Numerous researchers have investigated the effects of government subsidies on supply chains^[3]. For instance, Abhijit Barman performed a comparative study on optimal pricing strategies in scenarios with and without government subsidies. He concluded that these subsidies have the potential to considerably reduce the costs linked to green manufacturing.^[4] Kelei Xue similarly discovered that government subsidies have a positive effect on the development of environmentally friendly products. Furthermore, supply chain entities, particularly those managed by manufacturers, generally experience increased profitability.^[5] Jian Xue examined the effects of government subsidies on the pricing, demand, and profitability of energy-efficient products within supply chains.^[6] Mao Lu investigated how government subsidies affect manufacturers by constructing a differential game model. At the same time, Shan Chen and colleagues also developed a differential game model to analyze the influence of government subsidies on manufacturers.^[7] They created a differential game model to investigate the optimal production levels and appropriate government subsidy rates. Their findings revealed that under the government's utility guidelines, the profit of a centralized supply chain is lower compared to that of a decentralized supply chain.^[8] Peng He investigated the pricing strategies and the optimal structure of channels in a two-channel closed-loop supply chain. His research extended into determining the ideal subsidy levels across different channel configurations. Qingfeng Meng and colleagues integrated consumer preferences into their research^[9]. They examined cases with and without government subsidies and concluded that these financial aids can reduce green product prices. Nonetheless, there remains a scarcity of research on decision-making in dual-channel green supply chains that consider government subsidies and account for free-riding behavior.^[10] Discovered that a lower degree of free-riding can boost demand for the online channel, but it negatively impacts the free-rider's profit.^[11] Taiguang GAO developed a competitive framework for the Nash game between supply chains. He investigated how risk aversion and free-riding behavior affect supply chain decision-making. Xujin Pu^[2] studied the impact of free-riding behavior on promotional efforts. They discovered that with a higher degree of free-riding behavior, the promotional efforts by offline brick-and-mortar stores diminish, which leads to a reduction in the overall profit

of the supply chain.^[12] Examine the effects of service free-riding on decision-making within a dual-channel supply chain. They discover that a moderate level of service free-riding can be advantageous for decision-makers and can also promote the greening of the supply chain.^[1] They argue that while manufacturers may gain from consumer free-riding behavior, it negatively impacts the overall carbon emissions within the supply chain. Consequently, this study seeks to improve both theoretical and practical aspects of green supply chain management by analyzing decision-making processes in dual-channel green supply chains that receive government subsidies, with a special focus on the context of free-riding behavior. The research will utilize a combination of mathematical modeling and optimization techniques to identify strategies for maximizing corporate profits and environmental benefits, considering factors such as government subsidies, consumer preferences, and free-riding behavior.

To accomplish this objective, this study will develop a Stackelberg game model centered on two different scenarios: one with consumers exhibiting free-riding behaviors and one without. The model involves a manufacturer operating an online channel and a traditional retailer. The analysis will focus on optimal pricing, product greenness, and the profitability of supply chain members under both conditions. By thoroughly examining this topic, the research aims to offer decision support to businesses crafting green supply chain strategies and to facilitate sustainable development and environmental conservation. The subsequent sections will outline the methods used and present the research model pertinent to this study. Ultimately, the study will confirm the model's validity and practical application feasibility through numerical simulations.

2 Problem Description and Underlying Assumptions

This study develops a model for a two-channel green supply chain, featuring a manufacturer with an online sales channel and a retailer limited to offline transactions, as shown in Fig. 1. In this model, the manufacturer operates as a Stackelberg leader, while the retailer acts as a follower. In the offline retail channel, the manufacturer provides green products to the retailer at a wholesale price ω , and the retailer in turn offers these products to consumers at a retail price p_r , ensuring that $p_r \geq \omega$. On the other hand, in the direct sales channel, the manufacturer sells green products directly to consumers at a price p_m .

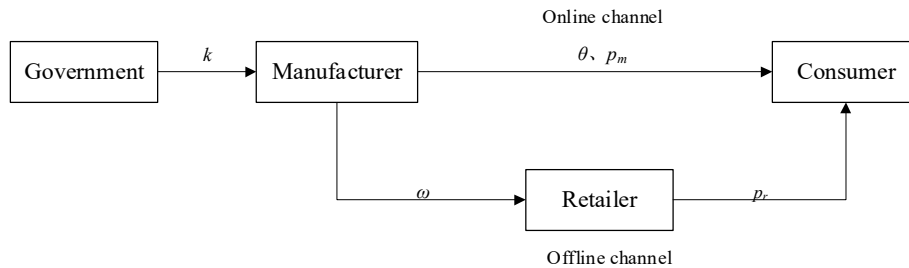


Fig. 1. Dual-channel green supply chain structure

Hypothesis 1: Referring to Ghosh and Shah (2012)^[13] hypothesis, The demand functions for manufacturers and retailers, which pertain to the retail prices of green products p_r and p_m and the environmental friendliness of a single unit product (θ), are modeled as linear functions. It's assumed that the demand for both the traditional and direct sales channels decreases with higher retail prices and increases with greater product greenness. Consequently, the demand functions for the online and offline channels are formulated as follows:

$$D_r = a\alpha + \beta\theta + ep_m - p_r \quad (1)$$

$$D_m = a(1 - \alpha) + \beta\theta - p_m + ep_r \quad (2)$$

where a represents the potential market demand for green products, and α signifies the market share proportion secured by retailers' offline channels. The parameter e ($0 < e \leq 1$) indicates the cross-price coefficient between the channels. The coefficient β ($\beta > 0$) quantifies the effectiveness of the greenness of a single unit of green product in expanding both online and offline channel demand.

Hypothesis 2: The unit production cost for the manufacturer to create a green product is denoted by c . For generality, let's express the manufacturing cost of the green product as $c(\theta) = \frac{\mu}{2}\theta^2$ ^[14], which is modeled as a quadratic function concerning the greenness of a single unit product. Here, μ represents the coefficient associated with the green manufacturing cost.

Hypothesis 3: To incentivize green production and research & development, the government provides subsidies to manufacturers based on the environmental friendliness of each unit of product. The subsidy amount per green product unit is represented by the coefficient k .

Hypothesis 4: The retailer engages in promotional activities, whereby the cost associated with these efforts is expressed by the function $h(s) = \frac{\eta}{2}s^2$ ^[15], where η represents the effort cost coefficient. The term τs indicates the amount of demand increase attributable to the free-riding behavior from the manufacturer's direct channel, whereas $(1 - \tau)s$ signifies the demand increase in the traditional channel.

Hypothesis 5: In the market, the product is exclusively produced by the manufacturer. The retailer is limited to purchasing the product wholesale from the manufacturer and is not permitted to acquire it for resale from the direct sales channel.

The article employs the superscript c to indicate centralized decision-making and the superscript d to signify decentralized decision-making. Similarly, cf and df represent centralized and decentralized decision-making under free-riding behavior. The subscript sc is used to denote the entire supply chain, while the superscript $*$ signifies the optimal decision variables. Table 1 offers a comprehensive explanation of the pertinent parameters and their meanings as referenced in this study.

Table 1. Relevant parameters and meanings.

model parameter	Parameter Meaning
D_m	Manufacturer online channel demand
D_r	Retailer offline channel demand
a	Potential market demand for the product

α	Proportion of market share captured by offline retail channels
k	Government Green Product Subsidies
c	Production cost per unit of product
e	Cross-price elasticity coefficient
β	Consumer Green Product Preferences
η	Retailer promotional effort cost factor
τ	Consumer free-rider factor
μ	Green manufacturing cost factor
Decision variables	Parameter Meaning
p_m	Online channel retail price
p_r	Offline channel retail price
ω	Wholesale price per unit of product
s	Level of promotional effort
θ	Greenness per unit of product

3 Model Construction and Solution

3.1 No Free-riding Situations

Centralized Decision-making.

In the absence of free-riding behavior, the retailer undertakes promotional activities, thereby attracting a market size of s , without experiencing any instances of free-rider behavior. The green supply chain includes a manufacturer focused on green products and a traditional retailer. This supply chain functions with a vertically integrated, centralized decision-making approach. The manufacturer, as the primary decision-maker, is responsible for setting prices for both online and offline sales, determining the greenness level of the products, and deciding on the extent of promotional efforts. The following sections outline the aggregated demand and profit functions for this supply chain setup.

$$D_t = a + 2\beta\theta + (e - 1)p_m + (e - 1)p_r + s \quad (3)$$

$$\pi_{sc}^c = (p_r - c)(a\alpha + \beta\theta + ep_m - p_r + s) + (p_m - c)(a(1 - \alpha) + \beta\theta + ep_r - p_m) - \frac{\theta^2\mu}{2} - \frac{s^2\eta}{2} + \theta k \quad (4)$$

In the centralized decision-making scenario without free-riding, the supply chain optimal online channel retail price p_m^{c*} and offline retail price p_r^{c*} and greenness per unit of product θ^{c*} and promotional effort level s^{c*} respectively:

$$p_m^{c*} = \frac{\mu(c(2(e^2-1)\eta+e+1)-a(\alpha+2\alpha(e-1)\eta+2\eta-1))+\beta(-\beta c+\beta\eta(-2a\alpha+a+4c(e+1))-2(e+1)\eta k+k)}{4(e^2-1)\eta\mu+\beta^2(4(e+1)\eta-1)+2\mu} \quad (5)$$

$$p_r^{c*} = \frac{2\mu(c(e^2-1)\eta+c+a\eta((\alpha-1)e-\alpha))+\beta(a(2\alpha-1)\beta\eta+\beta c(4(e+1)\eta-1)-2(e+1)\eta k)}{4(e^2-1)\eta\mu+\beta^2(4(e+1)\eta-1)+2\mu} \quad (6)$$

$$\theta^{c*} = \frac{\beta(-a\alpha + a - 2(e+1)\eta(a+2c(e-1)) + c(e-1)) + k(4(e^2-1)\eta+2)}{4(e^2-1)\eta\mu + \beta^2(4(e+1)\eta-1) + 2\mu} \quad (7)$$

$$s^{c*} = \frac{a(2\alpha-1)\beta^2 - 2\mu(c(e^2-1) + a(\alpha - \alpha e + e)) - 2\beta(e+1)k}{4(e^2-1)\eta\mu + \beta^2(4(e+1)\eta-1) + 2\mu} \quad (8)$$

Decentralized Decision-making.

Within a dual-channel green supply chain operating under a decentralized decision-making framework, the manufacturer and a traditional retailer participate in a Stackelberg game. In this setup, the manufacturer assumes the role of the leader, while the retailer acts as the follower. First, the manufacturer establishes the wholesale price, sets the retail price for the online channel, and determines the greenness level of each product unit. Afterward, the retailer sets its own retail price and determines the scale of its promotional efforts. The profit functions for both the manufacturer and the retailer in this configuration are as follows:

$$\pi_m^d = (\omega - c)(a\alpha + \beta\theta + ep_m - p_r + s) + (p_m - c)(a(1 - \alpha) + \beta\theta + ep_r - p_m) - \frac{\theta^2\mu}{2} + \theta k \quad (9)$$

$$\pi_r^d = (p_r - \omega)(a\alpha + \beta\theta + ep_m - p_r + s) - \frac{s^2\eta}{2} \quad (10)$$

In the case of decentralized decision making without free-riding, the supply chain optimal online channel retail price p_m^{d*} and offline channel retail price p_r^{d*} Wholesale price ω^{d*} Greenness per unit of product θ^{d*} and promotional effort levels s^{d*} respectively:

$$p_m^{d*} = \frac{2(e-1)\mu^2(c(e-1)(8(e+1)\eta-2e-1) - a(\alpha+8\eta(\alpha(e-1)+1)-1)) + D_1 + D_2}{(e+1)(\beta^4(2(e+3)^2\eta-1) + 4\beta^2(e-1)\mu(4(e+3)\eta-1) + 4(e-1)^2(8\eta-1)\mu^2)} \quad (11)$$

$$p_r^{d*} = \frac{\beta\mu(4\beta c + 2\eta(\beta c(e-1)(e+1)(e+3)(e+5) + D_5) - \beta e((\alpha-1)a + 3ce + c) + 2(e-1)ek) + D_3 + D_4}{4\beta^2(e^2-1)\mu(4(e+3)\eta-1) + \beta^4(e+1)(2(e+3)^2\eta-1) + 4(e-1)^2(e+1)(8\eta-1)\mu^2} \quad (12)$$

$$\omega^{d*} = \frac{\beta^3(ek - c\beta - ce\beta + (3+e)(-4(1+e)k + 2c(1+e)(3+e)\beta + (2+e)a(-1+2\alpha)\beta)\eta) + E_1 + E_2}{(1+e)\beta^4(-1+2(3+e)^2\eta) + 4(-1+e^2)\beta^2(-1+4(3+e)\eta)\mu + 4(-1+e)^2(1+e)(-1+8\eta)\mu^2} \quad (13)$$

$$\theta^{d*} = \frac{4(-1+e)k - \beta(c(-1+e)(3+e) + a(2+(-1+e)\alpha) + \frac{F_1}{F_2})}{(3+e)\beta^2 + 4(-1+e)\mu} \quad (14)$$

$$s^{d*} = \frac{(\beta^2 + 2(-1+e)\mu)(2(-1+e)k\beta + a(-1+2\alpha)\beta^2 + 2c(-1+e)^2\mu + 2(-1+e)a\alpha\mu)}{\beta^4(-1+2(3+e)^2\eta) + 4(-1+e)\beta^2(-1+4(3+e)\eta)\mu + 4(-1+e)^2(-1+8\eta)\mu^2} \quad (15)$$

(of which $D_1, D_2, D_3, D_4, D_5, E_1, E_2, F_1, F_2$ see appendix)

Proposition 1 In the no consumer free-rider scenario, the level of retailers' promotional effort and greenness per unit of product are positively related to consumers' green product preferences, regardless of whether the decision is centralized or decentralized.

That is $\frac{\partial s^{d*}}{\partial \beta} > 0$, $\frac{\partial s^{c*}}{\partial \beta} > 0$, $\frac{\partial \theta^{d*}}{\partial \beta} > 0$, $\frac{\partial \theta^{c*}}{\partial \beta} > 0$. In terms of price, the retail price in the online channel is higher than the retail price in the offline channel. When it comes to pricing, the retail prices in both the online and offline channels, as well as the wholesale price, rise as consumer preference for green products increases. Namely $\frac{\partial p_m^{c*}}{\partial \beta} > 0$, $\frac{\partial p_m^{d*}}{\partial \beta} > 0$, $\frac{\partial p_r^{c*}}{\partial \beta} > 0$, $\frac{\partial p_r^{d*}}{\partial \beta} > 0$, $\frac{\partial \omega^{d*}}{\partial \beta} > 0$.

Proposition 1 indicates that an increase in consumer preference for green products boosts the motivation for retailers to conduct promotional activities and encourages manufacturers to intensify green product production. As consumer preference for green products grows, retail prices escalate, resulting in consumers facing higher costs for purchasing eco-friendly products. Consequently, promoting consumer awareness about green products benefits both environmental protection and the firms within the supply chain.

Proposition 2: In scenarios involving both centralized and decentralized decision-making, the retail price and the level of greenness per unit of product in both online and offline channels are positively influenced by government subsidies for green products. Additionally, these factors experience further increases as the coefficient of consumer preference for green products rises. That is $\frac{\partial p_m^{c*}}{\partial k} > 0$, $\frac{\partial p_m^{d*}}{\partial k} > 0$, $\frac{\partial p_r^{d*}}{\partial k} > 0$, $\frac{\partial p_r^{c*}}{\partial k} > 0$, $\frac{\partial \theta^{c*}}{\partial k} > 0$, $\frac{\partial \theta^{d*}}{\partial k} > 0$. And $\frac{\partial^2 p_m^{c*}}{\partial k \partial \beta} > 0$, $\frac{\partial^2 p_m^{d*}}{\partial k \partial \beta} > 0$, $\frac{\partial^2 p_r^{c*}}{\partial k \partial \beta} > 0$, $\frac{\partial^2 p_r^{d*}}{\partial k \partial \beta} > 0$, $\frac{\partial^2 \theta^{c*}}{\partial k \partial \beta} > 0$, $\frac{\partial^2 \theta^{d*}}{\partial k \partial \beta} > 0$ wholesale price in the decentralized decision-making scenario increases with the increase in government subsidies for green products, i.e. $\frac{\partial \omega^{d*}}{\partial k} > 0$.

Proposition 2 asserts that enhanced government subsidies for manufacturers producing eco-friendly products result in elevated retail prices and increased product greenness. Therefore, consumers will encounter higher costs for more sustainable products. Additionally, the manufacturer's wholesale price is expected to go up due to the anticipation of greater benefits, which spurs an increase resulting from these subsidies. As the inclination of consumers towards green products strengthens, government subsidies provided to manufacturers will also escalate. This indicates that with a growing segment of green-conscious consumers in the marketplace, the government will likely increase subsidies for companies focusing on environmentally friendly production.

3.2 Consideration of Situations of Free-riding Behavior

Centralized Decision-making.

In the scenario that involves free-riding behavior, the traditional retailer engages in promotional efforts indicated by (s), which attracts a new cohort of consumers to the market. Among these new consumers, some will participate in free-riding behavior. This group benefits from the promotional services offered by the traditional retail channel but ultimately makes their purchases through the direct channel. After enjoying the promotional benefits in the traditional retail outlets, they may switch to the direct sales channels. Consequently, when free-riding behavior occurs, the increased demand in the direct sales channel is represented by τs (τ denotes the proportion of such hitchhikers).

Hence, the increased demand in the offline channel is $(1 - \tau)s$. Under centralized decision-making, the manufacturer simultaneously determines the retail prices for both online and offline channels, the extent of promotional efforts, and the greenness level of each product unit. In this situation, the supply chain's demand function and profit function are outlined as follows:

$$D_t^f = a + 2\beta\theta + (e - 1)p_m + (e - 1)p_r + s \quad (16)$$

$$\pi_{sc}^f = (p_r - c)(a\alpha + \beta\theta + ep_m - p_r + s(1 - \tau)) + (p_m - c)(a(1 - \alpha) + \beta\theta + ep_r - p_m + s\tau) - \frac{\theta^2\mu}{2} + \theta k - \frac{\eta s^2}{2} \quad (17)$$

The supply chain optimal pricing, greenness per unit product, and promotional effort for the centralized decision-making scenario in the presence of free-riding behavior are, respectively:

$$p_m^{cf*} = \frac{D_6 + a\mu(\alpha(-2e\eta + 2\eta + \tau - 1) - 2\eta + (\tau - 1)^2) + \beta k((\tau - 1)(2\tau - 1) - 2(e + 1)\eta)}{\beta^2(4(e + 1)\eta - (1 - 2\tau)^2) + 4(e - 1)\mu(e\eta + \eta - \tau^2 + \tau) + 2\mu} \quad (18)$$

$$p_r^{cf*} = \frac{a((-1 + 2\alpha)\beta^2\eta + \mu(-2e\eta + 2(-1 + e)\alpha\eta + (-1 + \alpha)\tau + \tau^2)) + D_7}{2\mu + \beta^2(4(1 + e)\eta - (1 - 2\tau)^2) + 4(-1 + e)\mu(\eta + e\eta + \tau - \tau^2)} \quad (19)$$

$$s^{cf*} = -\frac{a(2\alpha - 1)\beta^2(2\tau - 1) + 2c(e^2 - 1)\mu + 2a\mu(-2\alpha\tau + \alpha + ae(2\tau - 1) + e(-\tau) + e + \tau) + 2\beta(e + 1)k}{2\mu(2(e^2 - 1)\eta - 2(e - 1)\tau^2 + 2(e - 1)\tau + 1) + \beta^2(4(e + 1)\eta - (1 - 2\tau)^2)} \quad (20)$$

$$\theta^{cf*} = \frac{\beta(-c(e - 1)(4(e + 1)\eta - (1 - 2\tau)^2) - a(-2\alpha\tau + \alpha + 2(e + 1)\eta - 2\tau^2 + 3\tau - 1)) + D_8}{2\mu(2(e^2 - 1)\eta - 2(e - 1)\tau^2 + 2(e - 1)\tau + 1) + \beta^2(4(e + 1)\eta - (1 - 2\tau)^2)} \quad (21)$$

(of which D_6 D_7 D_8 see appendix)

Decentralized Decision-making.

In a dual-channel green supply chain scenario with decentralized decision-making, the manufacturer and the traditional retailer engage in a Stackelberg game, with the manufacturer as the leader and the retailer as the follower. First, the manufacturer sets the wholesale price, the retail price for the online channel, and the greenness level per product unit. Subsequently, the retailer determines its own retail price and the scale of its promotional efforts. At this point, the demand and profit functions for both the manufacturer and the retailer are established as follows:

$$D_m^{cf} = a(1 - \alpha) + \beta\theta + s\tau - p_m + ep_r \quad (22)$$

$$D_r^{cf} = a\alpha + \beta\theta + s(1 - \tau) + ep_m - p_r \quad (23)$$

$$\pi_m^{cf} = (\omega - c)(a\alpha + \beta\theta + ep_m - p_r + s(1 - \tau)) + (p_m - c)(a(1 - \alpha) + \beta\theta + ep_r - p_m + s\tau) - \frac{\theta^2\mu}{2} + \theta k \quad (24)$$

$$\pi_r^{cf} = (p_r - \omega)(a\alpha + \beta\theta + ep_m - p_r + s(1 - \tau)) - \frac{\eta s^2}{2} \quad (25)$$

In the situation of decentralized decision-making that includes free-riding behavior, the manufacturer and the retailer make the following optimal decisions, respectively:

$$S^{df*} = -\frac{(2(-1+e)k\beta + a(-1+2\alpha)\beta^2 + 2c(-1+e)^2\mu + 2(-1+e)a\alpha\mu)E_4}{\beta^4(2(3+e)^2\eta - (1-2\tau)^2) + 4(-1+e)^2\mu^2(8\eta - (-1+\tau)^2) + E_3} \quad (26)$$

$$p_m^{df*} = \frac{D_{10} - D_{11} + 2a(e-1)\mu^2(\alpha(-8(e-1)\eta + \tau - 1) - 8\eta + (\tau-1)^2)}{(e+1)(D_9 + \beta^4(2(e+3)^2\eta - (1-2\tau)^2))} \quad (27)$$

$$p_r^{df*} = \frac{e^2(2a((2\alpha-1)\beta^4\eta + (13\alpha-6)\beta^2\eta\mu + \mu^2(\alpha(4\eta + \tau - 1) - 8\eta + (\tau-1)^2)) + D_{14}) + D_{12} + D_{13}}{(e+1)(D_9 + \beta^4(2(e+3)^2\eta - (1-2\tau)^2))} \quad (28)$$

$$\omega^{df*} = \frac{e^2(a((2\alpha-1)\beta^4\eta + 4(3\alpha-2)\beta^2\eta\mu + 2\mu^2(\alpha(8\eta + \tau - 1) - 8\eta + (\tau-1)^2)) + D_{17}) + D_{15} + D_{16}}{(e+1)(D_9 + \beta^4(2(e+3)^2\eta - (1-2\tau)^2))} \quad (29)$$

$$\theta^{df*} = \frac{4(-1+e)k - \beta(c(-3+2e+e^2) + a(2+(-1+e)\alpha) - \frac{F_3}{F_4})}{(3+e)\beta^2 + 4(-1+e)\mu} \quad (30)$$

(where $D_9, D_{11}, D_{17}, E_3, E_4$ etc. see appendix)

Proposition 3: In situations where centralized decision-making is implemented, the retail price for the online channel positively correlates with the consumer free-rider coefficient. As this coefficient increases, there is a tendency for several factors to decline, including the retail price in the offline channel, the greenness of each unit of the product, and the retailer's promotional efforts. That is $\frac{\partial p_m^{df*}}{\partial \tau} > 0$, $\frac{\partial p_r^{df*}}{\partial \tau} < 0$, $\frac{\partial \theta^{df*}}{\partial \tau} < 0$, $\frac{\partial S^{df*}}{\partial \tau} < 0$.

Proposition 3 demonstrates that as the number of consumers engaging in free-rider behavior increases, manufacturers are inclined to increase prices in the online channel. This tendency occurs while they reduce their focus on producing green products in a bid to maximize profits. Meanwhile, retailers become less motivated to engage in promotional activities since more consumers switch to the online channel. Consequently, retailers tend to reduce retail prices in the offline channel with the aim of attracting and retaining customers.

Proposition 4: In scenarios involving decentralized decision-making, when $0 \leq \tau \leq \frac{1}{2\mu(\beta^2 + (-1+e)\mu)^2} (X_1 + X_2)$, the retail price in the online channel rises with an increase in the consumer free-rider coefficient. Conversely, when $\frac{1}{2\mu(\beta^2 + (-1+e)\mu)^2} (X_1 + X_2) \leq \tau \leq 1$, the retail price in the online channel declines as the hitchhiking coefficient increases (refer to Appendix for X_1, X_2). The wholesale price and the greenness of the unit product are negatively related to the consumer free-rider coefficient. Furthermore, as the free-rider coefficient climbs, both the retail prices in the offline channel and retailers' promotional effort coefficients decline.

Proposition 4 reveals that due to consumers' free-riding behavior, retailers fail to gain the anticipated rewards from their promotional activities. Consequently, retailers

are inclined to cut back on promotional expenditures and attract more customers by reducing retail prices in offline channels. Conversely, manufacturers gain from the free-riding behavior of consumers and tend to invest less in producing green products while increasing online retail prices to boost profits. As a significant portion of consumers engage in free-rider behavior, retailers further decrease their promotional efforts, which results in a reduction in market demand. Meanwhile, manufacturers respond by lowering both wholesale and online retail prices to stimulate market demand.

4 Numerical Simulation Analysis

Due to the complexity inherent in the equilibrium solution outcomes, in order to make the obtained results more intuitive, this paper simulates the simulation with the help of numerical examples. Referring to the related literature [9, 16], the basic parameters are set by default as: $\alpha = 400, \mu = 2, \eta = 4, \alpha = 0.6, \beta = 0.5, e = 0.2, k = 18, c = 25$, and $\tau = 0.4$. When considering the effect of a parameter on the equilibrium solution, it is set as a variable and the rest as constants.

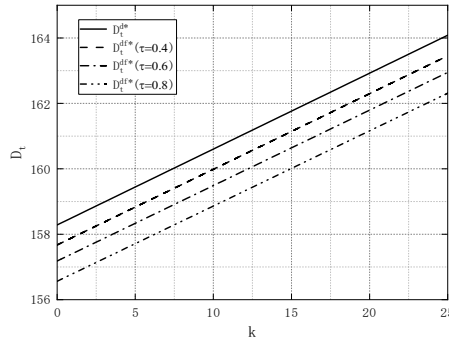


Fig. 2. D_t Trend with k .

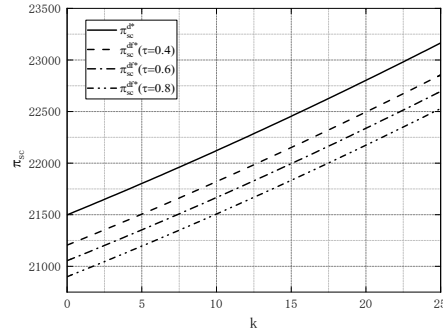


Fig. 3. π_{sc} Trend of variation with k .

An analysis of Figure 2 demonstrates that the aggregate demand within the supply chain is always higher when consumer free-riding behavior is absent, as opposed to when such behavior is present. Furthermore, as the consumer free-riding coefficient increases, the total demand in the supply chain decreases. Conversely, total demand increases as government subsidies rise. According to Figure 3, the supply chain achieves its maximum total profit in the absence of consumer free-riding behavior. Additionally, overall supply chain profit grows with increasing government subsidies but diminishes as the consumer free-riding coefficient increases.

By analyzing the information presented in Table 2, we can observe specific trends as the free-rider coefficient τ varies between 0.4 and 0.8 in both centralized and decentralized decision-making scenarios. The information in Table 2 shows that with an increase in the free-rider coefficient τ , there is a decline in both the greenness of each product unit and the retailer's promotional efforts across centralized and decentralized frameworks. This indicates that consumer free-riding behavior diminishes

manufacturers' motivation to create green products and results in reduced promotional activities by retailers. Furthermore, higher levels of free-riding correlate with a decrease in total demand and overall profit, underscoring the detrimental effect on the supply chain's performance.

Table 2. Comparison of optimal decisions with different free-rider coefficients.

optimal decision		hitchhik-			
		$\tau = 0.4$	$\tau = 0.6$	$\tau = 0.8$	
Model Type					
Consumer hitchhiking	Centralized decision-making	θ^{cf*}	84.49	84.15	83.93
		s^{cf*}	38.66	36.82	35.63
		D_t^{cf*}	214.55	240.46	239.76
	Decentralized decision-making	θ^{df*}	59.57	59.42	59.22
		s^{df*}	5.28	3.73	2.20
		D_t^{df*}	161.82	161.32	160.68
		π^{df*}	161.82	161.32	160.68

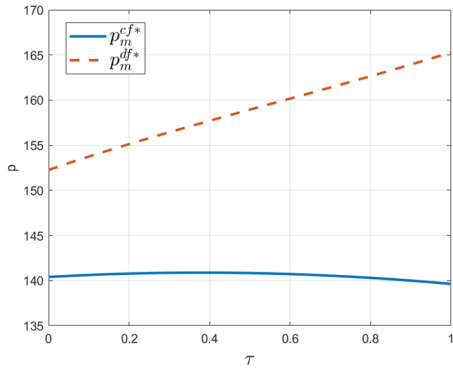


Fig. 4. p_m Trend with τ .

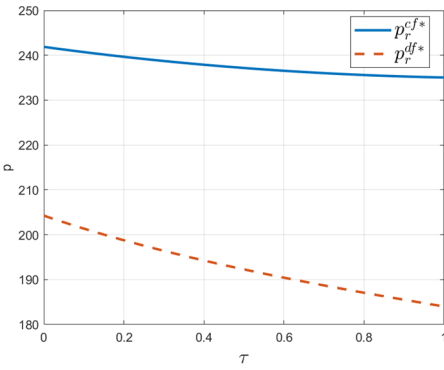


Fig. 5. p_r Trend with τ .

By analyzing Figure 4, it is evident that within a decentralized decision-making framework, the online channel's retail price escalates as the consumer free-riding coefficient grows. This price increase happens because a higher free-riding coefficient encourages consumers to shift from offline to online channels. Consequently, the manufacturer implements a strategy to hike the retail price to boost profits. In contrast, under centralized decision-making, unlike the conclusions drawn by scholars such as Abhijit Barman et al^[3], the retail prices in the online channel initially increase but then decrease when considering "free-riding" behavior. This pattern arises because manufacturers initially hike prices in response to increased demand resulting from consumer migration. However, when the free-riding coefficient exceeds a certain limit, manufacturers start lowering retail prices to attract customers, driven by an overall decrease in demand due

to reduced promotional efforts by retailers. As indicated in Figure 5, both in centralized and decentralized decision-making contexts, the retail price in the offline channel consistently drops as the free-riding coefficient increases. This price drop is a strategy used by the retailer to prevent losing customers and to attract more consumers.

Figures 6 and 7 show a significant difference in how consumer free-riding behavior impacts the profits of manufacturers and retailers. Unlike the conclusions found in literature [9], this study reveals that the manufacturer's profit (π_m) follows an inverted U-shaped pattern as the free-rider coefficient (τ) changes. This pattern indicates that free-riding behavior initially enhances the manufacturer's profit, but as the free-rider coefficient exceeds a certain threshold, the manufacturer's profit starts to decrease. Conversely, the retailer's profit (π_r) persistently falls as the free-rider coefficient increases. This observation suggests that in the market, limited consumer free-riding behavior can be advantageous for the manufacturer. However, if this behavior becomes widespread, it adversely affects both manufacturers and retailers in the supply chain.

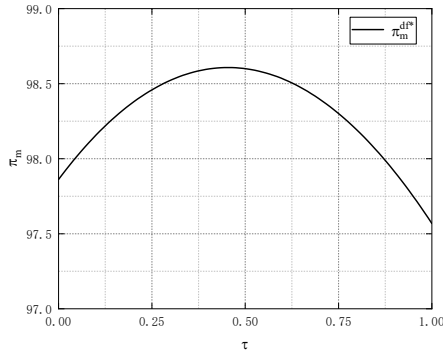


Fig. 6. π_m Trend with τ .

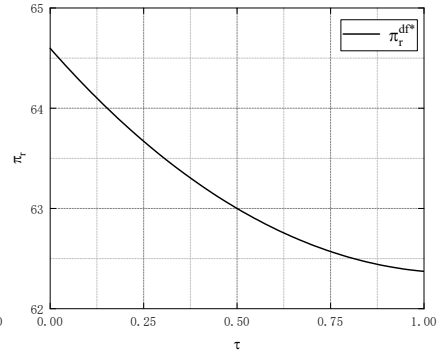


Fig. 7. π_r Trend with τ .

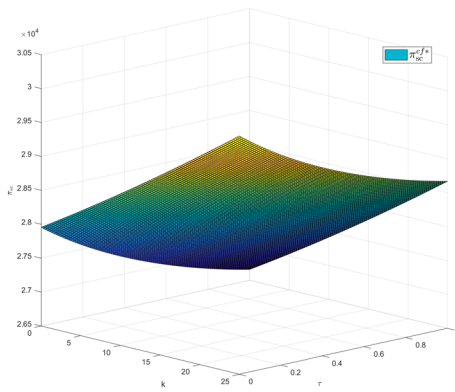


Fig. 8. π_{sc} Trends with k and τ .

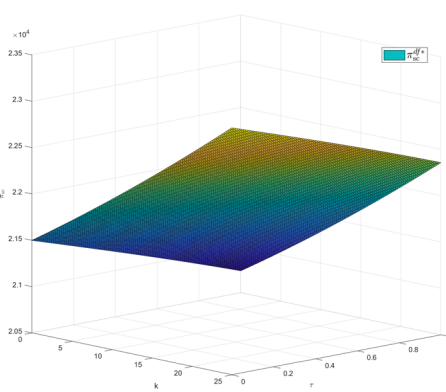


Fig. 9. π_{sc} Trend with τ and k .

Analyzing Figures 8 and 9 reveals that the overall profit (π_{sc}) of the supply chain decreases as the consumer free-riding coefficient (τ) increases, both under centralized

and decentralized decision-making scenarios. This observation suggests that consumer free-riding behavior negatively impacts the total profit of the supply chain. Additionally, these figures demonstrate the positive effect of government subsidies (k) on the supply chain's profitability. As government subsidies increase, the total profit of the supply chain follows suit. Consequently, raising government subsidies can help mitigate the adverse effects of consumer free-riding behavior and improve the supply chain's overall performance.

5 Conclusion

This research develops a model for a dual-channel supply chain, comprising online manufacturers and traditional retailers, and examines the effect of consumer "free-riding" behavior within the framework of government subsidies for manufacturers. Key findings are as follows: (1) Impact of Consumer "Free-Riding" Behavior: When consumer "free-riding" behavior is absent, both the demand and profit across the supply chain achieve peak levels. Such behavior notably reduces manufacturers' motivation for green production, retailers' promotional activities, and the overall profitability in the supply chain. (2) Price Change Trends: Numerical simulations reveal that under centralized decision-making, retail prices in the online channel initially climb and then fall as the free-riding coefficient rises, while retail prices in the offline channel consistently decline with an increasing free-riding coefficient. (3) Overall Demand and Profit: Whether under centralized or decentralized decision-making, the presence of consumer "free-riding" behavior results in higher overall demand and profit within the supply chain compared to its absence. For manufacturers, this behavior is initially beneficial but follows a U-shaped profit trend, declining with more prevalent free-riding; meanwhile, retailer profits consistently decrease. Additionally, greater government subsidies result in higher overall demand and profit.

Policy Recommendations:

(1) For policymakers: The government should consider providing increased funding and technical support to assist businesses in reducing the costs of green transformation, thereby promoting green consumption. Furthermore, more precise subsidy strategies should be developed to maximize the incentive effects of policies. (2) For business practices: Enterprises should actively engage in the wave of green development, particularly leveraging the synergy between online and offline channels. Manufacturers should focus on moderate innovation and production adjustments to adapt to changes in consumer behavior, while retailers need to enhance their promotional strategies to improve market competitiveness.

Future Research Directions:

Despite the valuable insights provided, this study does have certain limitations and future research could delve deeper into the following areas (1) Random demand: Introducing more scenarios involving random demand can enhance the model's applicability and real-world guidance. (2) Diverse government subsidy strategies: Exploring a wider range of dynamic government subsidy strategies and analyzing their long-term impact on supply chain decisions. (3) Consumer diversity: Considering the needs and

behavioral patterns of different consumer segments for a more comprehensive understanding of market dynamics.

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