







steinb13.txt	100	125	17	175	172	175	165
steinb14.txt	100	125	25	237	253	235	236
steinb15.txt	100	125	50	323	335	318	318
steinb16.txt	100	200	17	137	138	133	127
steinb17.txt	100	200	25	134	139	132	131
steinb18.txt	100	200	50	222	250	222	218

**Table 2.** Experimental algorithm results on the *steinc* graph group

Test	$n$	$m$	$ L $	MST-Steiner	SPT-Steiner	PD-Steiner	Node-based	Path-based	VNS
steinc1.txt	500	625	5	88	86	85	85	85	85
steinc2.txt	500	625	10	144	158	144	144	144	144
steinc3.txt	500	625	83	779	843	762	754	754	754
steinc4.txt	500	625	125	1114	1193	1085	1079	1079	1079
steinc5.txt	500	625	250	1599	1706	1583	1579	1579	1579
steinc6.txt	500	1000	5	60	56	55	55	55	55
steinc7.txt	500	1000	10	115	103	102	102	103	102
steinc8.txt	500	1000	83	531	597	516	509	509	509
steinc9.txt	500	1000	125	728	865	718	707	707	707
steinc10.txt	500	1000	250	1117	1327	1107	1093	1093	1093
steinc11.txt	500	2500	5	37	32	34	32	33	33
steinc12.txt	500	2500	10	49	46	48	46	46	46
steinc13.txt	500	2500	83	274	322	268	258	258	258
steinc14.txt	500	2500	125	337	417	332	323	323	323
steinc15.txt	500	2500	250	571	703	562	556	556	556
steinc16.txt	500	12500	5	13	12	12	11	11	11
steinc17.txt	500	12500	10	19	19	20	18	18	18
steinc18.txt	500	12500	83	125	146	123	116	116	115
steinc19.txt	500	12500	125	158	195	159	147	147	148
steinc20.txt	500	12500	250	269	339	268	267	268	268

**Table 3.** Experimental algorithm results on the *steind* graph group

Test	$n$	$m$	$ L $	MST-Steiner	SPT-Steiner	PD-Steiner	Node-based	Path-based	VNS
steind1.txt	1000	1250	5	107	107	107	106	106	106
steind2.txt	1000	1250	10	237	228	232	220	220	220
steind3.txt	1000	1250	167	1636	1771	1593	1565	1565	1565
steind4.txt	1000	1250	250	2012	2174	1957	1935	1935	1935
steind5.txt	1000	1250	500	3310	3511	3270	3250	3254	3250
steind6.txt	1000	2000	5	74	70	75	68	70	67
steind7.txt	1000	2000	10	105	111	103	103	103	103
steind8.txt	1000	2000	167	1138	1287	1104	1072	1077	1073
steind9.txt	1000	2000	250	1540	1773	1500	1448	1449	1448
steind10.txt	1000	2000	500	2163	2550	2141	2110	2111	2111
steind11.txt	1000	5000	5	31	29	31	29	29	29

steind12.txt	1000	5000	10	43	44	42	42	42	42
steind13.txt	1000	5000	167	531	643	518	501	502	502
steind14.txt	1000	5000	250	702	851	691	669	667	671
steind15.txt	1000	5000	500	1151	1437	1134	1117	1120	1116
steind16.txt	1000	25000	5	15	13	14	13	13	13
steind17.txt	1000	25000	10	25	25	23	23	23	23
steind18.txt	1000	25000	167	251	301	246	228	228	228
steind19.txt	1000	25000	250	344	424	334	313	317	318
steind20.txt	1000	25000	500	544	691	542	537	539	538

**Table 4.** Experimental algorithm results on the *steine* graph group

Test	$n$	$m$	$ L $	MST-Steiner	SPT-Steiner	PD-Steiner	VNS
steine1.txt	2500	3125	5	125	111	115	111
steine2.txt	2500	3125	10	244	214	227	214
steine3.txt	2500	3125	417	4232	4570	4118	4015
steine4.txt	2500	3125	625	5316	5675	5201	5101
steine5.txt	2500	3125	1250	8313	8976	8226	8128
steine6.txt	2500	5000	5	86	73	78	73
steine7.txt	2500	5000	10	165	150	159	145
steine8.txt	2500	5000	417	2809	3254	2726	2648
steine9.txt	2500	5000	625	3809	4474	3727	3608
steine10.txt	2500	5000	1250	5745	6847	5673	5600
steine11.txt	2500	12500	5	39	34	38	34
steine12.txt	2500	12500	10	73	68	69	67
steine13.txt	2500	12500	417	1370	1704	1332	1292
steine14.txt	2500	12500	625	1814	2304	1778	1735
steine15.txt	2500	12500	1250	2856	3626	2819	2784
steine16.txt	2500	62500	5	17	15	15	15
steine17.txt	2500	62500	10	27	27	26	25
steine18.txt	2500	62500	417	646	804	639	583
steine19.txt	2500	62500	625	809	1059	806	768
steine20.txt	2500	62500	1250	1358	1753	1357	1342

This section aims to compare the solution quality of VNS algorithm with the group of MST-Steiner, SPT-Steiner, PD-Steiner algorithm [15] and group of Node-based, Path-based algorithm [12].

With 20 sets of data in *steinb* group, the VNS algorithm offers a better solution quality at 72.2%, equivalent quality at 22.2% and worse quality at 5.6% in comparison with MST-Steiner algorithm. The VNS algorithm offers a better solution quality at 77.8%, equivalent quality at 16.7% and worse quality at 5.6% in comparison with SPT-Steiner algorithm. The VNS algorithm offers a better solution quality at 50.0%, equivalent quality at 44.4% and worse quality at 5.6% in comparison to PD-Steiner algorithm.

With 20 sets of data in *steinc* group, the VNS algorithm offers a better solution quality at 5%, equivalent quality at 80% and worse quality at 15% in comparison with Node-

based algorithm. The VNS algorithm offers a better solution quality at 10%, equivalent quality at 85% and worse quality at 5% in comparison with Path-based algorithm. The VNS algorithm offers a better solution quality at 95%, equivalent quality at 5% and worse quality at 0% in comparison with MST-Steiner algorithm. The VNS algorithm offers a better solution quality at 90%, equivalent quality at 5% and worse quality at 5% in comparison with SPT-Steiner algorithm. The VNS algorithm offers a better solution quality at 75%, equivalent quality at 25% and worse quality at 0% in comparison to PD-Steiner algorithm.

With 20 sets of data in *steind* group, the VNS algorithm offers a better solution quality at 10%, equivalent quality at 60% and worse quality at 30% in comparison with Node-based algorithm. The VNS algorithm offers a better

solution quality at 30%, equivalent quality at 60% and worse quality at 10% in comparison with Path-based algorithm. The VNS algorithm offers a better solution quality at 100%, equivalent quality at 0% and worse quality at 0% in comparison with MST-Steiner algorithm. The VNS algorithm offers a better solution quality at 90%, equivalent quality at 10% and worse quality at 0% in comparison with SPT-Steiner algorithm. The VNS algorithm offers a better solution quality at 85%, equivalent quality at 15% and worse quality at 0% in comparison to PD-Steiner algorithm.

With 20 sets of data in *steine* group, the VNS algorithm offers a better solution quality at 100%, equivalent quality at 0% and worse quality at 0% in comparison with MST-Steiner algorithm. The VNS algorithm offers a better solution quality at 75%, equivalent quality at 25% and worse quality at 0% in comparison with SPT-Steiner algorithm. The VNS algorithm offers a better solution quality at 95%, equivalent quality at 5% and worse quality at 0% in comparison to PD-Steiner algorithm.

## 4. Conclusions

In this paper, the VNS algorithm has been proposed to solve SMT problem in sparse graphs; The proposed algorithm has been experimentally implemented and evaluated using 78 sets of data as sparse graphs in the standard experimental datasets. The experiment outcomes show promising results in which the solution quality provided by the proposed algorithm is significantly improved compared to MST-Steiner, SPT-Steiner, PD-Steiner, Node-based and Path-based algorithm.

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