Application of Hexagonal Fuzzy Soft Matrices in The Diagnosis of Respiratory Tract Infections, Common Cold, and Pharyngitis

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Abstract. Hexagonal fuzzy numbers are an extension of pentagonal fuzzy numbers that allow a more flexible representation of uncertainty through asymmetric membership functions. In medical diagnosis, symptom intensity and patient conditions are often subjective and imprecise, making classical fuzzy representations insufficient. This paper proposes a hexagonal fuzzy soft matrix–based diagnostic model for respiratory tract infections, common cold, and pharyngitis. The novelty of this study lies in integrating hexagonal fuzzy number arithmetic with fuzzy soft matrix modeling, incorporating error variables to represent subjective uncertainty, and applying the model directly to medical diagnostic scoring. Interview and observation data are first transformed into numerical intervals and then mapped into hexagonal fuzzy numbers. These data are modeled as hexagonal fuzzy soft sets and represented in matrix form to compute diagnostic scores. The results show that the proposed approach can distinguish diseases with overlapping symptoms more sensitively than conventional fuzzy soft matrix methods, even under uncertain and limited data conditions.

Keywords: Hexagonal fuzzy numbers, fuzzy soft matrices, hexagonal fuzzy soft matrices, medical diagnosis.

1 Introduction

In the real world, problems involving uncertainty are often encountered, whether due to limited information or the existence of subjective data. One effective mathematical approach to dealing with such uncertainty is fuzzy set theory. Fuzzy set theory was first introduced by Zadeh [1]. Fuzzy sets differ from classical sets in that they assign membership degrees to each element in a continuous range $[0, 1]$, thus better representing vague or ambiguous concepts. Since its introduction, fuzzy theory has developed rapidly and has been widely applied in various fields, one of which is medical diagnosis.

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Along with the development of fuzzy theory, Molodtsov [2] introduced the concept of soft sets as a mathematical approach for handling parameterized uncertainty. Soft sets became the basis for the development of fuzzy soft sets, which are a combination of fuzzy sets and soft sets that can accommodate uncertainty while considering relevant parameters. Furthermore, to simplify the representation of data in tabular form and support the calculation process, fuzzy soft sets were developed into fuzzy soft matrices, The matrices representation of fuzzy soft sets was further developed by Çağman and Enginoğlu [3] in the form of fuzzy soft matrices. Rathika et al. [4] showed that this matrices representation allows symptom data to be processed systematically in medical diagnosis problems. The general form of fuzzy soft matrices is $\tilde{A} = [(\mu_{ij}^{A})]_{m\times n}, \quad i = 1,2,3,\ldots,m,\ j = 1,2,3,\ldots,n$ with μ_{ij}^{A} the fuzzy membership function.

In the medical field, fuzzy soft matrices have proven to play an important role in aiding the diagnosis process. Various studies show that this method can be used to analyze patient symptoms that often overlap between diseases. Rathika et al. [4] and Celik and Yamak [5] demonstrated that fuzzy soft matrices can be effectively applied to medical diagnosis problems involving uncertain symptom data. However, most of these studies still use simple fuzzy number representations, such as triangles and trapezoids. Several previous studies have shown that the application of fuzzy soft sets has great potential in helping the disease diagnosis process. This approach provides high flexibility in handling uncertain or incomplete medical data. Celik and Yamak [5] combined fuzzy arithmetic operations with fuzzy soft set theory to improve the handling of uncertainty in decision-making problems. Das and Kar [6] proposed a triangular fuzzy soft set model for multi-attribute decision making under a high degree of uncertainty. This model developed into a triangular fuzzy membership matrices that is more practical for use in medical diagnosis systems because it can directly connect between patient symptom conditions and their mathematical representation. Sangodapo et al. [7] proposed a revision method for fuzzy soft matrix–based decision making that improves diagnostic accuracy. Xiao et al. [8] extended fuzzy soft set theory by applying trapezoidal fuzzy soft sets in multicriteria decision making systems. Zulqarnain et al. [9] demonstrated that trapezoidal fuzzy soft sets can improve the accuracy of disease identification.

Fuzzy shapes such as triangles and trapezoids are relatively simple and easy to use, but they are not always capable of capturing the complexity of medical data. In real cases, patient symptoms can have different levels of intensity with asymmetrical ranges of values on the left and right, or even show two different centers. Hexagonal fuzzy numbers were introduced as a more flexible fuzzy representation by several researchers, including Sandhiy et al. [10]. Hexagonal fuzzy numbers have six parameters $(a, b, \alpha_1, \alpha_2, \beta_1, \beta_2)$ that describe two centers and the transition width on the left and right sides. With these six parameters, hexagonal fuzzy numbers can represent uncertainty more flexibly.

The application of hexagonal fuzzy numbers in fuzzy soft matrices gives rise to the concept of hexagonal fuzzy soft matrices, which are matrices whose elements are represented by hexagonal fuzzy numbers. This concept processes patient symptom data with greater precision, resulting in more accurate diagnoses. Sangodapo et al. [7] showed that hexagonal fuzzy matrices representations can provide more accurate medical decision support than conventional fuzzy models. This study develops a new algorithm by incorporating the parametric form of hexagonal fuzzy numbers to express membership values, as well as using hexagonal fuzzy number arithmetic operations. The arithmetic development follows the formulations proposed by Mashadi et al. [11, 12, 13, 14, 15] and further extended by Puspita et al. [16], ensuring that for any hexagonal fuzzy number $\tilde{a} \times \frac{1}{\tilde{a}} = \tilde{i}$.

A common problem in diagnosing infectious diseases is the similarity of symptoms between diseases. For example, Respiratory Tract Infections, the Common Cold, and Pharyngitis have similar initial symptoms, such as coughing, runny nose, fever, and sore throat. This condition makes it difficult for medical personnel to quickly distinguish between diseases based solely on initial observations. Therefore, the need for mathematical methods capable of handling uncertain symptoms with varying intensity levels has been emphasized by several researchers, including Ahmadi et al. [17], Zhang et al. [18], Lavanya and Akila [19], and Pal et al. [20].

Unlike existing studies that employ triangular or trapezoidal fuzzy numbers in fuzzy soft matrices–based medical diagnosis, this research introduces a hexagonal fuzzy soft matrices framework with asymmetric membership widths and an error variable. This framework enables a more realistic modeling of symptom intensity variations and subjective uncertainty. Furthermore, this study formulates a complete diagnostic scoring mechanism based on hexagonal fuzzy arithmetic and demonstrates its applicability through a real medical case study. These contributions distinguish the proposed approach from existing fuzzy diagnostic models.

2 Literature review

This section provides several definitions and basic concepts of hexagonal fuzzy soft matrices. Zadeh [1] introduced fuzzy set theory, which later formed the basis for the development of fuzzy soft sets. Molodtsov [2] subsequently proposed soft set theory as a parameter-based approach to decision making. The matrix representation of fuzzy soft sets was further developed by Çağman and Enginoğlu [3], and later extended by several researchers [21, 22, 23, 24, 25].

2.1 Fuzzy soft matrices

Definition 2.1 Let X be the universal set. The fuzzy set $\tilde{a}$ on X is defined as the function

$$\mu_{\tilde{a}} : X \to [0,1],$$

which is represented as

$$\tilde{a} = \{(x, \mu_{\tilde{a}}(x)) \mid x \in X\},$$

where $\mu_{\tilde{a}}$ is called the membership function.

Definition 2.2 Let X be the universal set and H be the parameter set. The power set of X is denoted by $PS(X)$. An ordered pair (F_A, H) is called a soft set with respect to X, so that it can be denoted as:

$$(F_A, H) = \{(h, F_A(h)) : h \in H, F_A(h) \in PS(X)\},$$

with $F_A : A \to PS(X)$ for $A \subseteq H$, and $F_A(h) = \emptyset$ if $h \notin A$.

Definition 2.3 Let X be the universal set and H the parameter set. Let $FS(X)$ denote the set of all fuzzy sets over X. The ordered pair $(\tilde{F}_A, H)$ is called a fuzzy soft set [26, 27] over X, so that it can be denoted:

$$(\tilde{F}_A, H) = \{(h, \tilde{F}_A(h)) : h \in H, \tilde{F}_A(h) \in FS(X)\}.$$

Definition 2.4 Let an ordered pair of soft sets on X be denoted by (F_A, H), so that a subset of $X \times H$ is uniquely defined as:

$$R_A = \{(x,h) : h \in A, x \in F_A(h)\}.$$

The relation (F_A, H) with characteristic function $\chi_{R_A} : X \times H \to \{0,1\}$ is given by:

$$\chi_{R_A}(x,h) = \begin{cases} 1, & \text{if } (x,h) \in R_A \\ 0, & \text{if } (x,h) \notin R_A \end{cases}$$

Let $X = \{x_1, x_2, ..., x_m\}$ and $H = \{h_1, h_2, ..., h_n\}$ with $A \subseteq H$, if $a_{ij} = \chi_{R_A}(x_i, h_j)$, then the matrix can be defined as follows:

$$\tilde{A}_{SM} = [a_{ij}]_{m\times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}.$$

$\tilde{A}_{SM}$ are called soft matrices $m \times n$ of fuzzy soft sets (F_A, H) towards X.

Definition 2.5 Let $(\tilde{F}_A, H)$ is given by X, is given by $X \times H$ is given as:

$$R_A = \{(x,h) : h \in A, x \in \tilde{F}_A(h)\}.$$

The form of the relation $(\tilde{F}_A, H)$ with membership function $\mu_{R_A} : X \times H \to [0,1]$ is written as:

$$\mu_{R_A}(x,h) = \begin{cases} \mu_A, & \text{if } (x,h) \in R_A \\ 0, & \text{if}(x,h) \notin R_A \end{cases}$$

Suppose $X = \{x_1, x_2, ..., x_m\}$ and $H = \{h_1, h_2, ..., h_n\}$ with $A \subseteq H$, if $\mu_{A_{ij}} = \mu_{R_A}(u_i, c_j)$, then the matrix can be defined as follows:

$$\tilde{A}_{FSM} = \left[\tilde{\mu}_{A_{ij}}\right]_{m\times n} = \begin{bmatrix} \tilde{\mu}_{11} & \tilde{\mu}_{12} & \cdots & \tilde{\mu}_{1n} \\ \tilde{\mu}_{21} & \tilde{\mu}_{22} & \cdots & \tilde{\mu}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\mu}_{m1} & \tilde{\mu}_{m2} & \cdots & \tilde{\mu}_{mn} \end{bmatrix}.$$

$\tilde{A}_{FSM}$ are called fuzzy soft matrices $m \times n$ of fuzzy soft sets $(\tilde{F}_A, H)$ towards X.

2.2 Hexagonal fuzzy soft matrices

Several researchers, including Savarimuthu et al. [28], Sudha et al. [29], Harun et al. [30], Mitlif et al. [31], and Gomathi et al. [32], have defined the concept and arithmetic operations of hexagonal fuzzy numbers. In this study, the concept of hexagonal fuzzy numbers is adopted following the formulation of Puspita et al. [16], which guarantees that for any hexagonal fuzzy number there exists a unique inverse producing the identity element, analogous to the identity inverse property of hexagonal fuzzy number matrices.

Definition 2.6 A hexagonal fuzzy number $\tilde{a}$ is a fuzzy number with six parameters:

$$\tilde{a} = (a, b, \alpha_1, \alpha_2, \beta_1, \beta_2),$$

where a, b are the centre points, α_1, α_2 are the left widths, and β_1, β_2 are the right widths that have the following membership functions:

$$\mu_{\tilde{a}}(x) = \begin{cases} \frac{1}{2} - \frac{1}{2} \cdot \frac{x-a+\alpha_1}{\alpha_2}, & a - \alpha_1 - \alpha_2 \leq x \leq a - \alpha_1 \\ 1 + \frac{1}{2} \cdot \frac{x-a}{\alpha_1}, & a - \alpha_1 \leq x \leq a \\ 1, & a \leq x \leq b \\ 1 - \frac{1}{2} \cdot \frac{x-b}{\beta_1}, & b \leq x \leq b + \beta_1 \\ \frac{1}{2} - \frac{1}{2} \cdot \frac{x-b-\beta_1}{\beta_2}, & b + \beta_1 \leq x \leq b + \beta_1 + \beta_2 \\ 0, & \text{others} \end{cases}$$

A set consisting of hexagonal fuzzy numbers is called a hexagonal fuzzy number set. The parametric form of a fuzzy hexagonal number $\tilde{a} = (a, b, \alpha_1, \alpha_2, \beta_1, \beta_2)$ can be written as $\tilde{a} = [\underline{a}_2(r), \underline{a}_1(r), \bar{a}_1(r), \bar{a}_2(r)]$ with

$$\underline{a}_2(r) = (a - (1-2r)\alpha_2 - \alpha_1) \tag{1}$$

$$\underline{a}_1(r) = (a - (1-r)2\alpha_1) \tag{2}$$

$$\bar{a}_1(r) = (b + (1-r)2\beta_1) \tag{3}$$

$$\bar{a}_2(r) = (b + (1-2r)\beta_2 + \beta_1) \tag{4}$$

The shape of the hexagonal fuzzy number curve is as follows

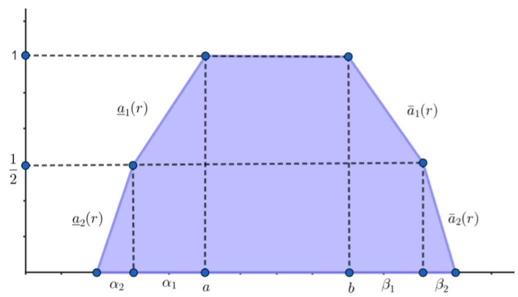

Fig. 1. Hexagonal fuzzy number curve.

Figure 1 illustrates the membership function of a hexagonal fuzzy number defined by six parameters $(a,b,\alpha_1,\alpha_2,\beta_1,\beta_2)$. The interval $[a,b]$ represents the core region where the membership degree is equal to one, indicating the most representative range of the observed value. The parameters α_1 and α_2 control the left transition region, determining how gradually the membership value increases from zero to one. Similarly, β_1 and β_2 regulate the right transition region, describing how the membership value decreases from one back to zero. This asymmetric structure allows different rates of transition on both sides of the core interval. In medical diagnosis, such flexibility is essential because symptom intensities often increase and decrease at different rates. By adjusting these parameters, the hexagonal fuzzy number can model subtle variations in symptom severity more accurately, thereby improving diagnostic sensitivity compared to triangular or trapezoidal fuzzy representations.

Radhika et al. [33] formulated membership and non-membership functions using triangular, trapezoidal, pentagonal, and other shapes to accurately represent the distribution of information in the system. Furthermore, for the hexagonal fuzzy number membership function in Definition 2.6, an error variable is added, whose membership is defined as follows.

Definition 2.7 A hexagonal fuzzy number $\tilde{a} = (a,b,\alpha_1,\alpha_2,\beta_1,\beta_2)$ with the addition of an error variable ε can be expressed as follows:

$$\mu_{\tilde{a}}(x) = \begin{cases} \frac{1}{2} - \frac{1}{2}\frac{x-a+\alpha_1}{\alpha_2} - \varepsilon, & a-\alpha_1-\alpha_2 \leq x < a-\alpha_1 \\ 1 + \frac{1}{2}\frac{x-a}{\alpha_1} - \varepsilon, & a-\alpha_1 \leq x < a \\ 1-\varepsilon, & a \leq x \leq b \\ 1 - \frac{1}{2}\frac{x-b}{\beta_1} - \varepsilon, & b < x \leq b+\beta_1 \\ \frac{1}{2} - \frac{1}{2}\frac{x-b-\beta_1}{\beta_2} - \varepsilon, & b+\beta_1 < x \leq b+\beta_1+\beta_2 \\ 0, & \text{others} \end{cases}$$

error variable ε denotes the uncertainty value for subjective data, generally used $0 < \varepsilon \leq 0.1$. The hexagonal fuzzy number curve containing the error variable is given in Figure 1.

Definition 2.8 Let X be the universal set and H the parameter set. Let $FS(X)$ enote the set of all fuzzy set on X with $A \subseteq H$. The ordered pair (F_A,H) is called a fuzzy soft set on X, so that it can be denoted as

$$(F_A, H) = \{(h, F_A(h)) : h \in H, F_A(h) \in FS(X)\}$$

with $F_A : A \to FS(X)$ such that $F_A(h) = \emptyset$ if $h \notin A$.

Definition 2.9 Let $HxFSS(X)$ denote the set of all hexagonal fuzzy soft sets. The ordered pair $(\tilde{F}_A, H)$ is a hexagonal fuzzy soft set towards X. Its complement is defined as:

$$(\tilde{F}_A^c, H) = \{(h, \tilde{F}_A^c(h)) : h \in H, \tilde{F}_A^c(h) = \tilde{i} - \mu_{\tilde{A}}(x)\},$$

with $\tilde{i} = (1,1,0,0,0,0)$ as the identity of the hexagonal fuzzy number.

Definition 2.10 Let the ordered pair of hexagonal soft sets on X be denoted by $(\tilde{F}_A, H)$, so that the subset of $X \times H$ is uniquely defined as

$$R_A = \{(x, h) : h \in A, x \in F_A(h)\}.$$

The form of the relation $(\tilde{F}_A, H)$ with characteristic function R_A can be written as

$$\mu_{R_A} : X \times H \to [0,1],$$

given by:

$$\mu_{\tilde{a}}(x) = \begin{cases} \frac{1}{2} - \frac{1}{2} \cdot \frac{x-a+\alpha_1}{\alpha_2}, & a-\alpha_1-\alpha_2 \leq x \leq a-\alpha_1 \\ 1 + \frac{1}{2} \cdot \frac{x-a}{\alpha_1}, & a-\alpha_1 \leq x \leq a \\ 1, & a \leq x \leq b \\ 1 - \frac{1}{2} \cdot \frac{x-b}{\beta_1}, & b \leq x \leq b+\beta_1 \\ \frac{1}{2} - \frac{1}{2} \cdot \frac{x-b-\beta_1}{\beta_2}, & b+\beta_1 \leq x \leq b+\beta_1+\beta_2 \\ 0, & \text{others} \end{cases}$$

Suppose $X = \{x_1, x_2, ..., x_m\}$ and $H = \{h_1, h_2, ..., h_n\}$ with $A \subseteq H$, if $\mu_{A_{ij}} = \mu_{R_A}(u_i, h_j)$, then the matrix can be defined as follows:

$$A_{HxFSM} = \left[\tilde{\mu}_{A_{ij}}\right]_{m \times n} = \begin{bmatrix} (\tilde{\mu}_{11}) & (\tilde{\mu}_{12}) & \cdots & (\tilde{\mu}_{1n}) \\ (\tilde{\mu}_{21}) & (\tilde{\mu}_{22}) & \cdots & (\tilde{\mu}_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ (\tilde{\mu}_{m1}) & (\tilde{\mu}_{m2}) & \cdots & (\tilde{\mu}_{mn}) \end{bmatrix}.$$

A_{HxFSM} is called the $m \times n$ hexagonal fuzzy soft matrix of the hexagonal fuzzy soft set $(\tilde{F}_A, H)$ towards X with $\mu_{A_{ij}}$ is a fuzzy hexagonal membership function.

Definition 2.11 The relation form $(\tilde{F}_A, H)$ with characteristic function R_A can be written as

$$\mu_{R_A} : X \times H \to [0,1]$$

which is given by:

$$\mu_{R_A}(x,h)=\begin{cases}\frac{1}{2}-\frac{1}{2}\frac{x-a+\alpha_1}{\alpha_2}-\varepsilon, & a-\alpha_1-\alpha_2\le x<a-\alpha_1\\ 1+\frac{1}{2}\frac{x-a}{\alpha_1}-\varepsilon, & a-\alpha_1\le x<a\\ 1-\varepsilon, & a\le x\le b\\ 1-\frac{1}{2}\frac{x-b}{\beta_1}-\varepsilon, & b<x\le b+\beta_1\\ \frac{1}{2}-\frac{1}{2}\frac{x-b-\beta_1}{\beta_2}-\varepsilon, & b+\beta_1<x\le b+\beta_1+\beta_2\\ 0, & \text{others}\end{cases}$$

Suppose $X=\{x_1,x_2,...,x_m\}$ and $H=\{h_1,h_2,...,h_n\}$ with $A\subseteq H$, if $\mu_{A_{ij}}=\mu_{R_A}(u_i,h_j)$, then the matrix can be defined as follows:

$$M_{HxFSM}=\left[(\tilde{\mu}_{M_{ij}})\right]_{m\times n}=\begin{bmatrix}(\tilde{\mu}_{11}) & (\tilde{\mu}_{12}) & \cdots & (\tilde{\mu}_{1n})\\ (\tilde{\mu}_{21}) & (\tilde{\mu}_{22}) & \cdots & (\tilde{\mu}_{2n})\\ \vdots & \vdots & \ddots & \vdots\\ (\tilde{\mu}_{m1}) & (\tilde{\mu}_{m2}) & \cdots & (\tilde{\mu}_{mn})\end{bmatrix}.$$

M_{HxFSM} is called an $m\times n$ hexagonal fuzzy soft matrix of hexagonal fuzzy soft sets $(\tilde{F}_A,H)$ towards X with $\mu_{M_{ij}}$ is a hexagonal fuzzy membership function of the form $(a,b,\alpha_1,\alpha_2,\beta_1,\beta_2)$ containing an error variable $0<\varepsilon\le 0.1$.

Definition 2.12 Suppose there are two hexagonal fuzzy numbers $\tilde{a}=(a,b,\alpha_1,\alpha_2,\beta_1,\beta_2)$ and $\tilde{b}=(c,d,\gamma_1,\gamma_2,\delta_1,\delta_2)$ then the following operations apply:

(i) $(\tilde{a}-\tilde{b})=(a-d,b-c,\alpha_1+\delta_1,\alpha_2+\delta_2,\beta_1+\gamma_1,\beta_2+\gamma_2)$

(ii) $(\tilde{a}+\tilde{b})=(a+c,b+d,\alpha_1+\gamma_1,\alpha_2+\gamma_2,\beta_1+\delta_1,\beta_2+\delta_2)$

(iii) $k\tilde{a}=\begin{cases}(ka,kb,k\alpha_1,k\alpha_2,k\beta_1,k\beta_2) & \text{jika } k\ge 0,\\ (kb,ka,-k\beta_2,-k\beta_1,-k\alpha_1,-k\alpha_2) & \text{jika } k\le 0.\end{cases}$

(iv) $(\tilde{a}\times\tilde{b})=\begin{array}{l}\big(a\cdot m(\tilde{b})+c\cdot m(\tilde{a})-m(\tilde{b})\cdot m(\tilde{a}),\\ \ b\cdot m(\tilde{b})+d\cdot m(\tilde{a})-m(\tilde{b})\cdot m(\tilde{a}),\\ \ \alpha_1\cdot m(\tilde{b})+\gamma_1\cdot m(\tilde{a}),\alpha_2\cdot m(\tilde{b})+\gamma_2\cdot m(\tilde{a}),\\ \ \beta_1\cdot m(\tilde{b})+\delta_1\cdot m(\tilde{a}),\beta_2\cdot m(\tilde{b})+\delta_2\cdot m(\tilde{a})\big)\end{array}$

(v) $\frac{\tilde{a}}{\tilde{b}} = \begin{pmatrix} \dfrac{a \cdot m(\tilde{b}) + 2m(\tilde{a}) \cdot m(\tilde{b}) - c \cdot m(\tilde{a}) - m(\tilde{a}) \cdot m(\tilde{b})}{(m(\tilde{b}))^2}, \\ \dfrac{b \cdot m(\tilde{b}) + 2m(\tilde{a}) \cdot m(\tilde{b}) - d \cdot m(\tilde{a}) - m(\tilde{a}) \cdot m(\tilde{b})}{(m(\tilde{b}))^2}, \\ \dfrac{\alpha_1 \cdot m(\tilde{b}) - \gamma_1 \cdot m(\tilde{a})}{(m(\tilde{b}))^2}, \dfrac{\alpha_1 \cdot m(\tilde{b}) - \gamma_2 \cdot m(\tilde{a})}{(m(\tilde{b}))^2}, \\ \dfrac{\beta_1 \cdot m(\tilde{b}) - \delta_1 \cdot m(\tilde{a})}{(m(\tilde{b}))^2}, \dfrac{\beta_2 \cdot m(\tilde{b}) - \delta_2 \cdot m(\tilde{a})}{(m(\tilde{b}))^2} \end{pmatrix}$

with $m(\tilde{a}) = \frac{a+b}{2}$ and $m(\tilde{b}) = \frac{c+d}{2}$.

Definition 2.13 Suppose there are two hexagonal fuzzy numbers with $\tilde{g} = (a, b, \alpha_1, \alpha_2, \beta_1, \beta_2)$,their parametric forms become $\tilde{a} = [\underline{a}_2(r), \underline{a}_1(r), \bar{a}_1(r), \bar{a}_2(r)]$ and $\tilde{h} = (c, d, \gamma_1, \gamma_2, \delta_1, \delta_2)$ its parametric form becomes $\tilde{b} = [\underline{b}_2(r), \underline{b}_1(r), \bar{b}_1(r), \bar{b}_2(r)]$ then the following operations apply:

(i) $\tilde{a}(r) \ominus \tilde{b}(r) = \left[\underline{a}_2(r) - \underline{b}_2(r),\ \underline{a}_1(r) - \underline{b}_1(r),\ \bar{a}_1(r) - \bar{b}_1(r),\ \bar{a}_2(r) - \bar{b}_2(r)\right]$

(ii) $\tilde{a}(r) \oplus \tilde{b}(r) = \left[\underline{a}_2(r) + \underline{b}_2(r),\ \underline{a}_1(r) + \underline{b}_1(r),\ \bar{a}_1(r) + \bar{b}_1(r),\ \bar{a}_2(r) + \bar{b}_2(r)\right]$

(iii) $k\tilde{a}(r) = \begin{cases} \left(k\bar{a}_2(r),\ k\bar{a}_1(r),\ k\underline{a}_1(r),\ k\underline{a}_2(r)\right), & \text{jika } k < 0, \\ \left(k\underline{a}_2(r),\ k\underline{a}_1(r),\ k\bar{a}_1(r),\ k\bar{a}_2(r)\right), & \text{jika } k \geq 0. \end{cases}$

(iv) $\tilde{a}(r) \otimes \tilde{b}(r) = (a_2(r), a_1(r), \bar{a}_1(r), \bar{a}_2(r)) \otimes (b_2(r), b_1(r), \bar{b}_1(r), \bar{b}_2(r))$

$$= \Big[a_2(r)m_2(\tilde{b}) + b_2(r)m_2(\tilde{a}) - m_2(\tilde{a})m_2(\tilde{b}),$$

$$a_1(r)m_1(\tilde{b}) + b_1(r)m_1(\tilde{a}) - m_1(\tilde{a})m_1(\tilde{b}),$$

$$\bar{a}_1(r)m_1(\tilde{b}) + \bar{b}_1(r)m_1(\tilde{a}) - m_1(\tilde{a})m_1(\tilde{b}),$$

$$\bar{a}_2(r)m_2(\tilde{b}) + \bar{b}_2(r)m_2(\tilde{a}) - m_2(\tilde{a})m_2(\tilde{b})\Big]$$

(v) $$\frac{\tilde{a}(r)}{\tilde{b}(r)} = \left[\frac{a_2(r)m_2(\tilde{b}) - b_2(r)m_2(\tilde{a}) + m_2(\tilde{a})m_2(\tilde{b})}{\left(m_2(\tilde{b})\right)^2},\right.$$

$$\frac{\bar{a}_1(r)m_1(\tilde{b}) - \bar{b}_1(r)m_1(\tilde{a}) + m_1(\tilde{a})m_1(\tilde{b})}{\left(m_1(\tilde{b})\right)^2},$$

$$\frac{a_1(r)m_1(\tilde{b}) - b_1(r)m_1(\tilde{a}) + m_1(\tilde{a})m_1(\tilde{b})}{\left(m_1(\tilde{b})\right)^2},$$

$$\left.\frac{\bar{a}_2(r)m_2(\tilde{b}) - \bar{b}_2(r)m_2(\tilde{a}) + m_2(\tilde{a})m_2(\tilde{b})}{\left(m_2(\tilde{b})\right)^2}\right]$$

with $m_1(\tilde{a}) = \frac{a+b}{2}$, $\quad m_2(\tilde{a}) = \frac{\underline{a}_1(1)+\bar{a}_1(1)}{2}$, $\quad m_1(\tilde{b}) = \frac{c+d}{2}$, $\quad m_2(\tilde{b}) = \frac{\underline{b}_1(1)+\bar{b}_1(1)}{2}$.

3 Materials and methods

The purpose of this study is to detect diseases in patients based on their main symptoms. In this study, fuzzy soft matrices are applied to medical diagnosis using the method proposed by Sangodapo et al. [7]. The first step is to determine the parameters required in the diagnosis process. After that, fuzzification is performed on the data from the interviews to obtain hexagonal fuzzy soft matrices in parametric form based on the concepts described earlier. Finally, membership values are calculated and diagnosis scores are determined to identify the diseases experienced by patients. The following are definitions for calculating membership values and diagnosis scores.

Definition 3.1 Let $X = [\tilde{x}(r)_{ij}]_{m\times n}$ be hexagonal fuzzy soft matrices with entries in the form of parametric hexagonal fuzzy numbers, namely

$$\tilde{x}(r)_{ij} = [\underline{x}_2(r), \underline{x}_1(r), \bar{x}_1(r), \bar{x}_2(r)].$$

The membership value corresponding to X is defined as

$$MV(X)(r) = [\delta(X)_{ij}]_{m\times n} = \left[((\bar{x}_1(r) + \bar{x}_2(r)) - ((\underline{x}_1(r) + \underline{x}_2(r))\right],$$

for $i = 1,\ldots,m$ and $j = 1,\ldots,n$.
Based on the membership values obtained using Definition 3.1, a definition of the diagnosis score from the membership values for $\tilde{X}_1, \tilde{X}_2, \tilde{X}_3$, and $\tilde{X}_4$ can be constructed as follows.

Definition 3.2 The diagnosis scores for hexagonal fuzzy soft matrices denoted by S_{K_1} and S_{K_2} indicate the potential for disease to attack the patient, and S_{K_2} indicates the strength of the patient's immune system. The representation is as follows:

$$S_{K_1} = [\delta(K_1)_{ij}] = MV(X_1)(r) - MV(X_3)(r)$$

and

$$S_{K_2} = [\delta(K_2)_{ij}] = MV(X_2)(r) - MV(X_4)(r).$$

The final diagnosis result using hexagonal fuzzy soft matrices is obtained from the difference between the diagnosis scores S_{K_1} and S_{K_2}. Let us denote (p_i, d_j) as the matrix entry with p_i being patient i and d_j being disease j, for every $i = 1,\ldots,m$ and $j = 1,\ldots,n$. If

$$S_{KD} = \max(S_{K_1}(p_i,d_j) - S_{K_2}(p_i,d_j))$$

occurs precisely for (p_i, d_j), then it can be concluded that the diagnosis for patient p_i is disease d_j.

These steps are summarized in Algorithm 1 as follows.

Table 1: Patient diagnosis score algorithm.

a)	Input Symptom set S, Disease set D, Patient set P, Symptom intensity intervals and output is diagnosed disease for each patient.
b)	Convert symptom and patient data into numerical intervals $[0,10]$
c)	Do the fuzzification process by entering the data obtained into the form of a hexagonal fuzzy number $\tilde{a}_{hexa} = (a, b, \alpha_1, \alpha_2, \beta_1, \beta_2)$.
d)	Construct hexagonal fuzzy soft sets for patients and diseases.
e)	Represent the fuzzy soft sets as hexagonal fuzzy soft matrices.
f)	Perform matrices multiplication using hexagonal fuzzy arithmetic.
g)	Convert the resulting matrices into parametric form.
h)	Compute membership values using the reduction formula.
i)	Calculate the diagnosis score SK_1 and SK_2.
j)	Determine the final diagnosis by selecting the maximum score difference.

4 Result and discussion

Based on information obtained from interviews with doctors and patients at a clinic in Pekanbaru (attached in Appendices 1 and 2), there were three patients represented by $P = \{p_1, p_2, p_3\}$. They exhibited symptoms such as cough, runny nose, fever, and sore throat, which are denoted as $S = \{s_1, s_2, s_3, s_4\}$. Suppose that the possible conditions associated with these symptoms are denoted by $D = \{d_1, d_2, d_3\}$, di mana d_1, d_2, and d_3 represent Respiratory Tract Infections, Common Cold, and Pharyngitis. Furthermore, Table 2 and Table 3 include data from provided by general practitioners showing real number intervals that describe how much influence the symptoms have on the disease. These intervals range from $[0, 10]$, where 0 indicates no influence of symptoms on the disease, and 10 indicates that the influence of symptoms on the disease is very high.

Table 2: Table of symptom influence levels on disease.

Symptoms	Interval of Symptom Influence Level		
	ISPA	*Common Cold*	Pharyngitis
Cough	7 – 9	5 – 7	6 – 8
Cold	7 – 9	6 – 9	5 – 6
Fever	6 – 8	4 – 6	6 – 8
Sore throat	5 – 7	3 – 5	8 – 10

Source: Keluargaku Primary Care Clinic

Table 3: Table of patient pain level intervals for symptoms.

Patient	Pain level for symptoms			
	Cough	Cold	Fever	Sore Throat
A	2 – 4	4 – 6	4 – 6	5 – 7
B	6 – 8	5 – 8	7 – 9	0 – 2
C	8 – 10	7 – 9	4 – 6	7 – 9

Source: Keluargaku Primary Care Clinic

Based on the algorithm obtained in the previous section, the diagnosis of the disease in the three patients will be continued as follows.

a) Do the fuzzification process by converting data from Table 2 and Table 3 into hexagonal fuzzy numbers $\tilde{a}_{hexa} = (a, b, \alpha_1, \alpha_2, \beta_1, \beta_2)$to obtain Table 4 and Table 5 as follows

Table 4: Table of symptom impact level intervals on disease.

Gejala	ISPA						Common Cold						Pharyngitis					
	a	b	α_1	α_2	β_1	β_2	a	b	α_1	α_2	β_1	β_2	a	b	α_1	α_2	β_1	β_2
Cough	7	9	0.5	0.5	0.5	0.5	5	7	0.5	0.5	0.5	0.5	6	8	0.5	0.5	0.5	0.5
Cold	7	9	0.5	0.5	0.5	0.5	6	9	0.5	0.5	0.5	0.5	5	6	0.5	0.5	0.5	0.5
Fever	6	8	0.5	0.5	0.5	0.5	4	6	0.5	0.5	0.5	0.5	6	8	0.5	0.5	0.5	0.5
Sore Throat	5	7	0.5	0.5	0.5	0.5	3	5	0.5	0.5	0.5	0.5	8	10	0.5	0.5	0.5	0.5

Table 5: Table of pain level intervals for patients regarding symptoms.

Patient	Cough						Cold						Fever						Sore Throat					
	a	b	α_1	α_2	β_1	β_2	a	b	α_1	α_2	β_1	β_2	a	b	α_1	α_2	β_1	β_2	a	b	α_1	α_2	β_1	β_2
A	2	4	0.5	0.5	0.5	0.5	4	6	0.5	0.5	0.5	0.5	4	6	0.5	0.5	0.5	0.5	5	7	0.5	0.5	0.5	0.5
B	6	8	0.5	0.5	0.5	0.5	5	8	0.5	0.5	0.5	0.5	7	9	0.5	0.5	0.5	0.5	0	2	1	1	1	1
C	8	10	1	1	1	1	7	9	0.5	0.5	0.5	0.5	4	6	0.5	0.5	0.5	0.5	7	9	0.5	0.5	0.5	0.5

b) Input the hexagonal fuzzy number data in Tables 2 and 3 into the hexagonal fuzzy number membership function defined in Definition 2.6 and constructed into a hexagonal fuzzy soft set form. The ordered pair of hexagonal fuzzy soft sets $(\tilde{F}, S)$ atas P on P in Table 5 represents the patient's pain level for symptoms expressed as

$$(\tilde{F}, S) = \left\{ \begin{array}{l} (s_1, \{(A, 0.753, 0.743, 0.436, 0.468, 0.748, 0.244), (B, 0.818, 0.783, 0.622, 0.587, 0.432, \\ 0.366), (C, 0.813, 0.8845, 0.7265, 0.6255, 0.4475, 0.3735)\}), \\ (s_2, \{(A, 0.637, 0.527, 0.529, 0.486, 0.511, 0.371), (B, 0.832, 0.783, 0.525, 0.656, 0.681, \\ 0.277), (C, 0.825, 0.757, 0.671, 0.567, 0.452, 0.371)\}), \\ (s_3, \{(A, 0.637, 0.527, 0.529, 0.486, 0.511, 0.371), (B, 0.825, 0.757, 0.671, 0.567, 0.452, \\ 0.371), (C, 0.637, 0.527, 0.529, 0.486, 0.511, 0.371)\}), \\ (s_4, \{(A, 0.712, 0.646, 0.549, 0.441, 0.648, 0.375), (B, 0.0644, 0.8235, 0.6925, 0.679, \\ 0.03075, 0.4785), (C, 0.825, 0.757, 0.671, 0.567, 0.452, 0.371)\}) \end{array} \right\}$$

After that, the ordered hexagonal fuzzy soft set pair $(\tilde{F}, D)$ on S given in Table 4 represents the influence on the disease, so it can be expressed as

$$(\tilde{F}, D) = \left\{ \begin{array}{l} (d_1, \{(s_1, 0.825, 0.757, 0.671, 0.567, 0.452, 0.371),\ (s_2, 0.825, 0.757, 0.671, 0.567, 0.452, 0.371), \\ \quad (s_3, 0.818, 0.783, 0.622, 0.587, 0.432, 0.366),\ (s_4, 0.712, 0.646, 0.549, 0.441, 0.648, 0.375)\}), \\ (d_2, \{(s_1, 0.712, 0.646, 0.549, 0.441, 0.648, 0.375),\ (s_2, 0.762, 0.679, 0.642, 0.518, 0.589, 0.377), \\ \quad (s_3, 0.637, 0.527, 0.529, 0.486, 0.511, 0.371),\ (s_4, 0.573, 0.548, 0.547, 0.445, 0.644, 0.342)\}), \\ (d_3, \{(s_1, 0.818, 0.783, 0.622, 0.587, 0.432, 0.366),\ (s_2, 0.804, 0.727, 0.617, 0.544, 0.563, 0.147), \\ \quad (s_3, 0.818, 0.783, 0.622, 0.587, 0.432, 0.366),\ (s_4, 0.721, 0.676, 0.652, 0.649, 0.546, 0.447)\}) \end{array} \right\}$$

Then, state the complement of the hexagonal fuzzy soft set $(\tilde{F}, S)$ and $(\tilde{F}, D)$ based on Definition 2.9, namely the hexagonal fuzzy soft complement denoted by $(\tilde{F}^c, S)$ and $(\tilde{F}^c, D)$ which states the non-possession of patients towards symptoms and the non-possession of symptoms towards diseases, so that it can be written as follows:

$$(\tilde{F}^c,S)=\left\{\begin{array}{l}(s_1,\{(A,(0.247,0.257,0.564,0.532,0.252,0.756)),(B,(0.182,0.217,0.378,0.413,0.568,0.634)),(C,(0.187,0.155,0.274,0.375,0.553,0.627))\}),\\(s_2,\{(A,(0.363,0.473,0.471,0.514,0.489,0.629)),(B,(0.168,0.217,0.475,0.344,0.319,0.723)),(C,(0.175,0.243,0.329,0.433,0.548,0.629))\}),\\(s_3,\{(A,(0.363,0.473,0.471,0.514,0.489,0.629)),(B,(0.175,0.243,0.329,0.433,0.548,0.629)),(C,(0.363,0.473,0.471,0.514,0.489,0.629))\}),\\(s_4,\{(A,(0.288,0.354,0.451,0.559,0.352,0.625)),(B,(0.936,0.177,0.308,0.321,0.969,0.522)),(C,(0.175,0.243,0.329,0.433,0.548,0.629))\})\end{array}\right\}$$

$$(\tilde{F}^c,D)=\left\{\begin{array}{l}(d_1,\{(s_1,(0.175,0.243,0.329,0.433,0.548,0.629)),(s_2,(0.175,0.243,0.329,0.433,0.548,0.629)),\\\quad(s_3,(0.182,0.217,0.378,0.413,0.568,0.634)),(s_4,(0.288,0.354,0.451,0.559,0.352,0.625))\}),\\(d_2,\{(s_1,(0.288,0.354,0.451,0.559,0.352,0.625)),(s_2,(0.238,0.321,0.358,0.482,0.411,0.623)),\\\quad(s_3,(0.363,0.473,0.471,0.514,0.489,0.629)),(s_4,(0.427,0.452,0.453,0.555,0.356,0.658))\}),\\(d_3,\{(s_1,(0.182,0.217,0.378,0.413,0.568,0.634)),(s_2,(0.196,0.273,0.383,0.456,0.437,0.853)),\\\quad(s_3,(0.182,0.217,0.378,0.413,0.568,0.634)),(s_4,(0.279,0.324,0.348,0.351,0.454,0.553))\})\end{array}\right\}$$

c) Represent the four hexagonal fuzzy soft sets in the previous step, namely $(\tilde{F},S)$, $(\tilde{F},D)$,$(\tilde{F}^c,S)$, and $(\tilde{F}^c,D)$ into hexagonal fuzzy soft matrices based on Definition 5,denoted by M_{HxFSM},N_{HxFSM}, M^c_{HxFSM} and N^c_{HxFSM} as follows.

$$N_{HxFSM}=\begin{bmatrix}(0.753,0.743,0.436,0.468,0.748,0.244)&(0.818,0.783,0.622,0.587,0.432,0.366)&(0.813,0.845,0.726,0.625,0.447,0.373)\\(0.637,0.527,0.529,0.486,0.511,0.371)&(0.832,0.783,0.525,0.656,0.681,0.277)&(0.825,0.757,0.671,0.567,0.452,0.371)\\(0.637,0.527,0.529,0.486,0.511,0.371)&(0.825,0.757,0.671,0.567,0.452,0.371)&(0.637,0.527,0.529,0.486,0.511,0.371)\\(0.712,0.646,0.549,0.441,0.648,0.375)&(0.644,0.823,0.692,0.679,0.307,0.478)&(0.825,0.757,0.671,0.567,0.452,0.371)\end{bmatrix}$$

$$N_{HxFSM}=\begin{bmatrix}(0.753,0.743,0.436,0.468,0.748,0.244)&(0.818,0.783,0.622,0.587,0.432,0.366)&(0.813,0.845,0.726,0.625,0.447,0.373)\\(0.637,0.527,0.529,0.486,0.511,0.371)&(0.832,0.783,0.525,0.656,0.681,0.277)&(0.825,0.757,0.671,0.567,0.452,0.371)\\(0.637,0.527,0.529,0.486,0.511,0.371)&(0.825,0.757,0.671,0.567,0.452,0.371)&(0.637,0.527,0.529,0.486,0.511,0.371)\\(0.712,0.646,0.549,0.441,0.648,0.375)&(0.644,0.823,0.692,0.679,0.307,0.478)&(0.825,0.757,0.671,0.567,0.452,0.371)\end{bmatrix}$$

$$N^c_{HxFSM}=\begin{bmatrix}(0.2470, 0.2570, 0.5640, 0.5320, 0.2520, 0.7560)&(0.1820, 0.2170, 0.3780, 0.4130, 0.5680, 0.6340)&(0.1870, 0.1155, 0.2735, 0.3750, 0.5525, 0.6265)\\(0.3630, 0.4730, 0.4710, 0.5140, 0.4890, 0.6290)&(0.1680, 0.2170, 0.4750, 0.3440, 0.3190, 0.7230)&(0.1750, 0.2430, 0.3290, 0.4330, 0.5480, 0.6290)\\(0.3630, 0.4730, 0.4710, 0.5140, 0.4890, 0.6290)&(0.1750, 0.2430, 0.3290, 0.4330, 0.5480, 0.6290)&(0.3630, 0.4730, 0.4710, 0.5140, 0.4890, 0.6290)\\(0.2880, 0.3540, 0.4510, 0.5590, 0.3520, 0.6250)&(0.9356, 0.1765, 0.3075, 0.3210, 0.9692, 0.5215)&(0.1750, 0.2430, 0.3290, 0.4330, 0.5480, 0.6290)\end{bmatrix}$$

$$M^c_{HxFSM}=\begin{bmatrix}(0.175,0.243,0.329,0.433,0.548,0.629)&(0.175,0.243,0.329,0.433,0.548,0.629)&(0.182,0.217,0.378,0.413,0.568,0.634)&(0.288,0.354,0.451,0.559,0.352,0.625)\\(0.288,0.354,0.451,0.559,0.352,0.625)&(0.238,0.321,0.358,0.482,0.411,0.623)&(0.363,0.473,0.471,0.514,0.489,0.629)&(0.427,0.452,0.453,0.555,0.356,0.658)\\(0.182,0.217,0.378,0.413,0.568,0.634)&(0.196,0.273,0.383,0.456,0.437,0.853)&(0.182,0.217,0.378,0.413,0.568,0.634)&(0.279,0.324,0.348,0.351,0.454,0.553)\end{bmatrix}$$

d) Then, perform the fourth matrix relation multiplication as follows

$$\sigma_{HxFSM_1}=[M_{HxFSM}\times N_{HxFSM}]=\begin{bmatrix}(2.171,1.787,3.187,2.838,3.137,1.999)&(2.496,2.311,3.888,3.598,2.807,2.292)&(2.457,2.112,3.867,3.326,2.923,2.248)\\(1.829,1.464,2.756,2.414,3.099,1.807)&(2.109,1.885,3.357,3.064,2.935,2.081)&(2.082,1.748,3.364,2.844,2.990,2.040)\\(2.159,1.810,3.190,2.982,3.125,1.914)&(2.477,2.332,3.887,3.750,2.793,2.165)&(2.441,2.134,3.868,3.491,2.915,2.125)\end{bmatrix}$$

which denotes patients with symptoms and diseases,

$$\sigma_{HxFSM_2}=[M_{HxFSM}\times N^c_{HxFSM}]$$

$$\begin{bmatrix}(1.009,1.156,2.388,2.385,1.908,2.545)&(1.088,0.622,1.845,1.754,2.426,2.361)&(0.722,0.801,1.693,1.886,2.113,2.289)\\(0.855,0.936,2.053,2.018,1.835,2.200)&(0.904,0.511,1.611,1.497,2.185,2.025)&(0.602,0.626,1.434,1.580,1.931,1.959)\\(1.002,1.159,2.388,2.450,1.913,2.477)&(1.095,0.631,1.880,1.870,2.403,2.355)&(0.719,0.804,1.697,1.929,2.114,2.258)\end{bmatrix}$$

which denotes patients with symptoms but no disease,

$$\sigma_{HxFSM_3} = [M^c_{HxFSM} \times N_{HxFSM}]$$

$$\begin{bmatrix} (0.568, 0.656, 1.447, 1.634, 1.872, 1.953) & (0.623, 0.835, 1.755, 2.024, 2.010, 2.333) & (0.643, 0.774, 1.723, 1.912, 1.932, 2.231) \\ (0.910, 0.979, 1.878, 2.058, 1.910, 2.145) & (1.010, 1.261, 2.286, 2.558, 1.914, 2.544) & (1.018, 1.138, 2.226, 2.394, 1.865, 2.439) \\ (0.580, 0.633, 1.444, 1.490, 1.884, 2.038) & (0.642, 0.814, 1.756, 1.872, 2.018, 2.460) & (0.659, 0.752, 1.722, 1.747, 1.940, 2.354) \end{bmatrix}$$

which denotes asymptomatic patients with the disease, and

$$\sigma_{HxFSM_4} = [M^c_{HxFSM} \times N^c_{HxFSM}]$$

$$\begin{bmatrix} (0.252, 0.401, 0.978, 1.143, 1.083, 1.503) & (0.373, 0.231, 0.801, 0.915, 1.135, 1.303) & (0.178, 0.274, 0.696, 0.856, 1.012, 1.212) \\ (0.406, 0.621, 1.313, 1.510, 1.156, 1.848) & (0.557, 0.342, 1.036, 1.171, 1.376, 1.640) & (0.298, 0.449, 0.956, 1.163, 1.194, 1.541) \\ (0.259, 0.398, 0.978, 1.078, 1.078, 1.571) & (0.366, 0.223, 0.767, 0.798, 1.158, 1.309) & (0.181, 0.270, 0.693, 0.813, 1.011, 1.242) \end{bmatrix}$$

indicating patients without symptoms and without disease.

e) After obtaining the results of multiplying the four matrix relations, convert the matrix entries into their parametric form based on equations $(1-4)$ denoted by $\sigma_{HxFSM_1}(r), \sigma_{HxFSM_2}(r), \sigma_{HxFSM_3}(r)$, and $\sigma_{HxFSM_4}(r)$ as follows.

$$\sigma_{HxFSM_1} = \begin{bmatrix} (-3.60+5.17r,\ -4.20+6.37r,\ 8.06-6.27r,\ 6.92-4.00r) & (-4.96+7.20r,\ -5.22+7.72r,\ 7.92-5.61r,\ 7.41-4.58r) & (-4.67+6.52r,\ -5.28+7.74r,\ 7.95-5.84r,\ 7.51-4.96r) \\ (-3.29+4.83r,\ -3.64+5.53r,\ 7.63-6.20r,\ 6.43-3.79r) & (-4.33+6.14r,\ -4.62+6.73r,\ 7.76-5.87r,\ 6.84-4.04r) & (-3.35+5.69r,\ -3.87+6.73r,\ 7.70-5.98r,\ 6.81-4.20r) \\ (-3.97+5.96r,\ -4.22+6.38r,\ 8.10-6.29r,\ 6.90-3.88r) & (-5.16+7.50r,\ -5.30+7.77r,\ 7.90-5.59r,\ 7.27-4.33r) & (-4.91+6.98r,\ -5.28+7.73r,\ 7.97-5.83r,\ 7.17-4.25r) \end{bmatrix}$$

$$\sigma_{HxFSM_2} = \begin{bmatrix} (-3.76+4.77r,\ -3.77+4.78r,\ 4.97-3.82r,\ 5.61-5.09r) & (-2.51+3.51r,\ -2.60+3.69r,\ 5.47-4.85r,\ 5.41-4.72r) & (-2.86+3.77r,\ -2.66+3.39r,\ 5.03-4.23r,\ 5.20-4.58r) \\ (-3.22+4.04r,\ -3.25+4.11r,\ 4.61-3.67r,\ 4.99-3.83r) & (-2.65+3.74r,\ -2.66+3.76r,\ 5.44-4.81r,\ 5.39-4.71r) & (-2.41+3.16r,\ -2.27+2.87r,\ 4.49-3.86r,\ 4.52-3.92r) \\ (-3.84+4.90r,\ -3.77+4.78r,\ 4.99-3.83r,\ 5.55-4.95r) & (-2.65+3.74r,\ -2.66+3.76r,\ 5.44-4.81r,\ 5.39-4.71r) & (-2.91+3.86r,\ -2.67+3.39r,\ 5.03-4.23r,\ 5.18-4.52r) \end{bmatrix}$$

$$\sigma_{HxFSM_3} = \begin{bmatrix} (-2.51+3.27r,\ -2.33+2.89r,\ 4.40-3.74r,\ 4.48-3.91r) & (-3.15+4.05r,\ -2.89+3.51r,\ 4.86-4.02r,\ 5.18-4.67r) & (-2.99+3.82r,\ -2.80+3.45r,\ 4.64-3.86r,\ 4.94-4.46r) \\ (-3.03+4.12r,\ -2.85+3.76r,\ 4.80-3.82r,\ 4.87-3.73r) & (-3.56+5.15r,\ -3.49+5.12r,\ 4.89-3.76r,\ 5.09-3.54r) & (-3.43+4.45r,\ -3.44+4.45r,\ 4.75-3.63r,\ 5.04-3.77r) \\ (-2.35+2.98r,\ -2.31+2.89r,\ 4.40-3.77r,\ 4.55-4.08r) & (-2.99+3.74r,\ -2.87+3.51r,\ 4.85-4.04r,\ 5.29-4.92r) & (-2.81+3.49r,\ -2.78+3.44r,\ 4.63-3.88r,\ 5.05-4.71r) \end{bmatrix}$$

$$\sigma_{HxFSM_4} = \begin{bmatrix} (-1.15+2.06r,\ -1.06+1.86r,\ 2.14-2.17r,\ 2.56-2.29r) & (-1.23+1.97r,\ -1.37+1.83r,\ 1.96-2.27r,\ 2.13-2.61r) & (-1.28+1.64r,\ -1.19+1.46r,\ 1.97-2.00r,\ 2.17-2.42r) \\ (-1.11+2.01r,\ -0.89+1.51r,\ 2.31-2.31r,\ 3.00-2.70r) & (-1.25+1.81r,\ -1.47+1.81r,\ 2.55-2.75r,\ 2.81-3.28r) & (-1.19+1.44r,\ -1.08+1.37r,\ 2.38-2.39r,\ 2.73-3.08r) \\ (-1.13+2.01r,\ -0.99+1.79r,\ 2.13-2.16r,\ 2.63-3.14r) & (-1.28+1.88r,\ -1.42+1.87r,\ 2.12-2.32r,\ 2.28-2.62r) & (-1.30+1.66r,\ -1.21+1.51r,\ 1.99-2.02r,\ 2.21-2.48r) \end{bmatrix}$$

f) Do matrix reduction on the parametric entries to obtain membership values and simplify the calculation based on Definition 3.1, denoted as $MV_{1HxFSM}(r), MV_{2HxFSM}(r)$, $MV_{3HxFSM}(r)$, and $MV_{4HxFSM}(r)$ so that it can be expressed as follows:

$$MV_{1HxFSM}(r) = \begin{bmatrix} (22.79-21.81r) & (25.52-25.11r) & (25.42-25.06r) \\ (20.99-20.35r) & (23.54-22.78r) & (21.73-22.60r) \\ (23.19-22.52r) & (25.63-25.19r) & (25.33-24.79r) \end{bmatrix}$$

$$MV_{2HxFSM}(r) = \begin{bmatrix} (21.80-21.02r) & (20.91-20.09r) & (21.29-20.53r) \\ (22.28-21.45r) & (21.19-20.46r) & (21.74-20.98r) \\ (21.91-21.06r) & (22.31-21.45r) & (21.84-21.00r) \end{bmatrix}$$

$$MV_{3HxFSM}(r) = \begin{bmatrix} (20.38-19.54r) & (22.18-21.30r) & (21.85-20.95r) \\ (21.40-20.58r) & (21.94-21.08r) & (21.53-20.74r) \\ (20.98-20.14r) & (22.28-21.42r) & (21.87-20.97r) \end{bmatrix}$$

$$MV_{4HxFSM}(r) = \begin{bmatrix} (9.47-9.08r) & (9.14-8.72r) & (9.03-8.61r) \\ (9.56-9.13r) & (9.46-9.02r) & (9.31-8.87r) \\ (9.28-8.84r) & (9.36-8.92r) & (9.28-8.83r) \end{bmatrix}$$

g) After obtaining the membership values of the hexagonal fuzzy soft matrices, the diagnosis scores are calculated using Definition 3.2 by subtracting $MV(\tilde{X}_1) - MV(\tilde{X}_3)$ and $MV(\tilde{X}_2) - MV(\tilde{X}_4)$.Then, by substituting $r = 0.2$ and $r = 0.5$, the diagnosis scores of the matrix with real number entries are obtained. The results are given as follows

$$S_{K_1} = MV(\tilde{X}_1) - MV(\tilde{X}_3) = \begin{bmatrix} (2.41-2.28r) & (3.34-3.81r) & (3.57-4.11r) \\ (-0.41+0.23r) & (1.61-1.69r) & (0.19-1.85r) \\ (2.22-2.38r) & (3.35-3.77r) & (3.46-3.82r) \end{bmatrix}$$

$$S_{K_2} = MV(\tilde{X}_2) - MV(\tilde{X}_4) = \begin{bmatrix} (12.33-11.93r) & (11.77-11.37r) & (12.26-11.92r) \\ (12.72-12.32r) & (11.73-11.44r) & (12.42-12.11r) \\ (12.63-12.22r) & (12.95-12.53r) & (12.56-12.16r) \end{bmatrix}$$

Substitute $r = 0.2$

$$S_{K_1} = MV(\tilde{X}_1) - MV(\tilde{X}_3) = \begin{bmatrix} 1.95 & 2.57 & 2.74 \\ -0.37 & 1.27 & -0.18 \\ 1.74 & 2.59 & 2.69 \end{bmatrix}$$

$$S_{K_2} = MV(\tilde{X}_2) - MV(\tilde{X}_4) = \begin{bmatrix} 9.94 & 9.50 & 9.87 \\ 10.25 & 9.44 & 10.00 \\ 10.18 & 10.44 & 10.13 \end{bmatrix}$$

Substitute $r = 0.5$

$$S_{K_1} = MV(\tilde{X}_1) - MV(\tilde{X}_3) = \begin{bmatrix} 1.27 & 1.43 & 1.51 \\ -0.30 & 0.76 & -0.74 \\ 1.03 & 1.58 & 1.82 \end{bmatrix}$$

$$S_{K_2} = MV(\tilde{X}_2) - MV(\tilde{X}_4) = \begin{bmatrix} 6.36 & 6.09 & 6.30 \\ 6.56 & 6.00 & 6.37 \\ 6.52 & 6.68 & 6.48 \end{bmatrix}$$

h) Thus, the final diagnosis result for the patient is obtained.
For $r = 0.2$

$$S_{KD} = \max[S_{K_1} - S_{K_2}] = \begin{bmatrix} -7.99 & -6.92 & -7.13 \\ -10.62 & -8.17 & -10.18 \\ -8.44 & -7.85 & -7.44 \end{bmatrix}$$

For $r = 0.5$

$$S_{KD} = \max[S_{K_1} - S_{K_2}] = \begin{bmatrix} -5.09 & -4.66 & -4.79 \\ -6.86 & -5.24 & -7.10 \\ -5.49 & -5.10 & -4.67 \end{bmatrix}$$

Consider the maximum value in each row of the S_{KD} matrix and based on the given disease diagnosis classification, it can be concluded that patient A in the first row and patient B in the second row have the common cold, and patient C in the third row has pharyngitis. The diagnosis results indicate that Patient A and Patient B are classified as having the common cold, while Patient C is diagnosed with pharyngitis. This outcome is consistent with the symptom intensity patterns, particularly sore throat dominance in Patient C, which is strongly associated with pharyngitis in the hexagonal fuzzy representation. The use of hexagonal fuzzy numbers allows asymmetric modeling of symptom intensity, which improves sensitivity to small variations. Furthermore, the inclusion of the error variable ε increases robustness against subjective uncertainty in patient reported data. Although the dataset is limited to three patients, the results demonstrate that the proposed method is effective as a proof of concept diagnostic support tool, rather than a clinical replacement.

5 Conclusion

This study proposes a hexagonal fuzzy soft matrices based approach for medical diagnosis under uncertainty. The integration of hexagonal fuzzy numbers with fuzzy soft matrices enables more flexible modeling of symptom intensity and subjective uncertainty. The results demonstrate that the proposed method can distinguish diseases with overlapping symptoms more sensitively than conventional fuzzy models. However, this study is limited by a small dataset and lacks clinical validation. Future research will focus on larger datasets, parameter optimization, and comparative analysis with existing diagnostic methods.

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