Application of Fuzzy Pentagonal Number Matrix with Robust Ranking Defuzzification Method in Disease Diagnosis

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Abstract. Among the arithmetic alternatives for Pentagonal Fuzzy Numbers (PFN) proposed by different authors, addition, subtraction, and scalar multiplication are generally defined in a similar manner. In contrast, multiplication and division are defined differently across the literature. However, the arithmetic offered for any pentagonal fuzzy number does not necessarily have an inverse, and the application of PFN matrix with fuzzy soft sets and fuzzy soft matrix is done separately. Therefore, this paper will present the arithmetic of pentagonal fuzzy numbers that yields an inverse and will directly apply the application of pentagonal fuzzy number matrix with fuzzy soft sets and fuzzy soft matrix simultaneously. By applying this concept, two pentagonal fuzzy number matrices were formed. The relationship between matrices is calculated, and the robust ranking values are used to predict the disease diagnosis. From the calculation example, the results obtained are consistent with the medical diagnosis.

Keywords: Application of PFN, fuzzy soft matrix, fuzzy soft sets, medical diagnosis.

1 Introduction

Fuzzy logic was first introduced by Zadeh [1] through his 1965 paper on fuzzy set theory. The term fuzzy refers to something vague or unclear. Fuzzy logic is not an imprecise logic, but rather a logic used to describe vagueness. Fuzzy logic is an extension of crisp logic. If in crisp logic the value of a true item is 1 and the value of a false item is 0, then in fuzzy logic an item can be both true and false based on its degree of membership, which ranges from 0 to 1.

A fuzzy set is a set of ordered pairs of an object and its membership value, and it is a new tool for solving problems related to sets that do not have a clear definition. Fuzzy sets then continued to develop, one of which became pentagonal fuzzy numbers. In general, a pentagonal fuzzy number is a subset of Riil Numbers that have five parameters as introduced in previous studies [2], [3], [4], [5], and [6]. Researchers define the arithmetic of fuzzy pentagonal numbers differently. Panda and Pal

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[4] as well as Pathinathan and Ponnivalavan [5] have defined the operations of addition, subtraction, multiplication, and scalar multiplication on pentagonal fuzzy numbers element-wise, but they did not define the division operation. The research reviewed by Mondal and Mandal [3] defines addition on pentagonal fuzzy numbers by adding elements element-wise and defines subtraction on pentagonal fuzzy numbers by subtracting elements in reverse order, but it does not define the multiplication and division operations. Arfina and Mashadi [2] as well as Tilopa and Mashadi [6] offer an alternative method for multiplication, division, and inverse arithmetic for any pentagonal fuzzy number, so that the goal of alternative multiplication, division, and inverse arithmetic is to ensure that any pentagonal fuzzy number $\tilde{a}_{PFN}$ exists uniquely $(\tilde{a}_{PFN})^{-1} = \frac{1}{\tilde{a}_{PFN}}$ such that $\tilde{a}_{PFN} \otimes \frac{1}{\tilde{a}_{PFN}} = \tilde{i}_{PFN}$. Furthermore, the constructed arithmetic is also used to determine the inverse of a pentagonal fuzzy number matrix using a modification of elementary row operations resulting in $\tilde{A}_{PFN} \otimes \frac{1}{\tilde{A}_{PFN}} = \tilde{I}_{PFN}$.

Aggarwal [7] define diagnosis as the process of identifying a disease based on a patient's symptoms, medical history, and physical test results. To determine the patient's illness, the doctor first asks about the symptoms the patient is experiencing. The symptoms experienced by the patient will point toward the diseases the patient might be suffering from. Certain diseases can be easily identified due to their characteristic symptoms that are not present in other diseases, but some diseases have similar symptoms, leading to problems and uncertainty in the disease identification process.

The similarity of symptoms between diseases is one factor that can lead to misdiagnosis in patients. It is not uncommon for more than one diagnosis to be required to obtain accurate results regarding the patient's illness. Common Cold, Pharyngitis, and Acute Respiratory Tract Infection (ARTI) are often misidentified because of the similarity of symptoms found in all three diseases, namely cough, runny nose, fever, and sore throat. Uncertainty also arises from the descriptions of symptom severity provided by patients. Therefore, to overcome the problem of uncertainty in diagnosing diseases, appropriate steps are needed to obtain accurate results.

As science and knowledge have advanced, the application of fuzzy concepts has extended to various fields, including the medical field for diagnosing diseases. Fuzzy logic was first applied in the medical field by Sanchez [8]. The method used by Sanchez [8] known as the Sanchez method and served as a reference for many subsequent studies. Sanchez's method was generalized by De et al. [9] to Intuitionistic Fuzzy Sets (IFS), which represent the degree of membership and non-membership of an element. Then Meenakshi and Kaliraja [10] applied the Sanchez Method to disease diagnosis using a representation of fuzzy interval number matrices. Based on previous research, the application of triangular fuzzy numbers in disease diagnosis is discussed. In research conducted by Elizabeth and Sujatha [11], by combining the concepts of soft sets and triangular fuzzy numbers, triangular fuzzy number matrices $\tilde{R}_O$ and $\tilde{R}_S$ were formed, representing the relationships between disease symptoms and patients, respectively. Then, the relationship between these matrices is calculated and their robust ranking is defuzzified to determine the disease suffered by each patient. Soft sets were first introduced by Molodtsov [12] as a mathematical tool to handle uncertainty caused by a lack of parameters.

This article is an extension of research conducted by Elizabeth and Sujatha [11]. In previous research, triangular fuzzy number arithmetic with the Mean defuzzification method was used to diagnose a disease. In this study, the pentagonal fuzzy number arithmetic used by Arfina and Mashadi [2] as well as Tilopa and Mashadi [6] which satisfies the identity property by writing the pentagonal fuzzy number in the form $\tilde{a}_{PFN} = (a_m, \alpha_{l1}, \alpha_{l2}, \beta_{r1}, \beta_{r2})$ with a_m is the center point, α_{l1} is the left spread distance from the center point a_m to the point $(a_m - \alpha_{l1})$, α_{l2} is the left spread distance from the point $(a_m - \alpha_{l1})$ to the point $(a_m - \alpha_{l1} - \alpha_{l2})$, β_{r1} is the right spread distance from the center point a_m to the point $(a_m + \beta_{r1})$, and β_{r2} is the right spread distance from the point $(a_m + \beta_{r1})$ to the point $(a_m + \beta_{r1} + \beta_{r2})$. On the other hand, the robust ranking defuzzification method has also proven effective in processing fuzzy logic based data as introduced in previous studies [11], [13], [14], [15], and [16]. This method allows for a more structured mapping of the relationship between symptoms and diseases, thus supporting a more systematic medical decision-making process. The data used is primary data sourced from doctor interviews and patient interviews at a clinic in Pekanbaru. The data collected consists of interval data on the severity of symptoms' impact on the disease and interval data on patients' pain levels related to the symptoms. This data is represented on an interval scale from 0 to 10, where 0 means no symptoms or no pain, and 10 means the symptoms are very severe or the patient's pain is very high. Then the data is presented in table form, which can be seen in Table 1 and Table 2.

This article consists of five parts. Part 1 provides an overview of the issues discussed in this article. Part 2 explains some of the theoretical foundations supporting this research. Part 3 contains an explanation of the core problem, which is the application of pentagonal fuzzy number matrices using the robust ranking method to diagnose Common Cold, Pharyngitis, and Acute Respiratory Tract Infections (ARTI). Then, Part 4 contains the conclusion of the problems that have been discussed. And finally, part 5 contains expressions of gratitude to those who supported this research.

2 Literature review

This section explains some basic definitions that are the main objects of this research, namely pentagonal fuzzy number, arithmetic of pentagonal fuzzy number, pentagonal fuzzy soft matrix, pentagonal fuzzy number matrix, robust ranking defuzzification method, and research methodology.

2.1 Pentagonal fuzzy number

According to Zadeh [1], a fuzzy set is an extension of the concept of a crisp set. In a fuzzy set, each element has a specific degree of membership. Unlike crisp sets, which use a characteristic function with values of 0 or 1, the membership function in fuzzy sets allows for values between 0 and 1. Thus, fuzzy sets encompass classical sets as a special case. The membership function of a fuzzy number is described as a curve that maps each point to a membership value within the range $[0,1]$. One method for determining these membership values is thru a function approach. Here are the definitions of fuzzy sets, pentagonal fuzzy numbers, and pentagonal fuzzy numbers in parametric form, as provided by Panda and Pal [4] as well as Pathinathan and Ponnivalavan [5].

Definition 2.1. A fuzzy set is defined as a membership function that takes values from a domain or universe and maps them into the interval [0,1]. A fuzzy set $\tilde{A}$ in the universe X is defined as follows:

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X\} \tag{1}$$

with $\mu_{\tilde{A}} : X \to [0,1]$ and $\mu_{\tilde{A}}(x)$ is its degree of membership.

Definition 2.2. A pentagonal fuzzy number denoted as $\tilde{a}_{PFN} = (a_m, \alpha_{l1}, \alpha_{l2}, \beta_{r1}, \beta_{r2})$ is a fuzzy set on $\mathbb{R}$ whose membership function must satisfy the following conditions:

1. $\tilde{a}(x)$ is upper semi continuous.
2. $\tilde{a}(x) = 0$ outside the interval $[a_m - \alpha_{l1} - \alpha_{l2}, a_m + \beta_{r1} + \beta_{r2}]$.
3. There exists a real number x in the interval $[a_m - \alpha_{l1} - \alpha_{l2}, a_m + \beta_{r1} + \beta_{r2}]$ such that
   - $\tilde{a}(x)$ is a monotonically non-decreasing function on $[a_m - \alpha_{l1} - \alpha_{l2}, a_m - \alpha_{l1}]$ and $[a_m - \alpha_{l1}, a_m]$.
   - $\tilde{a}(x)$ is a monotonically non-increasing function on $[a_m, a_m + \beta_{r1}]$ and $[a_m + \beta_{r1}, a_m + \beta_{r1} + \beta_{r2}]$.
4. $\tilde{a}(x) = 1$ for $x = a_m$.

The membership function of the pentagonal fuzzy number $\tilde{a}_{PFN}$ = $(a_m, \alpha_{l1}, \alpha_{l2}, \beta_{r1}, \beta_{r2})$ is defined by [2], [3], [4], [6], [17] as follows:

$$\mu_{\tilde{a}_{PFN}(x)} = \begin{cases} \frac{1}{2} + \frac{1}{2}\left(\frac{x - a_m + \alpha_{l1}}{\alpha_{l2}}\right), & a_m - \alpha_{l1} - \alpha_{l2} \leq x \leq a_m - \alpha_{l1} \\ 1 + \frac{1}{2}\left(\frac{x - a_m}{\alpha_{l1}}\right), & a_m - \alpha_{l1} \leq x \leq a_m \\ 1, & x = a_m \\ 1 - \frac{1}{2}\left(\frac{x - a_m}{\beta_{r1}}\right), & a_m \leq x \leq a_m + \beta_{r1} \\ \frac{1}{2} - \frac{1}{2}\left(\frac{x - a_m + \beta_{r1}}{\beta_{r2}}\right), & a_m + \beta_{r1} \leq x \leq a_m + \beta_{r1} + \beta_{r2} \\ 0, & x \text{ others} \end{cases} \tag{2}$$

The parametric form of a pentagonal fuzzy number can be written as $\tilde{a}_{PFN}(r) = [\underline{a}_2(r), \underline{a}_1(r), \overline{a}_1(r), \overline{a}_2(r)]$ with:

1. $\underline{a}_2(r) = a_m - (1-2r)\alpha_{l2} - \alpha_{l1}$
2. $\underline{a}_1(r) = a_m - (1-r)2\alpha_{l1}$
3. $\overline{a}_1(r) = a_m + (1-r)2\beta_{r1}$
4. $\overline{a}_2(r) = a_m + (1-2r)\beta_{r2} + \beta_{r1}$

A pentagonal fuzzy number on $\mathbb{R}$ defined as $\tilde{a}_{PFN}(r) = [\underline{a}_2(r), \underline{a}_1(r), \overline{a}_1(r), \overline{a}_2(r)]$ must satisfy the following conditions:

1. $\underline{a}_2(r)$ is right continuous and monotonically non-decreasing on $[0, 0.5]$

2. $\underline{a}_1(r)$ is continuous and monotonically non-decreasing on $[0.5,1]$

3. $\overline{a}_1(r)$ is continuous and monotonically non-increasing on $[0.5,1]$

4. $\overline{a}_2(r)$ is left continuous and monotonically non-increasing on $[0,0.5]$

The following illustrates the pentagonal fuzzy number in parametric form.

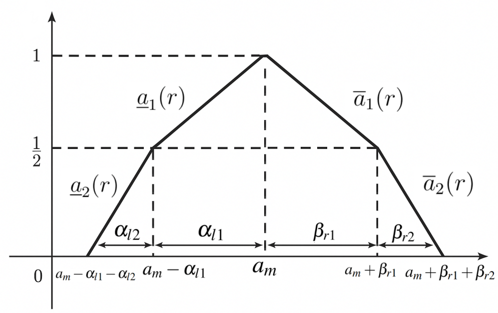

Fig. 1. Pentagonal fuzzy number $\tilde{a}_{PFN} = (a_m, \alpha_{l1}, \alpha_{l2}, \beta_{r1}, \beta_{r2})$

2.2 Arithmetic of pentagonal fuzzy number

Arithmetic of pentagonal fuzzy numbers involves mathematical steps such as addition, subtraction, multiplication, and division performed by Arfina and Mashadi [2] as well as Tilopa and Mashadi [6] considering the membership function within the interval $[0,1]$. The following will provide definitions of pentagonal fuzzy number arithmetic and pentagonal fuzzy number arithmetic in parametric form.

Definition 2.3. Suppose there are two pentagonal fuzzy numbers with $\tilde{a}_{PFN} = (a_m, \alpha_{l1}, \alpha_{l2}, \beta_{r1}, \beta_{r2})$ and $\tilde{b}_{PFN} = (b_m, \gamma_{l1}, \gamma_{l2}, \delta_{r1}, \delta_{r2})$, then the addition of two pentagonal fuzzy numbers is defined as follows:

$$\tilde{a}_{PFN} + \tilde{b}_{PFN} = (a_m + b_m, \alpha_{l1} + \gamma_{l1}, \alpha_{l2} + \gamma_{l2}, \beta_{r1} + \delta_{r1}, \beta_{r2} + \delta_{r2}) \tag{3}$$

Next, in parametric form, it will be expressed as follows:
Let $\tilde{a}_{PFN}(r) = [\underline{a}_2(r), \underline{a}_1(r), \overline{a}_1(r), \overline{a}_2(r)]$ and $\tilde{b}_{PFN}(r) = [\underline{b}_2(r), \underline{b}_1(r), \overline{b}_1(r), \overline{b}_2(r)]$, then

$$\tilde{a}_{PFN}(r) \oplus \tilde{b}_{PFN}(r) = [\underline{a}_2(r) + \underline{b}_2(r), \underline{a}_1(r) + \underline{b}_1(r), \overline{a}_1(r) + \overline{b}_1(r), \overline{a}_2(r) + \overline{b}_2(r)]$$

Definition 2.4. Suppose there are two pentagonal fuzzy numbers with $\tilde{a}_{PFN} = (a_m, \alpha_{l1}, \alpha_{l2}, \beta_{r1}, \beta_{r2})$ and $\tilde{b}_{PFN} = (b_m, \gamma_{l1}, \gamma_{l2}, \delta_{r1}, \delta_{r2})$, then the subtraction of two pentagonal fuzzy numbers is defined as follows:

$$\tilde{a}_{PFN} - \tilde{b}_{PFN} = (a_m - b_m, \alpha_{l1} + \delta_{r1}, \alpha_{l2} + \delta_{r2}, \beta_{r1} + \gamma_{l1}, \beta_{r2} + \gamma_{l2}) \tag{4}$$

Next, in parametric form, it will be expressed as follows:
Let $\tilde{a}_{PFN}(r) = [\underline{a}_2(r), \underline{a}_1(r), \overline{a}_1(r), \overline{a}_2(r)]$ and $\tilde{b}_{PFN}(r) = [\underline{b}_2(r), \underline{b}_1(r), \overline{b}_1(r), \overline{b}_2(r)]$, then

$$\tilde{a}_{PFN}(r) \ominus \tilde{b}_{PFN}(r) = [\underline{a}_2(r) - \underline{b}_2(r), \underline{a}_1(r) - \underline{b}_1(r), \overline{a}_1(r) - \overline{b}_1(r), \overline{a}_2(r) - \overline{b}_2(r)]$$

Definition 2.5. Suppose there are two pentagonal fuzzy numbers with $\tilde{a}_{PFN} = (a_m, \alpha_{l1}, \alpha_{l2}, \beta_{r1}, \beta_{r2})$ and $\tilde{b}_{PFN} = (b_m, \gamma_{l1}, \gamma_{l2}, \delta_{r1}, \delta_{r2})$, then the multiplication of two pentagonal fuzzy numbers is defined as follows:

$$\begin{aligned} \tilde{a}_{PFN} \times \tilde{b}_{PFN} = [a_m b_m, (\alpha_{l1} b_m + \gamma_{l1} a_m), (a_m \gamma_{l2} - \alpha_{l1} \gamma_{l2} + b_m \alpha_{l2} - \gamma_{l1} \alpha_{l2}), \\ (\beta_{r1} b_m + \delta_{r1} a_m), (b_m \beta_{r2} + \beta_{r1} \delta_{r2} + a_m \delta_{r2} + \delta_{r1} \beta_{r2})] \end{aligned} \tag{5}$$

Next, in parametric form, it will be expressed as follows:
Let $\tilde{a}_{PFN}(r) = [\underline{a}_2(r), \underline{a}_1(r), \overline{a}_1(r), \overline{a}_2(r)]$ and $\tilde{b}_{PFN}(r) = [\underline{b}_2(r), \underline{b}_1(r), \overline{b}_1(r), \overline{b}_2(r)]$, then

$$\begin{aligned} \tilde{a}_{PFN}(r) \otimes \tilde{b}_{PFN}(r) = [&\underline{a}_2(r) m_2(\tilde{b}_{PFN}) + \underline{b}_2(r) m_2(\tilde{a}_{PFN}) - m_2(\tilde{a}_{PFN}) m_2(\tilde{b}_{PFN}), \\ &\underline{a}_1(r) m_1(\tilde{b}_{PFN}) + \underline{b}_1(r) m_1(\tilde{a}_{PFN}) - m_1(\tilde{a}_{PFN}) m_1(\tilde{b}_{PFN}), \\ &\overline{a}_1(r) m_1(\tilde{b}_{PFN}) + \overline{b}_1(r) m_1(\tilde{a}_{PFN}) - m_1(\tilde{a}_{PFN}) m_1(\tilde{b}_{PFN}), \\ &\overline{a}_2(r) m_2(\tilde{b}_{PFN}) + \overline{b}_2(r) m_2(\tilde{a}_{PFN}) - m_2(\tilde{a}_{PFN}) m_2(\tilde{b}_{PFN})] \end{aligned}$$

with $m_1(\tilde{a}_{PFN}) = \frac{\underline{a}_1(1) + \overline{a}_1(1)}{2}$, $m_2(\tilde{a}_{PFN}) = \frac{\underline{a}_2\left(\frac{1}{2}\right) + \overline{a}_2\left(\frac{1}{2}\right)}{2}$, $m_1(\tilde{b}_{PFN}) = \frac{\underline{b}_1(1) + \overline{b}_1(1)}{2}$, and $m_2(\tilde{b}_{PFN}) = \frac{\underline{b}_2\left(\frac{1}{2}\right) + \overline{b}_2\left(\frac{1}{2}\right)}{2}$
In scalar calculations, let's say there exists $k \in \mathbb{R}$ and $\tilde{a}_{PFN}(r) = [\underline{a}_2(r), \underline{a}_1(r), \overline{a}_1(r), \overline{a}_2(r)]$, then

- For $k \geq 0$

$$k \cdot \tilde{a}_{PFN}(r) = [k\underline{a}_2(r), k\underline{a}_1(r), k\overline{a}_1(r), k\overline{a}_2(r)]$$

- For $k < 0$

$$k \cdot \tilde{a}_{PFN}(r) = [k\overline{a}_2(r), k\overline{a}_1(r), k\underline{a}_1(r), k\underline{a}_2(r)]$$

Definition 2.6. Suppose there are two pentagonal fuzzy numbers in parametric form with $\tilde{a}_{PFN}(r) = [\underline{a}_2(r), \underline{a}_1(r), \overline{a}_1(r), \overline{a}_2(r)]$ and $\tilde{b}_{PFN}(r) = [\underline{b}_2(r), \underline{b}_1(r), \overline{b}_1(r), \overline{b}_2(r)]$, then

$$\begin{aligned} \frac{\tilde{a}_{PFN}(r)}{\tilde{b}_{PFN}(r)} = \Bigg[&\frac{\overline{a}_2(r) m_2(\tilde{b}_{PFN}) - \overline{b}_2(r) m_2(\tilde{a}_{PFN}) + m_2(\tilde{a}_{PFN}) m_2(\tilde{b}_{PFN})}{(m_2(\tilde{b}_{PFN}))^2}, \\ &\frac{\overline{a}_1(r) m_1(\tilde{b}_{PFN}) - \overline{b}_1(r) m_1(\tilde{a}_{PFN}) + m_1(\tilde{a}_{PFN}) m_1(\tilde{b}_{PFN})}{(m_1(\tilde{b}_{PFN}))^2}, \\ &\frac{\underline{a}_1(r) m_1(\tilde{b}_{PFN}) - \underline{b}_1(r) m_1(\tilde{a}_{PFN}) + m_1(\tilde{a}_{PFN}) m_1(\tilde{b}_{PFN})}{(m_1(\tilde{b}_{PFN}))^2}, \\ &\frac{\underline{a}_2(r) m_2(\tilde{b}_{PFN}) - \underline{b}_2(r) m_2(\tilde{a}_{PFN}) + m_2(\tilde{a}_{PFN}) m_2(\tilde{b}_{PFN})}{(m_2(\tilde{b}_{PFN}))^2} \Bigg] \end{aligned}$$

with $m_1(\tilde{a}_{PFN}) = \frac{a_1(1)+\bar{a}_1(1)}{2}$, $m_2(\tilde{a}_{PFN}) = \frac{a_2\left(\frac{1}{2}\right)+\bar{a}_2\left(\frac{1}{2}\right)}{2}$, $m_1(\tilde{b}_{PFN}) = \frac{b_1(1)+\bar{b}_1(1)}{2}$, and $m_2(\tilde{b}_{PFN}) = \frac{b_2\left(\frac{1}{2}\right)+\bar{b}_2\left(\frac{1}{2}\right)}{2}$

2.3 Pentagonal fuzzy soft matrix

Fuzzy Soft Matrix (FSM) is a representation of Soft Set (SS), Soft Matrix (SM), and Fuzzy Soft Set (FSS) theories that can be applied to various uncertainty problems. The following definitions of soft sets are provided by Aktas and Cagman [18] as well as Babitha and Sunil [19], definitions of soft matrices by Cagman and Enginoglu [20] as well as Vijayabalaji and Ramesh [21], definitions of fuzzy soft sets by Cagman et al. [22] as well as Celik and Yamak [23], and definitions of fuzzy soft matrix by Borah et al. [24], Cagman and Enginoglu [25], as well as Neog and Dutta [26].

Definition 2.7. Let X be the universal set and E be the parameter set, $P(X)$ denotes the power set of X, and $A \subseteq E$. The ordered pair (F_A, E) is called a soft set of X, if and only if F maps E to $P(X)$. A soft set can be written as follows:

$$(F_A, E) = \{(e, F_A(e)) : e \in E, F_A(e) \in P(X)\} \tag{6}$$

Definition 2.8. Let the ordered pair (F_A, E) be a soft set of X, such that a subset of $X \times E$ is uniquely defined as follows:

$$R_A = \{(x, e) : e \in A, x \in F_A(e)\} \tag{7}$$

The form of the relation (F_A, E) with characteristic function R_A is written as

$$\chi_{R_A} : X \times E \to [0,1], \text{ given by : } \chi_{R_A}(x,e) = \begin{cases} 1, & \text{if } (x,e) \in R_A \\ 0, & \text{if } (x,e) \notin R_A \end{cases} \tag{8}$$

Let $X = \{x_1, x_2, \ldots, x_m\}$, $E = \{e_1, e_2, \ldots, e_n\}$ and $A \subseteq E$, if $a_{ij} = \chi_{R_A}(x_i, e_j)$, then the matrix can be defined as follows:

$$A_{SM} = [a_{ij}]_{m \times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \tag{9}$$

A_{SM} is called a soft matrix $m \times n$ of the soft set (F_A, E) over X.

Definition 2.9. Let X be the universal set and E be the parameter set. Let $\tilde{P}(X)$ denote the fuzzy power set of X and $A \subseteq E$. The ordered pair (F_A, E) is called a fuzzy soft set over X, if $F_A : A \to \tilde{P}(X)$ such that $F_A(e) = \emptyset$ if $e \notin A$, then the fuzzy soft set can be written as:

$$(F_A, E) = \{(e, F_A(e)) : e \in E, F_A(e) \in \tilde{P}(X)\} \tag{10}$$

Definition 2.10. Let the ordered pair (F_A, E) be a fuzzy soft set over X, then the subset of $X \times E$ is uniquely defined as follows:

$$R_A = \{(\mu_{R_A}(x,e), (x,e) : e \in A, x \in F_A(e)\} \tag{11}$$

The form of the relation (F_A, E) with characteristic function R_A is written as

$$\mu_{R_A} : X \times E \to [0,1], \text{ given by : } \mu_{R_A}(x,e) = \begin{cases} \mu_{F_A(e)}(x), & \text{if } (x,e) \in R_A \\ 0, & \text{if } (x,e) \notin R_A \end{cases} \tag{12}$$

with $\mu_{F_A(e)}(x)$ is the fuzzy membership value of x in $F_A(e)$.
Let $X = \{x_1, x_2, \ldots, x_m\}$, $E = \{e_1, e_2, \ldots, e_n\}$, and $A \subseteq E$, if $\mu_{ij} = \mu_{R_A}(x_i, e_j)$, then the matrix can be defined as follows:

$$A_{FSM} = [\mu_{ij}]_{m \times n} = \begin{bmatrix} \mu_{11} & \mu_{12} & \cdots & \mu_{1n} \\ \mu_{21} & \mu_{22} & \cdots & \mu_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \mu_{m1} & \mu_{m2} & \cdots & \mu_{mn} \end{bmatrix} \tag{13}$$

A_{FSM} is called a fuzzy soft matrix $m \times n$ of the fuzzy soft set (F_A, E) of X.

Definition 2.11. Let X be the universal set and E be the parameter set, $\tilde{P}(X)$ denotes the set of all pentagonal fuzzy numbers from X, and $A \subseteq E$. The ordered pair $(\tilde{F}_A, E)$ is called a pentagonal fuzzy soft set of X, if and only if F maps E to $\tilde{P}(X)$. Therefore, a pentagonal fuzzy soft set of X can be written as follows:

$$(\tilde{F}_A, E) = \{(e, \tilde{F}_A(e)) : e \in E, \tilde{F}_A(e) \in \tilde{P}(X)\} \tag{14}$$

Then, a pentagonal fuzzy number matrix can be formed from the pentagonal fuzzy soft set $(\tilde{F}_A, E)$, which is called a pentagonal fuzzy soft matrix and is defined as follows.

Definition 2.12. Let the ordered pair $(\tilde{F}_A, E)$ be a pentagonal fuzzy soft set over X, such that the subset of $X \times E$ is uniquely defined as follows:

$$R_A = \{\psi_{R_A}(x,e), (x,e) : e \in A, x \in \tilde{F}_A(e)\} \tag{15}$$

The form of the relation $(\tilde{F}_A, E)$ with characteristic function R is written as

$$\psi_R : X \times E \to \tilde{P}(X), \text{ given by : } \psi_{R_A}(x,e) = \begin{cases} \psi_{\tilde{F}_A(e)}(x), & \text{if } (x,e) \in R_A \\ (0,0,0,0,0), & \text{if } (x,e) \notin R_A \end{cases} \tag{16}$$

with $\psi_{\tilde{F}_A(e)}(x)$ is the pentagonal fuzzy number of x in $\tilde{F}_A(e)$.
Let $X = \{x_1, x_2, ..., x_m\}$, $E = \{e_1, e_2, ..., e_n\}$ and $A \subseteq E$, if $\tilde{a}_{ij} = \psi_R(x_i, e_j)$, then the matrix can be defined as follows:

$$\tilde{A}_{PFSM} = [\tilde{a}_{ij}]_{m \times n} = \begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \cdots & \tilde{a}_{1n} \\ \tilde{a}_{21} & \tilde{a}_{22} & \cdots & \tilde{a}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{m1} & \tilde{a}_{m2} & \cdots & \tilde{a}_{mn} \end{bmatrix} \tag{17}$$

$\tilde{A}_{PFSM}$ is called a fuzzy soft matrix $m \times n$ of the pentagonal fuzzy soft set $(\tilde{F}_A, E)$ of X.

2.4 Pentagonal fuzzy number matrix

A pentagonal fuzzy number matrix has the same concept as a real number matrix. A pentagonal fuzzy number matrix is a matrix where each element is a pentagonal fuzzy number. Its inter-element operations use the arithmetic of pentagonal fuzzy numbers, which was explained in Subsection 2.2. The definitions of pentagonal fuzzy number matrices and operations on pentagonal fuzzy number matrices given by Panda and Pal [4] as well as Pathinathan and Ponnivalavan [5] are as follows.

Definition 2.13. A pentagonal fuzzy number matrix of order $m \times n$ is defined as $\tilde{A}_{PFN} = (\tilde{a}_{ij})_{m\times n}$ with $\tilde{a}_{ij} = (a_{m_{ij}}, \alpha_{l1_{ij}}, \alpha_{l2_{ij}}, \beta_{r1_{ij}}, \beta_{r2_{ij}})$ is the ij-th element of $\tilde{A}_{PFN}$. Based on this definition, the form of a pentagonal fuzzy number matrix of order $m \times n$ is as follows:

$$\tilde{A}_{m\times n} = \begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \cdots & \tilde{a}_{1n} \\ \tilde{a}_{21} & \tilde{a}_{22} & \cdots & \tilde{a}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{m1} & \tilde{a}_{m2} & \cdots & \tilde{a}_{mn} \end{bmatrix} \tag{18}$$

Definition 2.14. Let $\tilde{A}_{PFN} = (\tilde{a}_{ij})_{m\times n}$ and $\tilde{B}_{PFN} = (\tilde{b}_{ij})_{m\times n}$ be two pentagonal fuzzy number matrices of the same order. Then the addition and subtraction operations are defined as follows:

- Addition

  $$\tilde{A}_{PFN} \oplus \tilde{B}_{PFN} = (\tilde{a}_{ij} \oplus \tilde{b}_{ij})_{m\times n}$$

  with $\tilde{a}_{ij} \oplus \tilde{b}_{ij}$ is the element in the i, j position of $\tilde{A}_{PFN} \oplus \tilde{B}_{PFN}$

- Subtraction

  $$\tilde{A}_{PFN} \ominus \tilde{B}_{PFN} = (\tilde{a}_{ij} \ominus \tilde{b}_{ij})_{m\times n} \tag{19}$$

  with $\tilde{a}_{ij} \ominus \tilde{b}_{ij}$ is the element in the i, j position of $\tilde{A}_{PFN} \ominus \tilde{B}_{PFN}$

In the addition and subtraction operations of pentagonal fuzzy number matrices, the operations between their elements are calculated using the arithmetic of addition and subtraction of pentagonal fuzzy numbers as explained in equations 3 and 4.

Definition 2.15. Let $\tilde{A}_{PFN} = (\tilde{a}_{ij})_{m\times p}$ and $\tilde{B}_{PFN} = (\tilde{b}_{ij})_{p\times n}$ be two pentagonal fuzzy number matrices. Then the multiplication operation of these two matrices is defined as follows:

$$\tilde{A}_{PFN} \otimes \tilde{B}_{PFN} = (\tilde{c}_{ij})_{m\times n} \tag{20}$$

with $\tilde{c}_{ij} = \sum_{k=1}^{p} \tilde{a}_{ik} \otimes \tilde{b}_{kj}$ for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$.
In calculating the multiplication of two pentagonal fuzzy matrices, the addition and multiplication

between pentagonal fuzzy numbers in the pentagonal fuzzy matrix multiplication operation are calculated using the arithmetic of pentagonal fuzzy numbers as explained in equations 3 and 5.

Definition 2.16. Let $\tilde{A}_{PFN} = (\tilde{a}_{ij})_{m\times n}$ be a pentagonal fuzzy number matrix. The complement of the matrix $\tilde{A}_{PFN}$ is

$$(\tilde{A}_{PFN})^c = (\tilde{I}_{PFN} - \tilde{A}_{PFN}) \tag{21}$$

with $\tilde{I}_{PFN} = [\tilde{i}_{ij}]_{m\times n}$ is a pentagonal fuzzy identity matrix with all entries being $\tilde{i}$ = (1,0,0,0,0).

Definition 2.17. Let $\tilde{A}_{PFN} = (\tilde{a}_{ij})_{m\times n}$ and $\tilde{B}_{PFN} = (\tilde{b}_{ij})_{m\times n}$ be two pentagonal fuzzy number matrices of the same order with $\tilde{a}_{ij} = (a_{m_{ij}}, \alpha_{l1_{ij}}, \alpha_{l2_{ij}}, \beta_{r1_{ij}}, \beta_{r2_{ij}})$ and $\tilde{b}_{ij} = (b_{m_{ij}}, \gamma_{l1_{ij}}, \gamma_{l2_{ij}}, \delta_{r1_{ij}}, \delta_{r2_{ij}})$. Then the maximum of matrices $\tilde{A}_{PFN}$ and $\tilde{B}_{PFN}$ is as follows:

$$\begin{aligned} max\{\tilde{A}_{PFN}, \tilde{B}_{PFN}\} &= (\sup\{\tilde{a}_{ij}, \tilde{b}_{ij}\})_{ij} \\ &= (\sup\{a_{m_{ij}}, b_{m_{ij}}\}, \sup\{\alpha_{l1_{ij}}, \gamma_{l1_{ij}}\}, \sup\{\alpha_{l2_{ij}}, \gamma_{l2_{ij}}\}, \\ &\quad \sup\{\beta_{r1_{ij}}, \delta_{r1_{ij}}\}, \sup\{\beta_{r2_{ij}}, \delta_{r2_{ij}}\})_{ij} \end{aligned} \tag{22}$$

2.5 Robust ranking defuzzification method

Defuzzification is the process of converting fuzzy values into crisp values. The defuzzification method used in this study is robust ranking defuzzification with α-cuts by Jayaraman and Jahirhussian [14] as well as Ngastiti et al. [27]. Because in this study, the fuzzy pentagonal number form $\tilde{a}_{PFN} = (a_m, \alpha_{l1}, \alpha_{l2}, \beta_{r1}, \beta_{r2})$ is used, the robust ranking of the parametric form of the fuzzy pentagonal number is given as follows.

Definition 2.18. The robust ranking for the parametric form of the pentagonal fuzzy number $\tilde{a}_{PFN}(r)$ $= [\underline{a}_2(r), \underline{a}_1(r), \overline{a}_1(r), \overline{a}_2(r)] = [a_m - (1-2r)\alpha_{l2} - \alpha_{l1}, a_m - (1-r)2\alpha_{l1}, a_m + (1-r)2\beta_{r1}, a_m + (1-$

$2r)\beta_{r2}+\beta_{r1}]$ with $0 \leq r \leq 1$ is as follows:

$$\begin{aligned}
\Re(\tilde{A}_{PFN}) &= \int_0^1 0.5\Big[\underline{a}_2(r),\underline{a}_1(r),\overline{a}_1(r),\overline{a}_2(r)\Big]dr \\
&= \int_0^1 0.5\Big[a_m-(1-2r)\alpha_{l2}-\alpha_{l1}+a_m-(1-r)2\alpha_{l1}+a_m+(1-r)2\beta_{r1}+a_m+(1-2r) \\
&\quad \beta_{r2}+\beta_{r1}\Big]dr \\
&= \int_0^1 0.5\Big[a_m-\alpha_{l2}+2r\alpha_{l2}-\alpha_{l1}+a_m-2\alpha_{l1}+2r\alpha_{l1}+a_m+2\beta_{r1}-2r\beta_{r1}+a_m+\beta_{r2}- \\
&\quad 2r\beta_{r2}+\beta_{r1}\Big]dr \\
&= 0.5\int_0^1 \Big[4a_m-\alpha_{l2}+2r\alpha_{l2}-3\alpha_{l1}+2r\alpha_{l1}+3\beta_{r1}-2r\beta_{r1}+\beta_{r2}-2r\beta_{r2}\Big]dr \\
&= 0.5\Big[4a_mr-\alpha_{l2}r+\frac{2\alpha_{l2}r^2}{2}-3\alpha_{l1}r+\frac{2\alpha_{l1}r^2}{2}+3\beta_{r1}r-\frac{2\beta_{r1}r^2}{2}+\beta_{r2}r-\frac{2\beta_{r2}r^2}{2}\Big]_0^1 \\
&= 0.5\Big[4a_m-\alpha_{l2}+\alpha_{l2}-3\alpha_{l1}+\alpha_{l1}+3\beta_{r1}-\beta_{r1}+\beta_{r2}-\beta_{r2}\Big] \\
&= 0.5\Big[4a_m-2\alpha_{l1}+2\beta_{r1}\Big] \\
&= \frac{[4a_m-2\alpha_{l1}+2\beta_{r1}]}{2} \\
\Re(\tilde{A}_{PFN}) &= 2a_m-\alpha_{l1}+\beta_{r1}
\end{aligned} \tag{23}$$

2.6 Research methodology

The research was conducted in the form of a literature study using materials from journals and textbooks related to this discussion. The stages of this research are as follows.

1. Data collection. The data used in this study is primary data obtained from clinic. The data collection process was carried out by interviewing doctors and several patients. The expected outcome is to obtain information about diseases with similar symptoms, and the data collected from patients includes the symptoms they experience, the level of pain they feel from the symptoms, and the diagnosis results provided by the doctor.

2. Fuzzification process. Fuzzy set formation is done by determining fuzzy variables and their sets. The fuzzification method for the interval $[p,q]$ with $0 \leq p < q \leq 10$ for pentagonal fuzzy number $\tilde{a}_{PFN} = (a_m, \alpha_{l1}, \alpha_{l2}, \beta_{r1}, \beta_{r2})$ is as follows:

   - $a_m = \frac{p+q}{2}$
   - $\alpha_{l1} = a_m - p$
   - $\alpha_{l2} = \frac{10-p}{10}$

- $\beta_{r1} = q - a_m$
- $\beta_{r2} = \frac{10-q}{10}$

3. Forming a pentagonal fuzzy soft set $(\tilde{F}_A, D)$ over S and a pentagonal fuzzy soft set $(\tilde{F}_B, S)$ over X.

4. Forming a pentagonal fuzzy soft matrix $(\tilde{F}_A, D)$ over S representing the symptom-disease relationship, denoted by $\tilde{P}$, and a pentagonal fuzzy soft matrix $(\tilde{F}_B, S)$ over X representing the patient-symptom relationship, denoted by $\tilde{Q}$.

5. Counting the matrix $(\tilde{P})_{mem}$ using the equation:

$$(\tilde{p}_{ij})_{m\times n} = \left(\frac{p_1}{k}, \frac{p_2}{k}, \frac{p_3}{k}, \frac{p_4}{k}, \frac{p_5}{k}\right)_{ij} \tag{24}$$

with k is the maximum of $(p_5)_{ij}$ in the matrix $\tilde{P}$
and the matrix $(\tilde{Q})_{mem}$ using the equation:

$$(\tilde{q}_{ij})_{m\times n} = \left(\frac{q_1}{l}, \frac{q_2}{l}, \frac{q_3}{l}, \frac{q_4}{l}, \frac{q_5}{l}\right)_{ij} \tag{25}$$

with l is the maximum $(q_5)_{ij}$ in the matrix $\tilde{Q}$.

6. Counting $((\tilde{P})_{mem})^c$ and $((\tilde{Q})_{mem})^c$.

7. Counting the inter-matrix relationship to obtain a disease diagnosis:

   (a) Counting $R_1 = (\tilde{Q})_{mem} \otimes (\tilde{P})_{mem}$, i.e. the relationship between patients and disease symptoms.

   (b) Counting $R_2 = (\tilde{Q})_{mem} \otimes ((\tilde{P})_{mem})^c$, which is called the relation of patients without the disease.

   (c) Counting $R_3 = ((\tilde{Q})_{mem})^c \otimes (\tilde{P})_{mem}$, which is called the relation of patients without symptoms of the disease.

   (d) Counting $R_4 = \max\{R_2, R_3\}$.

   (e) Counting $R_5 = R_1 \ominus R_4$.

   (f) Counting $R_6 = (\Re((r_5)_{ij}))_{m\times n}$, with matrix R_6 is the result of the robust ranking calculation for each element of matrix R_5 and represents the predicted diagnosis of the disease suffered by each patient.

8. Finding the maximum value in each row of matrix R_6. For example, if the maximum value in the i-th row is $(R_6)_{ij}$, then it can be concluded that the i-th patient is most likely suffering from the j-th disease.

3 Results and discussion

Diagnosis can be defined as the process of identifying a disease based on a patient's symptoms, medical history, and physical test results. There is a lot of uncertainty in disease diagnosis, including in the patient's description of the severity of symptoms. The similarity of symptoms between diseases also poses a challenge in medical diagnosis. Therefore, based on Elizabeth and Sujatha [11], the diagnosis prediction for each patient can be calculated using pentagonal fuzzy numbers.

3.1 Data source

In this study, primary data obtained from doctor interviews and patient interviews at the Keluargaku Primary Clinic in Pekanbaru were used. The data collected is in the form of interval values between 0 and 10, representing the severity of each symptom, where 0 means the symptom does not appear and 10 means the symptom is very severe. The diseases that are variables in this study are the Common Cold, Pharyngitis, and Acute Respiratory Tract Infection (ARTI). These diseases have similar symptoms, namely cough, runny nose, fever, and sore throat. These diseases are commonly suffered by patients at the Keluargaku Primary Clinic in Pekanbaru.

(i) The Common Cold is the most common upper respiratory tract illness affecting the public. According to Heikkinen and Jarvinen [28], the common cold can be caused by over 200 different genetic variants of the virus. The common cold is transmitted thru the saliva droplets of infected people or enters the body thru the eyes or mouth after touching contaminated surfaces. The common cold usually resolves within an average of 7-10 days.

(ii) Pharyngitis is an inflammation of the pharynx and surrounding tissues. Pharyngitis is caused by viral, bacterial, and fungal infections. According to Sykes et al. [29], pharyngitis caused by viruses can resolve on its own, while pharyngitis caused by bacteria or fungi requires antibiotics to heal.

(iii) ARTI is an Acute Respiratory Tract Infection caused by viral or bacterial infection. According to Gageldonk et al. [30], ARTI is a significant cause of morbidity in the general population. Most infections are caused by viruses, especially rhinoviruses, influenza viruses, and respiratory syncytial virus. ARTI can easily spread thru saliva droplets, contact with contaminated objects, and shaking hands with an infected person.

3.2 Processing real data into pentagonal fuzzy number matrices

To process primary data into a pentagonal fuzzy number matrix, the following steps are taken.

(i) Presenting an interval table of the level of influence of symptoms on the disease from doctor interview data, as well as an interval table of patients' pain levels for symptoms. The symptoms of the Common Cold, Pharyngitis, and Acute Respiratory Tract Infection (ARTI) are cough, runny nose, fever, and sore throat. From the results of doctor interviews at the Pratama Family Clinic in Pekanbaru, primary data was obtained and presented in Table 1 as follows:

Table 1: Interval of the Level of Influence of Symptoms on the Disease

Symptom	Disease		
	Common Cold	Pharyngitis	ARTI
Cough	5 – 7	6 – 8	6 – 8
Runny Nose	6 – 8	5 – 7	5 – 7
Fever	4 – 6	6 – 8	6 – 8
Sore Throat	3 – 5	6 – 8	5 – 7

Next, the patient was interviewed about symptoms of cough, runny nose, fever, and sore throat. The data collected consisted of interval values for the pain level of each symptom experienced by the patient. The value of that interval is between 0 and 10, with 0 meaning no pain and 10 meaning very high pain. Based on patient interviews, primary data was obtained and presented in Table 2 as follows:

Table 2: Pain Level Interval of Patients Regarding Symptoms

Patient	Symptom			
	Cough	Runny Nose	Fever	Sore Throat
Patient A	7 – 9	1 – 3	4 – 6	6 – 8
Patient B	6 – 8	6 – 8	7 – 9	1 – 3
Patient C	7 – 9	7 – 9	4 – 6	7 – 9

(ii) Converting each interval value in Table 1 and Table 2 into a pentagonal fuzzy number and form the pentagonal fuzzy number soft sets $(\tilde{F}_A, D)$ and $(\tilde{F}_B, S)$.

Let s_1, s_2, s_3, and s_4 represent the symptoms of cough, runny nose, fever, and sore throat, and let d_1, d_2, and d_3 be the diseases Common Cold, Pharyngitis, and Acute Respiratory Tract Infection (ARTI). Let the set x_1, x_2, and x_3 be patients A, B, and C. Let $\tilde{P}(S)$ be the set of all pentagonal fuzzy numbers from S and $\tilde{P}(X)$ be the set of all pentagonal fuzzy numbers from X. Suppose for the interval $[5,7]$ we obtain its midpoint, which is $a_m = 6$, its left spread, which is $\alpha_{l1} = 6-5 = 1$ and $\alpha_{l2} = \frac{10-5}{10} = 0.5$, and its right spread, which is $\beta_{r1} = 7-6 = 1$ and $\beta_{r2} = \frac{10-7}{10} = 0.3$, so that a pentagonal fuzzy number can be formed $\tilde{a}_{PFN} = (6,1,0.5,1,0.3)$. By taking the midpoint, left spread, and right spread, a pentagonal fuzzy number can be formed from each interval value in Table 1. Then, based on equation 14 by mapping the set of diseases to the set of all pentagonal fuzzy numbers from S, denoted by $\tilde{F}_A : D \to \tilde{P}(S)$, we obtain:
$\tilde{F}_A(d_1) = \{\langle s_1,(6,1,0.5,1,0.3)\rangle, \langle s_2,(7,1,0.4,1,0.2)\rangle, \langle s_3,(5,1,0.6,1,0.4)\rangle, \langle s_4,(4,1,0.7,1,0.5)\rangle\}$
$\tilde{F}_A(d_2) = \{\langle s_1,(7,1,0.4,1,0.2)\rangle, \langle s_2,(6,1,0.5,1,0.3)\rangle, \langle s_3,(7,1,0.4,1,0.2)\rangle, \langle s_4,(7,1,0.4,1,0.2)\rangle\}$
$\tilde{F}_A(d_3) = \{\langle s_1,(7,1,0.4,1,0.2)\rangle, \langle s_2,(6,1,0.5,1,0.3)\rangle, \langle s_3,(7,1,0.4,1,0.2)\rangle, \langle s_4,(6,1,0.5,1,0.3)\rangle\}$

Thus, a soft set of fuzzy pentagonal numbers $(\tilde{F}_A, D)$ can be formed as follows:

$$\begin{aligned}(\tilde{F}_A, D) = \{&(d_1, \{\langle s_1, (6,1,0.5,1,0.3)\rangle, \langle s_2, (7,1,0.4,1,0.2)\rangle, \langle s_3, (5,1,0.6,1,0.4)\rangle, \langle s_4, (4,1,0.7,1,0.5)\rangle\}),\\ &(d_2, \{\langle s_1, (7,1,0.4,1,0.2)\rangle, \langle s_2, (6,1,0.5,1,0.3)\rangle, \langle s_3, (7,1,0.4,1,0.2)\rangle, \langle s_4, (7,1,0.4,1,0.2)\rangle\}),\\ &(d_3, \{\langle s_1, (7,1,0.4,1,0.2)\rangle, \langle s_2, (6,1,0.5,1,0.3)\rangle, \langle s_3, (7,1,0.4,1,0.2)\rangle, \langle s_4, (6,1,0.5,1,0.3)\rangle\})\}\end{aligned}$$

Then, based on the data from Table 2, an ordered pair soft set of fuzzy pentagonal numbers $(\tilde{F}_B, S)$ from X was formed by mapping the set S to the set of all fuzzy pentagonal numbers from X, defined by: $\tilde{F}_B : S \to \tilde{P}(X)$. The mapping results obtained are as follows:
$\tilde{F}_B(s_1) = \{\langle x_1, (8,1,0.3,1,0.1)\rangle, \langle x_2, (7,1,0.4,1,0.2)\rangle, \langle x_3, (8,1,0.3,1,0.1)\rangle\}$
$\tilde{F}_B(s_2) = \{\langle x_1, (2,1,0.9,1,0.7)\rangle, \langle x_2, (7,1,0.4,1,0.2)\rangle, \langle x_3, (8,1,0.3,1,0.1)\rangle\}$
$\tilde{F}_B(s_3) = \{\langle x_1, (5,1,0.6,1,0.4)\rangle, \langle x_2, (8,1,0.3,1,0.1)\rangle, \langle x_3, (5,1,0.6,1,0.4)\rangle\}$
$\tilde{F}_B(s_4) = \{\langle x_1, (7,1,0.4,1,0.2)\rangle, \langle x_2, (2,1,0.9,1,0.7)\rangle, \langle x_3, (8,1,0.3,1,0.1)\rangle\}$
Thus, a soft set of fuzzy pentagonal numbers $(\tilde{F}_B, S)$ can be formed as follows:

$$\begin{aligned}(\tilde{F}_B, S) = \{&(s_1, \{\langle x_1, (8,1,0.3,1,0.1)\rangle, \langle x_2, (7,1,0.4,1,0.2)\rangle, \langle x_3, (8,1,0.3,1,0.1)\rangle\}),\\ &(s_2, \{\langle x_1, (2,1,0.9,1,0.7)\rangle, \langle x_2, (7,1,0.4,1,0.2)\rangle, \langle x_3, (8,1,0.3,1,0.1)\rangle\}),\\ &(s_3, \{\langle x_1, (5,1,0.6,1,0.4)\rangle, \langle x_2, (8,1,0.3,1,0.1)\rangle, \langle x_3, (5,1,0.6,1,0.4)\rangle\}),\\ &(s_4, \{\langle x_1, (7,1,0.4,1,0.2)\rangle, \langle x_2, (2,1,0.9,1,0.7)\rangle, \langle x_3, (8,1,0.3,1,0.1)\rangle\})\}\end{aligned}$$

(iii) Forming matrices from the pentagonal fuzzy soft sets $(\tilde{F}_A, D)$ and $(\tilde{F}_B, S)$.
Based on equation 15, the relation form of the fuzzy pentagonal number soft set $(\tilde{F}_A, D)$ is:

$$\begin{aligned}R_A = \{&\langle (6,1,0.5,1,0.3), (s_1, d_1)\rangle, \langle (7,1,0.4,1,0.2), (s_2, d_1)\rangle, \langle (5,1,0.6,1,0.4), (s_3, d_1)\rangle,\\ &\langle (4,1,0.7,1,0.5), (s_4, d_1)\rangle, \langle (7,1,0.4,1,0.2), (s_1, d_2)\rangle, \langle (6,1,0.5,1,0.3), (s_2, d_2)\rangle,\\ &\langle (7,1,0.4,1,0.2), (s_3, d_2)\rangle, \langle (7,1,0.4,1,0.2), (s_4, d_2)\rangle, \langle (7,1,0.4,1,0.2), (s_1, d_3)\rangle,\\ &\langle (6,1,0.5,1,0.3), (s_2, d_3)\rangle, \langle (7,1,0.4,1,0.2), (s_3, d_3)\rangle, \langle (6,1,0.5,1,0.3), (s_4, d_3)\rangle\}\end{aligned}$$

The relation R_A can be represented in Table 3 as follows:

Table 3: Relation Representation R_A

R_A	d_1	d_2	d_3
s_1	$(6,1,0.5,1,0.3)$	$(7,1,0.4,1,0.2)$	$(7,1,0.4,1,0.2)$
s_2	$(7,1,0.4,1,0.2)$	$(6,1,0.5,1,0.3)$	$(6,1,0.5,1,0.3)$
s_3	$(5,1,0.6,1,0.4)$	$(7,1,0.4,1,0.2)$	$(7,1,0.4,1,0.2)$
s_4	$(4,1,0.7,1,0.5)$	$(7,1,0.4,1,0.2)$	$(6,1,0.5,1,0.3)$

Thus, we obtain the matrix from the pentagonal fuzzy soft set $(\tilde{F}_A, D)$, denoted by $\tilde{P}$, as follows:

$$\tilde{P} = \begin{bmatrix} (6,1,0.5,1,0.3) & (7,1,0.4,1,0.2) & (7,1,0.4,1,0.2) \\ (7,1,0.4,1,0.2) & (6,1,0.5,1,0.3) & (6,1,0.5,1,0.3) \\ (5,1,0.6,1,0.4) & (7,1,0.4,1,0.2) & (7,1,0.4,1,0.2) \\ (4,1,0.7,1,0.5) & (7,1,0.4,1,0.2) & (6,1,0.5,1,0.3) \end{bmatrix}$$

Element $\tilde{p}_{ij}$ of $\tilde{P}$ represents the pain level interval of symptom i for disease j in a pentagonal fuzzy number.
Based on equation 15, the relation form of the pentagonal fuzzy soft set $(\tilde{F}_B, S)$ is:

$$\begin{aligned} R_B = \{ & \langle (8,1,0.3,1,0.1),(x_1,s_1)\rangle, \langle (7,1,0.4,1,0.2),(x_2,s_1)\rangle, \langle (8,1,0.3,1,0.1),(x_3,s_1)\rangle \\ & \langle (2,1,0.9,1,0.7),(x_1,s_2)\rangle, \langle (7,1,0.4,1,0.2),(x_2,s_2)\rangle, \langle (8,1,0.3,1,0.1),(x_3,s_2)\rangle \\ & \langle (5,1,0.6,1,0.4),(x_1,s_3)\rangle, \langle (8,1,0.3,1,0.1),(x_2,s_3)\rangle, \langle (5,1,0.6,1,0.4),(x_3,s_3)\rangle \\ & \langle (7,1,0.4,1,0.2),(x_1,s_4)\rangle, \langle (2,1,0.9,1,0.7),(x_2,s_4)\rangle, \langle (8,1,0.3,1,0.1),(x_3,s_4)\rangle \} \end{aligned}$$

The relation R_B can be represented in Table 4 as follows:

Table 4: Relation Representation R_B

R_B	s_1	s_2	s_3	s_4
x_1	$(8,1,0.3,1,0.1)$	$(2,1,0.9,1,0.7)$	$(5,1,0.6,1,0.4)$	$(7,1,0.4,1,0.2)$
x_2	$(7,1,0.4,1,0.2)$	$(7,1,0.4,1,0.2)$	$(8,1,0.3,1,0.1)$	$(2,1,0.9,1,0.7)$
x_3	$(8,1,0.3,1,0.1)$	$(8,1,0.3,1,0.1)$	$(5,1,0.6,1,0.4)$	$(8,1,0.3,1,0.1)$

Thus, we obtain the matrix from the pentagonal fuzzy soft set $(\tilde{F}_B, S)$, denoted by $\tilde{Q}$, as follows:

$$\tilde{Q} = \begin{bmatrix} (8,1,0.3,1,0.1) & (2,1,0.9,1,0.7) & (5,1,0.6,1,0.4) & (7,1,0.4,1,0.2) \\ (7,1,0.4,1,0.2) & (7,1,0.4,1,0.2) & (8,1,0.3,1,0.1) & (2,1,0.9,1,0.7) \\ (8,1,0.3,1,0.1) & (8,1,0.3,1,0.1) & (5,1,0.6,1,0.4) & (8,1,0.3,1,0.1) \end{bmatrix}$$

Element $\tilde{q}_{ij}$ of $\tilde{Q}$ represents the pain level of symptom i experienced by patient j in a pentagonal fuzzy number.

3.3 Determining disease diagnosis

To determine the patient's disease diagnosis based on Elizabeth and Sujatha [11], matrices $(\tilde{P})_{mem}$ and $(\tilde{Q})_{mem}$ were formed, as well as matrices $((\tilde{P})_{mem})^c$ and $((\tilde{Q})_{mem})^c$, and then the relationships between these matrices were calculated. Based on equations 24 and 25, the matrices $(\tilde{P})_{mem}$ and $(\tilde{Q})_{mem}$ were obtained as follows:

$$(\tilde{P})_{mem} = \begin{bmatrix} (0.857,0.143,0.071,0.143,0.042) & (1.000,0.143,0.057,0.143,0.028) & (1.000,0.143,0.057,0.143,0.028) \\ (1.000,0.143,0.057,0.143,0.028) & (0.857,0.143,0.071,0.143,0.042) & (0.857,0.143,0.071,0.143,0.042) \\ (0.714,0.143,0.085,0.143,0.057) & (1.000,0.143,0.057,0.143,0.028) & (1.000,0.143,0.057,0.143,0.028) \\ (0.571,0.143,0.100,0.143,0.071) & (1.000,0.143,0.057,0.143,0.028) & (0.857,0.143,0.071,0.143,0.042) \end{bmatrix}$$

$$(\tilde{Q})_{mem} = \begin{bmatrix} (1.000,0.125,0.037,0.125,0.012) & (0.250,0.125,0.112,0.125,0.087) & (0.625,0.125,0.075,0.125,0.050) & (0.875,0.125,0.050,0.125,0.025) \\ (0.875,0.125,0.050,0.125,0.025) & (0.875,0.125,0.050,0.125,0.025) & (1.000,0.125,0.037,0.125,0.012) & (0.250,0.125,0.112,0.125,0.087) \\ (1.000,0.125,0.037,0.125,0.012) & (1.000,0.125,0.037,0.125,0.012) & (0.625,0.125,0.075,0.125,0.050) & (1.000,0.125,0.037,0.125,0.012) \end{bmatrix}$$

The matrix $(\tilde{P})_{mem}$ represents the symptom-disease relationship, with element $\tilde{p}_{ij}$ representing level of symptom influence i for disease j. The matrix $(\tilde{Q})_{mem}$ represents the patient-symptom relationship, with element $\tilde{q}_{ij}$ representing the pain level of symptom j experienced by patient i. Then, based on equation 21, the matrices $((\tilde{P})_{mem})^c$ and $((\tilde{Q})_{mem})^c$ are obtained as follows:

$$((\tilde{P})_{mem})^c = \begin{bmatrix} (0.143,0.143,0.071,0.143,0.042) & (0.000,0.143,0.057,0.143,0.028) & (0.000,0.143,0.057,0.143,0.028) \\ (0.000,0.143,0.057,0.143,0.028) & (0.143,0.143,0.071,0.143,0.042) & (0.143,0.143,0.071,0.143,0.042) \\ (0.286,0.143,0.085,0.143,0.057) & (0.000,0.143,0.057,0.143,0.028) & (0.000,0.143,0.057,0.143,0.028) \\ (0.429,0.143,0.100,0.143,0.071) & (0.000,0.143,0.057,0.143,0.028) & (0.143,0.143,0.071,0.143,0.042) \end{bmatrix}$$

The matrix $((\tilde{P})_{mem})^c$ is the complement of the matrix $(\tilde{P})_{mem}$ and represents the non-symptom-disease relationship.

$$((\tilde{Q})_{mem})^c = \begin{bmatrix} (0.000,0.125,0.037,0.125,0.012) & (0.750,0.125,0.112,0.125,0.087) & (0.375,0.125,0.075,0.125,0.050) & (0.125,0.125,0.050,0.125,0.025) \\ (0.125,0.125,0.050,0.125,0.025) & (0.125,0.125,0.050,0.125,0.025) & (0.000,0.125,0.037,0.125,0.012) & (0.750,0.125,0.112,0.125,0.087) \\ (0.000,0.125,0.037,0.125,0.012) & (0.000,0.125,0.037,0.125,0.012) & (0.375,0.125,0.075,0.125,0.050) & (0.000,0.125,0.037,0.125,0.012) \end{bmatrix}$$

The matrix $((\tilde{Q})_{mem})^c$ is the complement of the matrix $(\tilde{Q})_{mem}$ and is called the non-patient-symptom matrix.

Next, the inter-matrix relationship is calculated as follows:

(i) $R_1 = (\tilde{Q})_{mem} \otimes (\tilde{P})_{mem}$

$$R_1 = \begin{bmatrix} (2.052,0.784,0.372,0.784,0.341) & (2.714,0.874,0.346,0.874,0.281) & (2.588,0.856,0.349,0.856,0.292) \\ (2.480,0.821,0.328,0.821,0.285) & (2.874,0.910,0.357,0.910,0.277) & (2.838,0.892,0.342,0.892,0.270) \\ (2.874,0.910,0.357,0.910,0.277) & (3.482,1.000,0.342,1.000,0.227) & (3.339,0.982,0.349,0.982,0.241) \end{bmatrix}$$

Matrix R_1 is called the patient-symptom-disease relation. Matrix R_1 is calculated using the operation of multiplying pentagonal fuzzy number matrices based on equation 20.

(ii) $R_2 = (\tilde{Q})_{mem} \otimes ((\tilde{P})_{mem})^c$

$$R_2 = \begin{bmatrix} (0.696,0.498,0.196,0.498,0.221) & (0.035,0.410,0.104,0.410,0.132) & (0.160,0.428,0.122,0.428,0.150) \\ (0.518,0.535,0.211,0.535,0.225) & (0.125,0.446,0.124,0.446,0.135) & (0.160,0.464,0.140,0.464,0.153) \\ (0.750,0.624,0.257,0.624,0.233) & (0.143,0.535,0.167,0.535,0.144) & (0.286,0.552,0.185,0.552,0.161) \end{bmatrix}$$

Matrix R_2 is called the patient-non-disease relation. Matrix R_2 is calculated using the operation of multiplying pentagonal fuzzy number matrices based on equation 20.

(iii) $R_3 = ((\tilde{Q})_{mem})^c \otimes (\tilde{P})_{mem}$

$$R_3 = \begin{bmatrix} (1.250,0.570,0.233,0.570,0.246) & (1.142,0.659,0.268,0.659,0.246) & (1.124,0.642,0.261,0.642,0.246) \\ (0.660,0.534,0.197,0.534,0.211) & (0.982,0.624,0.232,0.624,0.210) & (0.874,0.606,0.225,0.606,0.210) \\ (0.267,0.445,0.108,0.445,0.122) & (0.375,0.535,0.143,0.535,0.122) & (0.375,0.517,0.136,0.517,0.122) \end{bmatrix}$$

Matrix R_3 is called the patient-non-symptom relationship. Matrix R_3 is calculated using the pentagonal fuzzy number matrix multiplication operation based on equation 20.

(iv) $R_4 = max\{R_2, R_3\}$ with $(\tilde{r}_4)_{ij} = (a_m, \alpha_{l1}, \alpha_{l2}, \beta_{r1}, \beta_{r2})_{ij}$

$$R_4 = \begin{bmatrix} (1.250,0.570,0.233,0.570,0.246) & (1.142,0.659,0.268,0.659,0.246) & (1.124,0.642,0.261,0.642,0.246) \\ (0.660,0.535,0.211,0.535,0.225) & (0.982,0.624,0.232,0.624,0.210) & (0.874,0.606,0.225,0.606,0.210) \\ (0.750,0.624,0.257,0.624,0.233) & (0.375,0.535,0.167,0.535,0.144) & (0.375,0.552,0.185,0.552,0.161) \end{bmatrix}$$

Matrix R_4 is calculated based on equation 22.

(v) $R_5 = R_1 \ominus R_4$

$$R_5 = \begin{bmatrix} (0.802, 1.354, 0.618, 1.354, 0.574) & (1.572, 1.533, 0.592, 1.533, 0.549) & (1.464, 1.498, 0.595, 1.498, 0.553) \\ (1.820, 1.356, 0.553, 1.356, 0.496) & (1.892, 1.534, 0.567, 1.534, 0.509) & (1.964, 1.498, 0.552, 1.498, 0.495) \\ (2.124, 1.534, 0.590, 1.534, 0.534) & (3.107, 1.535, 0.486, 1.535, 0.394) & (2.964, 1.534, 0.510, 1.534, 0.426) \end{bmatrix}$$

Matrix R_5 is calculated by performing fuzzy pentagonal number matrix subtraction operations based on equation 19.

(vi) Then counting the robust ranking defuzzification for each element in matrix R_5. This matrix is denoted by R_6, and the largest element in each row represents the predicted diagnosis of the disease suffered by each patient. Let's assume the maximum value in row i is $(R_6)_{ij}$. Then it can be concluded that patient i is most likely suffering from disease j. Based on equation 23, the matrix R_6 is as follows:

$$R_6 = \begin{bmatrix} 1.604 & 3.144 & 2.928 \\ 3.640 & 3.784 & 3.928 \\ 4.248 & 6.214 & 5.928 \end{bmatrix}$$

Based on matrix R_6, the predicted diagnosis of the patient's illness is obtained. The maximum value in the first row is element $(R_6)_{12}$, so it can be concluded that the first patient suffers from Pharyngitis. The maximum value in the second row is element $(R_6)_{23}$, so that the second patient suffers from Acute Respiratory Tract Infection (ARTI). The maximum value in the third row is element $(R_6)_{32}$, so it can be concluded that the third patient suffers from Pharyngitis.

4 Conclusion

This study discusses the application of pentagonal fuzzy number matrices in disease diagnosis using the robust ranking defuzzification method. From various fuzzy pentagonal number arithmetics, the author uses the fuzzy pentagonal number arithmetic by Arfina and Mashadi [2] as well as Tilopa and Mashadi [6] which satisfies the identity multiplication $\tilde{a}_{PFN} \otimes \frac{1}{\tilde{a}_{PFN}} = \tilde{i}_{PFN}$ by $\tilde{i}_{PFN}$ is a fuzzy pentagonal number identity. The data used is obtained from doctor interviews and medical documentation, as well as patient interviews at the Keluargaku Primary Clinic in Pekanbaru. This includes interval data on the level of influence of symptoms on the disease and interval data on patients' pain levels for symptoms, ranging from 0 to 10. The results of applying the pentagonal fuzzy number matrix for disease diagnosis discussed in this study are limited to predictions that still require the opinion of a doctor as a medical expert as in real-world application, there are other factors that must be considered, and it is not uncommon to require more than one diagnosis to obtain accurate results.

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